Multiple Heterogeneous Vehicle Routing Problem allowing Simultaneous Delivery and Pick-up from Single Depot while minimizing the Distance travelled by all vehicles to complete the entire operation

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Introduction

In this problem, we have a single depot with multiple types of vehicles which shall cater to demands at the relief centres (nodes) as well as pickup evacuees.

Inputs required

1. Table 1. Indicating Depot-Vehicle Specifications

Sample Vehicle details at the Depot					
Vehicle Type [VT]	Number of this type of Vehicles [VN]	Capacity [VQ]	Variable Cost [VS]	Fixed Cost [VC]	Layers differentiating attributes w.r.t. OpenStreetMaps (Road Vehicle Compatibility for generating appropriate layers from the original Network)
V1	4	50	80	100	highway = Motorway, Trunk, Primary, Secondary, Tertiary
V2	1	100	60	500	highway = Motorway, Trunk
V3	6	25	100	50	highway = Motorway, Trunk, Primary, Secondary, Tertiary, Unclassified, Residential
V4	1	500	60	2500	highway = Motorway, Trunk, Primary

2. Table 2. Indicating PickUp & Delivery details at the Nodes

Sample Node Details				
Node Number	Longitude	Latitude	PickUp	Delivery
0 [Warehouse / Depot]	24	83	[p]	[d]
1	38	71	50	60
2	31	81	0	67
3	16	75	80	0
⋮	⋮	⋮	⋮	⋮
44	23	80	83	60
45	33	69	8	100

3. Table 3. Distance Matrices for each type of vehicle (i.e. for each k). Here certain distance matrices shall be repeated for multiple vehicles in the same category/type.

This formulation allows only single vehicles to visit each node similar to the original disaggregated formulation considering homogeneous vehicles as mentioned in Amico, Righini & Salani 2006 as well as for heterogeneous vehicles as in Avci Topaloglu 2016:-

Network layers are specific to vehicle types such that each type of vehicle may be imagined to travel in its Type-Layer. Each layer can be be further improved with their individual vehicle-type specific variable costs.

1. Notations, Sets & Decision Variables:

   (a) N refers to the set of all Relief Centres, where $|N| = n$

   (b) N_0 refers to the extended set of all Relief Centres as well as including the Depot, where $|N_0| = n + 1$

   (c) k refers to the set of all Vehicle Types [VT]

   (d) x_{ijk} is a binary variable which refers to the k^{th} type of vehicle going from node i to node j on layer k, when it equals 1. The network layers are generated for specific vehicles types according to their compatibility with the road.

   (e) y_{ijk} refers to the amount of pickup (humans) being carried by a vehicle of type k on its specific layer, also type k, between nodes i and j.

(f) Similarly z_{ijk} refers to the amount of delivery (Food, Water, Medicine, Sanitary Items) being carried by a vehicle type of k, on its specific layer k, between nodes i and j.

(g) C_{ijk} is the cost of vehicle of type k travelling from node i to node j. This may be imagined as the normal cost matrix/distance matrix being extended to different layers specific to vehicle types. These types will help in differentiating road types to check compatibility between vehicle and road. The general cost matrix will be generated by finding the shortest distance between two nodes in the specific layer. The initial network layer may be therefore filtered according to vehicle types so that the network layer for say Vehicle Type V2 may contain the road segments on which only V2 type vehicles will be able to travel. The cost matrix, which is to be generated will be the shortest distances for each of these specific layers which is further indicated by the suffix k in C_{ijk}.

2. Distance Minimization Objective Function:

$$\sum_{k \in VT} \sum_{i \in N_0} \sum_{j \in N_0} C_{ijk} x_{ijk} VS_k + \sum_{j \in N} \sum_{k \in VT} x_{0jk} VC_k$$

Here VS_k is a variable cost which may be compared to Distance dependant elements like fuel or emissions. VC_k is the fixed cost which may be considered as the per tour expense towards maintaining and sending a crew consisting of the driver and personnel helping in Disaster Relief which may be considered the same for similar types of vehicles. This objective may be altered to minimize only the total distance covered as shown below:-

$$\sum_{k \in VT} \sum_{i \in N_0} \sum_{j \in N_0} C_{ijk} x_{ijk}$$

3. Constraints:

(a) At most a single vehicle may attend any relief centre/node.

$$\sum_{j \in N_0} \sum_{k \in VT} x_{ijk} \leq 1 \qquad \forall i \in N$$

Removing this constraint shall not allow complete split delivery (for split delivery, each vehicle should have its own layer variables which is also necessary for time minimization), but allow some number of vehicles to tend to a node through different paths. This may also be considered since the result of this formulation without this constraint is better; and may be chosen as per real-scenarios by the operator.

(b) Ensuring the same number of each type of vehicles entering any node also leaves it.

$$\sum_{j \in N_0} (x_{ijk} - x_{jik}) = 0 \qquad \forall k \in VT \quad \& \quad \forall i \in N_0$$

(c) Ensuring at most VN_k vehicles (where VN denotes the Vehicle Number for the particular layer of vehicle type as per Table 1. above) are allowed to exit the depot for the specific vehicle layer types

$$\sum_{j \in N} x_{0jk} \leq VN_k \qquad \forall k \in VT$$

(d) Ensuring the pickup constraints are satisfied

$$\sum_{k \in VT} \sum_{j \in N_0} (y_{ijk} - y_{jik}) = p_i \qquad \forall i \in N$$

(e) Ensuring delivery constraints are satisfied

$$\sum_{k \in VT} \sum_{j \in N_0} (z_{jik} - z_{ijk}) = d_i \qquad \forall i \in N$$

(f) Limiting the vehicle capacities according to the vehicle types

$$y_{ijk} + z_{ijk} \leq x_{ijk} VQ_k \qquad \forall i, j \in N_0 \quad \& \quad k \in VT$$

(g) Variable Constraints

$$y_{ijk}, z_{ijk} \geq 0 \qquad \forall i, j \in N_0 \quad \& \quad k \in VT$$

$$x_{ijk} = \{0, 1\} \qquad \forall i, j \in N_0 \quad \& \quad k \in VT$$

(h) Additionally flow limitation constraints of PickUp and Delivery are provided to get the exact flows within the respective flow variables of pickup and delivery.

i. Assigning the value 0 to all outgoing pickup values from the Node

$$y_{0jk} = 0 \qquad \forall j \in N \quad \& \quad \forall k \in VT$$

ii. Assigning the value 0 to all incoming delivery values from the Node

$$z_{i0k} = 0 \qquad \forall i \in N \quad \& \quad \forall k \in VT$$

References

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