Facility Location developing Sustainable Practices and ensuring development of Green Supply Chain

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Existing Facility location problems focus on the cost aspect of location, transportation and inventory management. Here we suggest development of a new dimension of sustainability in the supply chain, i.e. including Green Sustainable Supply Chains to ensure eternal preservation of our civilization.

Introduction

The present problem is a generalised problem involving the optimization of an existing supply chain with the single primary objective of adhering to United Nations Sustainable Developmental Goal 13 which is positive action against Climate Change. This may be similarly implemented on a larger scale on existing MRO supply chains like those of Grainger and McMaster Carr.

Notations and Sets

1. C refers to the set of all Customers, where $|C| = c$
2. B refers to the set of all Branches, where $|B| = b$
3. DC refers to the set of all Distribution Centres, where $|DC| = dc$
4. S refers to the set of all Suppliers, where $|S| = s$
5. K refers to the set of all products which are being demanded by the Customers

Inputs required

The distance matrices required are given below. All individual distances with each distance matrix consists of the shortest distance (Euclidean in this case).

1. $C_{ji}^{S->DC}$ refers to the distance matrix indicating the Euclidean distances from the Suppliers to the Distribution Centres, where $i \in$DC $\quad and \quad$ j∈S.

2. $C_{ji}^{DC->B}$ refers to the distance matrix indicating the Euclidean distances from the Distribution Centres to the Branches, where i ∈B *and* j∈DC.

3. $C_{ji}^{B->C}$ refers to the distance matrix indicating the Euclidean distances from the Branches to the Customers, where i ∈C *and* j∈B.

4. $C_{ji}^{S->B}$ refers to the distance matrix indicating the Euclidean distances from the Suppliers to the Branches, where i ∈B *and* j∈S.

5. $C_{ji}^{DC->C}$ refers to the distance matrix indicating the Euclidean distances from the Distribution Centres to the Customers, where i ∈C *and* j∈DC.

6. $Demand_{i,k}$ refers to the amount of Product k required by Customer i

7. $Supply_{i,k}$ refers to the amount of Product k available to be supplied by Supplier i

8. $Branch-Fixed-Emission_i$ refers to the minimum amount of emissions which will be done by a Branch even without any inventory due to upkeep and maintenance, where i ∈B

9. $DC-Fixed-Emission_i$ refers to the minimum amount of emissions which will be done by a Distribution Centre even without any inventory due to upkeep and maintenance, where i ∈DC

10. $Branch-Variable-Emission_i$ refers to the amount of emissions per unit of inventory due to daily operations and storage, where i ∈B

11. $DC-Variable-Emission_i$ refers to the amount of emissions per unit of inventory due to daily operations and storage, where i ∈DC

12. $Branch-Capacity_{i,k}$ refers to the amount of inventory holding and handling capacity of Branch i for product k

13. $DC-Capacity_{i,k}$ refers to the amount of inventory holding and handling capacity of Distribution Centre i for product k

Decision Variables

The following are the required Decision variables:-

1. $x_{ij}^{S->DC}$ is the binary decision variable which refers to whether the path from a Supplier to a Distribution Centre is selected, where i ∈S *and* j∈DC.

2. $x_{ij}^{DC->B}$ is the binary decision variable which refers to whether the path from a Distribution Centre to a Branch is selected, where i ∈DC *and* j∈B.

3. $x_{ij}^{B->C}$ is the binary decision variable which refers to whether the path from a Branch to a Customer is selected, where i ∈B *and* j∈C.

4. $x_{ij}^{S->B}$ is the binary decision variable which refers to whether the path from a Supplier to a Branch is selected, where $\quad i$ ∈S $\quad and \quad$ j∈B.

5. $x_{ij}^{DC->C}$ is the binary decision variable which refers to whether the path from a Distribution Centre to a Customer is selected, where $\quad i$ ∈DC $\quad and \quad$ j∈C.

6. $y_{ijk}^{S->DC}$ is the continuous decision variable which refers to the amount of product k flowing from a Supplier to a Distribution Centre, where $\quad k$ ∈K, $\quad$ i∈S $\quad and \quad$ j∈DC.

7. $y_{ijk}^{DC->B}$ is the continuous decision variable which refers to the amount of product k flowing from a Distribution Centre to a Branch, where $\quad k$ ∈K, $\quad$ i∈DC $\quad and \quad$ j∈B.

8. $y_{ijk}^{B->C}$ is the continuous decision variable which refers to the amount of product k flowing from a Branch to a Customer, where $\quad k$ ∈K, $\quad$ i∈B $\quad and \quad$ j∈C.

9. $y_{ijk}^{S->B}$ is the continuous decision variable which refers to the amount of product k flowing from a Supplier to a Branch, where $\quad k$ ∈K, $\quad$ i∈S $\quad and \quad$ j∈B.

10. $y_{ijk}^{DC->C}$ is the continuous decision variable which refers to the amount of product k flowing from a Distribution Centre to a Customer, where $\quad k$ ∈K, $\quad$ i∈DC $\quad and \quad$ j∈C.

11. z_i^{DC} is the binary decision variable which refers to whether a Distribution Centre i is selected in this supply chain while satisfying the customer demands. The value 0 indicates that the Distribution Centre is closed for temporarily shut down and need not be operated. However this assumption does not include the added emissions or costs which may be incurred when the facility shall be needed again in the future.

12. z_i^{B} is the binary decision variable which refers to whether a Branch i is selected in this supply chain while satisfying the customer demands. The value 0 indicates that the Branch is closed for temporarily shut down and need not be operated. However this assumption does not include the added emissions or costs which may be incurred when the facility shall be needed again in the future.

Objective Function

The average freight truck in the U.S. emits 161.8 grams of CO2 per ton-mile. Here we assume a constant E which is the variable unit emission per unit of quantity of product being transported per unit distance of the transportation. Also FE is the Fixed Emission for per unit distance of the transportation which may be considered equivalent to the per unit emission of the empty vehicle per unit distance travelled. We consider that already established transportation network exists between each subsequent levels of the flow of the products towards the customer. We further assume that a single type of Vehicle is used for transportation between each such subsequent layer. This may be a bad approximation and may be further improved by considering multiple heterogeneous vehicle routing problem allowing split deliveries with simultaneous delivery and pickup. However presently we assume that for each shortest path (i.e. for each

Euclidean distance as indicated in the various shortest path distance matrices), a vehicle of capacity Q is used for transportation and may travel only once from its origin to destination and then return back empty.
The below objective function adds all the emissions and minimises them. The process of obtaining the objective has been elaborated step-wise.

1. Fixed Emission from transportation between Supplier and Distribution Centres is

$$\sum_{i \in S} \sum_{j \in DC} 2 * C_{ij}^{S->DC} x_{ij}^{S->DC} * FE$$

   the above equation is multiplied by 2 since the empty vehicle has to return back.

2. Variable Emission from transportation between Supplier and Distribution Centres

$$\sum_{i \in S} \sum_{j \in DC} \sum_{k \in K} C_{ij}^{S->DC} * E * y_{ijk}^{S->DC}$$

3. Total Emission due to transportation between Supplier and Distribution Centres

$$\sum_{i \in S} \sum_{j \in DC} C_{ij}^{S->DC} (x_{ij}^{S->DC} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{S->DC})$$

Total Emission due to transportation

$$\sum_{i \in S} \sum_{j \in DC} C_{ij}^{S->DC} (x_{ij}^{S->DC} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{S->DC})$$

+

$$\sum_{i \in S} \sum_{j \in B} C_{ij}^{S->B} (x_{ij}^{S->B} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{S->B})$$

+

$$\sum_{i \in DC} \sum_{j \in B} C_{ij}^{DC->B} (x_{ij}^{DC->B} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{DC->B})$$

+

$$\sum_{i \in DC} \sum_{j \in C} C_{ij}^{DC->C} (x_{ij}^{DC->C} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{DC->C})$$

+

$$\sum_{i \in B} \sum_{j \in C} C_{ij}^{B->C} (x_{ij}^{B->C} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{B->C})$$

Similarly calculating the Branch emissions involves the following steps:-

1. The Fixed Emissions due to maintaining a Branch is

$$\sum_{i \in B} z_i^B * Branch - Fixed - Emission_i$$

2. The Variable Emissions due to maintaining a Branch is the sum of the emissions due to all products

$$\sum_{i \in B} Branch - Variable - Emission_i * (\sum_{k \in K} (\sum_{j \in S} y_{jik}^{S->B} + \sum_{j \in DC} y_{jik}^{DC->B}))$$

3. The Fixed Emissions due to maintaining a Distribution Centre is

$$\sum_{i \in DC} z_i^{DC} * DC - Fixed - Emission_i$$

4. The Variable Emissions due to maintaining a Distribution Centre is the sum of the emissions due to all products

$$\sum_{i \in DC} DC - Variable - Emission_i * (\sum_{k \in K} \sum_{j \in S} y_{jik}^{S->DC})$$

Therefore the total Emission due to transportation as well as inventory holding and handling is

$$\sum_{i \in S} \sum_{j \in DC} C_{ij}^{S->DC} (x_{ij}^{S->DC} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{S->DC})$$

$+$

$$\sum_{i \in S} \sum_{j \in B} C_{ij}^{S->B} (x_{ij}^{S->B} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{S->B})$$

$+$

$$\sum_{i \in DC} \sum_{j \in B} C_{ij}^{DC->B} (x_{ij}^{DC->B} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{DC->B})$$

$+$

$$\sum_{i \in DC} \sum_{j \in C} C_{ij}^{DC->C} (x_{ij}^{DC->C} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{DC->C})$$

$+$

$$\sum_{i \in B} \sum_{j \in C} C_{ij}^{B->C} (x_{ij}^{B->C} * FE * 2 + \sum_{k \in K} E * y_{ijk}^{B->C})$$

$+$

$$\sum_{i \in B} z_i^{B} * Branch - Fixed - Emission_i$$

$+$

$$\sum_{i \in B} Branch - Variable - Emission_i * (\sum_{k \in K} (\sum_{j \in S} y_{jik}^{S->B} + \sum_{j \in DC} y_{jik}^{DC->B}))$$

$+$

$$\sum_{i \in DC} z_i^{DC} * DC - Fixed - Emission_i$$

$+$

$$\sum_{i \in DC} DC - Variable - Emission_i * (\sum_{k \in K} \sum_{j \in S} y_{jik}^{S->DC})$$

Constraints

1. The supply of each product from Suppliers to Distribution Centres and Branches should be less than the amount of Supply available for each product

$$\sum_{j \in DC} y_{ijk}^{S->DC} + \sum_{j \in B} y_{ijk}^{S->B} \leq Supply_{i,k} \qquad \forall k \in K \quad \& \quad \forall i \in S$$

2. The supply of each product from Distribution Centres to Branches and Customers should be less than the amount of inventory available at the respective DCs for each product

$$\sum_{j \in B} y_{ijk}^{DC->B} + \sum_{j \in C} y_{ijk}^{DC->C} \leq \sum_{j \in S} y_{jik}^{S->DC} \qquad \forall k \in K \quad \& \quad \forall i \in DC$$

3. The supply of each product from Branches to Customers should be less than the amount of inventory available at the respective Branches for each product

$$\sum_{j \in C} y_{ijk}^{B->C} \leq \sum_{j \in S} y_{jik}^{S->B} + \sum_{j \in DC} y_{jik}^{DC->B} \qquad \forall k \in K \quad \& \quad \forall i \in B$$

4. The aggregate of same products sent to each Distribution Centre from all Suppliers should be less than the respective inventory holding and handling capacity for each product

$$\sum_{i \in S} y_{ijk}^{S->DC} \leq DC - Capacity_{j,k} \qquad \forall k \in K \quad \& \quad \forall j \in DC$$

5. The aggregate of same products sent to each Branch from all Distribution Centres and Suppliers should be less than the respective inventory holding and handling capacity for each product

$$\sum_{i \in DC} y_{ijk}^{DC->B} + \sum_{i \in S} y_{ijk}^{S->B} \leq Branch{-}Capacity_{j,k} \qquad \forall k \in K \quad \& \quad \forall j \in B$$

6. The aggregate of same products sent to each Customer from all Branches and Distribution Centres should be equal to the demand

$$\sum_{i \in B} y_{ijk}^{B->C} + \sum_{i \in DC} y_{ijk}^{DC->C} == Demand_{j,k} \qquad \forall k \in K \quad \& \quad \forall j \in C$$

7. Each transportation link between two similar points is carrying all products which need to be transported between them. The capacity of each

transportation link is limited to Q due to the limited capacity of the vehicle being used.

$$\sum_{k \in K} y_{ijk}^{S->DC} \leq x_{ij}^{S->DC} * Q \qquad \forall i \in S \quad \& \quad \forall j \in DC$$

$$\sum_{k \in K} y_{ijk}^{S->B} \leq x_{ij}^{S->B} * Q \qquad \forall i \in S \quad \& \quad \forall j \in B$$

$$\sum_{k \in K} y_{ijk}^{DC->B} \leq x_{ij}^{DC->B} * Q \qquad \forall i \in DC \quad \& \quad \forall j \in B$$

$$\sum_{k \in K} y_{ijk}^{DC->C} \leq x_{ij}^{DC->C} * Q \qquad \forall i \in DC \quad \& \quad \forall j \in C$$

$$\sum_{k \in K} y_{ijk}^{B->C} \leq x_{ij}^{B->C} * Q \qquad \forall i \in B \quad \& \quad \forall j \in C$$

We need the binary variable x's to calculate the fixed emissions which is being done in the same routes during product aggregation for transportation. In case the vehicle capacity is not required or is unobtainable, we need to use a very big value for Q for allowing product flows in only the selected paths.

8. We need to open the Branches which are to be used while satisfying the customers demands. Therefore the links which are incoming into or out-going from any Branch is considered for understanding whether the Branch needs to be kept open.

$$\sum_{j \in S} x_{ji}^{S->B} + \sum_{j \in DC} x_{ji}^{DC->B} + \sum_{j \in C} x_{ij}^{B->C} \leq z_i^B * M \qquad \forall i \in B$$

M is an arbitrary big value.

9. We need to open the Distribution Centres which are being used while satisfying the customers demands. Therefore the links which are incoming into or out-going from any DC is considered for understanding whether the DC needs to be kept open.

$$\sum_{j \in S} x_{ji}^{S->DC} + \sum_{j \in B} x_{ij}^{DC->B} + \sum_{j \in C} x_{ij}^{DC->C} \leq z_i^{DC} * M \qquad \forall i \in DC$$

M is an arbitrary big value.

Possible Expansion of the Problem considered

1. Including Vehicles as another layers mVRPSDP with Split Deliveries allowing better emission control

2. Including already existing inventories in both Branches and Distribution Centres

3. The capacity constraints of facilities are considered as if all the handling and holding is happening at the same time. However we need to include the time dimension and that will allow instantaneous inventory management.

4. Incorporating the feature of

5. How to calculate the emission of the extra inventory that remains in the Facility even after-all products have been supplied from that facility? In this case it will be zero, but we need to keep a minimum inventory and therefore understand how to calculate this. We therefore and some time component to differentiate these inventories and calculate emission for every subsequent new time-slot afresh.