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PRACTICAL GRID MODELING

Contents

List of Figures

List of Tables

Introduction

This book aims at explaining grid modelling from a practical perspective, providing state of the art models, algorithms as well as implementation hints and examples.

The goal is to provide a refference document for researchers and computer engineers in the field of electric systems modelling, so that the task of building your own simulator or extend the already available open source ones becomes feasible.

General network model

Magnitudes and units

As the electric energy is mostly distributed in altern current, electrical magnitudes are waves that vary their polarity (positive and negative value) and amplitude in time. because of this, the electrical magnitudes are expressed by complex numbers to denote the position of the value in the two-dimensional plane amplitude-time.

The units in electrical grid modelling are represented in the tables 1 and 2.

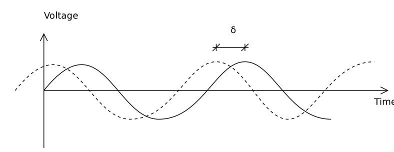

Figure 1: Voltage delay.

Table 1: Electrical magnitudes and their units.

Magnitude	Unit	Recomended user input unit
Voltage	V (Volt)	kV (kilo-Volt)
Current	A (Ampere)	kA (kilo-Ampere)
Potencia	VA (Volt-Ampere)	MVA (Mega-Volt-Ampere)
Active power	W (Watt)	MW (Mega-Watt)
Reactive power	VAr (Volt-Ampere-reactive)	$MVAr$ (Mega-Volt-Ampere-reactive)
Impedance	Ω (Ohm)	Ω (Ohm) or per-unit
Admittance	S (Siemens)	S (Siemens) or per-unit

The figure 1 shows two voltage waves. The one represented with a dotted line is delayed an angle δ with respect to the reference voltage wave represented by the plain black line. Since both waves are periodical, both can be represented as "phasors" or vectors indicating the magnitude's value and angle in the complex rectangular plane as depicted in the figure 2. Figures 1 and 2 are equivalent representations.

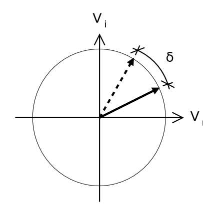

Figure 2: Voltage delay in phasor representation in the complex plane.

Table 2: Magnitudes and their real and imaginary complex components.

Magnitude	Real part	Imaginary part	Relation
S (Power)	P (Active power)	Q (Reactive power)	$S = P + jQ$
V (Voltage)	V_r (Real voltage)	V_i (Imaginary voltage)	$V = V_r + jV_i$
Expressed as			
	V_m (Voltage module)		
	δ (Voltage angle)		$V = V_m \cdot e^{\delta}$
I (Current)	I_r (Real current)	I_i (Imaginary current)	$I = I_r + jI_i$
Z (Impedance)	R (Resistance)	X (Inductance)	$Z = R + jX$
Y (Admittance)	G (Conductance)	B (Susceptance)	$Y = G + jB$

Components connection and their conversions

Let us assume a three-phase grid. The phases of the grid are denoted by the names of A, B and C. There are two main connection types that arise: Star and Delta.

The star and delta connections provide the ground to introduce the *phase* and *line* voltages. The phase to neutral voltage is called *phase voltage*, those are V_A, V_B and V_C. The phase to phase voltage is called *line voltage*, those are V_{AB}, V_{AC} and V_{BC}.

The delta connection is depicted in the figure 3. The delta conection has no neutral.

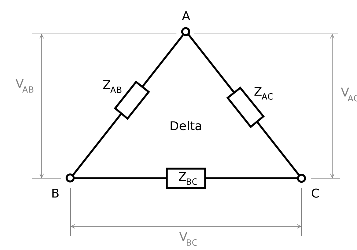

Figure 3: Delta connection scheme.

The star connection is depicted in the figure 4. The star conection does have neutral (N).

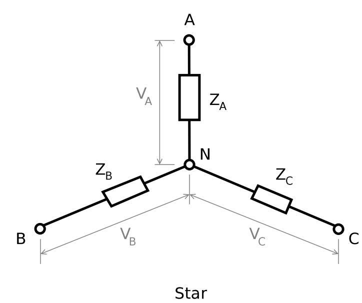

Figure 4: Star connection scheme.

Delta to Star The transformation of a three phase connected shunt element in Delta to it's Star equivalent is:

$$Elm_{Star} = D \times Elm_{Delta} \tag{1}$$

Where D is:

$$D = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{2}$$

For instance, considering the impedances transformation from figures 3 and 4:

$$\begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} Z_{AB} \\ Z_{AC} \\ Z_{BC} \end{bmatrix} \tag{3}$$

Star to Delta The transformation of a three phase connected shunt element in Star to it's Delta equivalent is:

$$Elm_{Delta} = D^{-1} \times Elm_{Star} \tag{4}$$

Where D^{-1} is:

$$D^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{5}$$

For instance, considering the impedances transformation from figures 3 and 4:

$$\begin{bmatrix} Z_{AB} \\ Z_{AC} \\ Z_{BC} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} \tag{6}$$

Per unit system

In an electrical grid there are multiple levels of voltage. This situation intruduces discontinuities in the numerical methods used to solve power flows and state estimations among others, making them have an unstable convergence behaviour. To avoid this, the per unit system is introduced. A side effect of the per unit representation is to have a very convenient way of visualizing the grid magnitudes, all referenced to their base. In the per unit system, all the voltages are expressed in terms of their nominal value. In this case, all the grid voltage values are around one. For instance, a voltage value of 0.98 means that the voltage is 98 % of the nominal voltage value at that point.

For most exchange formats in computer programs, the element's magnitudes are expressed with a mix of actual units and per unit values. Regardless of this, a practical way of converting any electrical magnitude to its per unit equivalent is presented.

First, we must choose an arbitrary value of power base conversion. This value can be seen as the grid's nominal power, even though that concept is not related to any phisical quantity, but it is rather a numerical aritifice.

The base power is most commonly chosen to be $S_{Base} = 100MVA$.

Magnitude	Base
V (Voltage)	V_{Base}: terminal's nominal voltage.
S (Power)	S_{Base}: Arbitrary value.
I (Current)	$I_{Base} = S_{Base}/Vline_{Base} = S_{Base}/(V_{Base} \cdot \sqrt{3})$
Z (Impedance)	$Z_{Base} = S_{Base}/V_{Base}^2$
Y (Admittance)	$Y_{Base} = 1/Z_{Base}$

Table 3: Electrical magnitudes and their per unit base.

Sequence components simplification

Charles L. Fortescue presented in 1918 his famous article [1] in which he describes how to represent a three-phase element in the so-called *sequence components*.

The main use of this technique is to reduce the amount of impedances needed to represent a line or transformer from usually nine, to three (or even two) if the element is considered to be balanced. An element is considered balance if the impedance in all it's phases is equal and

[1] Charles L Fortescue. Method of symmetrical co-ordinates applied to the solution of polyphase networks. *Transactions of the American Institute of Electrical Engineers*, 37(2):1027–1140, 1918

the phase-to-phase coupling impedances are also equal. This is an assumption that is commonly made for transmission grids (very high voltage) and distribution grids in high voltage. This advance allowed the popularization of the single-line diagrams in which every line represents a a number of wires transmitting power in a balanced scheme.

Fortesue defined a transformation matrix A_s and it's inverse as:

$$A_s = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{7}$$

$$A_s^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{8}$$

Where a is the transformation eigenvector:

$$a = 1^{120_{deg}} = 1 \cdot e^{j\frac{2}{3}\pi} = 1 \cdot cos\left(\frac{2}{3}\pi\right) + 1j \cdot sin\left(\frac{2}{3}\pi\right) \tag{9}$$

$$a^2 = 1^{-120_{deg}} = 1 \cdot e^{-j\frac{2}{3}\pi} = 1 \cdot cos\left(\frac{2}{3}\pi\right) - 1j \cdot sin\left(\frac{2}{3}\pi\right) \tag{10}$$

Then, any 3x3 impedance matrix representing the rectangular ABC three-phase impedance of an element (line, transformer, capacitor, etc.) can be transformed to a sequence equivalent using the formula:

$$Z_{seq} = A_s^{-1} \times Z_{ABC} \times A_s \tag{11}$$

Example Consider the following impedance matrix of a three-phase line. Example from [2].

[2] William H Kersting. *Distribution system modeling and analysis*. CRC press, 2012

$$Z_{ABC} = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix}$$

Using equation 11, we obtain the sequence impedance matrix:

$$Z_{seq} = \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & 0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & 0.0723j0.0060 \\ 0.0256 + j0.0115 & -0.0723j0.0059 & 0.3061 + j0.6270 \end{bmatrix}$$

For the sequence matrix, the non diagonal elements are neglected. Using only the three diagonal elements as:

$$\begin{aligned} Z_0 &= 0.7735 + j1.9373 \\ Z_1 &= 0.3061 + j0.6270 \\ Z_2 &= 0.3061 + j0.6270 \end{aligned}$$

Observe that Z_1 and Z_2 are identical (with the shown numerical precission). It is very common in utilities to store only Z_0 and Z_1 to define a line. The balanced element assumption is very common and should be carefully used.

Bulding Z_{ABC} from the sequence components

Once the complete 3x3 impedance matrix has been reduced to the sequence components and only those have been stored in the utility database, to obtain the full 3x3 matrix might be necessary to perform unbalanced calculations. Of course we will not be able to obtain the exact original Z_{ABC} from the previous example, but an approximation is better than just having the sequence values.

$$Z_{ABC_{approx}} = \frac{1}{3}\begin{bmatrix} 2Z_1 + Z_0 & Z_0 - Z_1 & Z_0 - Z_1 \\ Z_0 - Z_1 & 2Z_1 + Z_0 & Z_0 - Z_1 \\ Z_0 - Z_1 & Z_0 - Z_1 & 2Z_1 + Z_0 \end{bmatrix} \tag{12}$$

Example We need to compute two values, before assambling the 3x3 matrix:

$$\frac{1}{3}(Z_1 + Z_0) = \frac{1}{3}(2 \cdot (0.3061 + j0.6270) + (0.7735 + j1.9373)) = 0.4619 + j1.0638$$

$$\frac{1}{3}(Z_0 - Z_1) = \frac{1}{3}((0.7735 + j1.9373) - (0.3061 + j0.6270)) = 0.1558 + j0.4368$$

The the approximated impedance matrix is:

$$Z_{ABC_{approx}} = \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix}$$

We observe that the calculation outcome is a fair approximation of the original Z_{ABC} and that the approximated matrix is symmetric, matching the balanced assumption, which the original impedance matrix did not fully comply with.

The branch element

To the effect of most calculations run in operation of a electrical system, the lines, transformers, and any other element that connects two nodes are represented by the so-called Π model.

Π *Model*

The series admittance Y_{serie} is computed as the inverse of Z_{ABC}. From the line's calculation it is obtained the series impedance matrix Z_{ABC} and the shunt admittance matrix Y_{sh}.

The pi model is composed by a series admittance and a shunt admittance divided in two, connected at the sending and receiving terminals (primary and secondary). To accomodate the transformers and voltage regulators and the possibility of at the sending and/or receiving terminals, two per-unit transformers are included as well.

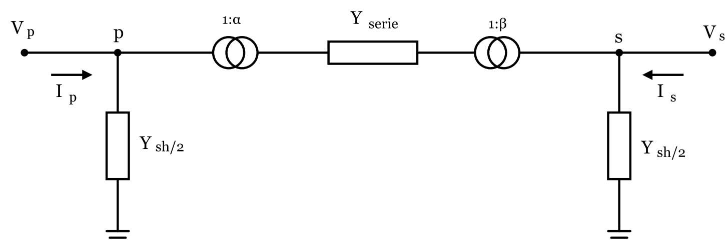

Figure 5: General branch model.

The generalized admittance matrix that corresponds to the Π model is:

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix} \tag{13}$$

Where:

Table 4: Equations of the generalized Π model.

Y_{pp}	Y_{ps}	Y_{sp}	Y_{ss}
$\frac{Y_{series}+\frac{Y_{sh}}{2}}{\alpha^2}$	$\frac{-Y_{series}}{\alpha\beta}$	$\frac{-Y_{series}}{\alpha\beta}$	$\frac{Y_{series}+\frac{Y_{sh}}{2}}{\beta^2}$

The formula 13 will be used to compute the network admittance matrices.

Line

The line element, utilizes the branch model as it has been formulated, only that the use of the tap ratio relations α and β is not necessary.

Example Let's assume that we have already computed the series impedance and the shunt admittance of a three-phase line. The nominal voltage at the line terminals is $66kV$ and we choose the base power to be $S_{base} = 100MVA$.

$$Z_{ABC} = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega$$

$$Y_{sh} = \begin{bmatrix} j5.6712 & -j1.8362 & -j0.7034 \\ -j1.8362 & j5.9774 & -j1.1690 \\ -j0.7034 & -j1.1690 & j5.3911 \end{bmatrix} \cdot 10^{-6} S$$

The first thing we need to do is to compute the base magnitudes:

$$Z_{base} = \frac{100MVA}{(66kV)^2} = 0.022956841 \quad \Omega$$

$$Y_{base} = \frac{1}{Z_{base}} = 43.56 \quad S$$

Then we must obtain the line series admittance matrix Y_{series} by inverting the 3x3 matrix Z_{ABC}. The we divide the resulting matrix by Y_{base}:

$$Y_{series} = \left(\frac{Z_{ABC}}{Z_{base}} \right)^{-1}$$

$$Y_{series} = \begin{bmatrix} 0.0112 - j0.0232 & -0.0054 + j0.008 & -0.0020 + j0.0052 \\ -0.0054 + j0.008 & 0.0125 - j0.024 & -0.0033 + j0.0063 \\ -0.0020 + j0.0052 & -0.0033 + j0.0063 & 0.0100 - j0.0224 \end{bmatrix} \quad p.u.$$

Dividing the shunt admittance by the base admittance we obtain the per unit shunt admittance:

$$Y_{sh} = \begin{bmatrix} j13.0193 & -j4.2153 & -j1.6148 \\ -j4.2153 & j13.7222 & -j2.6837 \\ -j1.6148 & -j2.6837 & j12.3763 \end{bmatrix} \cdot 10^{-8} \quad p.u.$$

Now we need to find the branch model admittances Y_{pp}, Y_{ps}, Y_{sp} and Y_{ss}.

$$Y_{pp} = Y_{ss} = Y_{series} + Y_{sh}/2 = \begin{bmatrix} 0.0112 - j0.0232 & -0.0054 + j0.008 & -0.0020 + j0.0052 \\ -0.0054 + j0.008 & 0.0125 - j0.024 & -0.0033 + j0.0063 \\ -0.0020 + j0.0052 & -0.0033 + j0.0063 & 0.0100 - j0.0224 \end{bmatrix} \quad p.u.$$

$$Y_{ps} = Y_{sp} = -Y_{series} = \begin{bmatrix} -0.0112 + j0.0232 & 0.0054 - j0.008 & 0.0020 - j0.0052 \\ 0.0054 - j0.008 & -0.0125 + j0.024 & 0.0033 - j0.0063 \\ 0.0020 - j0.0052 & 0.0033 - j0.0063 & -0.0100 + j0.0224 \end{bmatrix} \quad p.u.$$

If we convert the three-phase matrices into positive sequence values we obtain:

$$Y_{pp} = Y_{ss} = 0.0148 - j0.0296$$

$$Y_{ps} = Y_{sp} = -0.0148 + j0.0296$$

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} 0.0148 - j0.0296 & -0.0148 + j0.0296 \\ -0.0148 + j0.0296 & 0.0148 - j0.0296 \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix}$$

Transformer

Voltage regulator

The bus and it's connected elements

Topology analysis and consolidation

Islands detection

Calculation of the bus connected phases

Calculation of the admittance matrices

Calculation of the voltage, power and current vectors

Power flow

Z-Matrix

Newton-Raphson

Levenberg-Marquardt

Holomorphic embedding

Time series power flow

Stochastic power flow

Cumulative Density Function (CDF)

Monte Carlo

Latin Hypercube

State estimation

Short-circuit

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Charles L Fortescue. Method of symmetrical co-ordinates applied to the solution of polyphase networks. *Transactions of the American Institute of Electrical Engineers*, 37(2):1027–1140, 1918.

William H Kersting. *Distribution system modeling and analysis*. CRC press, 2012.