Proof of redundancy of Equation 9 of Mustafa Avci and Seyda Topaloglu

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1. Flow limitation constraints of PickUp are provided to get the exact flow of pickup by assigning the value 0 to all outgoing pickup values from the Node [Equation 5 of the paper]

$$y_{0ik} = 0 \qquad\qquad \forall i \in N \quad \& \quad \forall k \in VT$$

2. Ensuring the PickUp constraints are satisfied [Equation 7 of the paper]

$$\sum_{k \in VT} \sum_{j \in N_0} (y_{ijk} - y_{jik}) = p_i \qquad\qquad \forall i \in N$$

Adding all the PickUp constraints together:-

$$\sum_{i \in N} \sum_{k \in VT} \sum_{j \in N_0} (y_{ijk} - y_{jik}) = \sum_{i \in N} p_i$$

Splitting only the ranges in the summation, we get

$$\sum_{i \in N} \sum_{k \in VT} \sum_{j \in N} (y_{ijk} - y_{jik}) + \sum_{i \in N} \sum_{k \in VT} \sum_{j \in 0} (y_{ijk} - y_{jik}) = \sum_{i \in N} p_i$$

The left hand term on the LHS is zero:-

$$0 + \sum_{i \in N} \sum_{k \in VT} (y_{i0k} - y_{0ik}) = \sum_{i \in N} p_i$$

$$\sum_{i \in N} \sum_{k \in VT} y_{i0k} - \sum_{i \in N} \sum_{k \in VT} y_{0ik} = \sum_{i \in N} p_i$$

$$\sum_{i \in N} \sum_{k \in VT} y_{i0k} - 0 = \sum_{i \in N} p_i$$

The last term in the LHS is zero since all individual terms are zero (from Point 1).

3. Ultimately we reach the constraint as has been used again separately in Equation 9 of the paper since each decision variable y_{0ik} is zero as per the flow limitation constraints from Point 1 above.

$$\sum_{i \in N} \sum_{k \in VT} y_{i0k} = \sum_{i \in N} p_i$$

4. Therefore it is possible to obtain the constraint of Equation 9 from other constraints (Equations 5 and 7), thereby proving its redundancy. Similarly, the constraint as indicated in Equation 10 of the paper may also be proved to be redundant.