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PRACTICAL GRID MODELING

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Introduction

This book aims at explaining grid modelling from a practical perspective, providing state of the art models, algorithms as well as implementation hints and examples.

The goal is to provide a refference document for researchers and computer engineers in the field of electric systems modelling, so that the task of building your own simulator or extend the already available open source ones becomes feasible.

The book assumes that the reader has a basic understanding of matrices, vectors and complex numbers. The notation used in the book is intended to be clear and accesible as much as it is practicly possible.

General network model

The electrical grid can be assimilated to a graph. A graph is a mathematical object composed of nodes and edges (or branches). From a calculation point of view, a node is the place where the voltage is calculated given power and currnt injections, and a branch is the place where the current and power that flows through it are computed given the nodes voltage.

In the general network model, the the neutral wire and the earth "wire" are embedded into the three-phase equivalent using Kron's reduction. See [1] for a comprehensive explanation or simply [2] for practical application.

A node has power injection or consumption devices attached to it. Examples are loads, generators, capacitor banks or any other device that injects or consumes power from the grid.

A branch has attached to it devices that modify the flow through the branch. Such devices are known as FACTS (Flexible Alternating Current Transmission Systems).

A real life grid can be composed of several isolated groups of nodes (islands). It is impossible to calculate magnitudes of several islands at once in the same numerical process. Hence, the maximal calculation object is the island circuit: a graph that does not contain further islands inside. In practice, a grid is split in its islands, each island is computed separatelly and the resuts are merged back to provide a consistent analysis interface of the whole grid.

[1] Florian Dorfler and Francesco Bullo. Kron reduction of graphs with applications to electrical networks. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 60(1):150–163, 2013

[2] William H Kersting. *Distribution system modeling and analysis*. CRC press, 2012

n-phase modeling

The electrical grid calculations are mostly done in what is called "positive sequence equivalent. This is a single phase equivalent of a three-phase grid. In practice the positive sequence equivalent is applied to branches with one, two or three phases, this simplifies the modeling, but makes the ralistic analysis of the grid much harder since the phases inbalance has been neglected.

In this book, three-phase models will be presented, but the aplication of the numerical methods will be done in phase-by-phase basis. This is possible using the models presented by Vieira, Freitas and Morelato [3]. In their work, the authors pick the diagonal of the elements admittance matrices to form single-phase grids, that are simulated independently. The magnetic coupling effects are included in the single-phase admittance matrices as shunt elements. This allows a very flexible model of the grid while retaining the calculation accuracy of a full-blown n-phase admittance matrix.

So, because of the latter, the models will be introduced in three-phase and whenever needed in positive sequence equivalents, but the numerical methods will consider only one phase at the time. In practice we will sove one numerical problem per phase, and when all the phases are simulated, we will merge the results intro n-phase structures.

[3] JCM Vieira, W Freitas, and A Morelato. Phase-decoupled method for three-phase power-flow analysis of unbalanced distribution systems. *IEE Proceedings-Generation, Transmission and Distribution*, 151(5):568–574, 2004

Magnitudes and units

As the electric energy is mostly distributed in altern current, electrical magnitudes are waves that vary their polarity (positive and negative value) and amplitude in time. because of this, the electrical magnitudes are expressed by complex numbers to denote the position of the value in the two-dimensional plane amplitude-time.

The units in electrical grid modelling are represented in the tables 1 and 2.

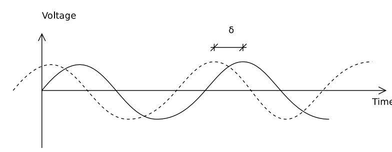

Figure 1: Voltage delay.

Table 1: Electrical magnitudes and their units.

Magnitude	Unit	Recomended user input unit
Voltage	V (Volt)	kV (kilo-Volt)
Current	A (Ampere)	kA (kilo-Ampere)
Potencia	VA (Volt-Ampere)	MVA (Mega-Volt-Ampere)
Active power	W (Watt)	MW (Mega-Watt)
Reactive power	VAr (Volt-Ampere-reactive)	$MVAr$ (Mega-Volt-Ampere-reactive)
Impedance	Ω (Ohm)	Ω (Ohm) or per-unit
Admittance	S (Siemens)	S (Siemens) or per-unit

The figure 1 shows two voltage waves. The one represented with a dotted line is delayed an angle δ with respect to the reference voltage wave represented by the plain black line. Since both waves are periodical, both can be represented as "phasors" or vectors indicating the magnitude's value and angle in the complex rectangular plane as depicted in the figure 2. Figures 1 and 2 are equivalent representations.

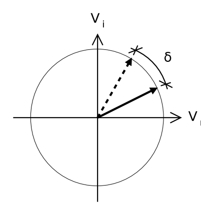

Figure 2: Voltage delay in phasor representation in the complex plane.

Table 2: Magnitudes and their real and imaginary complex components.

Magnitude	Real part	Imaginary part	Relation
S (Power)	P (Active power)	Q (Reactive power)	$S = P + jQ$
V (Voltage)	V_r (Real voltage)	V_i (Imaginary voltage)	$V = V_r + jV_i$
Expressed as			
	V_m (Voltage module)		
	δ (Voltage angle)		$V = V_m \cdot e^{\delta}$
I (Current)	I_r (Real current)	I_i (Imaginary current)	$I = I_r + jI_i$
Z (Impedance)	R (Resistance)	X (Inductance)	$Z = R + jX$
Y (Admittance)	G (Conductance)	B (Susceptance)	$Y = G + jB$

Components connection and their conversions

Let us assume a three-phase grid. The phases of the grid are denoted by the names of A, B and C. There are two main connection types that arise: Wye (like the letter "y") and Delta.

The wye and delta connections provide the ground to introduce the *phase* and *line* voltages. The phase to neutral voltage is called *phase voltage*, those are V_A, V_B and V_C. The phase to phase voltage is called *line voltage*, those are V_{AB}, V_{AC} and V_{BC}.

The delta connection is depicted in the figure 3. The delta conection has no neutral.

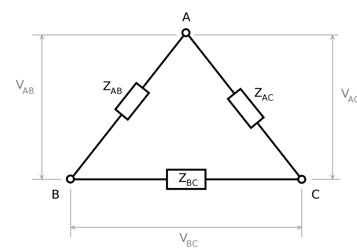

Figure 3: Delta connection scheme.

The wye connection is depicted in the figure 4. The star conection does have neutral (N). The wye connection is also known as star connection.

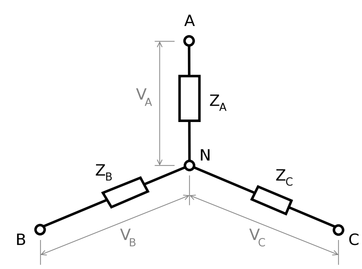

Figure 4: Wye connection scheme.

Delta to Wye ($\Delta \rightarrow Y$) The transformation of a three phase connected shunt element in Delta to it's Wye equivalent is:

$$Elm_{Wye} = D \times Elm_{Delta} \tag{1}$$

Where D is:

$$D = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{2}$$

For instance, considering the impedances transformation from figures 3 and 4:

$$\begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} Z_{AB} \\ Z_{AC} \\ Z_{BC} \end{bmatrix} \tag{3}$$

Wye to Delta ($Y \rightarrow \Delta$) The transformation of a three phase connected shunt element in Wye to it's Delta equivalent is:

$$Elm_{Delta} = D^{-1} \times Elm_{Wye} \tag{4}$$

Where D^{-1} is:

$$D^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{5}$$

For instance, considering the impedances transformation from figures 3 and 4:

$$\begin{bmatrix} Z_{AB} \\ Z_{AC} \\ Z_{BC} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} \tag{6}$$

Per unit system

In an electrical grid there are multiple levels of voltage. This situation intruduces discontinuities in the numerical methods used to solve power flows and state estimations among others, producing an unstable convergence behaviour. To avoid this, the per unit system is introduced. A side effect of the per unit representation is to have a very convenient way of visualizing the grid magnitudes, all referenced to their base. In the per unit system, all the voltages are expressed in terms of their nominal value. In this case, all the grid voltage values are around one. For instance, a voltage value of 0.98 means that the voltage is 98 % of the nominal voltage value at that point.

For most exchange formats in computer programs, the element's magnitudes are expressed with a mix of actual units and per unit values. Regardless of this, a practical way of converting any electrical magnitude to its per unit equivalent is presented.

First, we must choose an arbitrary value of power base conversion. This value can be seen as the grid's nominal power, even though that concept is not related to any phisical quantity, but it is rather a numerical aritifice.

The base power is most commonly chosen to be $S_{Base} = 100MVA$.

Table 3: Electrical magnitudes and their per unit base.

Magnitude	Base
V (Voltage)	V_{Base}: terminal's nominal voltage.
S (Power)	S_{Base}: Arbitrary value.
I (Current)	$I_{Base} = S_{Base}/Vline_{Base} = S_{Base}/(V_{Base} \cdot \sqrt{3})$
Z (Impedance)	$Z_{Base} = S_{Base}/V_{Base}^2$
Y (Admittance)	$Y_{Base} = 1/Z_{Base}$

Sequence components simplification

Charles L. Fortescue presented in 1918 his famous article [4] in which he describes how to represent a three-phase element in the so-called *sequence components*.

[4] Charles L Fortescue. Method of symmetrical co-ordinates applied to the solution of polyphase networks. *Transactions of the American Institute of Electrical Engineers*, 37(2):1027–1140, 1918

The main use of this technique is to reduce the amount of impedances needed to represent a line or transformer from usually nine, to three (or even two) if the element is considered to be balanced. An element is considered balance if the impedance in all it's phases is equal and the phase-to-phase coupling impedances are also equal. This is an assumption that is commonly made for transmission grids (very high voltage) and distribution grids in high voltage. This advance allowed the popularization of the single-line diagrams in which every line represents a a number of wires transmitting power in a balanced scheme.

Fortesue defined a transformation matrix A_s and it's inverse as:

$$A_s = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{7}$$

$$A_s^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{8}$$

Where a is the transformation eigenvector:

$$a = 1^{120_{deg}} = 1 \cdot e^{j\frac{2}{3}\pi} = 1 \cdot cos\left(\frac{2}{3}\pi\right) + 1j \cdot sin\left(\frac{2}{3}\pi\right) \tag{9}$$

$$a^2 = 1^{-120_{deg}} = 1 \cdot e^{-j\frac{2}{3}\pi} = 1 \cdot cos\left(\frac{2}{3}\pi\right) - 1j \cdot sin\left(\frac{2}{3}\pi\right) \tag{10}$$

Then, any 3x3 impedance matrix representing the rectangular ABC three-phase impedance of an element (line, transformer, capacitor, etc.) can be transformed to a sequence equivalent using the formula:

$$Z_{seq} = A_s^{-1} \times Z_{ABC} \times A_s \tag{11}$$

Example Consider the following impedance matrix of a three-phase line. Example from [5].

[5] William H Kersting. *Distribution system modeling and analysis*. CRC press, 2012

$$Z_{ABC} = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix}$$

Using equation 11, we obtain the sequence impedance matrix:

$$Z_{seq} = \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & 0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & 0.0723j0.0060 \\ 0.0256 + j0.0115 & -0.0723j0.0059 & 0.3061 + j0.6270 \end{bmatrix}$$

For the sequence matrix, the non diagonal elements are neglected. Using only the three diagonal elements as:

$$\begin{aligned} Z_0 &= 0.7735 + j1.9373 \\ Z_1 &= 0.3061 + j0.6270 \\ Z_2 &= 0.3061 + j0.6270 \end{aligned}$$

Observe that Z_1 and Z_2 are identical (with the shown numerical precission). It is very common in utilities to store only Z_0 and Z_1 to define a line. The balanced element assumption is very common and should be carefully used.

Bulding Z_{ABC} from the sequence components

Once the complete 3x3 impedance matrix has been reduced to the sequence components and only those have been stored in the utility database, the obtaining of the full 3x3 matrix might be necessary to perform unbalanced calculations. Of course we will not be able to obtain the exact original Z_{ABC} from the reduced sequence components, but the approximation is fair.

The approximated full impedance matrix is obtained from the sequence components as:

$$Z_{ABC_{approx}} = \frac{1}{3}\begin{bmatrix} 2Z_1 + Z_0 & Z_0 - Z_1 & Z_0 - Z_1 \\ Z_0 - Z_1 & 2Z_1 + Z_0 & Z_0 - Z_1 \\ Z_0 - Z_1 & Z_0 - Z_1 & 2Z_1 + Z_0 \end{bmatrix} \tag{12}$$

Example We need to compute two values, before assambling the 3x3 matrix:

$$\frac{1}{3}(2 \cdot Z_1 + Z_0) = \frac{1}{3}(2 \cdot (0.3061 + j0.6270) + (0.7735 + j1.9373)) = 0.4619 + j1.0638$$

$$\frac{1}{3}(Z_0 - Z_1) = \frac{1}{3}((0.7735 + j1.9373) - (0.3061 + j0.6270)) = 0.1558 + j0.4368$$

The the approximated impedance matrix is:

$$Z_{ABC_{approx}} = \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix}$$

We observe that the calculation outcome is a fair approximation of the original Z_{ABC} and that the approximated matrix is symmetric, matching the balanced assumption, which the original impedance matrix did not fully comply with. Therefore, is the original Z_{ABC} was reduced assuming a balanced impedance distribution when that was not the case, if we build the approximated matrix, we will never know if it represents the reality.

The branch element

To the effect of most calculations run in operation of a electrical system, the lines, transformers, and any other element that connects two nodes are represented by the so-called Π model.

Π *Model*

The pi model is composed by a series admittance Y_{serie} and a shunt admittance Y_{sh} divided in two. The shunt admittances are connected at the sending and receiving terminals (primary and secondary). To accomodate the possibility of regulating the voltage at the sending and/or receiving terminals, two per-unit transformers are included as well. The per unit transformers are modeled with the *tap ratio* parameters α and β.

In the case of lines, the series admittance Y_{serie} is computed as the inverse of Z_{ABC}. From the line's calculation it is obtained the series impedance matrix Z_{ABC} and the shunt admittance matrix Y_{sh}.

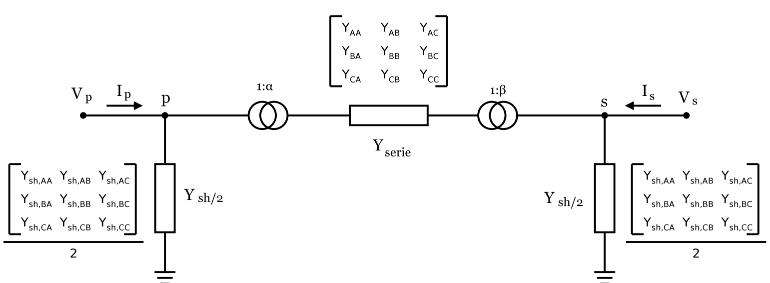

Figure 5: General branch model.

The generalized admittance matrix that corresponds to the Π model is:

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix} \tag{13}$$

Where:

Y_{pp}	Y_{ps}	Y_{sp}	Y_{ss}
$\frac{Y_{series}+\frac{Y_{sh}}{2}}{\alpha^2}$	$\frac{-Y_{series}}{\alpha\beta}$	$\frac{-Y_{series}}{\alpha\beta}$	$\frac{Y_{series}+\frac{Y_{sh}}{2}}{\beta^2}$

Table 4: Equations of the generalized Π model.

The formula 13 will be used to compute the network admittance matrices.

Line

The line element, utilizes the branch model as it has been formulated, only that the use of the tap ratio relations α and β is not necessary.

Overhead lines

Underground cables

Example Let's assume that we have already computed the series impedance and the shunt admittance of a three-phase line. The nominal voltage at the line terminals is $66kV$ and we choose the base power to be $S_{base} = 100MVA$.

$$Z_{ABC} = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega$$

$$Y_{sh} = \begin{bmatrix} j5.6712 & -j1.8362 & -j0.7034 \\ -j1.8362 & j5.9774 & -j1.1690 \\ -j0.7034 & -j1.1690 & j5.3911 \end{bmatrix} \cdot 10^{-6} S$$

The first thing we need to do is to compute the base magnitudes:

$$Z_{base} = \frac{100MVA}{(66kV)^2} = 0.022956841 \quad \Omega$$

$$Y_{base} = \frac{1}{Z_{base}} = 43.56 \quad S$$

Then we must obtain the line series admittance matrix Y_{series} by inverting the 3x3 matrix Z_{ABC}. The we divide the resulting matrix by Y_{base}. Analogously we can invert the per unit impedance matrix:

$$Y_{series} = \left(\frac{Z_{ABC}}{Z_{base}}\right)^{-1} \quad p.u.$$

$$Y_{series} = \begin{bmatrix} 0.0112 - j0.0232 & -0.0054 + j0.008 & -0.0020 + j0.0052 \\ -0.0054 + j0.008 & 0.0125 - j0.024 & -0.0033 + j0.0063 \\ -0.0020 + j0.0052 & -0.0033 + j0.0063 & 0.0100 - j0.0224 \end{bmatrix} \quad p.u.$$

Dividing the shunt admittance by the base admittance we obtain the per unit shunt admittance:

$$Y_{sh} = \begin{bmatrix} j13.0193 & -j4.2153 & -j1.6148 \\ -j4.2153 & j13.7222 & -j2.6837 \\ -j1.6148 & -j2.6837 & j12.3763 \end{bmatrix} \cdot 10^{-8} \quad p.u.$$

Now we need to find the branch model admittances Y_{pp}, Y_{ps}, Y_{sp} and Y_{ss}.

$$Y_{pp} = Y_{ss} = Y_{series} + Y_{sh}/2 = \begin{bmatrix} 0.0112 - j0.0232 & -0.0054 + j0.008 & -0.0020 + j0.0052 \\ -0.0054 + j0.008 & 0.0125 - j0.024 & -0.0033 + j0.0063 \\ -0.0020 + j0.0052 & -0.0033 + j0.0063 & 0.0100 - j0.0224 \end{bmatrix} \quad p.u.$$

$$Y_{ps} = Y_{sp} = -Y_{series} = \begin{bmatrix} -0.0112 + j0.0232 & 0.0054 - j0.008 & 0.0020 - j0.0052 \\ 0.0054 - j0.008 & -0.0125 + j0.024 & 0.0033 - j0.0063 \\ 0.0020 - j0.0052 & 0.0033 - j0.0063 & -0.0100 + j0.0224 \end{bmatrix} \quad p.u.$$

If we convert the three-phase matrices into positive sequence values we obtain:

$$Y_{pp} = Y_{ss} = 0.0148 - j0.0296$$

$$Y_{ps} = Y_{sp} = -0.0148 + j0.0296$$

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} 0.0148 - j0.0296 & -0.0148 + j0.0296 \\ -0.0148 + j0.0296 & 0.0148 - j0.0296 \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix}$$

Transformer

The transformer model implements the branch model as well, but the model admittances Y_{pp}, Y_{ps}, Y_{sp} and Y_{ss} vary depending on the connections types at the primary and at the secondary.

The most common transformer connections at the terminals are:

- Delta (Δ)
- Wye (Y)
- Grounded Wye (the neutral is grounded) (Yg)

Primary	Secondary	Y_{pp}	Y_{ss}	Y_{ps} and Y_{sp}
Yg	Yg	$\frac{1}{\alpha^2}Y_I$	$\frac{1}{\beta^2}Y_I$	$-\frac{1}{\alpha\beta}Y_I$
Yg	Y	$\frac{1}{3\alpha^2}Y_{II}$	$\frac{1}{3\beta^2}Y_{II}$	$-\frac{1}{3\alpha\beta}Y_{II}$
Yg	Δ	$\frac{1}{\alpha^2}Y_I$	$\frac{1}{\beta^2}Y_{II}$	$\frac{1}{\alpha\beta}Y_{III}$
Y	Yg	$\frac{1}{3\alpha^2}Y_{II}$	$\frac{1}{3\beta^2}Y_{II}$	$-\frac{1}{3\alpha\beta}Y_{II}$
Y	Y	$\frac{1}{3\alpha^2}Y_{II}$	$\frac{1}{\beta^2}Y_{II}$	$\frac{1}{\alpha\beta}Y_{III}$
Δ	Δ	$\frac{1}{\alpha^2}Y_{II}$	$\frac{1}{\beta^2}Y_{II}$	$-\frac{1}{\alpha\beta}Y_{II}$
Δ	Yg	$\frac{1}{\alpha^2}Y_{II}$	$\frac{1}{\beta^2}Y_I$	$\frac{1}{\alpha\beta}Y_{III}$

Table 5: Three-phase transformer impedances of the branch model.

The table 5 lays out the branch admittances for every pair of connections. The source for this model is the excellent book by J.Arrillaga[6].

[6] Jos Arrillaga and CP Arnold. *Computer analysis of power systems*. Wiley Online Library, 1990

$$Y_I = \begin{bmatrix} y_t & 0 & 0 \\ 0 & y_t & 0 \\ 0 & 0 & y_t \end{bmatrix} \tag{14}$$

$$Y_{II} = \begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix} \tag{15}$$

$$Y_{III} = \begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \\ y_t & 0 & -y_t \end{bmatrix} \tag{16}$$

y_t is the transformer winding per unit admittance. It is given in values per units from the transformer especifications sheet. Usually it is given either directly as magnetizing resistance r_m and inductance x_m, in which case:

$$y_t = \frac{3}{r_l + jx_l} \tag{17}$$

Or it is given as the "short circuit study" values. This is a more complete case.

Transformer definition from the short circuit study

In order to get the series impedance and shunt admittance of the transformer to match the branch model, it is advised to transform the specification sheet values of the device into the desired values. The values to take from the specs sheet are:

- S_n: Nominal power in MVA.
- U_{hv}: Voltage at the high-voltage side in kV.
- U_{lv}: Voltage at the low-voltage side in kV.
- U_{sc}: Short circuit voltage in %.
- P_{cu}: Copper losses in kW.
- I_0: No load current in %.
- GX_{hv1}: Reactance contribution to the HV side. Value from 0 to 1.
- GR_{hv1}: Resistance contribution to the HV side Value from 0 to 1.

Then, the series and shunt impedances are computed as follows:

Nominal impedance HV (Ohm):

$$Zn_{hv} = U_{hv}^2 / S_n \tag{18}$$

Nominal impedance LV (Ohm):

$$Zn_{lv} = U_{lv}^2 / S_n \tag{19}$$

Short circuit impedance (p.u.):

$$z_{sc} = U_{sc} / 100 \tag{20}$$

Short circuit resistance (p.u.):

$$r_{sc} = \frac{P_{cu}/1000}{S_n} \tag{21}$$

Short circuit reactance (p.u.):

$$x_{sc} = \sqrt{z_{sc}^2 - r_{sc}^2} \tag{22}$$

HV resistance (p.u.):

$$r_{cu,hv} = r_{sc} \cdot GR_{hv1} \tag{23}$$

LV resistance (p.u.):

$$r_{cu,lv} = r_{sc} \cdot (1 - GR_{hv1}) \tag{24}$$

HV shunt reactance (p.u.):

$$xs_{hv} = x_{sc} \cdot GX_{hv1} \tag{25}$$

LV shunt reactance (p.u.):

$$xs_{lv} = x_{sc} \cdot (1 - GX_{hv1}) \tag{26}$$

Shunt resistance (p.u.):

$$r_{fe} = \frac{Sn}{P_{fe}/1000} \tag{27}$$

Magnetization impedance (p.u.):

$$z_m = \frac{1}{I_0/100} \tag{28}$$

Magnetization reactance (p.u.):

$$x_m = \frac{1}{\sqrt{\frac{1}{z_m^2} - \frac{1}{r_{fe}^2}}} \tag{29}$$

If the content of the square root is negative, set the magnetization impedance to zero.

The final complex calculated parameters in per unit are:

Magnetizing impedance (or series impedance):

$$z_{series} = Z_m = r_{sc} + j \cdot x_{sc} \quad (30)$$

The series admittance is [p.u.]:

$$y_{series} = \frac{1}{z_{series}} \quad (31)$$

Leakage impedance (or shunt impedance):

$$Z_l = r_{fe} + j \cdot x_m \quad (32)$$

Shunt admittance [p.u.]:

$$y_{shunt} = 1/Z_l \quad (33)$$

The series admittance for the three-phase model [p.u.]:

$$y_t = \frac{3}{z_{series}} \quad (34)$$

we divide the impedance by 3, to reflect the three phases.

Example Let's consider a distribution transformer with the following nameplate characteristics:

- Primary connection: Δ
- Secondary connection: Yg
- $S_n = 0.5MVA$
- $U_{hv} = 20kV$
- $U_{lv} = 0.4kV$
- $U_{sc} = 6\%$
- $p_{cu} = 6kW$
- $p_{fe} = 1.4kW$
- $I_0 = 0.28\%$
- $GR_{hv} = 0.5$
- $GX_{hv} = 0.5$

First we obtain the impedance value, from the short circuit study:

$$Zn_{hv} = 800 \quad \Omega$$
$$Zn_{lv} = 0.3200 \quad \Omega$$
$$z_{sc} = 0.0600 \quad p.u.$$
$$r_{sc} = 0.0120 \quad p.u.$$
$$x_{sc} = 17.0103 \quad p.u.$$
$$r_{cu,hv} = 0.0060 \quad p.u.$$
$$r_{cu,lv} = 0.0060 \quad p.u.$$
$$x_{s,hv} = 8.5052 \quad p.u.$$
$$x_{s,lv} = 8.5052 \quad p.u.$$
$$r_{fe} = 357.1429 \quad p.u.$$
$$z_m = 357.1429 \quad p.u.$$
$$x_m = 24296003999.8084 \quad p.u.$$
$$z_{series} = 0.0120 + j17.0103 \quad p.u.$$
$$y_{series} = 0.0000 - j0.0588 \quad p.u.$$
$$y_{shunt} = 0.0000 - j0.0000 \quad p.u.$$
$$y_t = 0.0001 - j0.1764 \quad p.u.$$

Once the we have obtained the y_t value, we start building the apropiate branch impedance matrices Y_{pp}, Y_{ps}, Y_{sp} and Y_{ss}.

$$Y_I = \begin{bmatrix} 0.0001 - j0.1764 & 0.0000 + j0.0000 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0000 + j0.0000 & 0.0001 - j0.1764 \end{bmatrix}$$

$$Y_{II} = \begin{bmatrix} 0.0002 - j0.3527 & -0.0001 + j0.1764 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & 0.0002 - j0.3527 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & -0.0001 + j0.1764 & 0.0002 - j0.3527 \end{bmatrix}$$

$$Y_{III} = \begin{bmatrix} -0.0001 + j0.1764 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & -0.0001 + j0.1764 & 0.0001 - j0.1764 \\ 0.0001 - j0.1764 & 0.0000 + j0.0000 & -0.0001 + j0.1764 \end{bmatrix}$$

According to the transformer $\Delta \rightarrow Yg$ connection, we build the branch model admittances using the formulas for the table 5:

$$Y_{pp} = \begin{bmatrix} 0.0002 - j0.3527 & -0.0001 + j0.1764 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & 0.0002 - j0.3527 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & -0.0001 + j0.1764 & 0.0002 - j0.3527 \end{bmatrix}$$

$$Y_{ss} = \begin{bmatrix} 0.0001 - j0.1764 & 0.0000 + j0.0000 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0000 + j0.0000 & 0.0001 - j0.1764 \end{bmatrix}$$

$$Y_{ps} = Y_{sp} =$$

$$\begin{bmatrix} -0.0001 + j0.1764 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & -0.0001 + j0.1764 & 0.0001 - j0.1764 \\ 0.0001 - j0.1764 & 0.0000 + j0.0000 & -0.0001 + j0.1764 \end{bmatrix}$$

There is no computational need to assemble the 6x6 element impedance matrix in a computer program. The branch model matrices are added to the full circuit matrix instead.

Voltage regulator

Since all the electrical models are in per-unit values, the voltage regulator is modeled in the exact same way as a transformer.

The bus and it's connected elements

The substation

Types of buses

Load

Voltage controlled generator

Battery

Capacitor banks

Topology analysis and consolidation

Power flow

Z-Matrix

Newton-Raphson

Levenberg-Marquardt

Holomorphic embedding

Time series power flow

Stochastic power flow

Cumulative Density Function (CDF)

Monte Carlo

Latin Hypercube

State estimation

Short-circuit

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