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Analyzing the effect of fluid rheology on coherent flow structures using dynamic mode decomposition technique

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Certificate

This is to certify that the Capstone project report entitled "Analyzing the effect of fluid rheology on coherent flow structures using dynamic mode decomposition (DMD) technique", submitted by Sana Raffi, is a record of research work carried out under my guidance and supervision. To the best of my knowledge and belief, the work presented in this report is original and has not been submitted, either in part or full, for the award of any other degree, diploma, fellowship, associateship or similar title of any university or institution. In my opinion, the Project Report has reached the standard fulfilling the requirements of the regulations relating to the Capstone project.

11/05/23

Signature

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Abstract

In the present study we have extracted the coherent flow structures in the flow past a cylinder in laminar vortex shedding regime. To do so, we have done the numerical analysis of non Newtonian power law model having flow behaviour index, n=0.4 (shear thinning) and n=1.4 (for shear thickening fluid) at a fixed Reynolds number of Re=100. The dynamic mode decomposition technique has been used in this analysis to decompose the data to the lower order dimension. We find that the DMD is successful in effectively capturing the change in the vortex pattern for both shear thinning and shear thickening fluids. Moreover, the flow dynamics/structures are extracted with few modes only. The comparison of these modes for both the fluid is also presented in this work. From the modes, it can be seen that the structures pertaining to shear thinning fluids fade at a faster rate as compared to the shear thickening fluids and have less spanwise expanse. This suggests that the dissipation of vortices is rapid in shear thinning fluids as compared to the shear thickening ones.

1 Introduction

In the domain of fluid dynamics, the study of flow past a cylinder is the most widely explored area of interest. Many theoretical, experimental and numerical studies have been conducted till now to get more knowledge about the various aspects such as force acting on the body, transition of fluid with Reynolds number, wake formation, vortex shedding etc. associated with this geometry. The study of this flow problem is fundamental and has significant applications in different industries such as aircraft design, distillation columns, heat exchangers etc. [5]. Sufficient amount of literature is available on the flow past a cylinder for the Newtonian fluids. However, the attention of the researchers is shifting from the Newtonian to complex non-Newtonian fluids such shear thinning, shear thickening, viscoelastic fluids. This is because majority of the fluids do not display Newtonian characteristics, on the contrary, they are non Newtonian in nature.

In this project we aim to study in detail the coherent flow structures in the laminar vortex shedding regime of shear thinning and shear thickening fluids. These are considered to be the simplest and most common type of non-Newtonian fluids and are generally represented by a power law model (shear dependent viscosity). Apart from this, there are also other complex models of non-Newtonian shear thinning and shear thickening fluids, however in present study we restrict our attention to the most simple model. We resort to the Computational Fluid Dynamics(CFD) to solve the flow problem and use data-driven analysis to extract and analyze the coherent fluid structures.

With the rapid advances in technology huge amount of data can be generated from Computational Fluid Dynamics(CFD) simulations with the help of advanced computational power. Processing of such a huge amount of data for studying important flow characteristics proves to be very challenging. Moreover, not all the flow data is important because only specific fluid structures play significant role in the flow, and impact the quantities such as the forces acting on the body. Recently, mathematical tools such as Dynamic mode decomposition (DMD) and proper orthogonal decomposition (POD) have been developed which can be used to extract important flow features (coherent flow structures) from huge amount of data by decomposing them into modes. Among these two, in most of the cases, DMD is preferred, because it can give both the temporal and spatial coherent flow structures as compared to the POD which gives only spatial coherent structures of a flow field [6]. In this study, we are utilising DMD to study the coherent flow structures of flow past a cylinder for power law fluid in laminar vortex shedding regime.

2 Background

In the literature, many researchers worked on the different aspects of Newtonian and non-Newtonian fluid flows past a stationary cylinder. An important parameter used in these studies is the Reynolds number (Re), which is the ratio of inertial forces to the viscous forces in the flow field. Using this non-dimensional number, the transition to different flow regimes is controlled. For Newtonian fluids, creeping flow regime exits at a very low Reynolds number of Re<<1 , and as the Re is increased, at $Re \sim 6$ the transition of flow happens from creeping flow to laminar flow with two symmetrical steady vortices formed behind the cylinder[10]. On further increasing Re, the length of recirculation region also increases [7] till the Reynolds number of 40-47. In two dimensional flow, the transition to unsteady periodic flow (laminar vortex shedding) will happen at $Re \sim 47$ [12]. Finally, a three dimensional wake forms when the Reynolds number goes over $Re \sim 300$ [3].

Sivakumar, Bharti, and Chabra carried out numerical simulations to determine the lift and drag coefficients for the Power Law fluid as well as to investigate the vortices and instantaneous stream lines of flow past a stationary cylinder [14]. The results for both shear thinning and shear thickening fluids were investigated and the Reynolds number was varied from creeping flow to Re=140. The study of flow past a rotating cylinder was also conducted to illustrate how viscosity variation affects the drag and lift coefficient at different Reynolds number [11]. For viscoelastic fluids, Hulsen, Fattal & Kupferman presented the results of the variation of drag coefficient with Weissenberg number (Wi) which is the ratio of elastic force to the viscous force [9]. A very recent work by Hamid *et al.*, utilized the dynamic mode decomposition technique to analyze the coherent structures which are formed in the laminar flow of viscoelastic fluids past a fixed cylinder.

3 Problem Statement and Governing Equations

The problem that we are considering is the study of coherent flow features using dynamic mode decomposition (DMD) for the power law flow past a circular cylinder of diameter d in the laminar vortex shedding regime. The flow is assumed to be two dimensional and unconfined with a non-Newtonian power law fluid approaching the cylinder surface with an uniform velocity U_∞ as shown in the Figure 1. Since we are considering the two dimensional flow, we can neglect u_z and $\frac{\partial()}{\partial z}$. The dimension of our rectangular domain is $L_u + L_d$ (length) and H_d (height) where $L_u = 20d$ (upstream length), $L_d = 60d$ (downstream length) and $H_d = 40d$, see Figure 2.

Like all fluids, the flow in this domain is governed by the continuity and momentum equations which are given below:

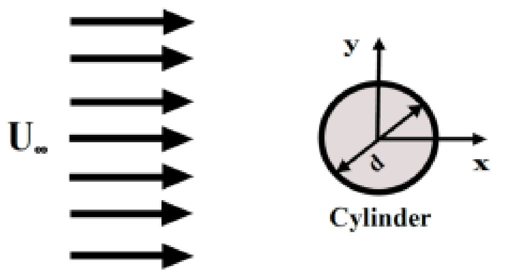

Figure 1: Schematic of the problem [8]

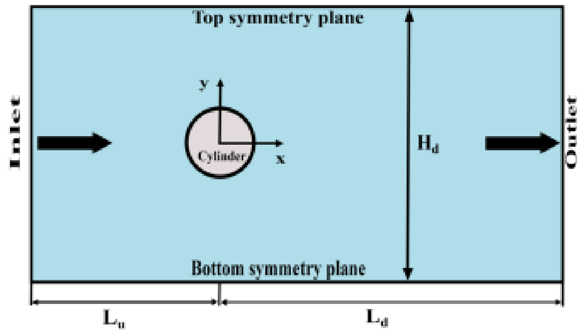

Figure 2: Computational domain [8]

Continuity equation

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

Momentum equation

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} \tag{2}$$

The power law model is defined as:

$$\tau = m(\frac{\partial u}{\partial y})^n \tag{3}$$

Reynolds number (Re) for the power law fluid is defines as:

$$Re = \frac{\rho d^n U_\infty^{(n-2)}}{m} \tag{4}$$

Here, τ is the shear stress, n is the power law index, and m is the flow consistency index. The total drag coefficient C_D, is the sum of pressure drag coefficient and frictional drag coefficient.

$$C_D = \frac{2F_D}{\rho U_\infty^2 d} = C_{DP} + C_{DF} \tag{5}$$

where, F_D is the total drag force, and ρ is the density.

The boundary conditions of this problem is listed below:

- At the inlet boundary: Uniform flow

  $u_x = 1 \quad u_y = 0 \quad \frac{\partial p}{\partial x} = 0$

- At the outlet boundary: Neumann type boundary condition is used except for pressure

  $\frac{\partial u_x}{\partial x} = 0 \quad \frac{\partial u_y}{\partial x} = 0 \quad p = 0$

- On the circular cylinder: No slip condition

  $u_x = 0 \quad u_y = 0$

- Top and bottom plane: Symmetry boundary condition

  $\frac{\partial u_x}{\partial y} = 0 \quad \frac{\partial u_y}{\partial y} = 0 \quad \frac{\partial p}{\partial y} = 0$

4 Methods and Materials

4.1 Numerical method

The governing equations of the problem of flow past a cylinder has been solved with the help of finite volume method (FVM). This can be done using a computational fluid dynamics (CFD) open-source tool called OpenFOAM. OpenFOAM (Open-source Field Operation And Manipulation) is a free, open-source CFD software based on the C++ toolbox from the OpenFOAM foundation of numerical analysis mainly concerning with the solution of continuum mechanics problem [1]. The solver called rheoFoam is available which has been used for the power law fluid. There are various factors such as grid density, domain size and time step size which can affect the results of simulation, so we have conducted a proper study to determine the best combination these factors to get accurate results with optimal computational cost. For finding the optimal number of grids, we did the grid independent test. In this test we generated our domain with different number of elements and compared the results of these grids. Finalising of grid elements was done on the basis of the accuracy of results with minimum computational cost. In our problem, number of elements chosen was G1=25000, G2=50,000 and G3=1,00,000 and the simulation was carried out with the extreme condition of shear thinning fluid having n=0.4 at Re=100. The results of G2 and G3 were found to be in excellent match, with an neglegible error of 0.53% between the average drag coefficients. For better accuracy, we chose the grid with higher number of elements, i.e G3.

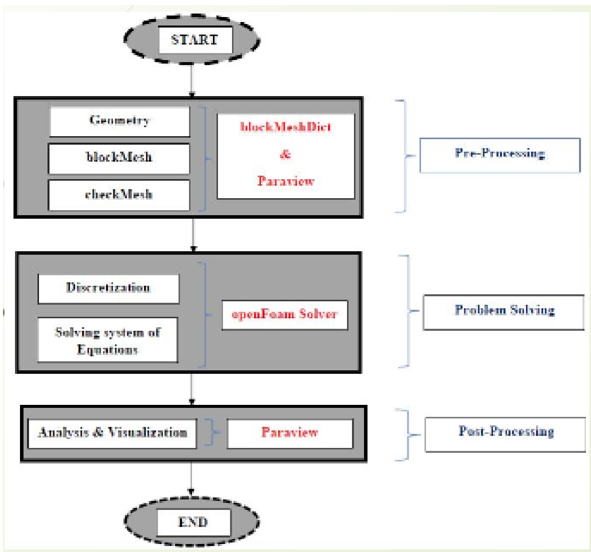

Figure 3: Steps in numerical simulation

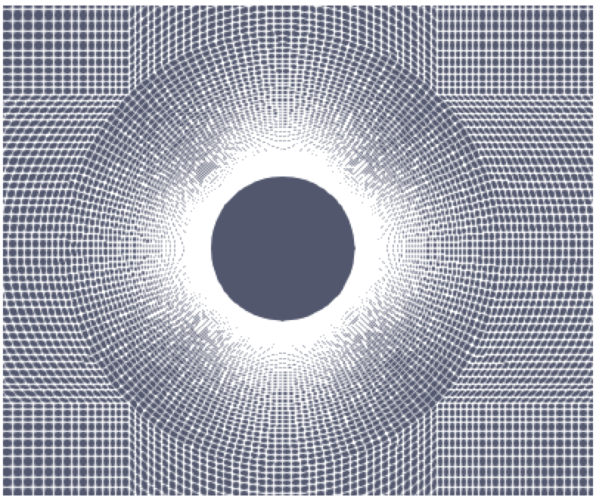

Figure 4: Typical grid in present study

4.2 Dynamic Mode Decomposition (DMD)

Dynamic Mode Decomposition (DMD) is a data driven methos which can extract important flow features from the spatiotemporal data that we generated through numerical simulation and experiments. First, we have to distribute our data into two consecutive temporal data sets by assuming a linear model for the dynamics. Thereafter, we have perform the singular value decomposition (SVD) to obtain the eigen vectors and eigen values of these data sets [13]. The generated eigen vectors represt the eigen flow field and corresponding eigen values (DMD modes) are represent the characteristic frequency and growth/decay rate of the structures.

In our problem, we are studying the vortex shedding phenomenon, therefore, we generate the spatiotemporal vorticity data from our simulation and convert the snapshots into a matrix form such that a column in the data matrix represents each state. thsi can be written as,

$$X_1^M = \begin{bmatrix} x(t_1) & x(t_2) & ..... & x(t_M) \end{bmatrix} \in R^{N\times M} \tag{6}$$

DMD assumes a linear mapping $A \in R^{N\times N}$between the two consecutive snapshots, which is constant over the data sequence, i.e.,

$$x_{j+1} = Ax_j \tag{7}$$

$$X_1^M = \begin{bmatrix} x_1 & Ax_1 & A^2x_1 & ..... & A^{M-1}x_1 \end{bmatrix} \tag{8}$$

The whole set can be written in the form given below,

$$X_2^M = AX_1^{M-1} \tag{9}$$

In Figure 6, we graphically demonstrate the methodology to carry out the DMD analysis.

$$X_t = \begin{bmatrix} | & | & & | \\ \vec{x}_1 & \vec{x}_2 & \cdots & \vec{x}_{m-1} \\ | & | & & | \end{bmatrix} \quad \text{and} \quad X_{t+1} = \begin{bmatrix} | & | & & | \\ \vec{x}_2 & \vec{x}_3 & \cdots & \vec{x}_m \\ | & | & & | \end{bmatrix}$$

Linear Operator $X_{t+1} = AX_t$

Singular Value Decomposition (SVD) $X_t = U\Sigma V^T$

Pseudo-inversion $A = X_{t+1}V\Sigma^{-1}U^T$

Similarity Transform $S = U^T AU = U^T X_{t+1}V\Sigma^{-1}$

Eigen Mode Decomposition $SW = W\Lambda$

DMD modes $\Phi = UW$

Figure 5: DMD algorithm

As shown, the snapshots are first assembled in a given sequence and are later decomposed into the modes and their time dynamics. This technique can also be employed to regenerate the flow field at later time steps and in lower dimensions as presented in the explained figure.

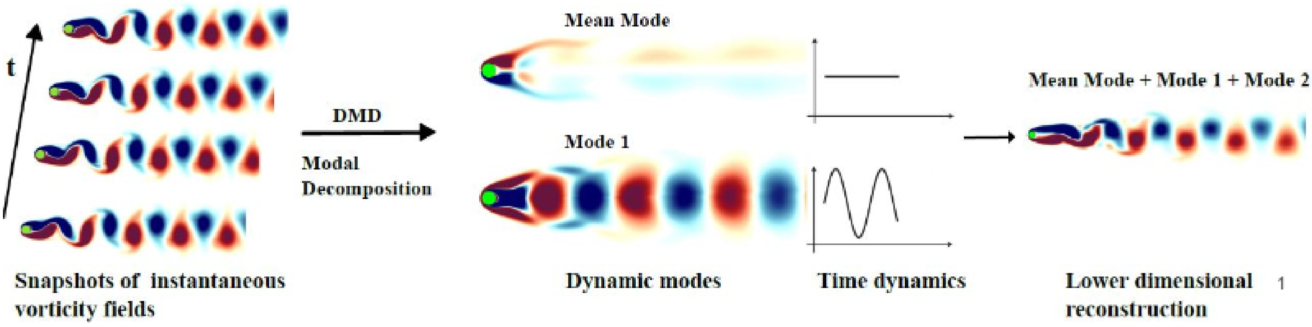

Figure 6: DMD methodology [8]

5 Results and Discussion

5.1 Validation

Before solving the problem we are considering, various validations of flow past a cylinder was done . Comparison of drag coefficient of power law fluid was done with the Bharti et al.(2006) [4]. From the table 1given below, it can be observed that the drag coefficient of present simulation is showing close agreement with the previous literature. Additionally, validation is also done for the flow past a cylinder for viscoelastic creeping fluid which is shown in the table 2 [2]. The results are showing a good match.

Re	n	Present	Bharti et al.(2006)	
		C_D	C_D	Error(%)
Re=20	0.6	1.810	1.955	7.4
	0.8	1.931	1.990	2.96
	1	2.027	2.045	0.88
	1.4	2.297	2.183	5.22
Re=40	0.6	1.30	1.276	1.89
	0.8	1.418	1.417	0.07
	1	1.516	1.529	0.85
	1.4	1.484	1.544	3.89

Table 1: Error analysis of drag coefficient for power law fluid

Wi	Present (C_D)	M. Alves et al.(2001) (C_D)	Error(%)
0.025	132.0295	132.227	0.149
0.1	130.1782	130.355	0.136
0.4	120.3868	120.607	0.182
0.7	116.8259	117.323	0.424
1	116.9635	118.518	1.312

Table 2: Error analysis of drag coefficient for viscoelastic fluid

5.2 Dynamic Mode Decomposition (DMD) analysis

In our problem we employed dynamic mode decomposition (DMD) to extract coherent flow structures in shear thinning and shear thickening fluid. In order to do this analysis, we took 150 snapshots of vorticity fields along z-direction with a time interval 0.1 units in the periodic shedding regime of all these snapshots. We decompose the data into 21 DMD modes. The mean mode corresponding to maximum amplitude, and the frequency of zero is shown for both fluids in the Figure 7. The zero frequency of mean mode is implies that it is not varying throughout the flow. The mean structures of shear thinning fluid are very less intense and are restricted to the cylinder surface, which is in contrast to the shear thickening fluid shown in the figure 7.a. Mode 1 signifies the vortex shedding phenomenon in both fluids. Again, shear thickening structures are more prominent and extend to the large distance in the downstream of the cylinder. This indicates that the vortices of shear thickening fluid will remain intact till larger distances as compared to the shear thinning fluids.

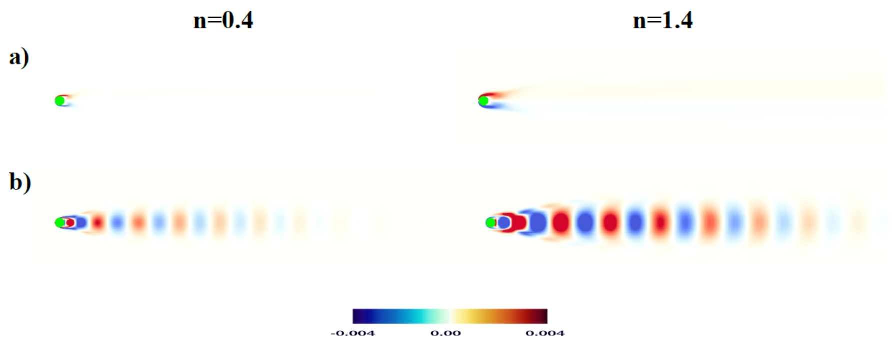

Figure 7: DMD modes of power law fluid

6 Conclusions

In conclusion, we have extracted the coherent flow structures of the power law fluid having flow behaviour index, n=0.4 (shear thinning) and n=1.4 (for shear thickening fluid) at a fixed Reynolds number of Re=100. This has been achieved using recently developed dynamic mode decomposition technique. DMD successfully captured the change in the vortex pattern for both shear thinning and shear thickening fluids with the help of very few modes. The comparison of these modes for both the fluids revealed that the structures pertaining to shear thinning fluids fade quickly as compared to the shear thickening fluids and have less spanwise expanse. This indicates that the dissipation of vortices is rapid in shear thinning fluids as compared to the shear thickening ones.

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