

SANTIAGO PEÑATE VERA

PRACTICAL GRID MODELING

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Contents

<i>Introduction</i>	9
<i>Notation</i>	11
<i>General network model</i>	13
<i>Magnitudes and units</i>	15
<i>The branch element</i>	21
<i>The bus and it's connected elements</i>	29
<i>Topology analysis and consolidation</i>	31
<i>Power flow</i>	33
<i>Time series power flow</i>	35
<i>Stochastic power flow</i>	37
<i>State estimation</i>	39

Short-circuit 41

Bibliography 43

List of Figures

1	Voltage delay.	15
2	Voltage delay in phasor representation in the complex plane.	15
3	Delta connection scheme.	16
4	Wye connection scheme.	16
5	General branch model.	21

List of Tables

1	Electrical magnitudes and their units.	15
2	Magnitudes and their real and imaginary complex components.	15
3	Electrical magnitudes and their per unit base.	17
4	Equations of the generalized Π model.	21
5	Three-phase transformer impedances of the branch model.	24

Introduction

This book aims at explaining grid modelling from a practical perspective, providing state of the art models, algorithms as well as implementation hints and examples.

The goal is to provide a reference document for researchers and computer engineers in the field of electric systems modelling, so that the task of building your own simulator or extend the already available open source ones becomes feasible.

The book assumes that the reader has a basic understanding of matrices, vectors and complex numbers. The notation used in the book is intended to be clear and accesible as much as it is practiclly possible.

Most of the formulas presented will imply vectorized operations such as matrix multiplications or element-wise multiplications. This notation makes it much easier to implement the numerical methods, and in case of scripting languages such as python, enables the use of fast linear algebra libraries. This is key in the implementation of electrical systems simulators, since most articles and books provide the formulas using summations, which implementation in languages other than c or fortran would make the simulator prohibitelly slow in comparison with the vectorized version.

Vectorized formulas also allow a much simpler debug and maintenance of the produced software.

Notation

- $A \times B \rightarrow$ Matrix-Matrix dot product.
- $A \times b \rightarrow$ Matrix-Vector dot product.
- $A \cdot B \rightarrow$ Matrix-Matrix element wise multiplication.
- $a \cdot b \rightarrow$ vector-vector element wise multiplication.
- $A^{-1} \rightarrow$ Matrix inverse.
- $A^{-1} \times b \rightarrow$ Solve the linear system where A is the coefficient matrix and b is the free terms vector. Never perform the inverse of A and then multiply it by b . Instead use a linear system solver.

General network model

The electrical grid can be assimilated to a graph. A graph is a mathematical object composed of nodes and edges (or branches). From a calculation point of view, a node is the place where the voltage is calculated given power and current injections, and a branch is the place where the current and power that flows through it are computed given the nodes voltage.

In the general network model, the the neutral wire and the earth "wire" are embedded into the three-phase equivalent using Kron's reduction. See ¹ for a comprehensive explanation or simply ² for practical application.

A node has power injection or consumption devices attached to it. Examples are loads, generators, capacitor banks or any other device that injects or consumes power from the grid.

A branch might have devices attached that modify the flow through the branch. Such devices are known as FACTS (Flexible Alternating Current Transmission Systems).

A real life grid can be composed of several isolated groups of nodes (islands). It is impossible to calculate magnitudes of several islands at once in the same numerical process. Hence, the maximal calculation object is the island circuit: a graph that does not contain further islands inside. In practice, a grid is split in its islands, each island is computed separately and the results are merged back to provide a consistent analysis interface of the whole grid.

n-phase modeling

The electrical grid calculations are mostly done in what is called "positive sequence equivalent. This is a single phase equivalent of a three-phase grid. In practice the positive sequence equivalent is applied to branches with one, two or three phases, this simplifies the modeling, but makes the realistic analysis of the grid much harder since the phases imbalance has been neglected.

In this book, three-phase models will be presented, but the application of the numerical methods will be done in phase-by-phase

¹ Florian Dorfler and Francesco Bullo. Kron reduction of graphs with applications to electrical networks. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 60(1):150–163, 2013

² William H Kersting. *Distribution system modeling and analysis*. CRC press, 2012

basis. This is possible using the models presented by Vieira, Freitas and Morelato³. In their work, the authors pick the diagonal of the elements admittance matrices to form single-phase grids, that are simulated independently. The magnetic coupling effects are included in the single-phase admittance matrices as shunt elements. This allows a very flexible model of the grid while retaining the calculation accuracy of a full-blown n-phase admittance matrix.

So, because of the latter, the models will be introduced in three-phases and whenever needed in positive sequence equivalents, but the numerical methods will consider only one phase at the time. In practice we will solve one numerical problem per phase, and when all the phases are simulated, we will merge the results into n-phase structures.

³ JCM Vieira, W Freitas, and A Morelato. Phase-decoupled method for three-phase power-flow analysis of unbalanced distribution systems. *IEE Proceedings-Generation, Transmission and Distribution*, 151(5):568–574, 2004

Magnitudes and units

As the electric energy is mostly distributed in altern current, electrical magnitudes are waves that vary their polarity (positive and negative value) and amplitude in time. because of this, the electrical magnitudes are expressed by complex numbers to denote the position of the value in the two-dimensional plane amplitude-time.

The units in electrical grid modelling are represented in the tables 1 and 2.

Magnitude	Unit	Recomended user input unit
Voltage	V (Volt)	kV (kilo-Volt)
Current	A (Ampere)	kA (kilo-Ampere)
Potencia	VA (Volt-Ampere)	MVA (Mega-Volt-Ampere)
Active power	W (Watt)	MW (Mega-Watt)
Reactive power	VAr (Volt-Ampere-reactive)	$MVAr$ (Mega-Volt-Ampere-reactive)
Impedance	Ω (Ohm)	Ω (Ohm) or per-unit
Admittance	S (Siemens)	S (Siemens) or per-unit

The figure 1 shows two voltage waves. The one represented with a dotted line is delayed an angle δ with respect to the reference voltage wave represented by the plain black line. Since both waves are periodical, both can be represented as "phasors" or vectors indicating the magnitude's value and angle in the complex rectangular plane as depicted in the figure 2. Figures 1 and 2 are equivalent representations.

Magnitude	Real part	Imaginary part	Relation
S (Power)	P (Active power)	Q (Reactive power)	$S = P + jQ$
V (Voltage)	V_r (Real voltage)	V_i (Imaginary voltage)	$V = V_r + jV_i$
Expressed as	V_m (Voltage module) δ (Voltage angle)		$V = V_m \cdot e^{j\delta}$
I (Current)	I_r (Real current)	I_i (Imaginary current)	$I = I_r + jI_i$
Z (Impedance)	R (Resistance)	X (Inductance)	$Z = R + jX$
Y (Admittance)	G (Conductance)	B (Susceptance)	$Y = G + jB$

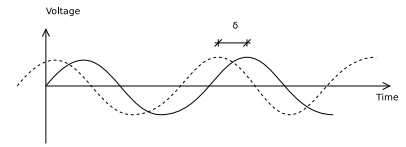


Figure 1: Voltage delay.

Table 1: Electrical magnitudes and their units.

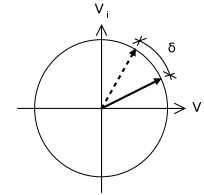


Figure 2: Voltage delay in phasor representation in the complex plane.

Table 2: Magnitudes and their real and imaginary complex components.

Components connection and their conversions

Let us assume a three-phase grid. The phases of the grid are denoted by the names of A, B and C. There are two main connection types that arise: Wye (like the letter "y") and Delta.

The wye and delta connections provide the ground to introduce the *phase* and *line* voltages. The phase to neutral voltage is called *phase voltage*, those are V_A , V_B and V_C . The phase to phase voltage is called *line voltage*, those are V_{AB} , V_{AC} and V_{BC} .

The *delta connection* is depicted in the figure 3. The delta connection has no neutral.

The *wye connection* is depicted in the figure 4. The star connection does have neutral (N). The wye connection is also known as star connection.

Delta to Wye ($\Delta \rightarrow Y$) The transformation of a three phase connected shunt element in Delta to it's Wye equivalent is:

$$Elm_{Wye} = D \times Elm_{Delta} \quad (1)$$

Where D is:

$$D = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (2)$$

For instance, considering the impedances transformation from figures 3 and 4:

$$\begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} Z_{AB} \\ Z_{AC} \\ Z_{BC} \end{bmatrix} \quad (3)$$

Wye to Delta ($Y \rightarrow \Delta$) The transformation of a three phase connected shunt element in Wye to it's Delta equivalent is:

$$Elm_{Delta} = D^{-1} \times Elm_{Wye} \quad (4)$$

Where D^{-1} is:

$$D^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (5)$$

For instance, considering the impedances transformation from figures 3 and 4:

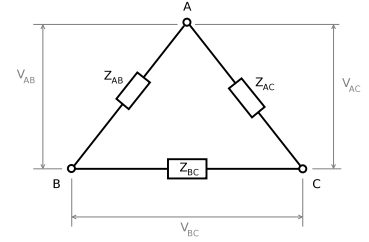


Figure 3: Delta connection scheme.

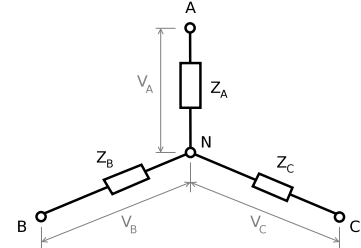


Figure 4: Wye connection scheme.

$$\begin{bmatrix} Z_{AB} \\ Z_{AC} \\ Z_{BC} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} \quad (6)$$

Per unit system

In an electrical grid there are multiple levels of voltage. This situation introduces discontinuities in the numerical methods used to solve power flows and state estimations among others, producing an unstable convergence behaviour. To avoid this, the per unit system is introduced. A side effect of the per unit representation is to have a very convenient way of visualizing the grid magnitudes, all referenced to their base. In the per unit system, all the voltages are expressed in terms of their nominal value. In this case, all the grid voltage values are around one. For instance, a voltage value of 0.98 means that the voltage is 98 % of the nominal voltage value at that point.

For most exchange formats in computer programs, the element's magnitudes are expressed with a mix of actual units and per unit values. Regardless of this, a practical way of converting any electrical magnitude to its per unit equivalent is presented.

First, we must choose an arbitrary value of power base conversion. This value can be seen as the grid's nominal power, even though that concept is not related to any physical quantity, but it is rather a numerical artifice.

The base power is most commonly chosen to be $S_{Base} = 100\text{MVA}$.

Magnitude	Base
V (Voltage)	V_{Base} : terminal's nominal voltage.
S (Power)	S_{Base} : Arbitrary value.
I (Current)	$I_{Base} = S_{Base} / V_{lineBase} = S_{Base} / (V_{Base} \cdot \sqrt{3})$
Z (Impedance)	$Z_{Base} = S_{Base} / V_{Base}^2$
Y (Admittance)	$Y_{Base} = 1 / Z_{Base}$

Table 3: Electrical magnitudes and their per unit base.

Sequence components simplification

Charles L. Fortescue presented in 1918 his famous article ⁴ in which he describes how to represent a three-phase element in the so-called *sequence components*.

The main use of this technique is to reduce the amount of impedances needed to represent a line or transformer from usually nine, to three (or even two) if the element is considered to be balanced. An element is considered balance if the impedance in all its phases is equal and

⁴ Charles L Fortescue. Method of symmetrical co-ordinates applied to the solution of polyphase networks. *Transactions of the American Institute of Electrical Engineers*, 37(2):1027–1140, 1918

the phase-to-phase coupling impedances are also equal. This is an assumption that is commonly made for transmission grids (very high voltage) and distribution grids in high voltage. This advance allowed the popularization of the single-line diagrams in which every line represents a number of wires transmitting power in a balanced scheme.

Fortesue defined a transformation matrix A_s and it's inverse as:

$$A_s = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad (7)$$

$$A_s^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (8)$$

Where a is the transformation eigenvector:

$$a = 1^{120_{deg}} = 1 \cdot e^{j\frac{2}{3}\pi} = 1 \cdot \cos\left(\frac{2}{3}\pi\right) + 1j \cdot \sin\left(\frac{2}{3}\pi\right) \quad (9)$$

$$a^2 = 1^{-120_{deg}} = 1 \cdot e^{-j\frac{2}{3}\pi} = 1 \cdot \cos\left(\frac{2}{3}\pi\right) - 1j \cdot \sin\left(\frac{2}{3}\pi\right) \quad (10)$$

Then, any 3x3 impedance matrix representing the rectangular ABC three-phase impedance of an element (line, transformer, capacitor, etc.) can be transformed to a sequence equivalent using the formula:

$$Z_{seq} = A_s^{-1} \times Z_{ABC} \times A_s \quad (11)$$

Example Consider the following impedance matrix of a three-phase line. Example from ⁵.

⁵ William H Kersting. *Distribution system modeling and analysis*. CRC press, 2012

$$Z_{ABC} = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix}$$

Using equation 11, we obtain the sequence impedance matrix:

$$Z_{seq} = \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & 0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & 0.0723j0.0060 \\ 0.0256 + j0.0115 & -0.0723j0.0059 & 0.3061 + j0.6270 \end{bmatrix}$$

For the sequence matrix, the non diagonal elements are neglected. Using only the three diagonal elements as:

$$Z_0 = 0.7735 + j1.9373$$

$$Z_1 = 0.3061 + j0.6270$$

$$Z_2 = 0.3061 + j0.6270$$

Observe that Z_1 and Z_2 are identical (with the shown numerical precision). It is very common in utilities to store only Z_0 and Z_1 to define a line. The balanced element assumption is very common and should be carefully used.

Bulding Z_{ABC} from the sequence components

Once the complete 3x3 impedance matrix has been reduced to the sequence components and only those have been stored in the utility database, the obtaining of the full 3x3 matrix might be necessary to perform unbalanced calculations. Of course we will not be able to obtain the exact original Z_{ABC} from the reduced sequence components, but the approximation is fair.

The approximated full impedance matrix is obtained from the sequence components as:

$$Z_{ABC_{approx}} = \frac{1}{3} \begin{bmatrix} 2Z_1 + Z_0 & Z_0 - Z_1 & Z_0 - Z_1 \\ Z_0 - Z_1 & 2Z_1 + Z_0 & Z_0 - Z_1 \\ Z_0 - Z_1 & Z_0 - Z_1 & 2Z_1 + Z_0 \end{bmatrix} \quad (12)$$

Example We need to compute two values, before assambling the 3x3 matrix:

$$\frac{1}{3}(2 \cdot Z_1 + Z_0) = \frac{1}{3}(2 \cdot (0.3061 + j0.6270) + (0.7735 + j1.9373)) = 0.4619 + j1.0638$$

$$\frac{1}{3}(Z_0 - Z_1) = \frac{1}{3}((0.7735 + j1.9373) - (0.3061 + j0.6270)) = 0.1558 + j0.4368$$

The the approximated impedance matrix is:

$$Z_{ABC_{approx}} = \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix}$$

We observe that the calculation outcome is a fair approximation of the original Z_{ABC} and that the approximated matrix is symmetric, matching the balanced assumption, which the original impedance matrix did not fully comply with. Therefore, is the original Z_{ABC} was reduced assuming a balanced impedance distribution when that was not the case, if we build the approximated matrix, we will never know if it represents the reality.

The branch element

To the effect of most calculations run in operation of a electrical system, the lines, transformers, and any other element that connects two nodes are represented by the so-called Π model.

Π Model

The pi model is composed by a series admittance Y_{serie} and a shunt admittance Y_{sh} divided in two. The shunt admittances are connected at the sending and receiving terminals (primary and secondary). To accomodate the possibility of regulating the voltage at the sending and/or receiving terminals, two per-unit transformers are included as well. The per unit transformers are modeled with the *tap ratio* parameters α and β .

In the case of lines, the series admittance Y_{serie} is computed as the inverse of Z_{ABC} . From the line's calculation it is obtained the series impedance matrix Z_{ABC} and the shunt admittance matrix Y_{sh} .

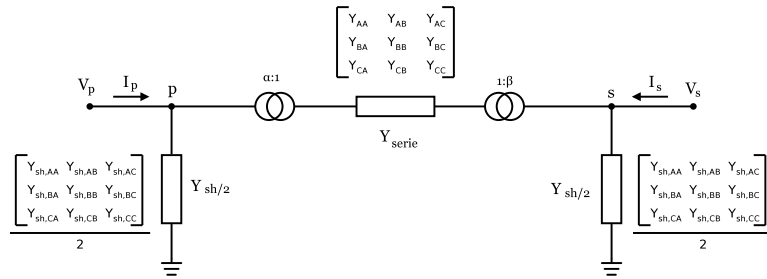


Figure 5: General branch model.

The generalized admittance matrix that corresponds to the Π model is:

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix} \quad (13)$$

Where:

Y_{pp}	Y_{ps}	Y_{sp}	Y_{ss}
$\frac{Y_{series} + \frac{Y_{sh}}{2}}{\alpha^2}$	$\frac{-Y_{series}}{\alpha\beta}$	$\frac{-Y_{series}}{\alpha\beta}$	$\frac{Y_{series} + \frac{Y_{sh}}{2}}{\beta^2}$

Table 4: Equations of the generalized Π model.

The formula 13 will be used to compute the network admittance matrices.

Line

The line element, utilizes the branch model as it has been formulated, only that the use of the tap ratio relations α and β is not necessary.

Overhead lines

Underground cables

Example Let's assume that we have already computed the series impedance and the shunt admittance of a three-phase line. The nominal voltage at the line terminals is $66kV$ and we choose the base power to be $S_{base} = 100MVA$.

$$Z_{ABC} = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega$$

$$Y_{sh} = \begin{bmatrix} j5.6712 & -j1.8362 & -j0.7034 \\ -j1.8362 & j5.9774 & -j1.1690 \\ -j0.7034 & -j1.1690 & j5.3911 \end{bmatrix} \cdot 10^{-6} S$$

The first thing we need to do is to compute the base magnitudes:

$$Z_{base} = \frac{100MVA}{(66kV)^2} = 0.022956841 \quad \Omega$$

$$Y_{base} = \frac{1}{Z_{base}} = 43.56 \quad S$$

Then we must obtain the line series admittance matrix Y_{series} by inverting the 3×3 matrix Z_{ABC} . Then we divide the resulting matrix by Y_{base} . Analogously we can invert the per unit impedance matrix:

$$Y_{series} = \left(\frac{Z_{ABC}}{Z_{base}} \right)^{-1} \quad p.u.$$

$$Y_{series} = \begin{bmatrix} 0.0112 - j0.0232 & -0.0054 + j0.008 & -0.0020 + j0.0052 \\ -0.0054 + j0.008 & 0.0125 - j0.024 & -0.0033 + j0.0063 \\ -0.0020 + j0.0052 & -0.0033 + j0.0063 & 0.0100 - j0.0224 \end{bmatrix} \quad p.u.$$

Dividing the shunt admittance by the base admittance we obtain the per unit shunt admittance:

$$Y_{sh} = \begin{bmatrix} j13.0193 & -j4.2153 & -j1.6148 \\ -j4.2153 & j13.7222 & -j2.6837 \\ -j1.6148 & -j2.6837 & j12.3763 \end{bmatrix} \cdot 10^{-8} \text{ p.u.}$$

Now we need to find the branch model admittances Y_{pp} , Y_{ps} , Y_{sp} and Y_{ss} .

$$Y_{pp} = Y_{ss} = Y_{series} + Y_{sh}/2 = \begin{bmatrix} 0.0112 - j0.0232 & -0.0054 + j0.008 & -0.0020 + j0.0052 \\ -0.0054 + j0.008 & 0.0125 - j0.024 & -0.0033 + j0.0063 \\ -0.0020 + j0.0052 & -0.0033 + j0.0063 & 0.0100 - j0.0224 \end{bmatrix} \text{ p.u.}$$

$$Y_{ps} = Y_{sp} = -Y_{series} = \begin{bmatrix} -0.0112 + j0.0232 & 0.0054 - j0.008 & 0.0020 - j0.0052 \\ 0.0054 - j0.008 & -0.0125 + j0.024 & 0.0033 - j0.0063 \\ 0.0020 - j0.0052 & 0.0033 - j0.0063 & -0.0100 + j0.0224 \end{bmatrix} \text{ p.u.}$$

If we convert the three-phase matrices into positive sequence values we obtain:

$$Y_{pp} = Y_{ss} = 0.0148 - j0.0296$$

$$Y_{ps} = Y_{sp} = -0.0148 + j0.0296$$

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} 0.0148 - j0.0296 & -0.0148 + j0.0296 \\ -0.0148 + j0.0296 & 0.0148 - j0.0296 \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix}$$

Transformer

The transformer model implements the branch model as well, but the model admittances Y_{pp} , Y_{ps} , Y_{sp} and Y_{ss} vary depending on the connections types at the primary and at the secondary.

The most common transformer connections at the terminals are:

- Delta (Δ)
- Wye (Y)
- Grounded Wye (the neutral is grounded) (Yg)

Primary	Secondary	Y_{pp}	Y_{ss}	Y_{ps} and Y_{sp}
Yg	Yg	$\frac{1}{\alpha^2} Y_I$	$\frac{1}{\beta^2} Y_I$	$-\frac{1}{\alpha\beta} Y_I$
Yg	Y	$\frac{1}{3\alpha^2} Y_{II}$	$\frac{1}{3\beta^2} Y_{II}$	$-\frac{1}{3\alpha\beta} Y_{II}$
Yg	Δ	$\frac{1}{\alpha^2} Y_I$	$\frac{1}{\beta^2} Y_{II}$	$\frac{1}{\alpha\beta} Y_{III}$
Y	Yg	$\frac{1}{3\alpha^2} Y_{II}$	$\frac{1}{3\beta^2} Y_{II}$	$-\frac{1}{3\alpha\beta} Y_{II}$
Y	Y	$\frac{1}{3\alpha^2} Y_{II}$	$\frac{1}{\beta^2} Y_{II}$	$\frac{1}{\alpha\beta} Y_{III}$
Δ	Δ	$\frac{1}{\alpha^2} Y_{II}$	$\frac{1}{\beta^2} Y_{II}$	$-\frac{1}{\alpha\beta} Y_{II}$
Δ	Yg	$\frac{1}{\alpha^2} Y_{II}$	$\frac{1}{\beta^2} Y_I$	$\frac{1}{\alpha\beta} Y_{III}$

Table 5: Three-phase transformer impedances of the branch model.

The table 5 lays out the branch admittances for every pair of connections. The source for this model is the excellent book by J.Arrillaga⁶.

⁶ Jos Arrillaga and CP Arnold. *Computer analysis of power systems*. Wiley Online Library, 1990

$$Y_I = \begin{bmatrix} y_t & 0 & 0 \\ 0 & y_t & 0 \\ 0 & 0 & y_t \end{bmatrix} \quad (14)$$

$$Y_{II} = \begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix} \quad (15)$$

$$Y_{III} = \begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \\ y_t & 0 & -y_t \end{bmatrix} \quad (16)$$

y_t is the transformer winding per unit admittance. It is given in values per units from the transformer specifications sheet. Usually it is given either directly as magnetizing resistance r_m and inductance x_m , in which case:

$$y_t = \frac{3}{r_l + jx_l} \quad (17)$$

Or it is given as the "short circuit study" values. This is a more complete case.

Transformer definition from the short circuit study

In order to get the series impedance and shunt admittance of the transformer to match the branch model, it is advised to transform the specification sheet values of the device into the desired values. The values to take from the specs sheet are:

- S_n : Nominal power in MVA.
- U_{hv} : Voltage at the high-voltage side in kV.
- U_{lv} : Voltage at the low-voltage side in kV.
- U_{sc} : Short circuit voltage in %.
- P_{cu} : Copper losses in kW.
- I_0 : No load current in %.
- $G_{X_{hv1}}$: Reactance contribution to the HV side. Value from 0 to 1.
- $G_{R_{hv1}}$: Resistance contribution to the HV side Value from 0 to 1.

Then, the series and shunt impedances are computed as follows:

Nominal impedance HV
(Ohm):

$$Z_{n_{hv}} = U_{hv}^2 / S_n \quad (18)$$

Nominal impedance LV
(Ohm):

$$Z_{n_{lv}} = U_{lv}^2 / S_n \quad (19)$$

Short circuit impedance (p.u.):

$$z_{sc} = U_{sc} / 100 \quad (20)$$

Short circuit resistance (p.u.):

$$r_{sc} = \frac{P_{cu} / 1000}{S_n} \quad (21)$$

Short circuit reactance (p.u.):

$$x_{sc} = \sqrt{z_{sc}^2 - r_{sc}^2} \quad (22)$$

HV resistance (p.u.):

$$r_{cu,hv} = r_{sc} \cdot G_{R_{hv1}} \quad (23)$$

LV resistance (p.u.):

$$r_{cu,lv} = r_{sc} \cdot (1 - G_{R_{hv1}}) \quad (24)$$

HV shunt reactance (p.u.):

$$x_{s_{hv}} = x_{sc} \cdot G_{X_{hv1}} \quad (25)$$

LV shunt reactance (p.u.):

$$x_{s_{lv}} = x_{sc} \cdot (1 - G_{X_{hv1}}) \quad (26)$$

Shunt resistance (p.u.):

$$r_{fe} = \frac{S_n}{P_{fe} / 1000} \quad (27)$$

Magnetization impedance
(p.u.):

$$z_m = \frac{1}{I_0 / 100} \quad (28)$$

Magnetization reactance (p.u.):

$$x_m = \frac{1}{\sqrt{\frac{1}{z_m^2} - \frac{1}{r_{fe}^2}}} \quad (29)$$

If the content of the square root is negative, set the magnetization impedance to zero.

The final complex calculated parameters in per unit are:

Magnetizing impedance (or series impedance):

$$z_{series} = Z_m = r_{sc} + j \cdot x_{sc} \quad (30)$$

The series admittance is [p.u.]:

$$y_{series} = \frac{1}{z_{series}} \quad (31)$$

Leakage impedance (or shunt

impedance):

$$Z_l = r_{fe} + j \cdot x_m \quad (32)$$

Shunt admittance [p.u.]:

$$y_{shunt} = 1/Z_l \quad (33)$$

The series admittance for the three-phase model [p.u.]:

$$y_t = \frac{3}{z_{series}} \quad (34)$$

we divide the impedance by 3, to reflect the three phases.

Example Let's consider a distribution transformer with the following nameplate characteristics:

- Primary connection: Δ
- Secondary connection: Yg
- $S_n = 0.5MVA$
- $U_{hv} = 20kV$
- $U_{lv} = 0.4kV$
- $U_{sc} = 6\%$
- $p_{cu} = 6kW$
- $p_{fe} = 1.4kW$
- $I_0 = 0.28\%$
- $GR_{hv} = 0.5$
- $GX_{hv} = 0.5$

First we obtain the impedance value, from the short circuit study:

$$\begin{aligned} Z_{n_{hv}} &= 800 \quad \Omega & x_{s,lv} &= 8.5052 \quad p.u. \\ Z_{n_{lv}} &= 0.3200 \quad \Omega & r_{fe} &= 357.1429 \quad p.u. \\ z_{sc} &= 0.0600 \quad p.u. & z_m &= 357.1429 \quad p.u. \\ r_{sc} &= 0.0120 \quad p.u. & x_m &= 24296003999.8084 \quad p.u. \\ x_{sc} &= 17.0103 \quad p.u. & z_{series} &= 0.0120 + j17.0103 \quad p.u. \\ r_{cu,hv} &= 0.0060 \quad p.u. & y_{series} &= 0.0000 - j0.0588 \quad p.u. \\ r_{cu,lv} &= 0.0060 \quad p.u. & y_{shunt} &= 0.0000 - j0.0000 \quad p.u. \\ x_{s,hv} &= 8.5052 \quad p.u. & y_t &= 0.0001 - j0.1764 \quad p.u. \end{aligned}$$

Once the we have obtained the y_t value, we start building the appropriate branch impedance matrices Y_{pp} , Y_{ps} , Y_{sp} and Y_{ss} .

$$Y_I = \begin{bmatrix} 0.0001 - j0.1764 & 0.0000 + j0.0000 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0000 + j0.0000 & 0.0001 - j0.1764 \end{bmatrix}$$

$$Y_{II} = \begin{bmatrix} 0.0002 - j0.3527 & -0.0001 + j0.1764 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & 0.0002 - j0.3527 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & -0.0001 + j0.1764 & 0.0002 - j0.3527 \end{bmatrix}$$

$$Y_{III} = \begin{bmatrix} -0.0001 + j0.1764 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & -0.0001 + j0.1764 & 0.0001 - j0.1764 \\ 0.0001 - j0.1764 & 0.0000 + j0.0000 & -0.0001 + j0.1764 \end{bmatrix}$$

According to the transformer $\Delta \rightarrow Yg$ connection, we build the branch model admittances using the formulas for the table 5:

$$Y_{pp} = \begin{bmatrix} 0.0002 - j0.3527 & -0.0001 + j0.1764 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & 0.0002 - j0.3527 & -0.0001 + j0.1764 \\ -0.0001 + j0.1764 & -0.0001 + j0.1764 & 0.0002 - j0.3527 \end{bmatrix}$$

$$Y_{ss} = \begin{bmatrix} 0.0001 - j0.1764 & 0.0000 + j0.0000 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & 0.0000 + j0.0000 & 0.0001 - j0.1764 \end{bmatrix}$$

$$Y_{ps} = Y_{sp} =$$

$$\begin{bmatrix} -0.0001 + j0.1764 & 0.0001 - j0.1764 & 0.0000 + j0.0000 \\ 0.0000 + j0.0000 & -0.0001 + j0.1764 & 0.0001 - j0.1764 \\ 0.0001 - j0.1764 & 0.0000 + j0.0000 & -0.0001 + j0.1764 \end{bmatrix}$$

There is no computational need to assemble the 6x6 element impedance matrix in a computer program. The branch model matrices are added to the full circuit matrix instead.

Voltage regulator

Since all the electrical models are in per-unit values, the voltage regulator is modeled in the exact same way as a transformer.

The bus and it's connected elements

The substation

Types of buses

Load

Voltage controlled generator

Battery

Capacitor banks

Topology analysis and consolidation

Islands detection

Calculation of the bus connected phases

Calculation of the admittance matrices

Calculation of the voltage, power and current vectors

Power flow

Z-Matrix

Newton-Raphson

Levenberg-Marquardt

Holomorphic embedding

Time series power flow

Stochastic power flow

Cumulative Density Function (CDF)

Monte Carlo

Latin Hypercube

State estimation

Short-circuit

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