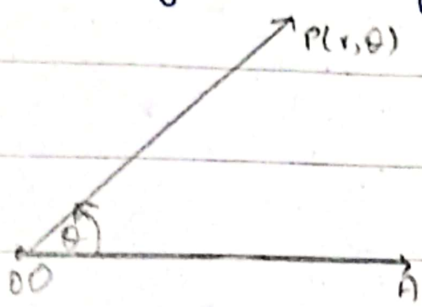
Chapter : 10
10.5
Polar Coordinates:-

To define polar coordinate:

i) Fix an origin O and an initial ray OA from O.

ii) Each point P can be located by assigning it to a polar coordinate pair (r, θ)

iii) r gives directed distance from O to P and θ gives the directional angle from the initial ray OA to ray OP



(r, θ) → directed angle.

↓ directed distance

Angles have two direction

i) Counter-clockwise ⇒ θ is positive

ii) Clockwise ⇒ θ is negative.

Questions related to polar coordinates can be asked in the paper in any of the following ways.

i) Write all versions / other labels / both infinite chain / polar co-ordinates of $(2, \frac{\pi}{6})$

ii) 3 – 4 points will be given and ~~the~~ it is required to check whether all of ~~these are version of~~ same point or not?

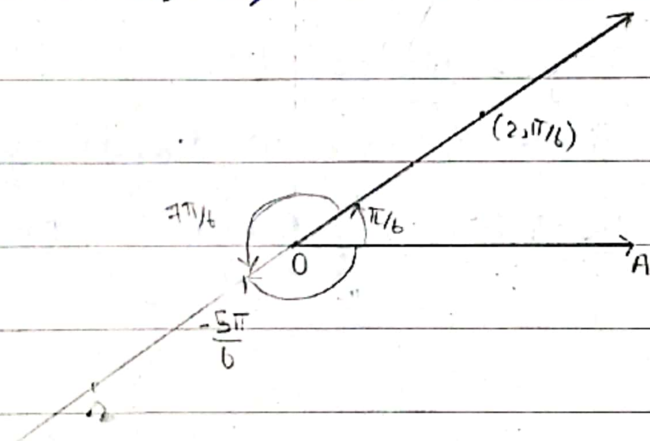
Similarly

r is positive $\longrightarrow$ directed distance in forward direction

r is negative $\rightarrow$ directed distance in backward / opposite direction

Example : 1

Find all the polar coordinates of point $P(2, \pi/6)$



$\frac{\pi}{6} + \pi = \frac{7\pi}{6}$

$-\frac{\pi}{2} - \frac{\pi}{3}$

$= \frac{-3\pi - 2\pi}{6} = \frac{-5\pi}{6}$

For $r = 2$

list of angles

$= \frac{\pi}{6}, \frac{\pi}{6} \pm 2\pi + \frac{\pi}{6} \pm 4\pi + \cdots$

$= \frac{\pi}{6} + 2n\pi$ where $n = 0, \pm 1, \pm 2, \pm 3, \cdots$

For $r = -2$

List of angles are

$-\frac{5\pi}{6}, -\frac{5\pi}{6} \pm 2\pi, -\frac{5\pi}{6} \pm 4\pi + \cdots$

$= -\frac{5\pi}{6} + 2n\pi$ where $n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \cdots$

The corresponding coordinate pairs of P are

$$\left(2, \frac{\pi}{6} + 2n\pi\right) \qquad n = 0, \pm 1, \pm 2, \ldots$$

and

$$\left(-2, -\frac{5\pi}{6} + 2n\pi\right) \qquad n = 0, \pm 1, \pm 2, \ldots$$

When $n = 0$, formula gives

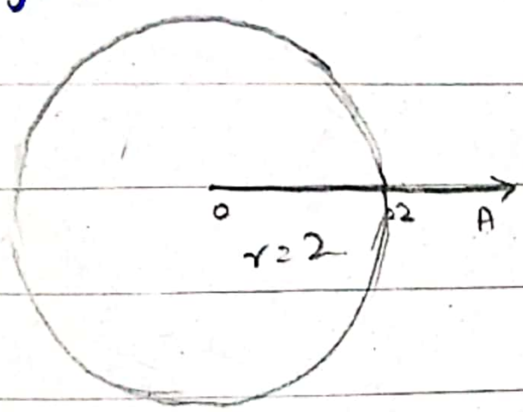
$$\left(2, \frac{\pi}{6}\right) \text{ and } \left(-2, \frac{-5\pi}{6}\right)$$

when $n = 1$ formula gives

$$\left(2, \frac{13\pi}{6}\right) \text{ and } \left(-2, \frac{7\pi}{6}\right) \text{ and so on.}$$

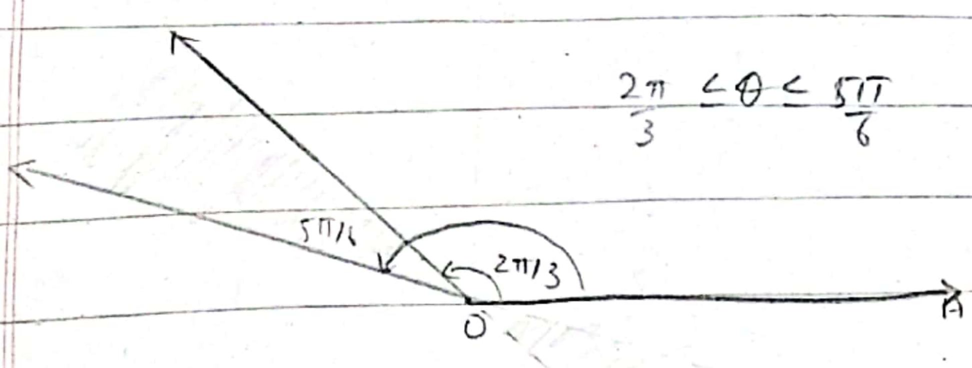
Graphs in polar coordinates

i) Draw graph of $r = 2$



Example 3

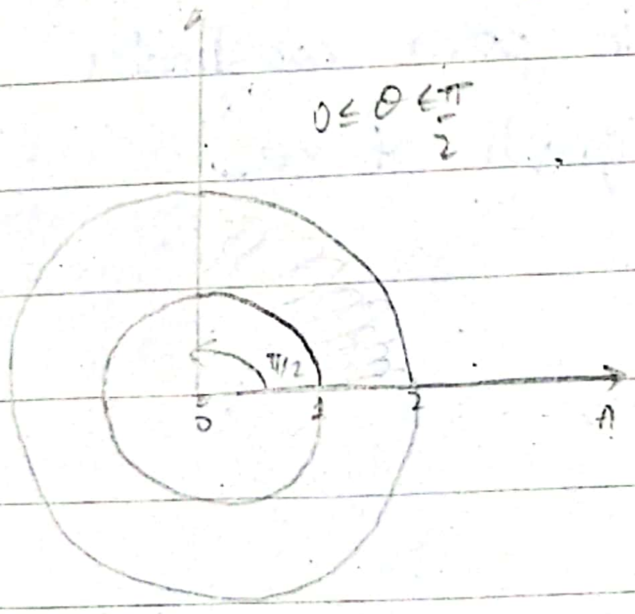
Part a) Draw graph of $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$



Example : 03

Part (A)

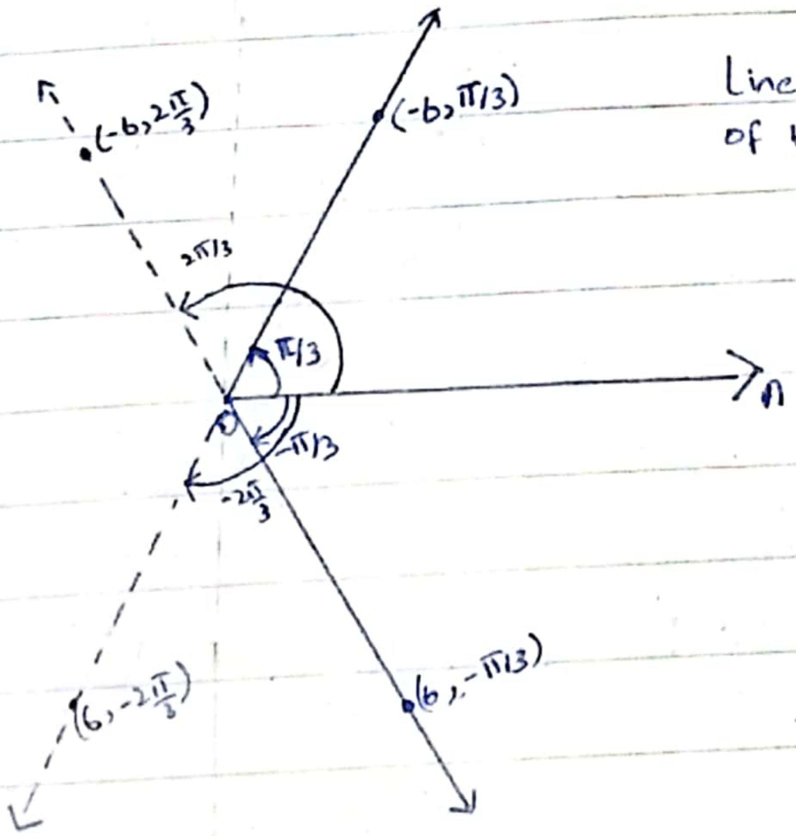
$1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$



M T W T F S (Pardon)

Q Find whether the line of inclination of the given points is same or not?

$(-6, \pi/3)$, $(6, -\pi/3)$



Line of inclination of both points : not same

$\frac{\pi}{6} + \frac{\pi}{2}$

$\frac{\pi + 3\pi}{6} = \frac{4\pi}{6}$

$= \frac{2\pi}{3}$

$-\frac{\pi}{2} - \frac{\pi}{6}$

For $(-6, \frac{\pi}{3})$ the infinite chains are

$\left(-6, \frac{\pi}{3} + 2n\pi\right)$ $n = 0, \pm 1, \pm 2, \dots$

$\left(6, -\frac{2\pi}{3} + 2n\pi\right)$ $n = 0, \pm 1, \pm 2, \dots$

For $(6, -\frac{\pi}{3})$ the infinite chains are

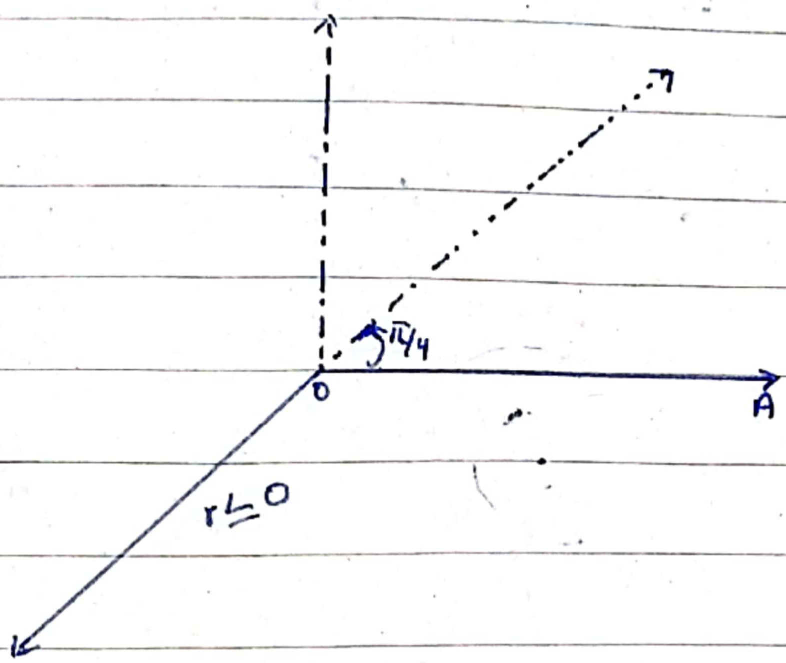
$\left(6, -\frac{\pi}{3} + 2n\pi\right)$ $n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \dots$

$\left(-6, \frac{2\pi}{3} + 2n\pi\right)$ $n = 0, \pm 1, \pm 2, \pm 3, \pm 4 \dots$

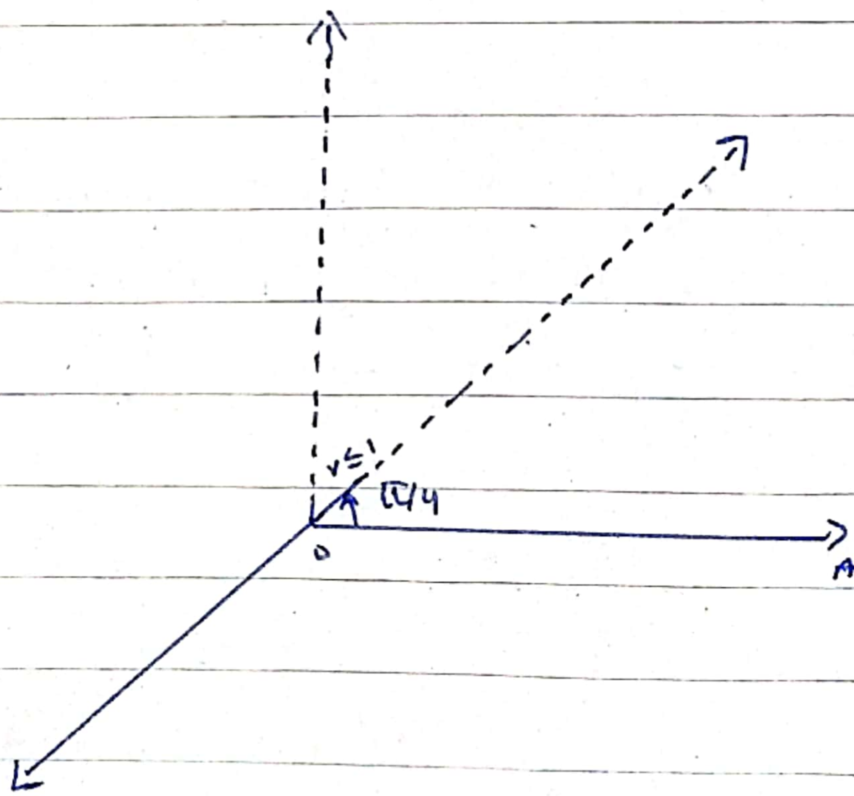
So the line of inclination is not same for both points

Draw graph

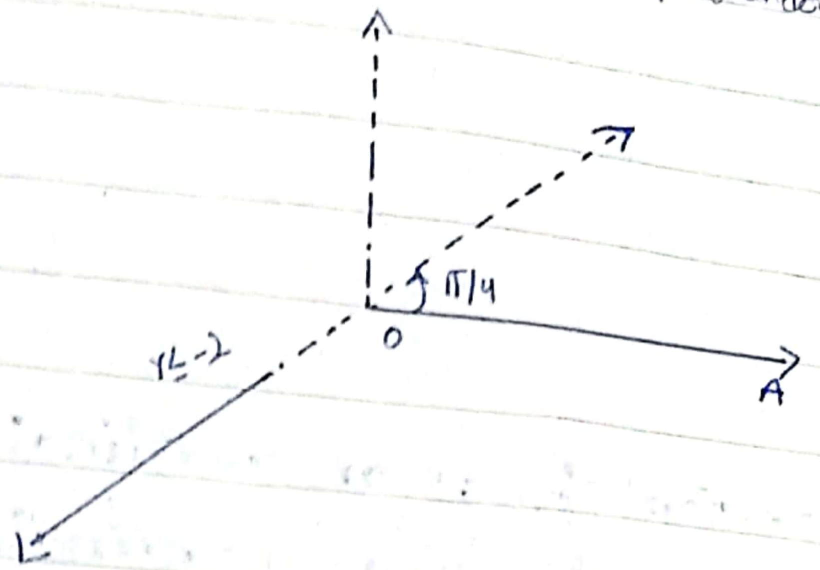
$r \le 0 \quad , \quad \theta = \frac{\pi}{4}$ (Random)



If $r \le 1 \quad , \theta = \pi/4$ (Random)



If $r \leq -2$ and $\theta = \pi/4$ (Random)



Conversion of x-y coordinate equation to polar coordinate equation.

To convert x-y coordinate equation to polar equation, we simply replace

x with $r\cos\theta$

y with $r\sin\theta$

x^2+y^2 with r^2

Convert into polar coordinate equation.

$$x^2-4x+4+y^2=4 \quad \text{(Random)}$$

$$= x^2-4x+\cancel{4}+y^2-\cancel{4}=0$$

Date:

$$x^2+y^2-4x=0$$

Replacing x^2+y^2 with r^2 and x with $r\cos\theta$

$$r^2-4r\cos\theta=0$$

$$r^2=4r\cos\theta \quad \text{or} \quad r=4\cos\theta$$

Conversion of polar coordinates equation to x-y coordinate equation

To convert polar coordinate equation to xy-coordinate equation, we simply replace

$r\cos\theta$ with x

$r\sin\theta$ with y

and r^2 with x^2+y^2.

Convert into xy-coordinate equation.

$r=1-\cos\theta$ (Random)

Multiplying both sides by r

$$r^2=r(1-\cos\theta)$$

$$r^2=r-r\cos\theta$$

M T W T F S Date:

Replace r^2 with x^2+y^2

and $r\cos\theta$ with x

$$x^2+y^2 = r - x$$

$$x^2+y^2+x = r$$

$$r = x^2+y^2+x$$

Taking square on both sides.

$$r^2 = (x^2+y^2+x)^2$$

Using formula

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$r^2 = x^4+y^4+x^2+2x^2y^2+2xy^2+2x^3$$

Replacing r^2 with x^2+y^2

$$x^2+y^2 = x^4+y^4+\cancel{x^2}+2x^2y^2+2xy^2+2x^3$$

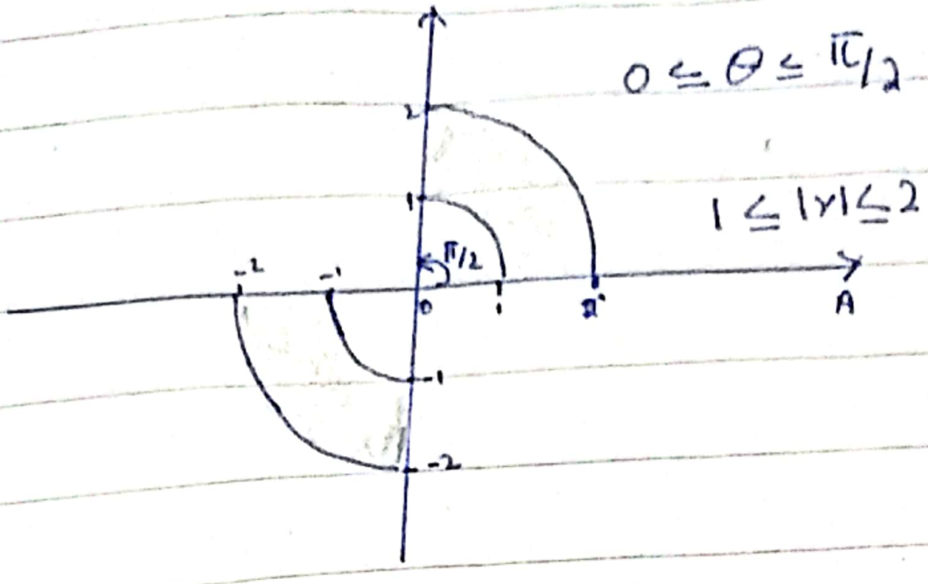
$$= x^4+y^4+2x^3+2x^2y^2+2xy^2-y^2$$

Exercise 10.5 Q22

Q Graph the following

$0 \le \theta \le \pi/2$, $1 \le |r| \le 2$

If there is $1 \le |r| \le 2$ it mean we will draw graph for both $1 \le r \le 2$ and $-1 \ge r \ge -2$



Exercise 10.6

Graphing the polar coordinate equations
(10 Marks long Question)

Following are the steps to draw a graph of a polar coordinate equation.

Step 1 : Check symmetry

Step 2 : Create Table.

Step 3 : Plot the points

Step 4 : Join them to draw graph

Symmetry checking :-

To check the symmetry of the graph, we do three tests to check

i) symmetry about x- axis

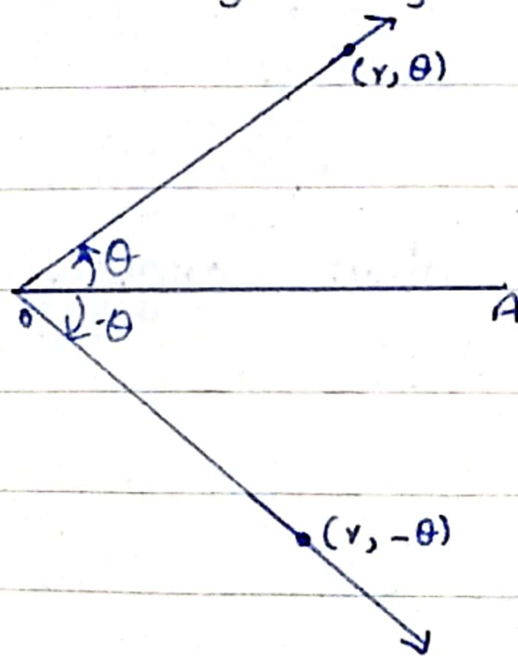
ii) symmetry about y - axis

iii) symmetry about origin

Symmetry about x- axis :-

To check the symmetry about x- axis we put the mirror point about x-axis in the given equation, If the mirror point satisfies the equation, it means the graph holds symmetry about x-axis.

[To find mirror image, we just travel θ but in clock wise direction (for- x-axis only)]

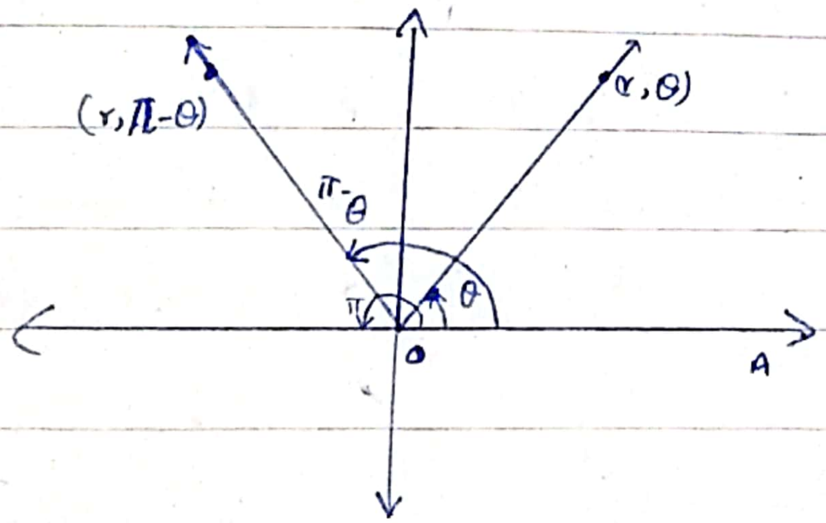


Mirror point about x-axis of (r,θ) is $(r,-\theta)$

Symmetry about y-axis

To check symmetry about y-axis, we put the mirror point about y-axis in the equation, If the mirror point satisfies the equation, the graph holds the symmetry about y-axis.

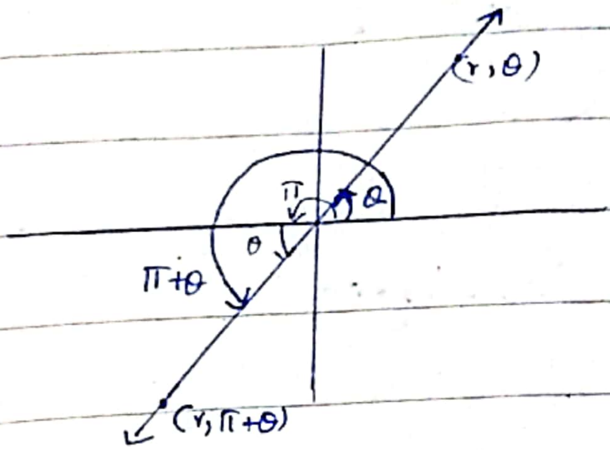
Mirror point along y-axis can be found by travelling in the direction $\pi-\theta$ in counter clockwise direction



Symmetry about origin.

To check symmetry about origin, we put the mirror point about origin in the equation, If the mirror point satisfies, the graph is symmetric about origin.

Mirror point about origin is found by traveling to $\pi+\theta$ in counter clockwise direction.



~~Erro~~ Table:-

To create table we simply take the values of θ according to the symmetry of the graph. Put the values of θ in the equation and solve to get value of r. Hence a point in form (r,θ) is created.

Graph:-

Join the points to draw the curve.

Polar points can be differentiated in two ways.

i) different lines of inclination.

ii) forward of one is backward of other.
Backward of one is forward of other.

Exercise 10.6 Q22

Q **Draw graph of $r = 1-\cos\theta$**

or

Draw a cardoid

$r = 1-\cos\theta$ (i)

Symmetry checking:-

i) Symmetry about x-axis

~~r=1-~~ Put $(r, -\theta)$ in equation (i)

$r = 1-\cos(-\theta)$

As $\cos(-\theta) = \cos\theta$

$r = 1-\cos\theta$

Symmetry holds

ii) Symmetry about y-axis:-

Put $(r, \pi-\theta)$ in equation (i)

$r = 1-\cos(\pi-\theta)$

Using formula $\cos(A-B) = \cos A\cos B + \sin A\sin B$

~~r=~~ $r = 1-(\cos\pi\cos\theta + \sin\pi\sin\theta)$

$r = 1-((-1)\cos\theta) + 0.\sin\theta)$

$r = 1+\cos\theta + 0$

$r = 1+\cos\theta$

Symmetry does not hold.

(iii) Symmetry about origin :-

Put $(r, \pi+\theta)$ in eq (i)

$$r = 1 - \cos(\pi + \theta)$$

Using formula

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$r = 1 - (\cos\pi \cos\theta - \sin\pi \sin\theta)$$

$$r = 1 - ((-1)\cos\theta - 0 \cdot \sin\theta)$$

$$r = 1 + \cos\theta = 0$$

$$r = 1 + \cos\theta$$

Table

θ	$r = 1 - \cos\theta$
$0^\circ = 0$	0
$30 = \pi/6$	0.13
$45^\circ = \pi/4$	0.29
$60^\circ = \pi/3$	0.5
$90^\circ = \pi/2$	1
$120^\circ = 2\pi/3$	1.5
$150^\circ = 5\pi/6$	1.86
$180^\circ = \pi$	2

Graph:-