Neuroscience

Homework1 report

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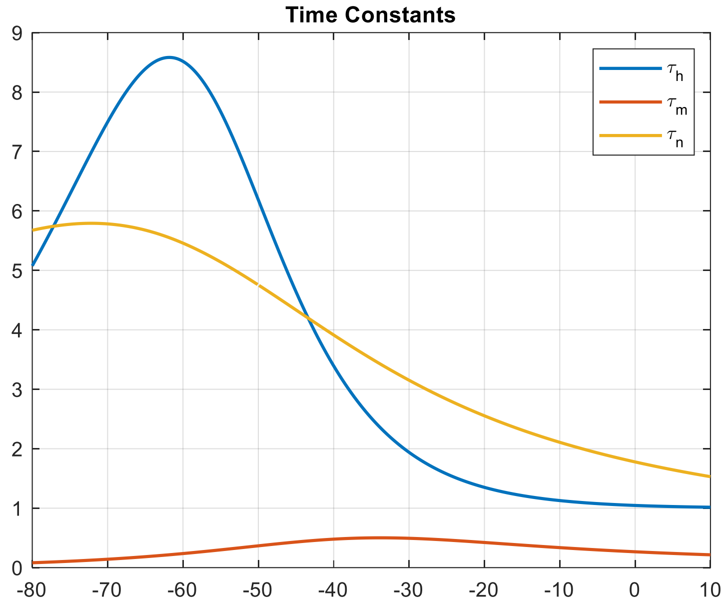
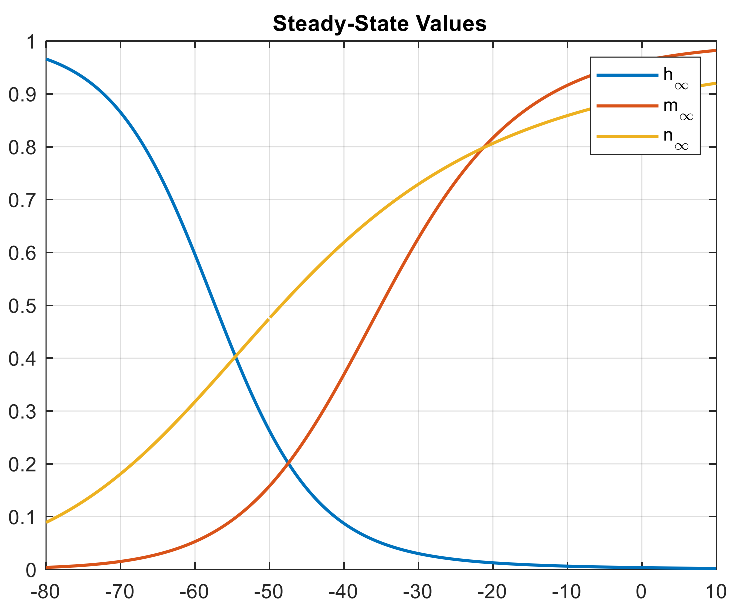
1. Hodgkin-Huxley model

1.1



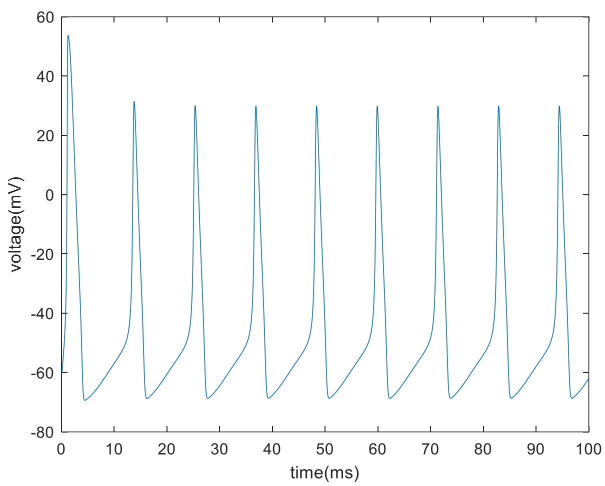


The explanation of $[Na^+]$ & $[K^+]$ channels:

In this part the whole cycle will be described step by step

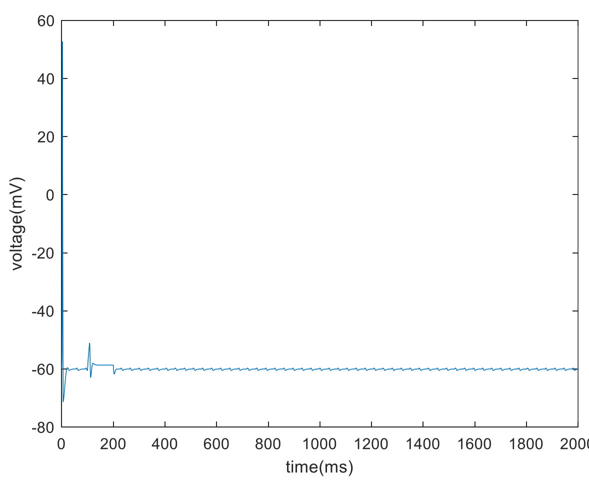
1. Rest voltage.
2. Entering Na from the synapse.
3. Increase in voltage.
4. The $[Na^+]$ channel opens. ($m_\infty \to 1$)
5. The h channel starts to close the $[Na^+]$ channel. ($h_\infty \to 0$)
6. In the point where, V = 10/20 mV, the $[Na^+]$ channel closes.
7. By increasing the voltage and closing the $[Na^+]$ channel, n channel starts to become open.
8. The voltage starts to fall. ($n_\infty \to 1$)

1.2

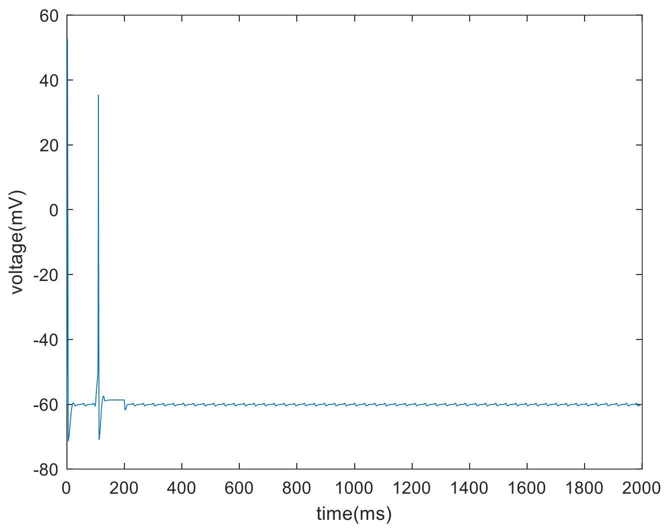


1.3

In the current = 2.171 mA the neuron cannot start the spiking process.

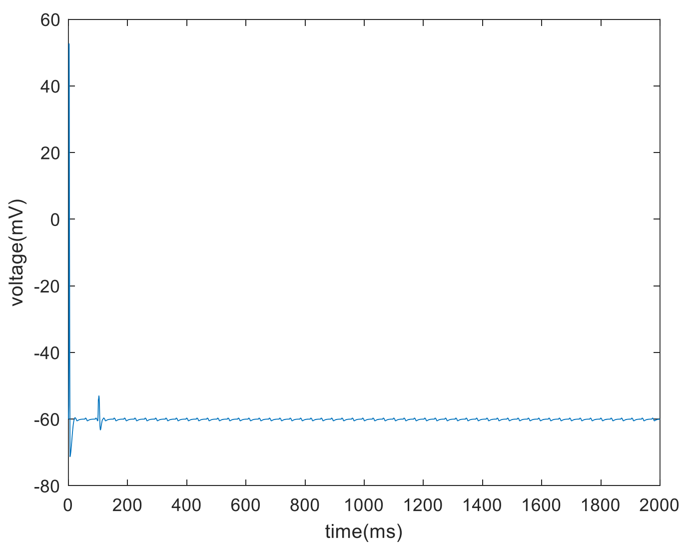


In the current I = 2.172:

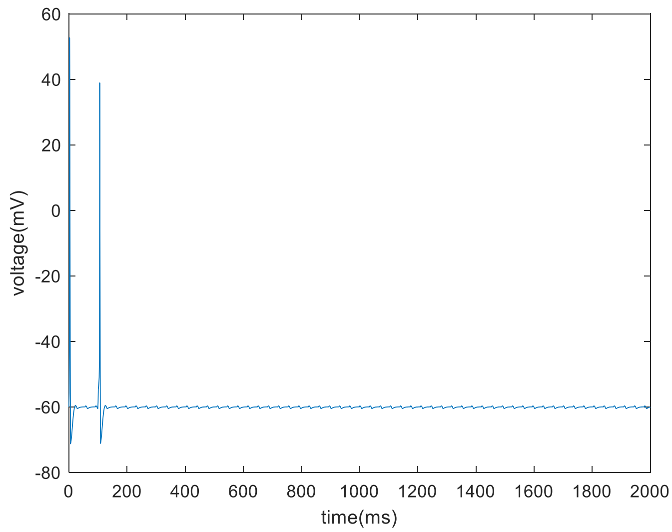


1.4

When the input current is given as $k = 1000, I(k, k + 121) = 6$ then the plot would become:



While if $k = 1000, I(k, k + 122) = 6$ the plot would be:

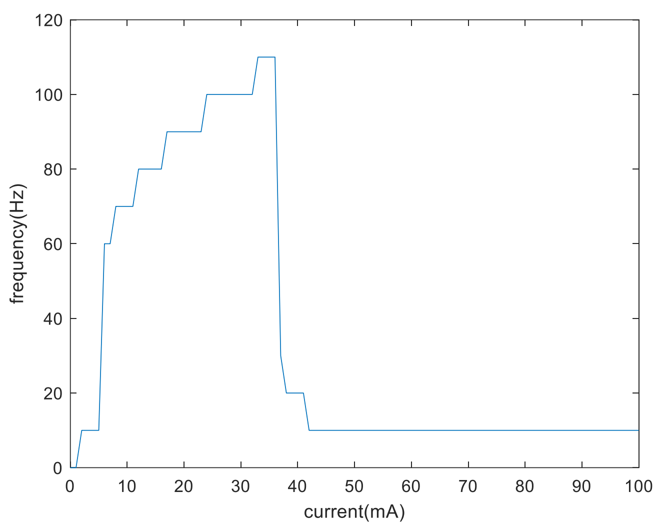


So the minimum time for giving the input to the system will be:

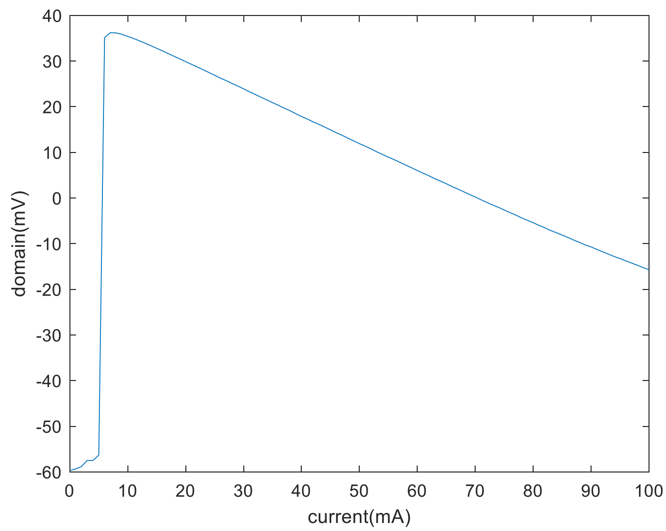
$$t = 121 \times dt = 121 \times 0.01 = 1.21\ ms$$

1.5

The frequency plot:



The domain plot:

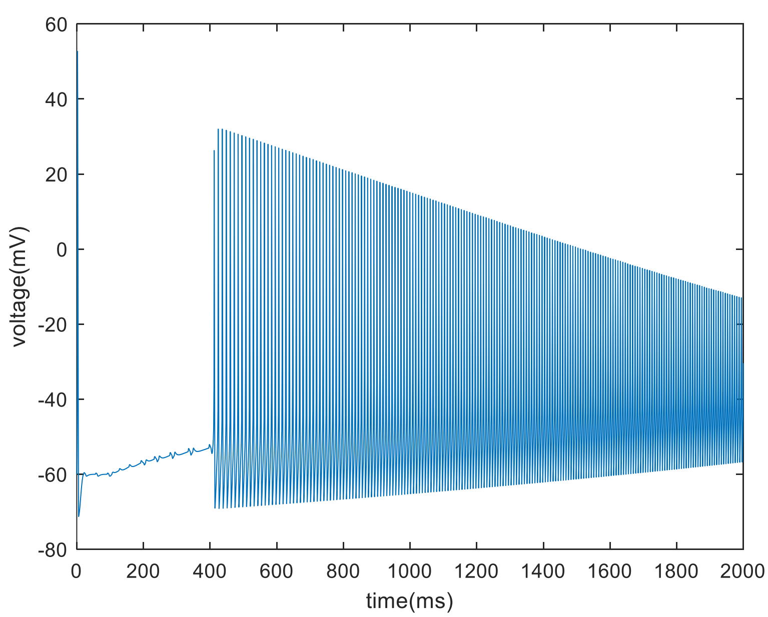


1.6

As it is seen in the plot of the previous part after a while the domain of the spikes will decrease after a threshold.

1.7

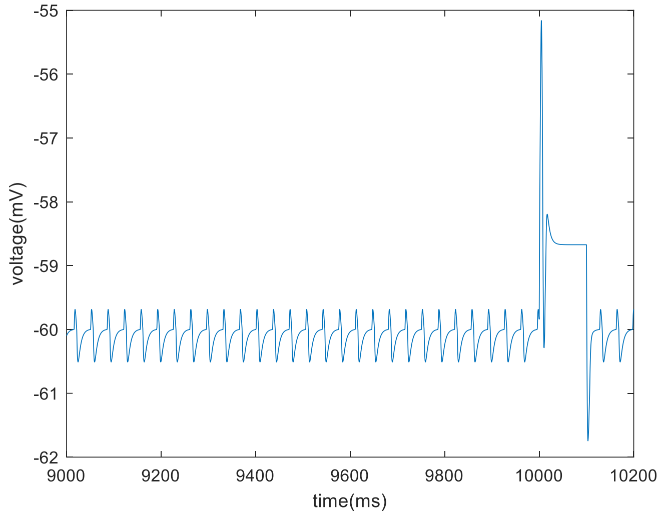
The current varies between 0.1 to 100 mA:

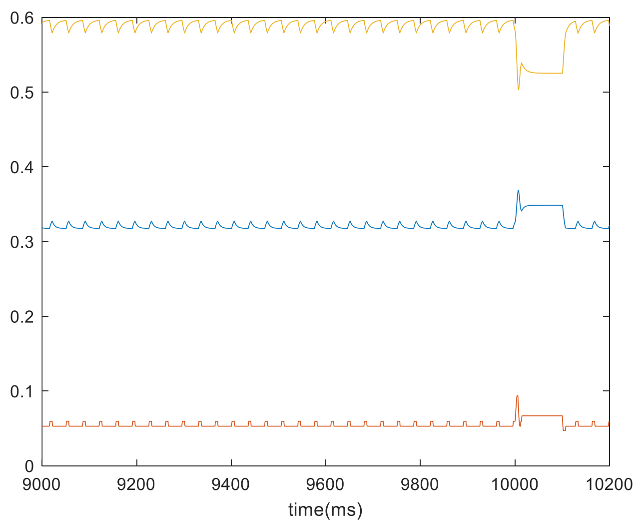


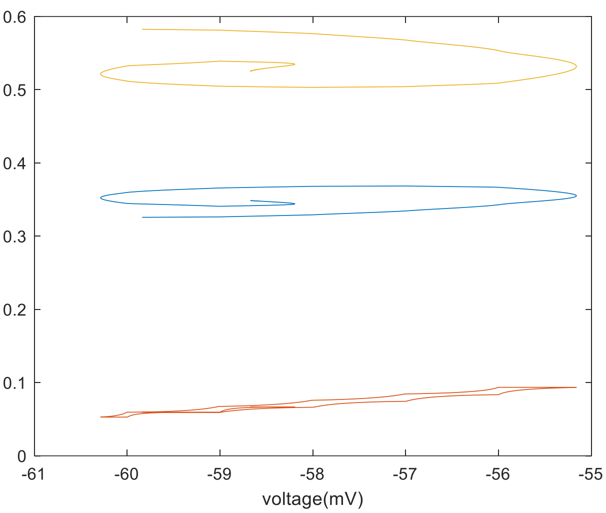
At first the maximum voltage for spiking rise and after reaching to the max voltage it starts to fall.

1.8

The plot for when we insert a small current input (the first plot is the voltage and the second one is for n, m, h).

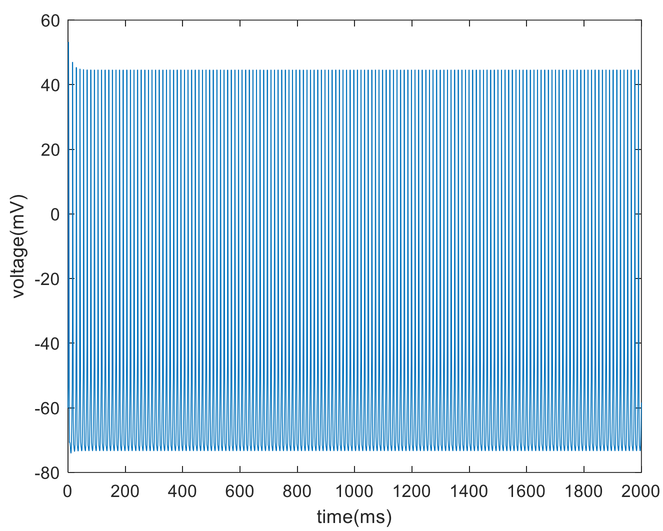


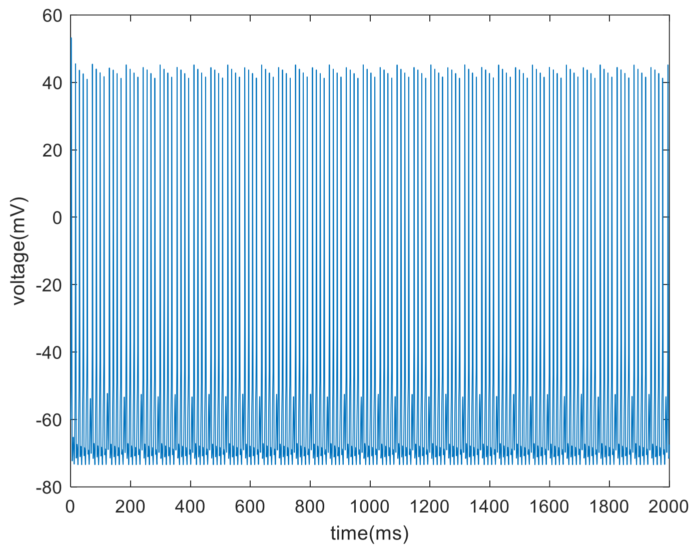




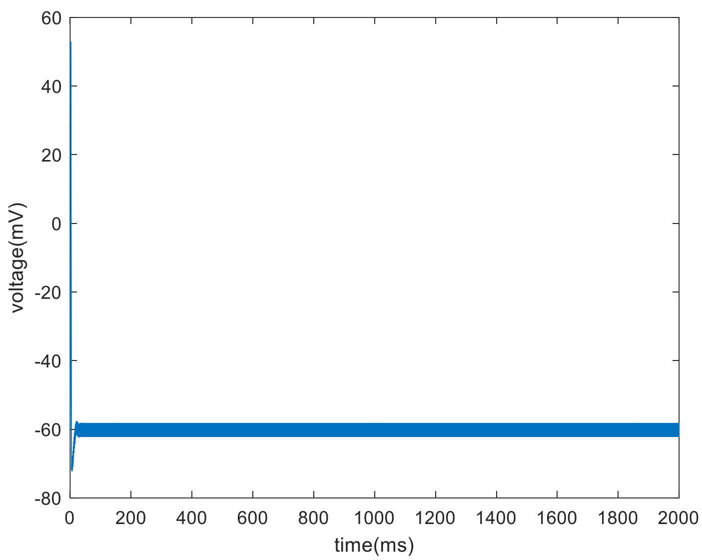
1.9

Sinusoidal input:

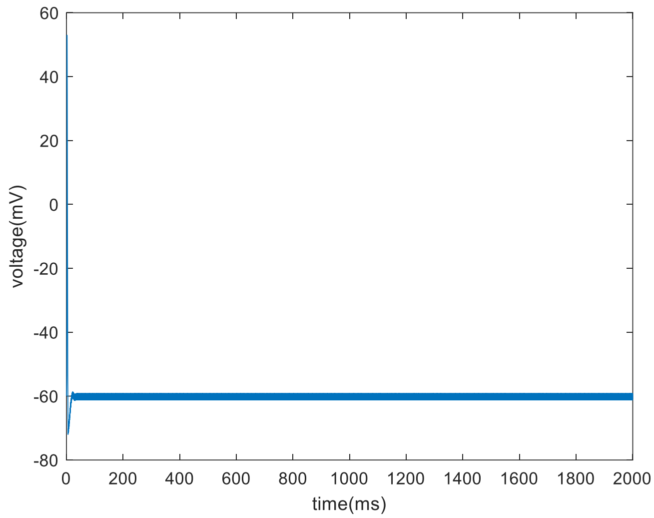




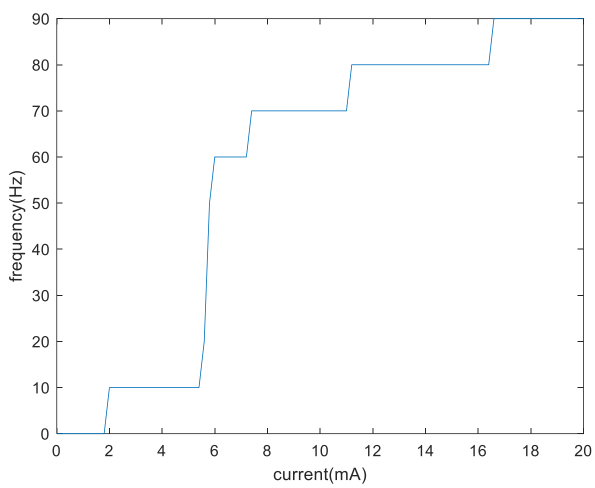
60
40
20
0
-20
-40
-60
-80
voltage(mV)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
time(ms)



60
40
20
0
-20
-40
-60
-80
voltage(mV)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
time(ms)



1.10



2. Izhikevich model

2.1

u is a function for simulating the $[K^+]$ channels which will decrease the amount of voltage.

2.2

C: is the minimum voltage or the rest voltage

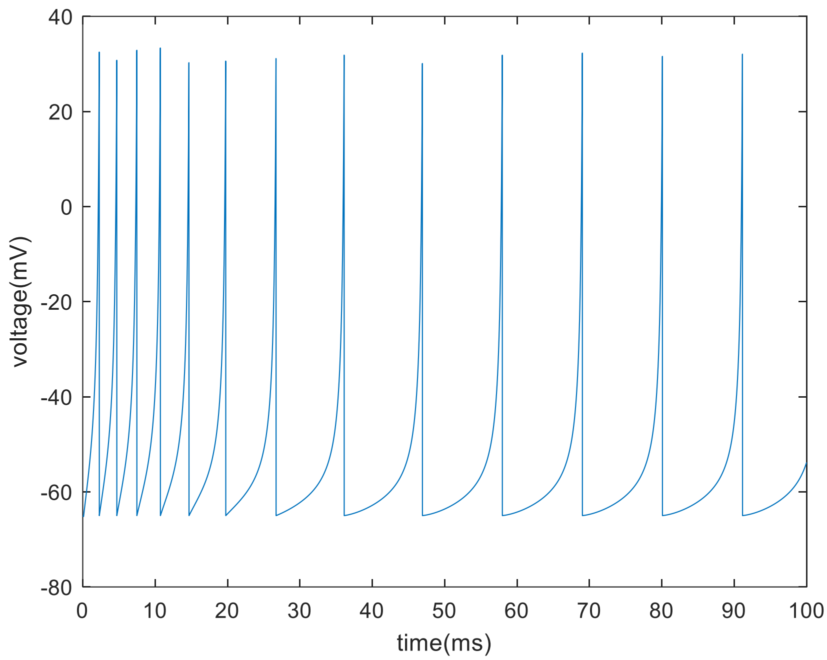
A: Describes time scale of recovery variable

B: Sensitivity of recovery variable

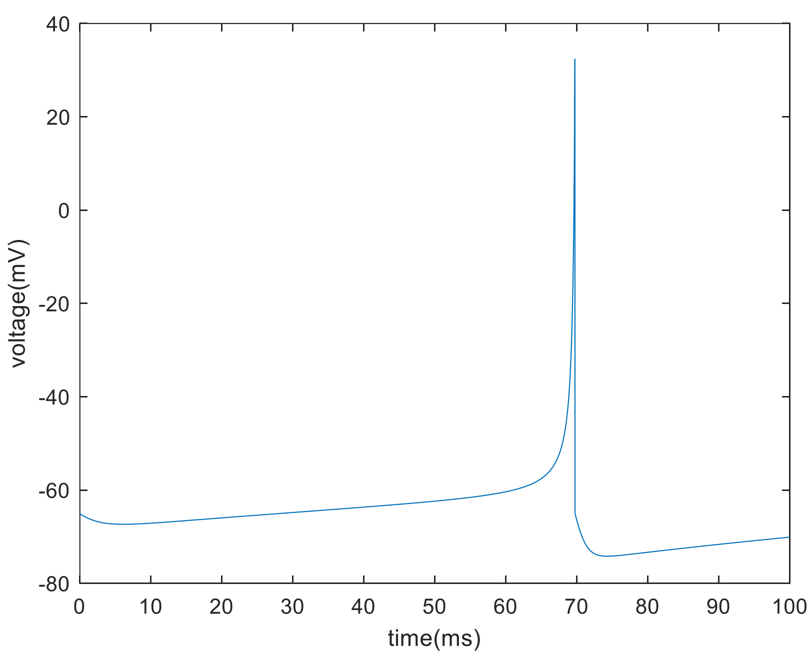
D: After-spike reset value of u

2.3

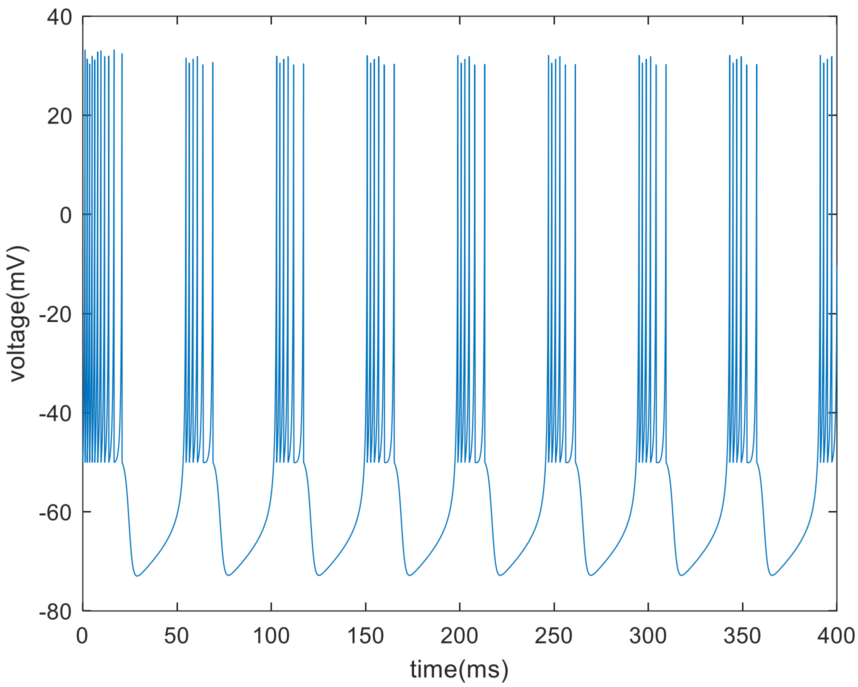
2.3.1 Tonic spiking



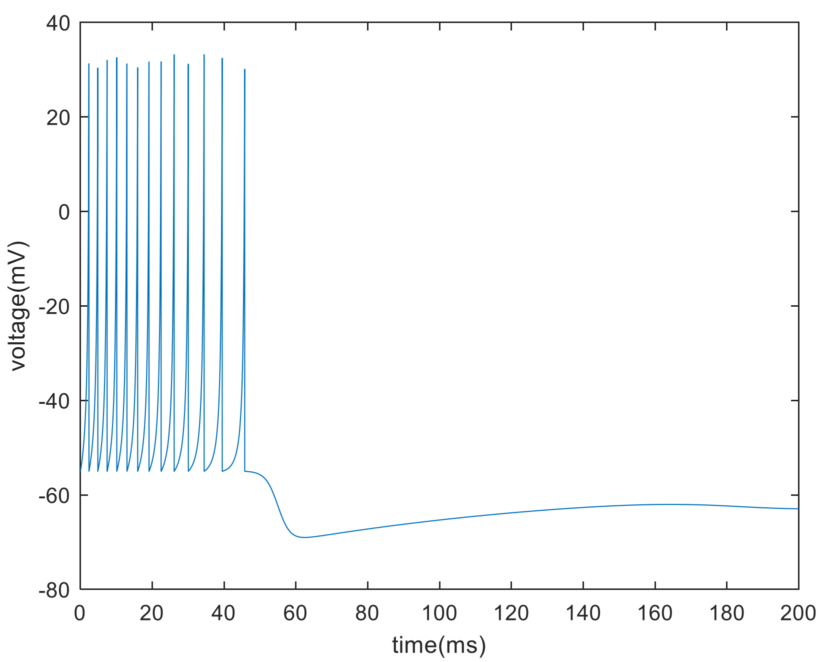
2.3.2 Phasic spiking



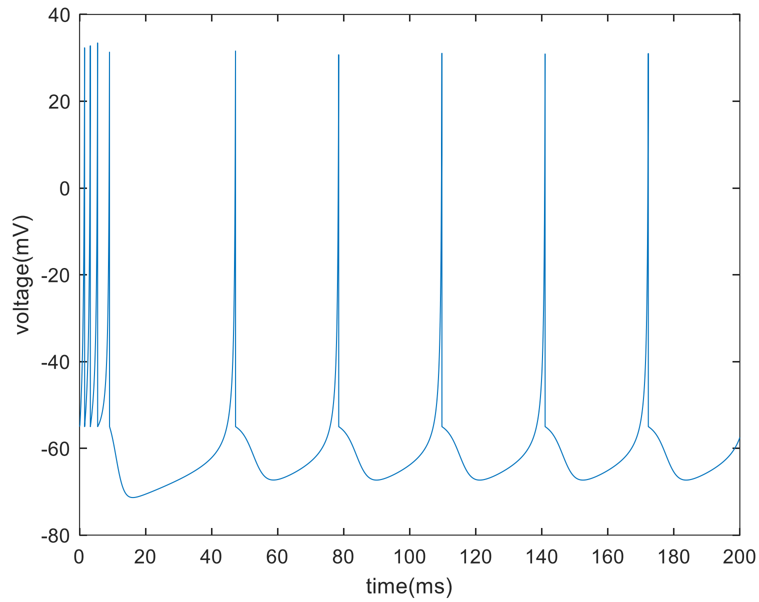
2.3.3 Tonic bursting



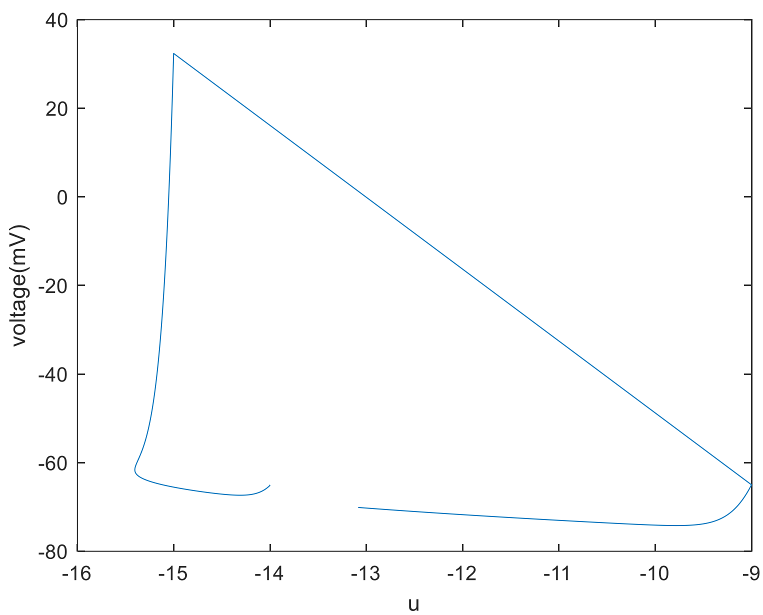
2.3.4 Phasic bursting



2.3.5 Mixed model



2.4

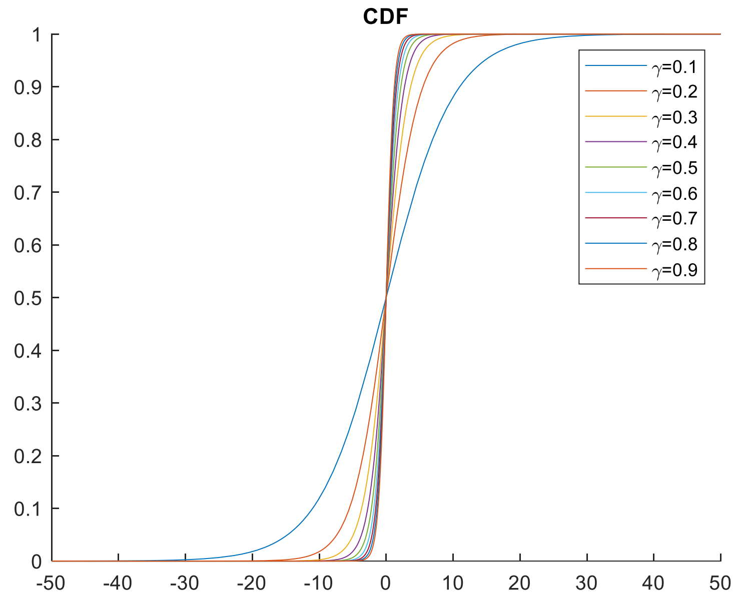


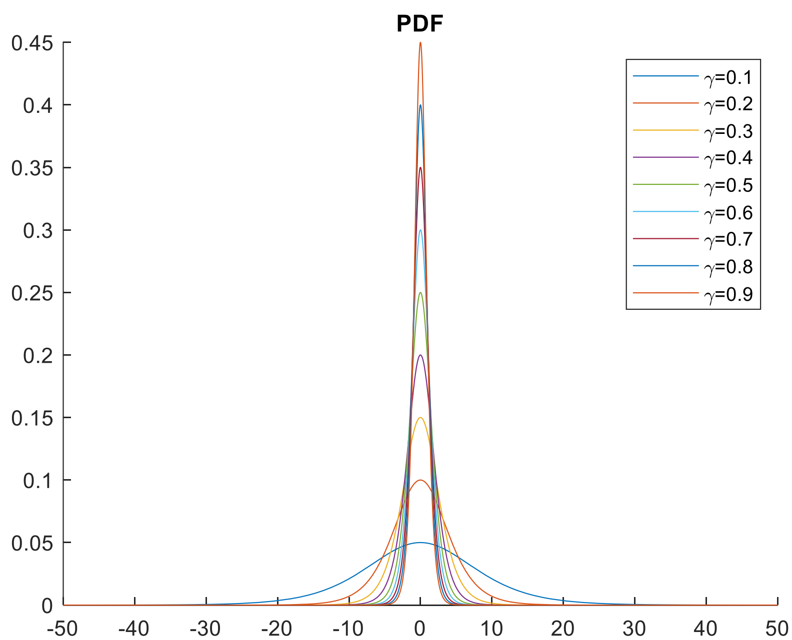
3. Noisy output model

3.1

Computing the PDF function:

$$f(x) = \frac{dF(x)}{dx} \rightarrow f(x) = \frac{4\gamma\beta}{(\varepsilon^{\gamma x} + e^{-\gamma x})^2}$$

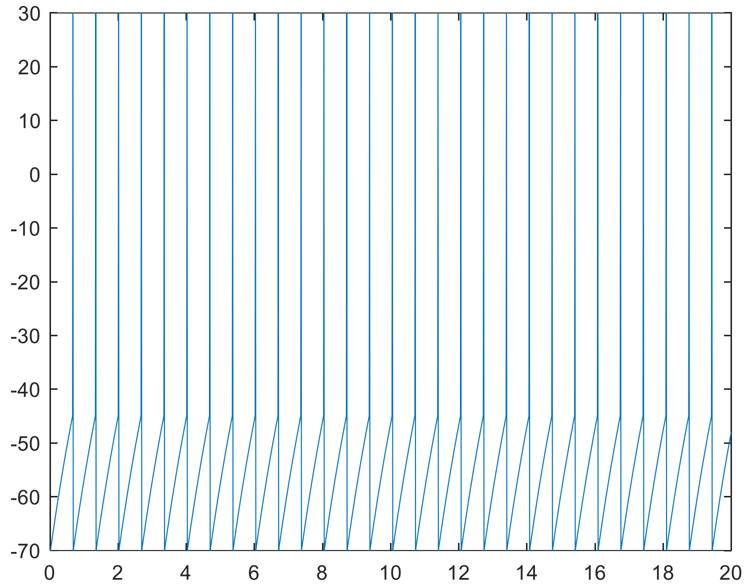




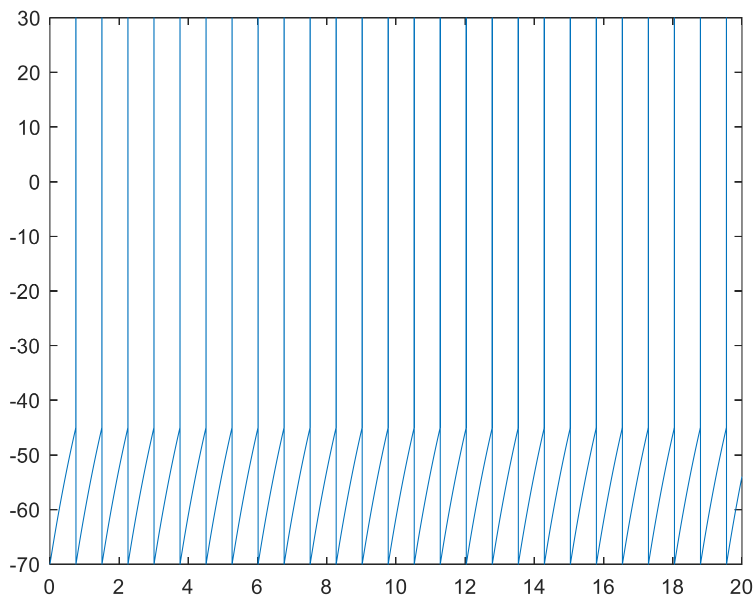
With an investigation on the above plots in can be realized that the most probable state is when the difference of threshold value and the voltage is zero and by increasing the difference the probability decrease.

3.2

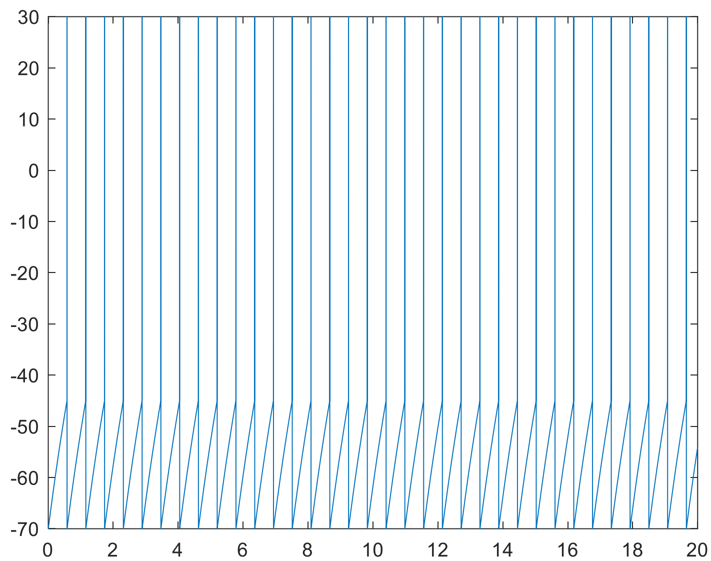
RI = 20mV



RI = 10mV



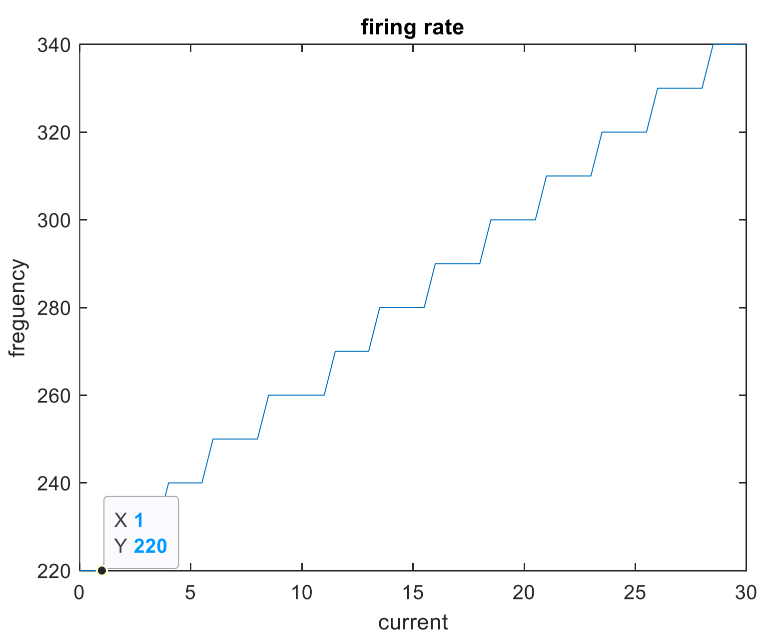
RI = 30mV



By increasing the amount of RI the frequency of spikes will rise.

3.3

3.4



4. Integrate and fire models

4.1

Perfect integrate and fire model:

It is also called non-leaky integrate and fire model. In this model a neuron is represented by two parameters it's **voltage** and the **input current** which are related as the equation bellow:

$$I(t) = C\frac{dV(t)}{dt}$$

Leaky integrate and fire model:

This model is also called passive integrate and fire model. In this model all active membrane conductance are ignored, including, for the moment, synaptic inputs, and the entire membrane conductance is modeled as a single passive leakage term:

$$i_m \equiv \overline{g_L}(V - E_L)$$

This model neuron behaves like an electric circuit consisting of a resistor and a capacitor in parallel.

$$C_m\frac{dV}{dt} = -i_m + \frac{I_e}{A} = -\overline{g_L}(V - E_L) + \frac{I_e}{A}$$

$$if\ r_m = \frac{1}{\overline{g_L}}\ and\ \frac{r_m}{A} = R_m\ and\ C_m r_m = \tau_m$$

$$\rightarrow \tau_m\frac{dv}{dt} = E_L - V + R_m I_e$$

In the above equation E_L is the reset potential.

From the equations $V(t)$ can easily be computed:

$$V(t) = E_L + R_m I_e + (V(0) - E_L - R_m I_e)\exp(-t\ /\ \tau_m)$$

Comparing two models:

The perfect integrate and fire model describes neither adaption nor leakage; thus, if the model once receives an under-threshold current, it will retain that voltage boost forever until another input later makes it fire. This characteristic is clearly not representing observed neuronal behavior. In the leakage integrate and fire model this shortcoming is somehow solved.

5. Investigation on bifurcations

5.1

Finding the fixed point:

$$\dot{x} = x^2 + a$$

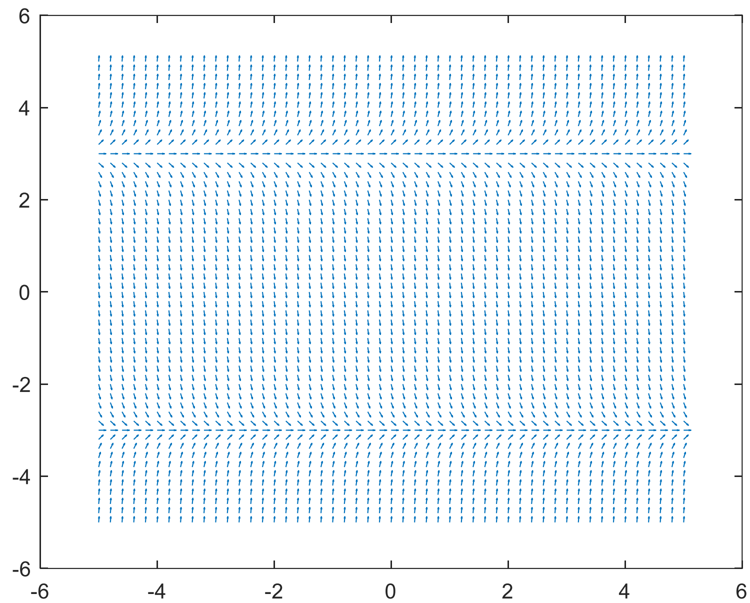
$$if\ \dot{x} = 0 \rightarrow x^2 + a = 0 \rightarrow x^2 = -a$$

$$\begin{cases} a < 0\ \rightarrow x = \pm\sqrt{-a} \\ a = 0\ \rightarrow x = 0 \\ a > 0 \rightarrow no\ fixed\ points \end{cases}$$

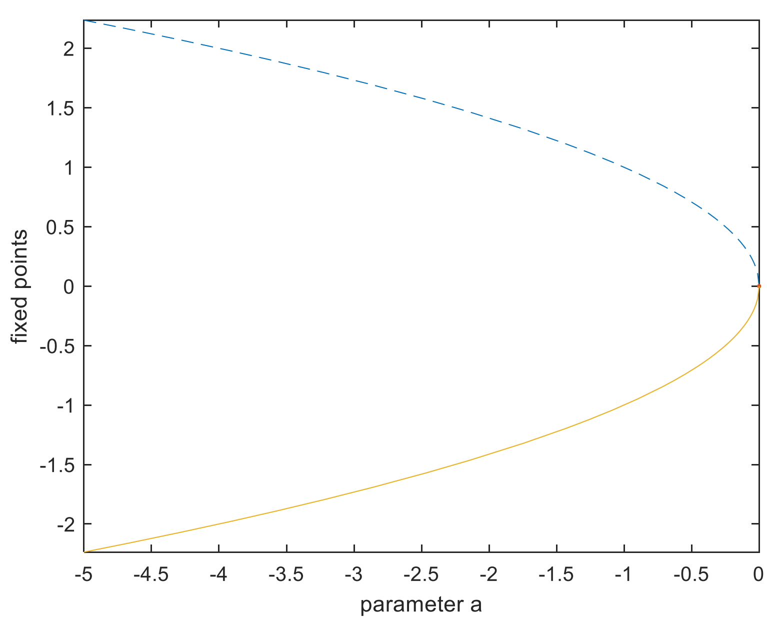
$$\begin{cases} x = +\sqrt{-a} \rightarrow not\ stable \\ x = 0 \ \rightarrow stable \\ x = -\sqrt{-a} \rightarrow stable \end{cases}$$

From the above lines it can be understand that in x =0 the number of fixed points has changed, thus, there is the point which saddle-node bifurcation takes place.

Also for finding the stability we can have a quick look to the field direction plot for an example a = -9:



The plot of fixed points values-parameter a:



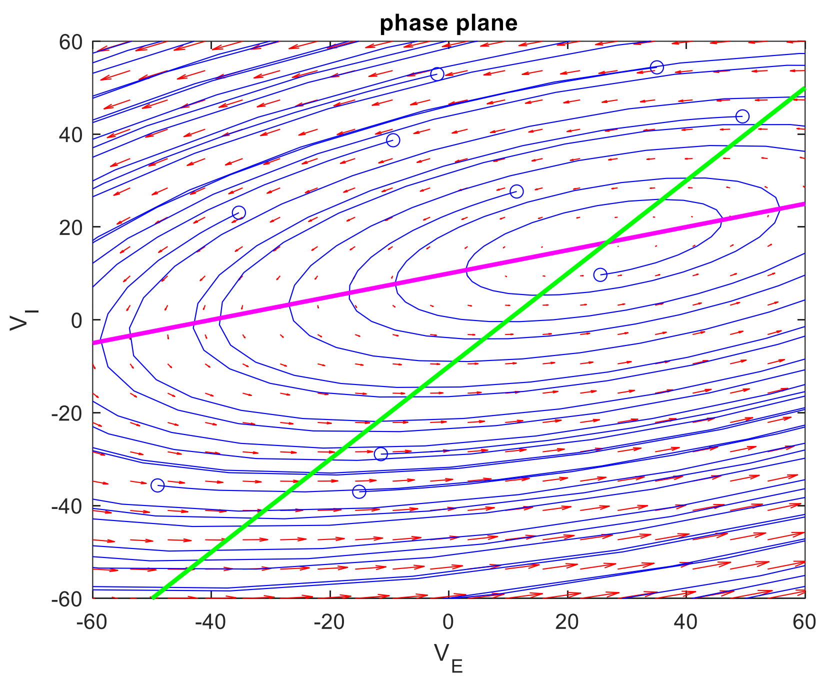
5.2

The number of fixed points of the differential equation $\dot{x} = f(x)$ is equal to the number of answers for the equation $f(x) = 0$, and the bifurcation takes place when this number changes. Thus, we are looking for the point that the number of a function's routs changes.

We know that between each pair of successive routs there is a point where $f'(x) = 0$; changing a constant in a function is similar to moving the x axis vertically and that's how that the number of routs will decrease by reaching to the point $f'(x) = 0$, as the pair of successive routs mentioned previously convert to one rout in $f'(x) = 0$.

6. Phase plane

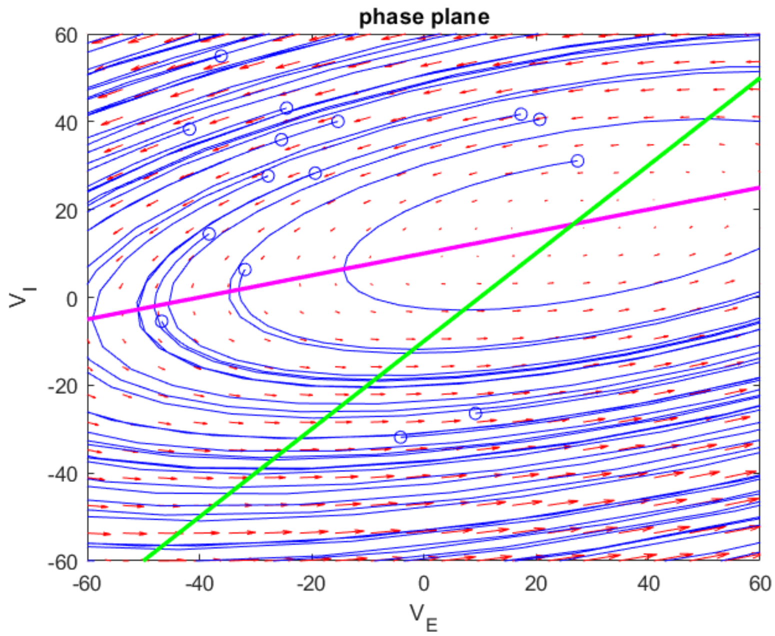
The first output of the code without any changes:



6.1

The phase plane for noip = 15:

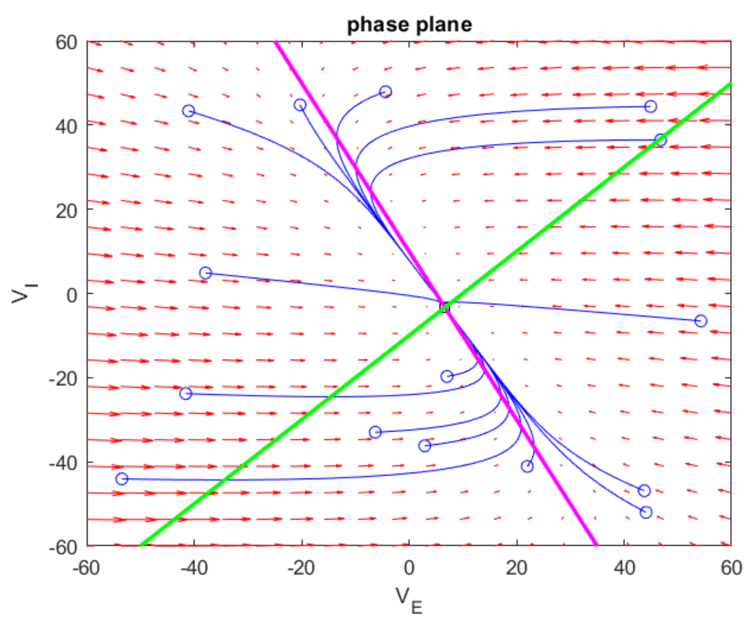
As we can see in the phase plane the type of the fixed point is stable spiral. Because the solutions are circular converge at the fixed point.



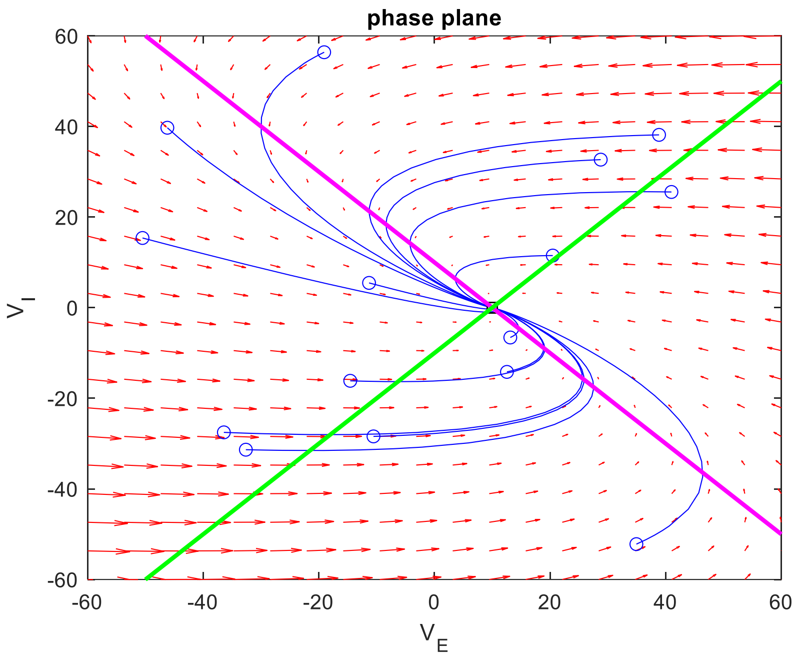
6.2

The effect of varying M_{EE} on the phase plane:

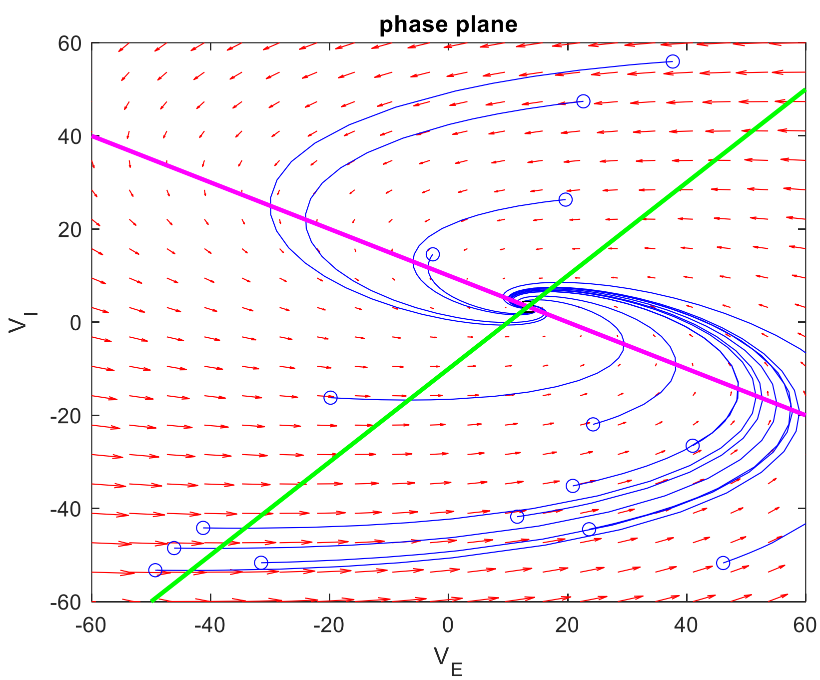
$$M_{EE} = -1$$



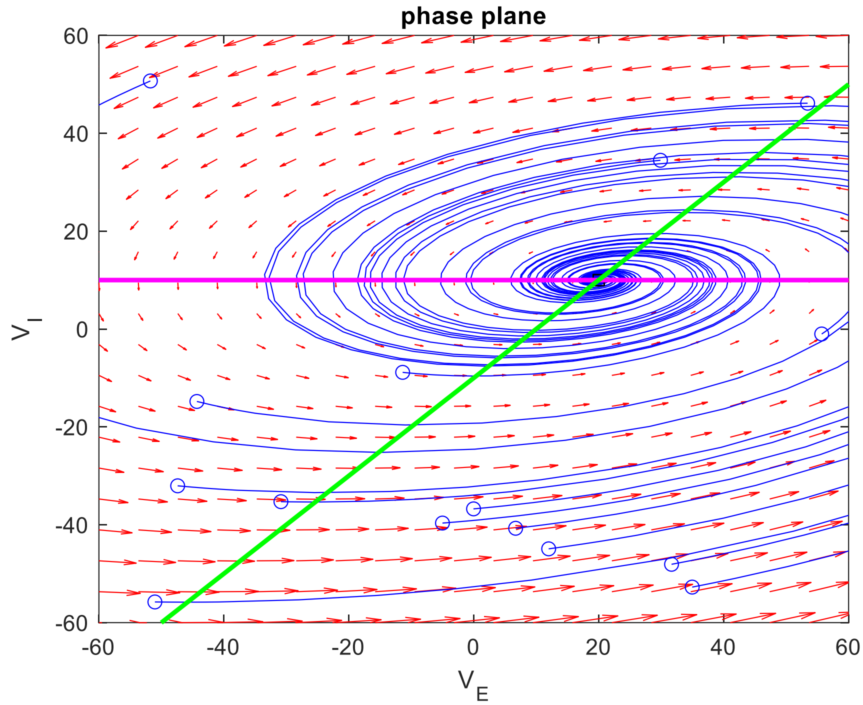
$$M_{EE} = 0$$


phase plane
V_I
V_E
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

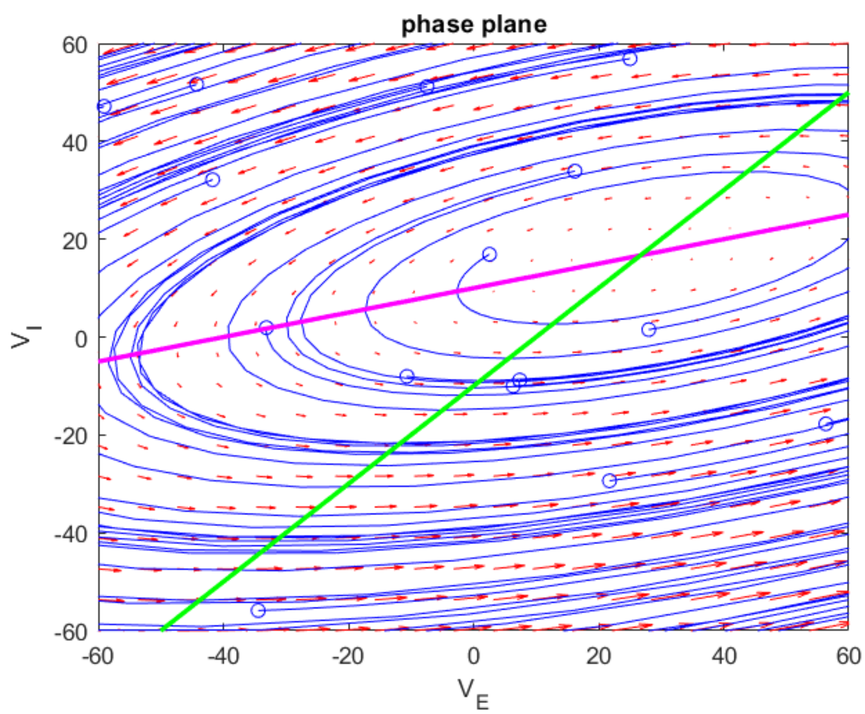
$M_{EE} = 0.5$


phase plane
V_I
V_E
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

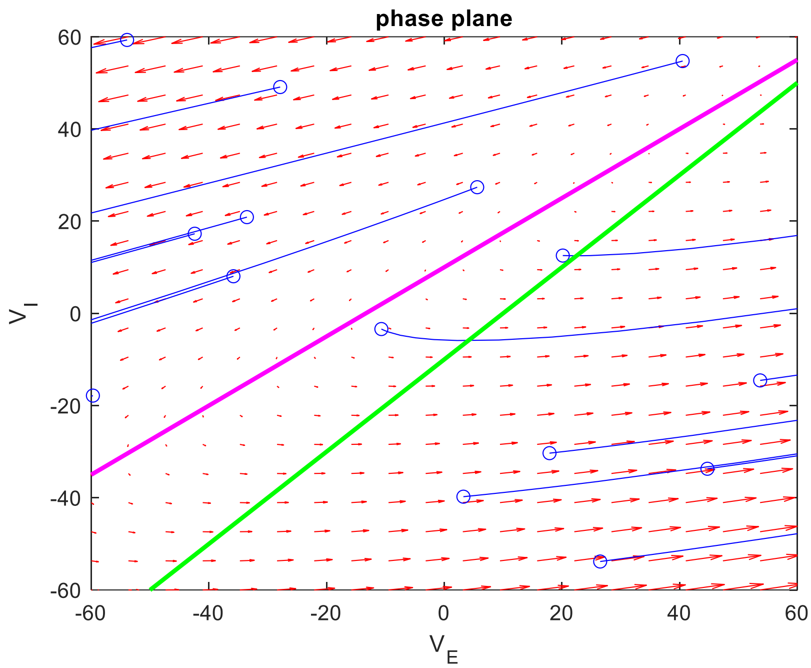
$M_{EE} = 1$



$M_{EE} = 1.25$



$M_{EE} = 1.75$

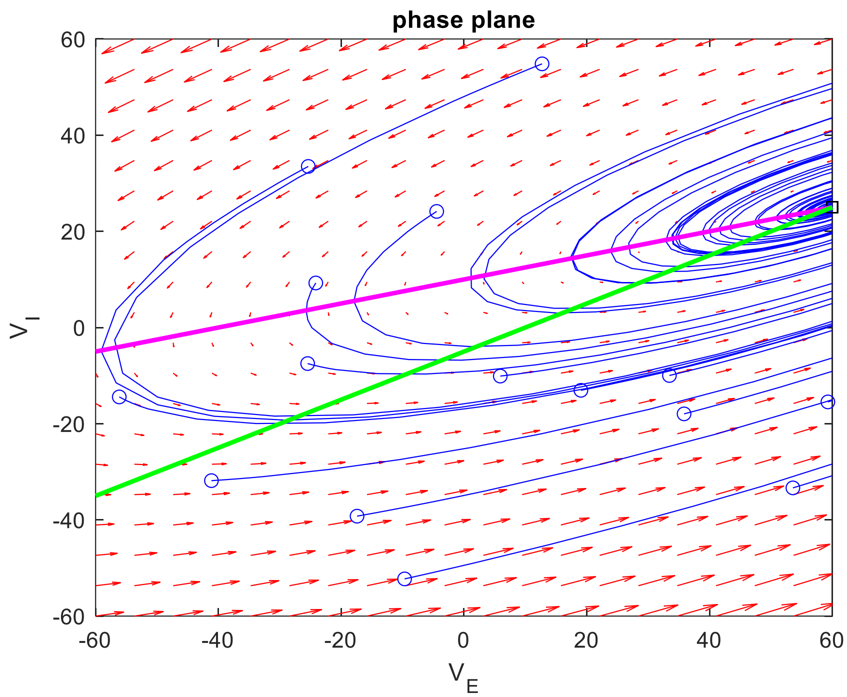


By increasing the amount of this parameter the random points will converge more slightly at the fixed point. And also the place of fixed point will change.

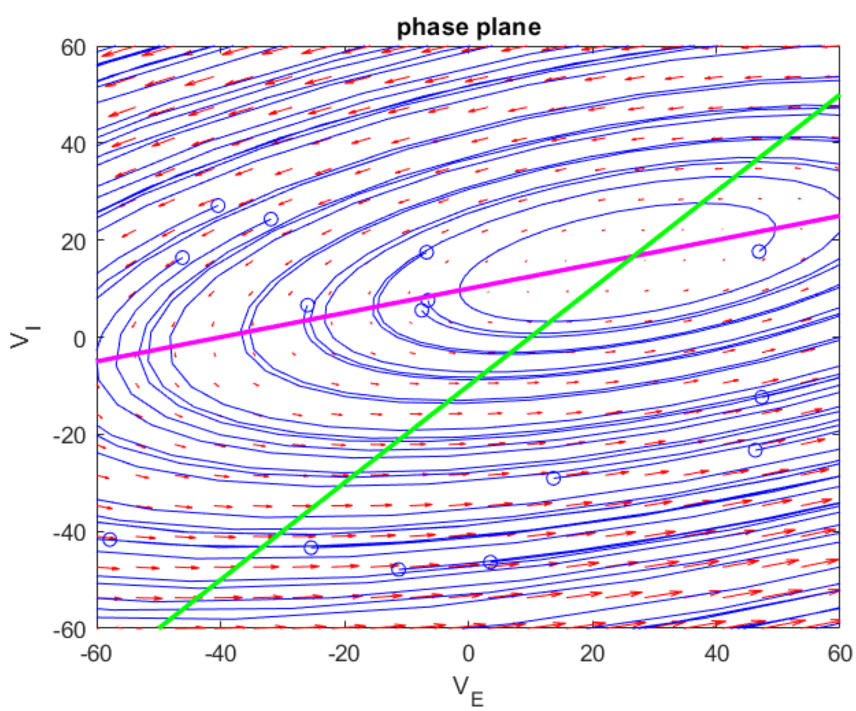
6.3

1. The effect of varying M_{II}:

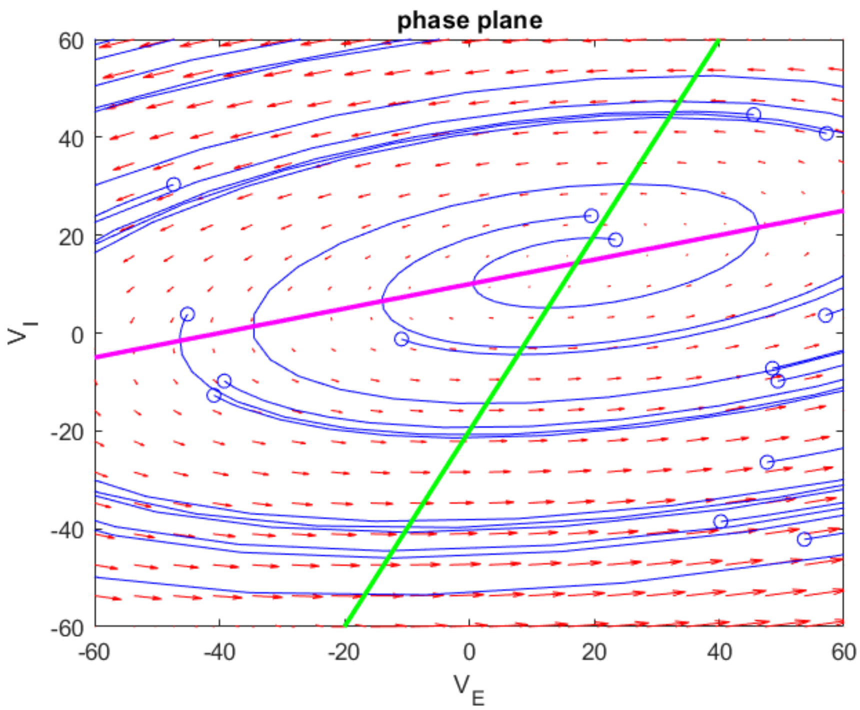
$$M_{II} = -1$$


phase plane
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
V_I
V_E

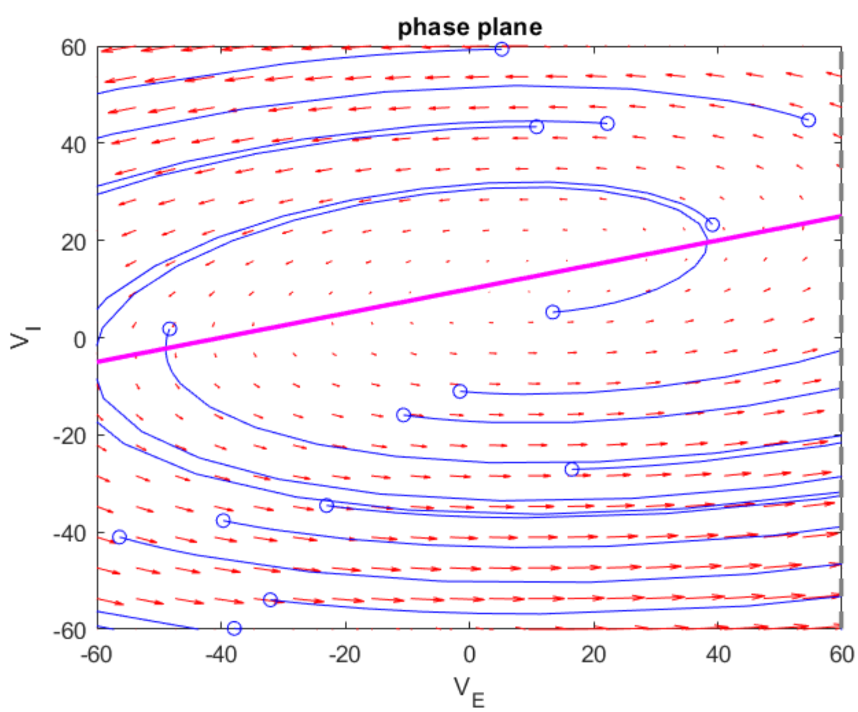
$M_{II} = 0$


phase plane
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
V_I
V_E

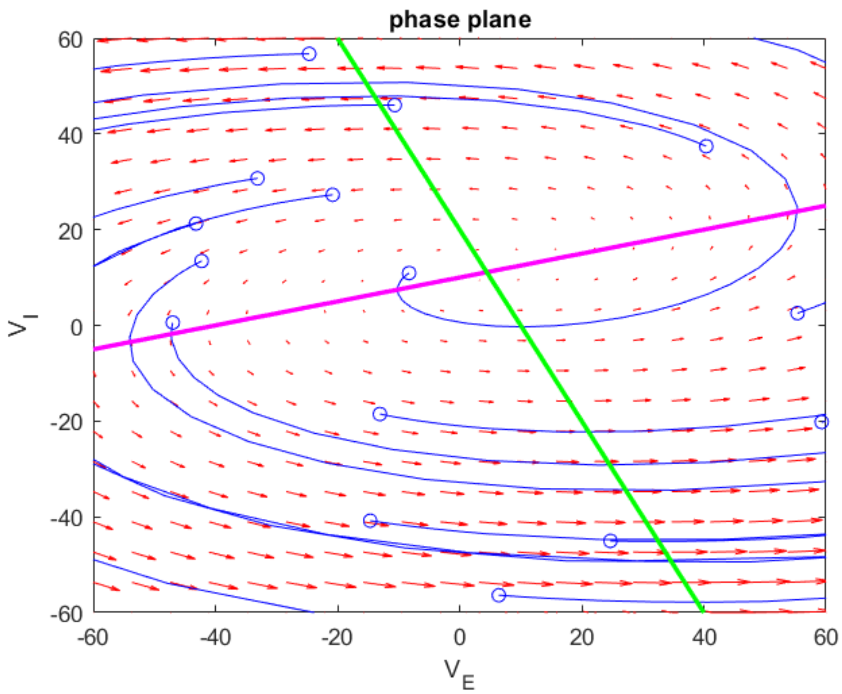
$M_{II} = 0.5$


phase plane
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
V_I
V_E

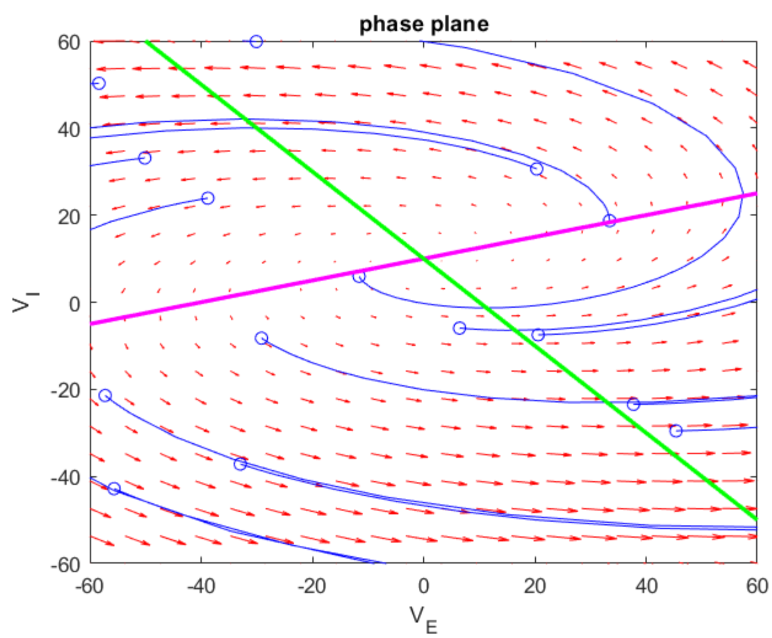
$M_{II} = 1$


phase plane
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
V_I
V_E

$M_{II} = 1.5$



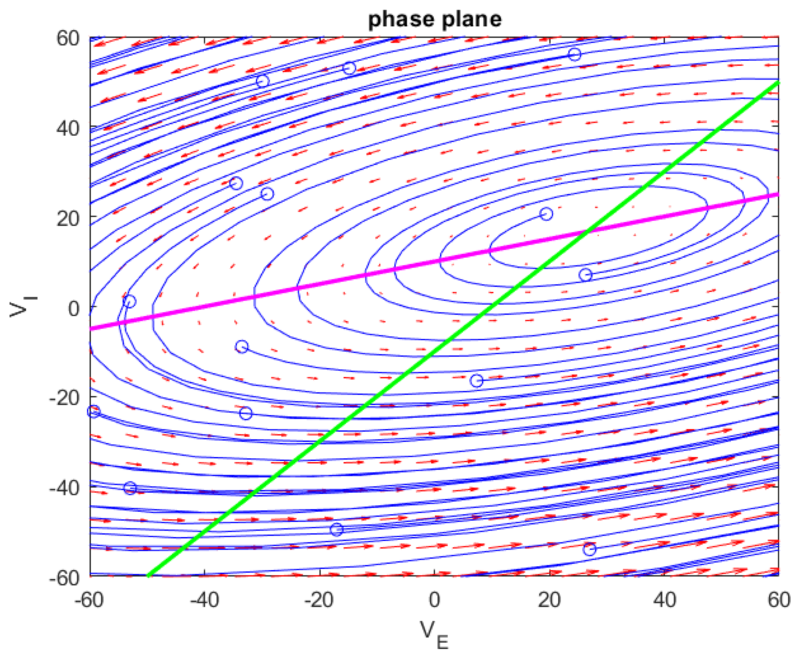
$$M_{II} = 2$$



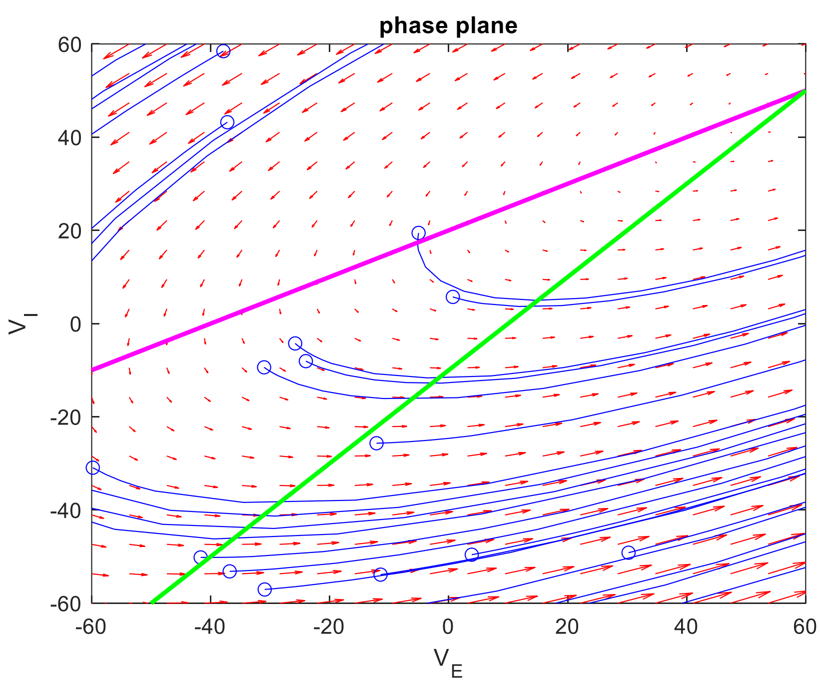
It is similar to the previous part.

2. The effect of varying M_{EI}:

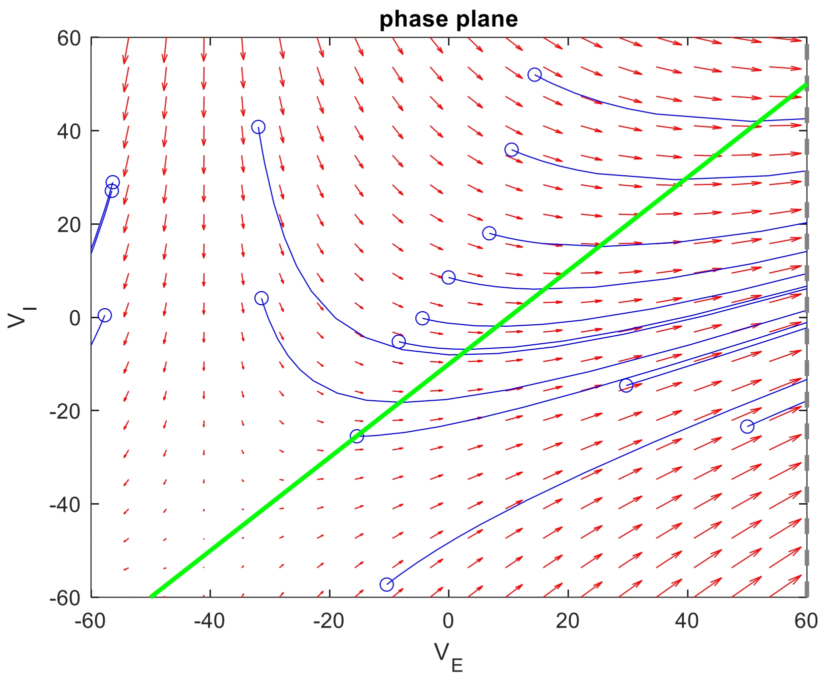
$M_{EI} = -1$



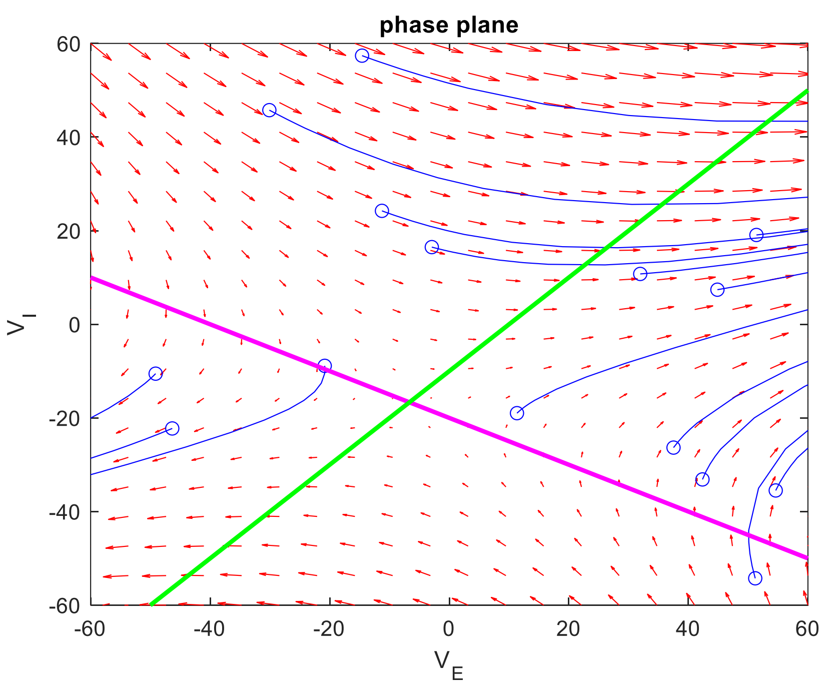
$M_{EI} = -0.5$



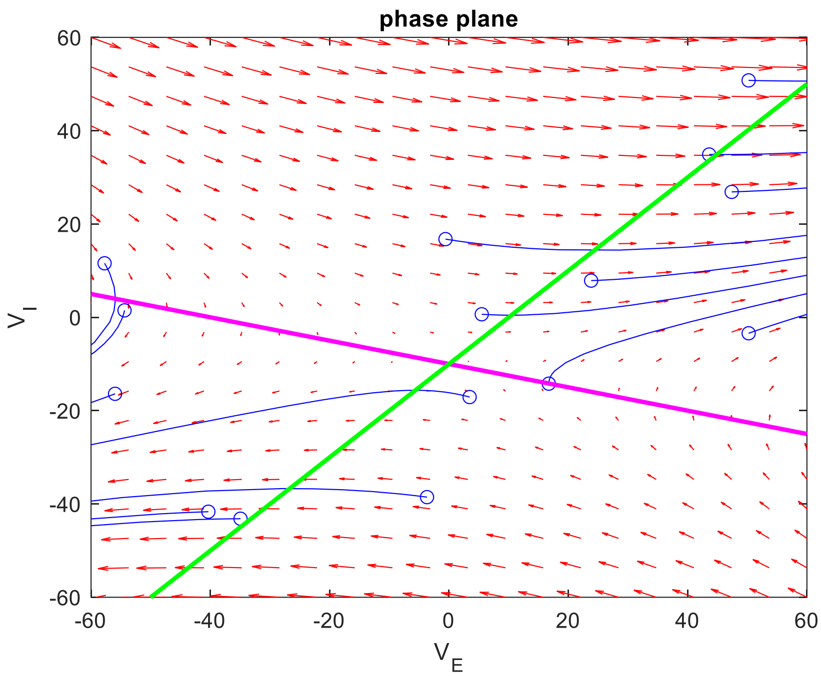
$M_{EI} = 0$


phase plane
V_I
V_E
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

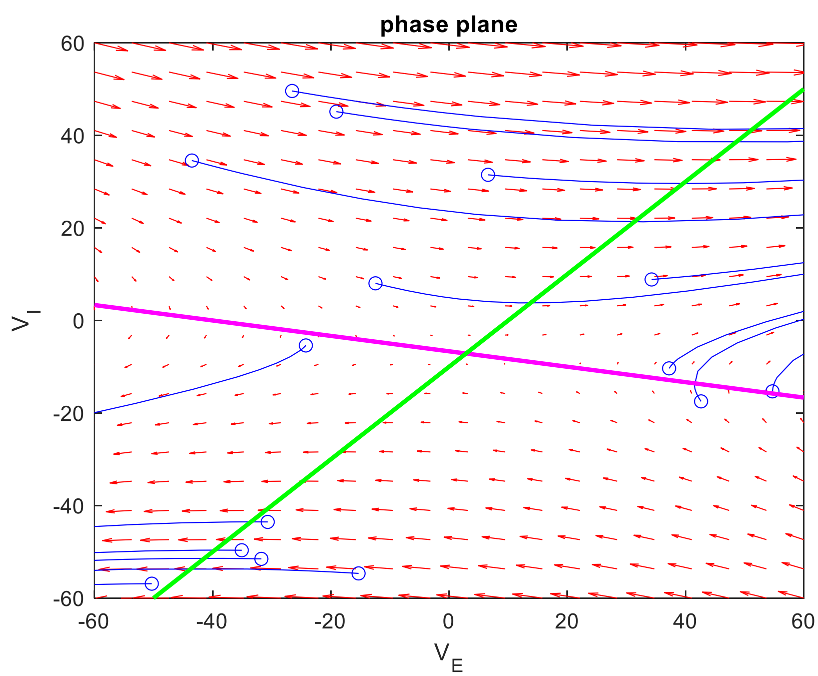
$M_{EI} = 0.5$


phase plane
V_I
V_E
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

$M_{EI} = 1$

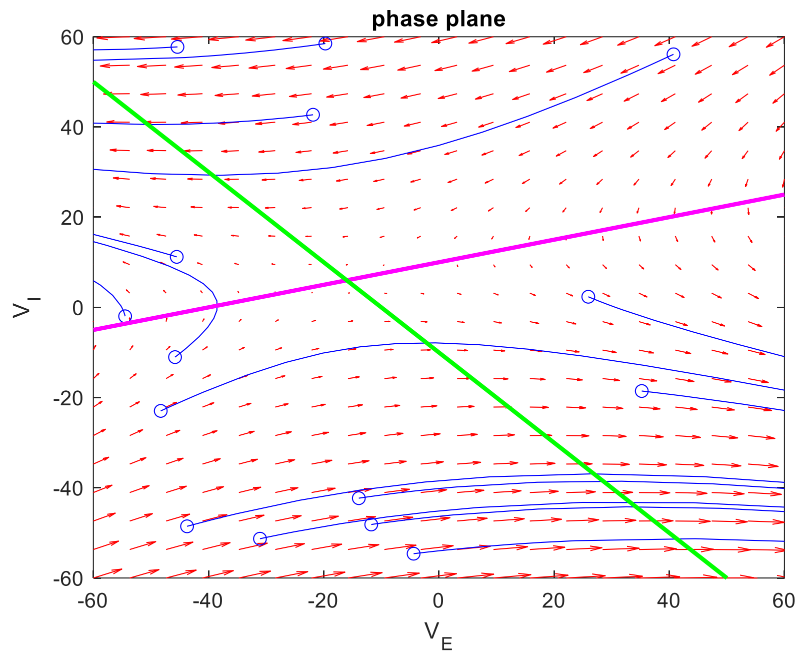


$M_{EI} = 1.5$

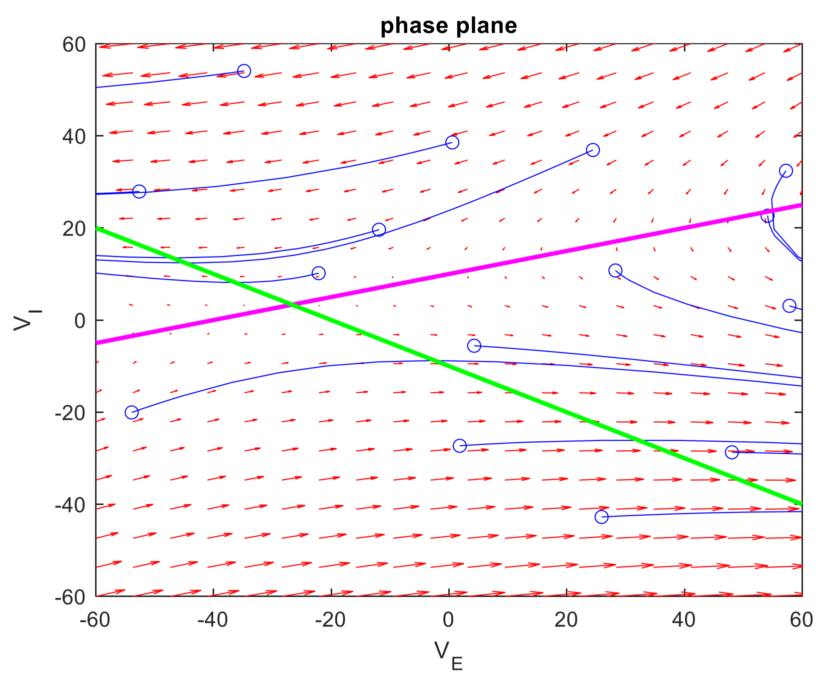


3. The effect of varying M_{IE}:

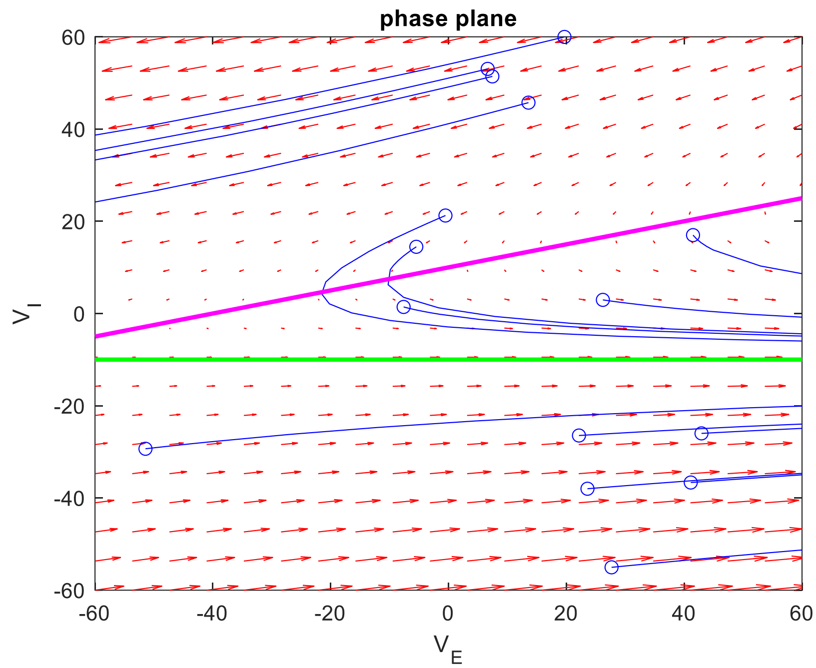
$M_{IE} = -1$



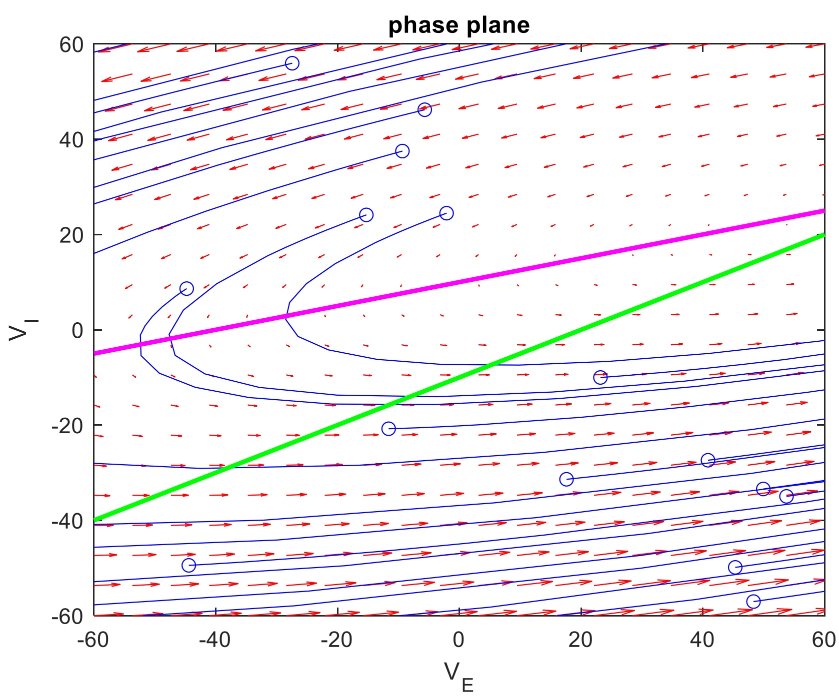
$M_{IE} = -0.5$



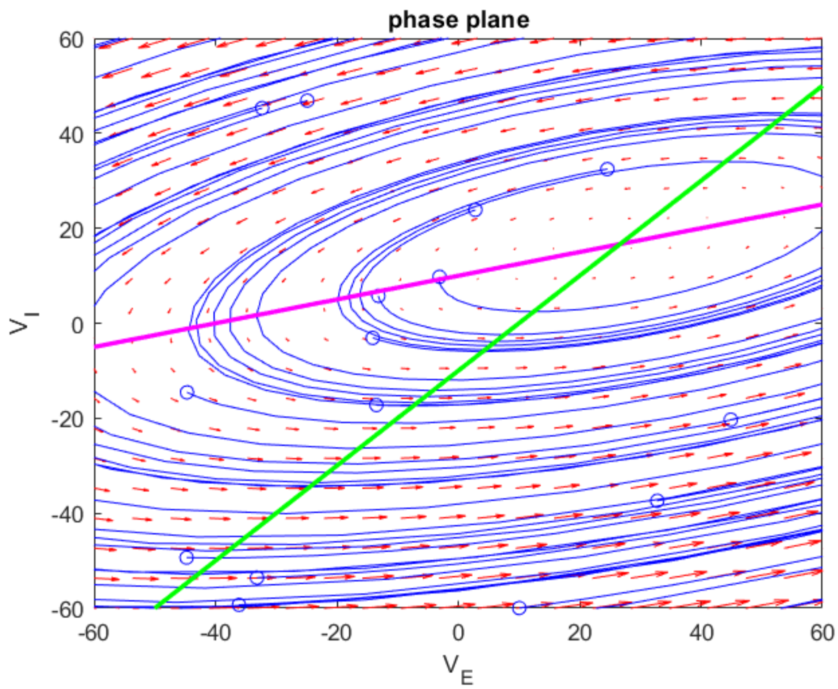
$M_{IE} = 0$


phase plane
V_I
V_E
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

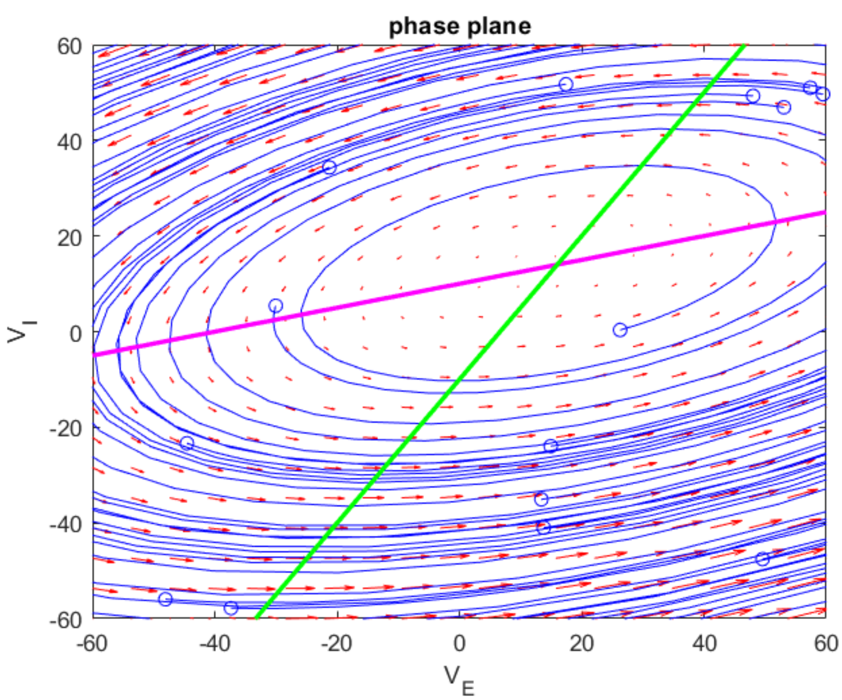
$M_{IE} = 0.5$


phase plane
V_I
V_E
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

$M_{IE} = 1$



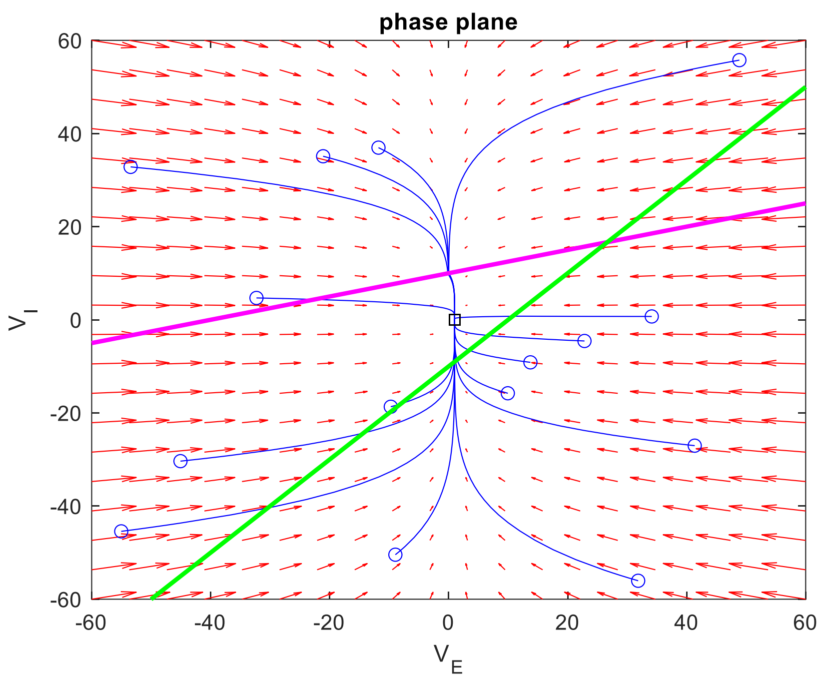
$M_{IE} = 1.5$



6.4

6.5

After changing the function to the new one, the phase plane convert to the image below, the fixed point type changed to a saddle stable one.



7. Statistical test

7.1

t-test:

t-test is a tool for evaluating the means of one or two populations using hypothesis testing. This test is for the normal populations. There are three main types of t-tests:

1. One sample/one mean t-test: used to evaluate a single group differs from a known value (μ_0). There is a test statistic (W) defined as bellow:

$$W = \frac{\bar{X} - \mu_0}{S \,/\, \sqrt{n}}$$

   Where variable S is defined as:

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

   Also the test statistic will be compared with the t-value which can be read from the t-distribution. This value involves both α (significance level) and the degree of freedom from data (n-1).

2. Independent two sample t-test: used to evaluate two groups differ from each other. In this case the range can be defined as:

$$(\mu_1 - \mu_2) = (\overline{x_1} - \overline{x_2}) \mp t_{\alpha/2, n_1+n_2-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Where s_P is defined as:

$$s_p^2 = \frac{1}{n_1 + n_2 - 2}[\sum_{i=1}^{n_1}(X_{1i} - \overline{X_1})^2 + \sum_{i=1}^{n_2}(X_{2i} - \overline{X_2})^2]$$

3. Paired/dependent samples t-test: a significant difference in paired measurements.

p-value:

P-value is the lowest significance level that results in rejecting the null hypothesis. Without p-value we only reported an "accept" or a "reject" decision as the conclusion of a hypothesis test. However, we can provide more information using P-values. In other words, we could indicate how close the decision was. More specifically, we can ask What is the lowest significance level α that results in rejecting the null hypothesis? That the answer is P-value.

Computing P-value:

Consider a hypothesis test for choosing between H_0 and H_1. Let W be the test statistic, and w_1 be the observed value of W.

1. Assume H_0 is true.
2. The P-value is P (type I error) when the test threshold c is chosen to be c = w_1.

other types of statistical tests:

1. Z-test
2. Likelihood ratio test(LRT):
   Let X_1, X_2, X_3, …, X_n be a random sample from a distribution with a parameter θ. Suppose that we have observed $X_1 = x_1$, $X_2 = x_2$, …, $X_n = x_n$. To decide between two simple hypotheses
   $H_0 = \theta_0$,
   $H_1 = \theta_1$,
   we define
   $$\lambda(x_1, x_2, \ldots, x_n) = \frac{L(x_1, x_2 \ldots, x_n; \theta_0)}{L(x_1, x_2, \ldots, x_n; \theta_1)}$$
   To perform a likelihood ratio test (LRT), we choose a constant c. We reject H_0 if $\lambda <$ c and accept it if $\lambda >$ c. The value of c can be chosen based on the desired α.
3. Chi-square test
4. Binomial test

Sources:

1. Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems, Peter Dayan and Larry Abbott, MIT Press, 2005

2. https://en.wikipedia.org/wiki/Biological_neuron_model
3. Introductory Statistics, Thomas-H-Wonnacott, Ronald-J-Wonnacott, 1969
4. Introduction to Probability, Statistics, and Random Processes, Hossein Pishro-Nik, University of Massachusetts Amherst