Advanced Topics in Neuroscience

Homework1(Neural Encoding)

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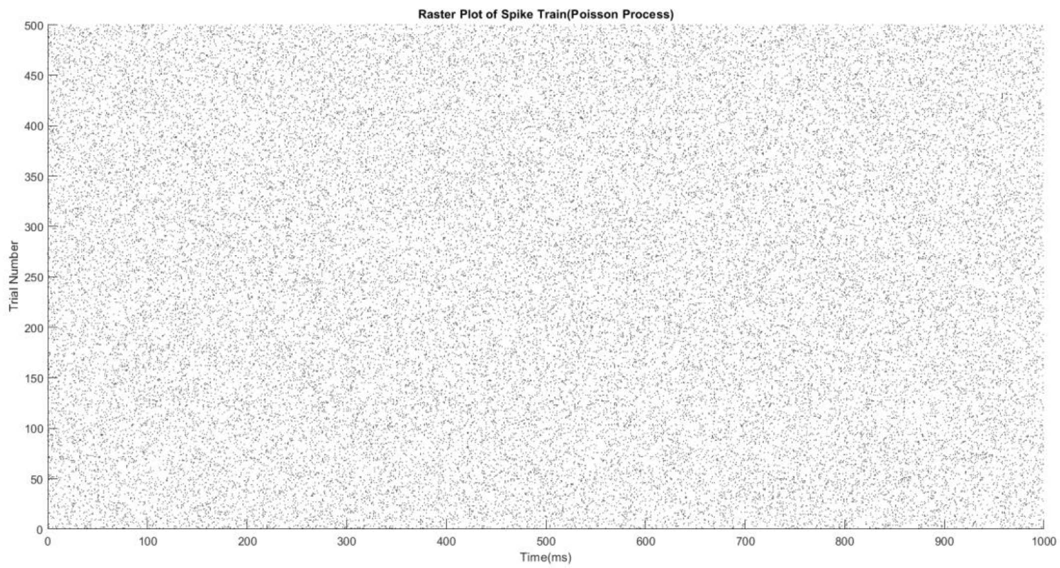
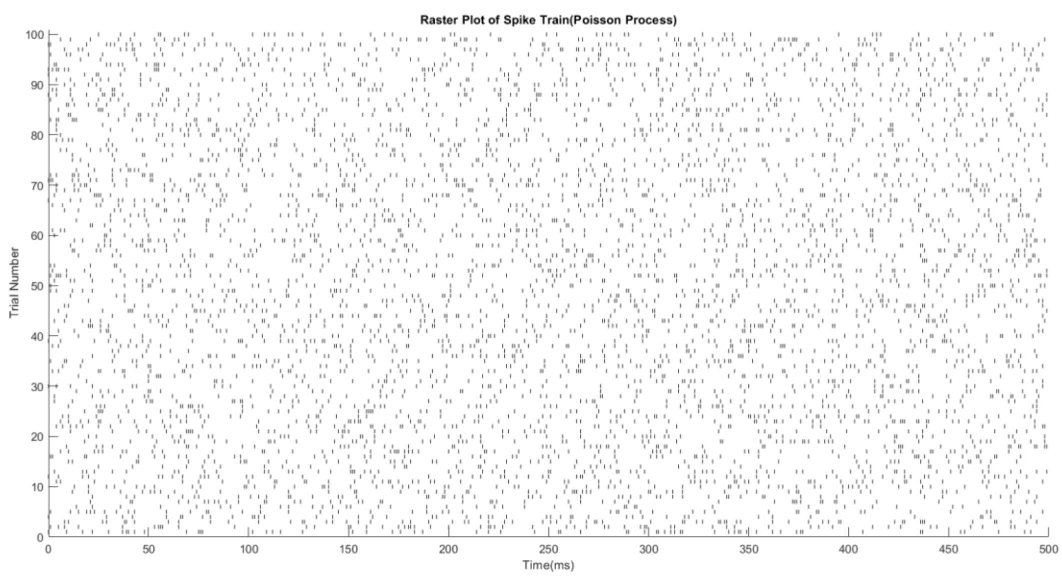
Integrate and Fire Neuron

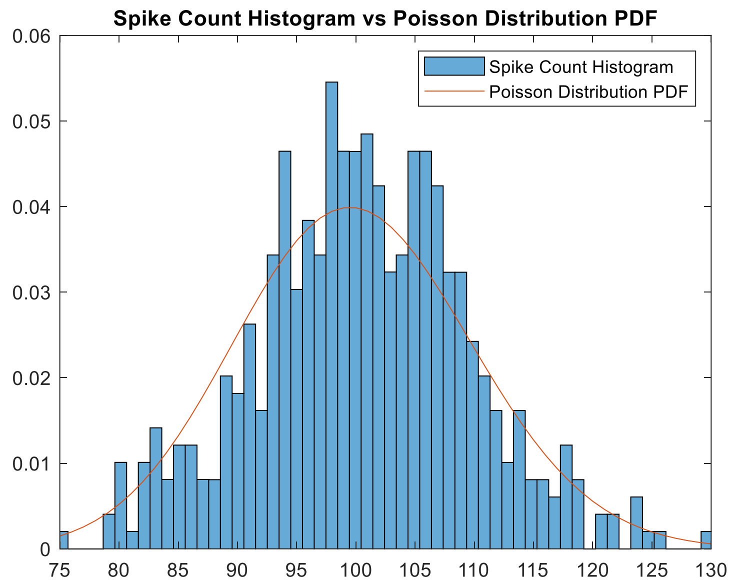
a. Here is the raster plot of spike train. ($firing\ rate = 100$, $\Delta\tau = 1ms$, $number\ of\ trials = 500$, $time\ duration = 1000ms$)



Here is another raster plot of spike train. ($firing\ rate = 100$, $\Delta\tau = 1ms$, $number\ of\ trials = 100$, $time\ duration = 500ms$)

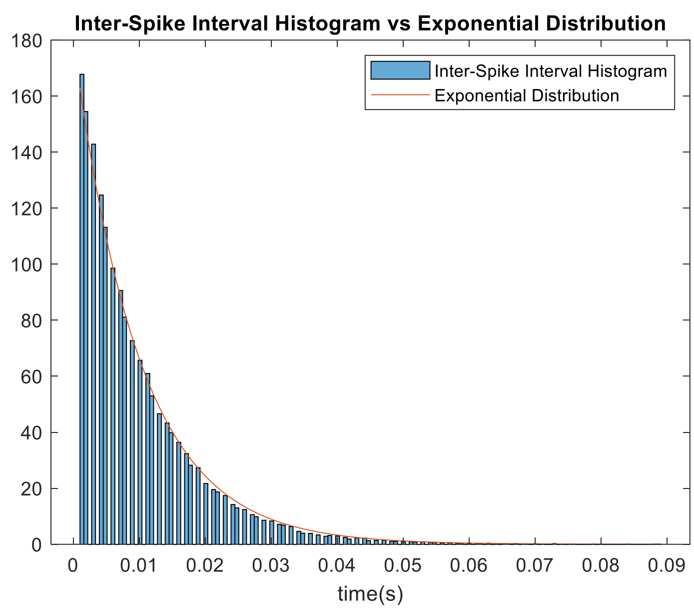


b. The figure below shows both the generated spike count histogram and the Poisson process PDF:



c. The figure below shows both the generated spike Inter-Spike Interval(ISI) histogram and the Poisson process PDF:

- This figure is computed by PDF mode of histogram.

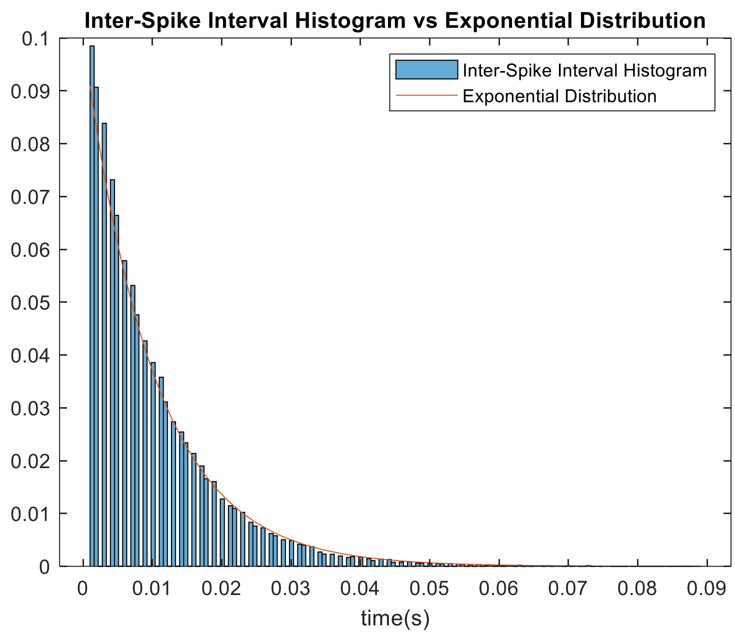


Note: the exponential distribution has the function as below:

$$\begin{cases} a\lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

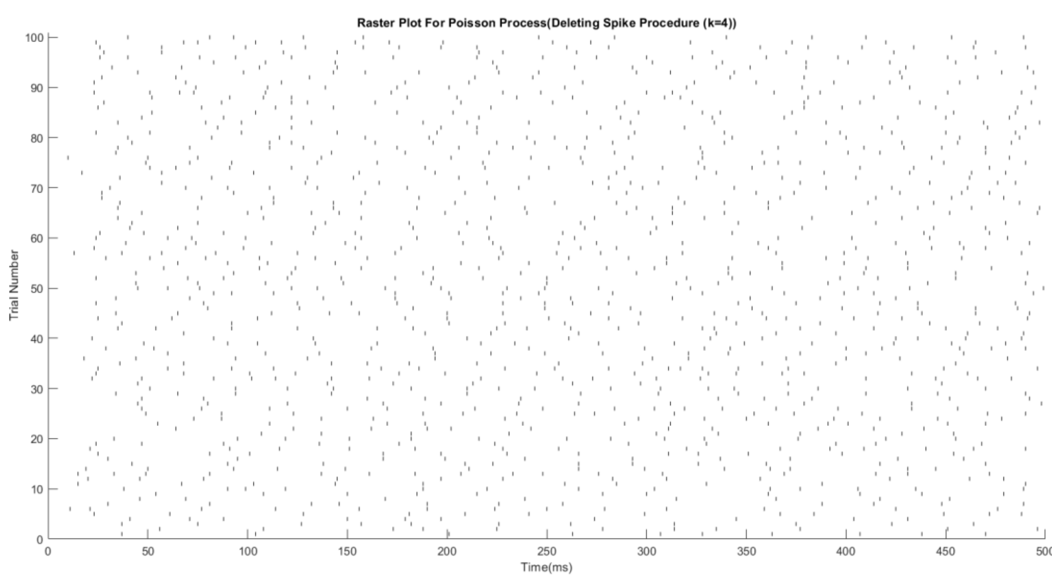
Where $\lambda = 100$ *and* $a = 1.8$

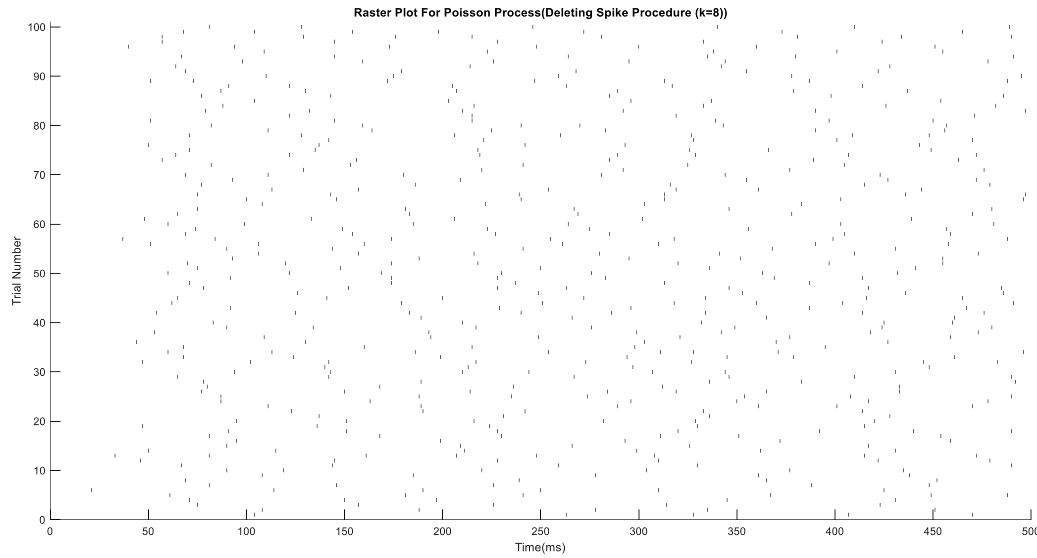
- This figure is computed by probability mode of histogram, thus, all the values are less than one and also the total integrate is equal to one. (the exponential PDF has been normalized)

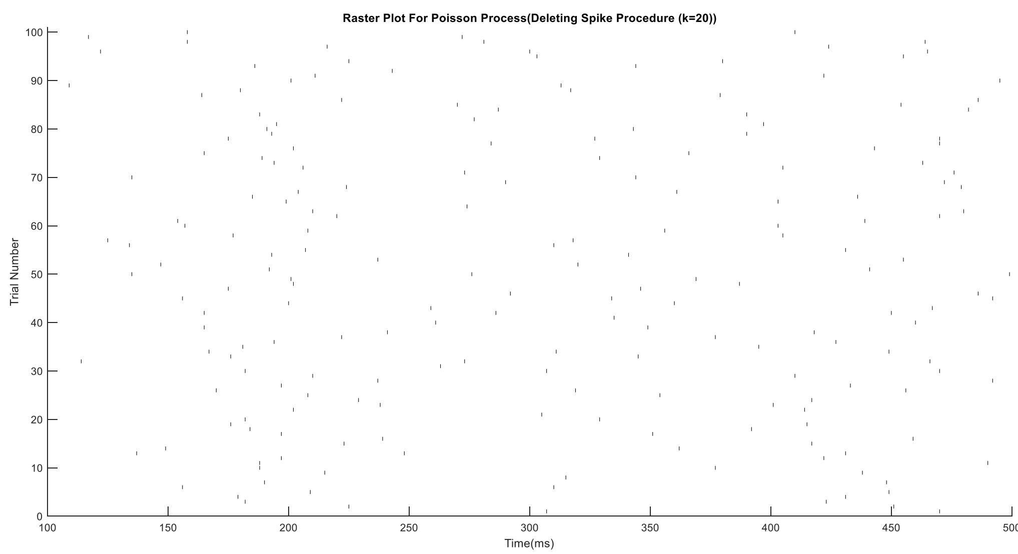


In this part we will repeat the previous sections for stimulation of integration over postsynaptic input.

I. In this section figures show raster plot for three different k value.

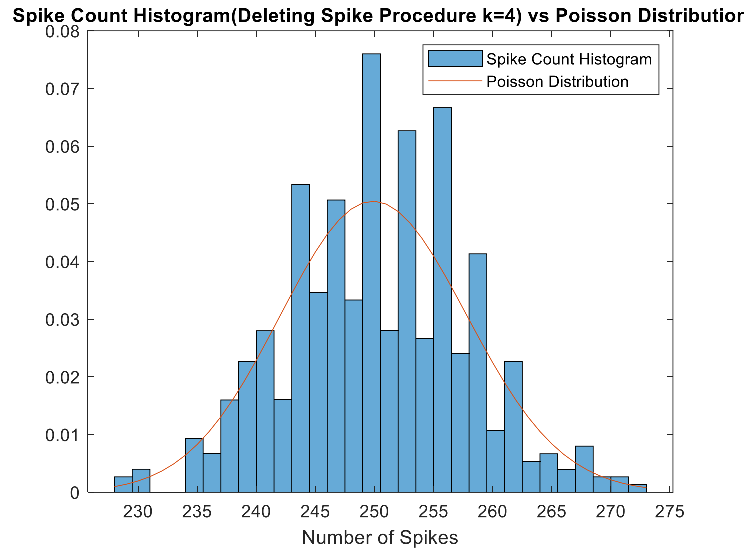


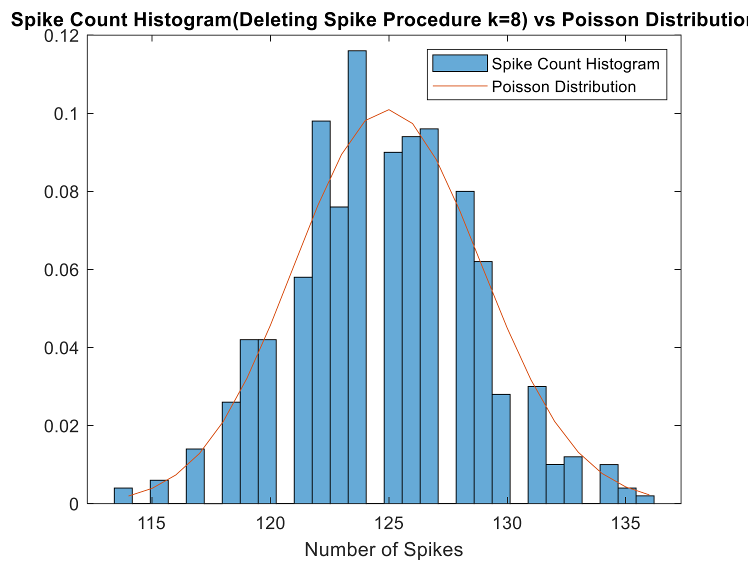


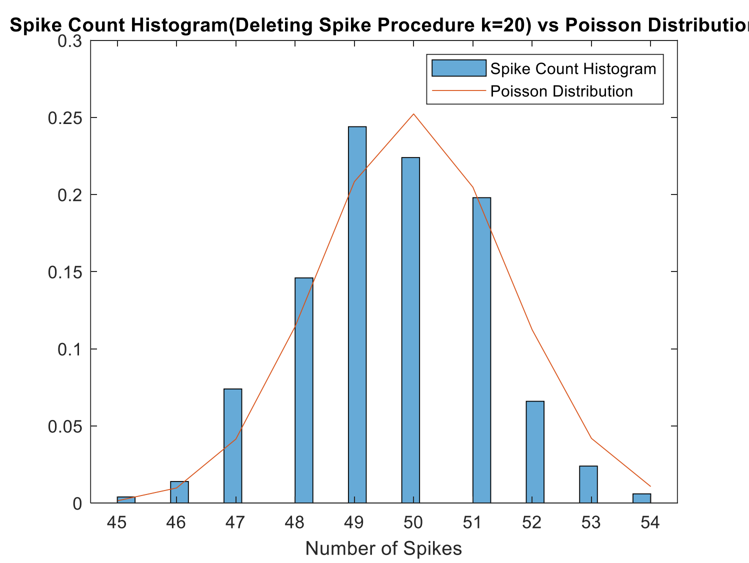


II. In this section we try to plot the spike count histogram for different values of k, comparing to the Poisson distribution's pdf. (the stimulation runs for 10 seconds)
Note: the distribution for this new definition of spike count (with integration), is as the Poisson distribution with another constant:

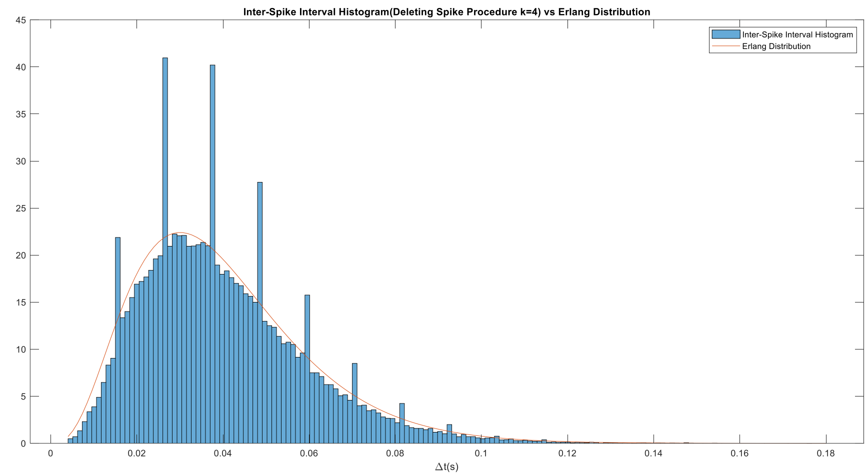
$$g(x,\lambda) = f\left(\frac{x}{k},\lambda\right); f\ is\ the\ previous\ poisson\ pdf, k\ is\ the\ intgration\ constant$$

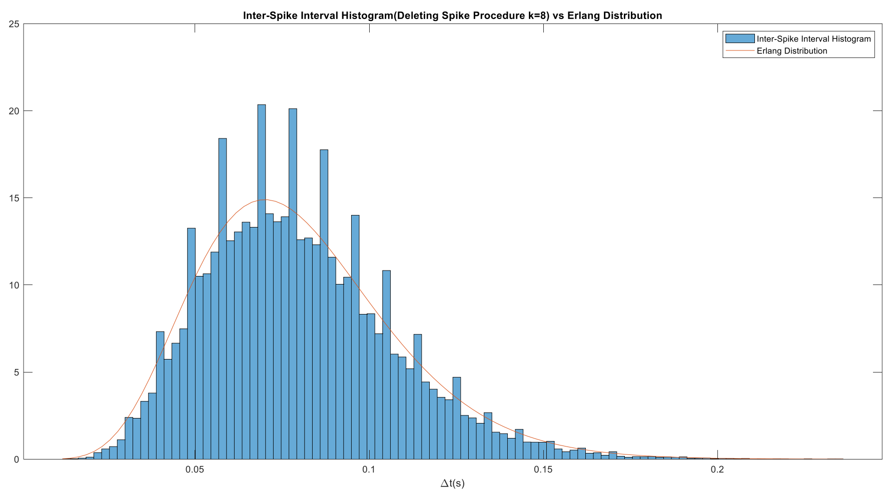

Spike Count Histogram(Deleting Spike Procedure k=4) vs Poisson Distributior
Spike Count Histogram
Poisson Distribution
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
230
235
240
245
250
255
260
265
270
275
Number of Spikes

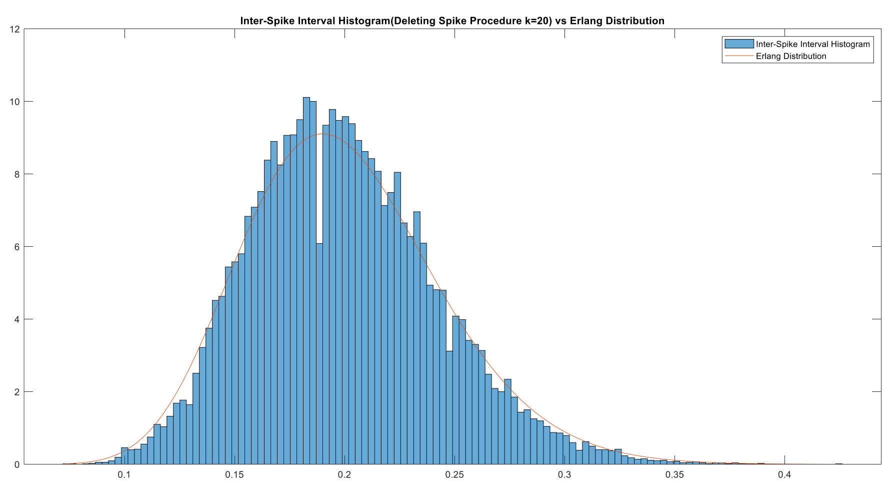

Spike Count Histogram(Deleting Spike Procedure k=8) vs Poisson Distributior
Spike Count Histogram
Poisson Distribution
0
0.02
0.04
0.06
0.08
0.1
0.12
115
120
125
130
135
Number of Spikes


Spike Count Histogram(Deleting Spike Procedure k=20) vs Poisson Distributio
Spike Count Histogram
Poisson Distribution
0
0.05
0.1
0.15
0.2
0.25
0.3
45
46
47
48
49
50
51
52
53
54
Number of Spikes

III. In this section we plot the ISI histogram for different values of k compare to the Erlang distribution.



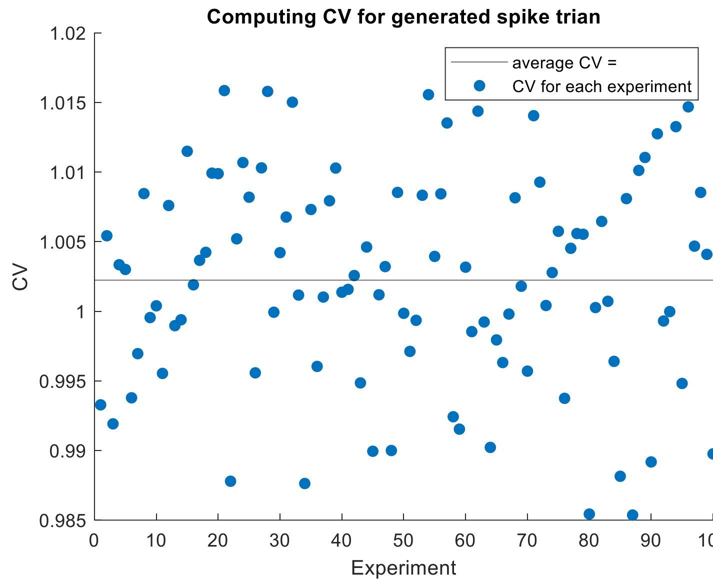




Note: the distribution of ISIs is same as Erlang distribution with the below function:

$$f(x;k,\lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} \quad for\ x \geq 0$$

d. In this section we try to generate the spike train with 500 trials for 100 times and calculate CV for each try then compute the average of all CVs and show them in the below figure: (the time step for this section is equal to 0.04ms)



As it is observed the reached value is closed to 1 which is the expected CV for a Poisson process. (the reached value = 1.002)

e. In this section we try to prove that CV of the Erlang distribution is equal to $\frac{1}{\sqrt{K}}$.

$$computing\ mean:$$

$$we\ assume\ that\ for\ k-1, E[X] = \frac{k-1}{\lambda};$$

$$then\ we\ try\ to\ prove\ for\ k\ it\ will\ be\ eaqual\ to\ \frac{k}{\lambda}.(induction)$$

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx \rightarrow for\ Erlang: E[X] = \int_{0}^{\infty} x\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}dx$$

$$= -\frac{\lambda^k x^{k-1} e^{-\lambda x}}{\lambda(k-1)!} + \int_{0}^{\infty} \frac{\lambda^k k x^{k-1} e^{-\lambda x}}{\lambda(k-1)!}dx$$

$$= -\frac{\lambda^{k-1} x^{k-1} e^{-\lambda x}}{(k-1)!} + \frac{k}{k-1}\int_{0}^{\infty} \frac{\lambda^{k-1} x^{k-1} e^{-\lambda x}}{(k-2)!}dx \ \ (i)$$

$$we\ assume\ that\ for\ k-1, E[X] = \frac{k-1}{\lambda} \rightarrow \int_{0}^{\infty} x\frac{\lambda^{k-1} x^{k-1} e^{-\lambda x}}{(k-2)!}dx = \frac{k-1}{\lambda} \ \ (ii)$$

$$(i),(ii) \rightarrow E[X] = -\frac{\lambda^k x^{k-1} e^{-\lambda x}}{\lambda(k-1)!}\Bigg|_{x=0}^{x=\infty} + \frac{k}{k-1} \times \frac{k-1}{\lambda} = 0 + \frac{k}{\lambda} = \frac{k}{\lambda}$$

$$\textit{and also for the first step}(k=1)\textit{we know that mean of exponential distribution is equal to } \frac{1}{\lambda}.\,(E\left[\lambda e^{-\lambda x}\right] = \frac{1}{\lambda})$$

$$\textit{computing standard deviation}:$$

$$var[X] = \int_{-\infty}^{\infty} (x - E[X])^2 f(x)dx \rightarrow \textit{for Erlang}: E[X] = \int_{0}^{\infty} \left(x - \frac{k}{\lambda}\right)^2 \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} dx$$

$$= \int_0^\infty x^2 \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} dx + \int_0^\infty \left(\frac{k}{\lambda}\right)^2 \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} dx - \int_0^\infty 2x \frac{k}{\lambda} \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} dx$$

$$= \frac{k}{\lambda} \int_0^\infty \frac{\lambda^{k+1} x^{k+1} e^{-\lambda x}}{(k)!} dx + \left(\frac{k}{\lambda}\right)^2 \frac{\lambda}{k-1} \int_0^\infty \frac{\lambda^{k-1} x^{k-1} e^{-\lambda x}}{(k-2)!} dx - \frac{k}{\lambda} \int_0^\infty \frac{\lambda^k x^k e^{-\lambda x}}{(k-1)!} dx$$

$$= \frac{k}{\lambda} E[X; k+1] + \left(\frac{k}{\lambda}\right)^2 \frac{\lambda}{k-1} E[X; k-1] - 2\frac{k}{\lambda} E[X; k]$$

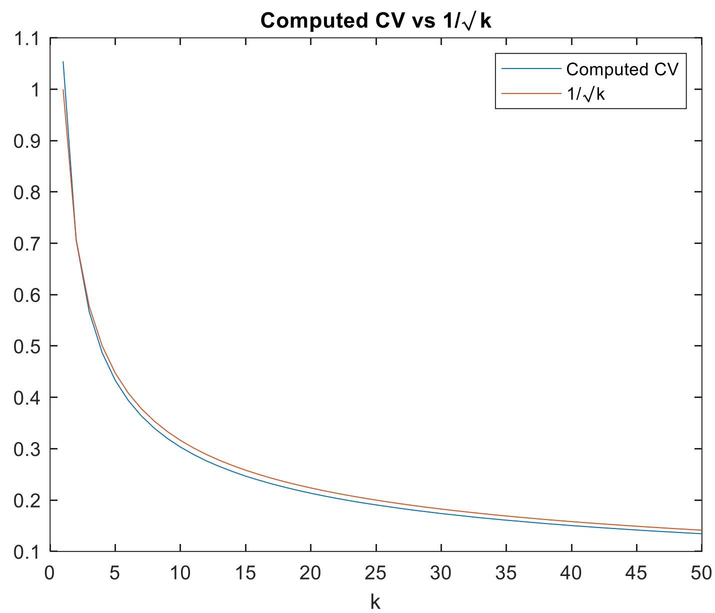
$$= \frac{k}{\lambda} \frac{k+1}{\lambda} + \left(\frac{k}{\lambda}\right)^2 \frac{\lambda}{k-1} \frac{k-1}{\lambda} - 2\frac{k}{\lambda} \times \frac{k}{\lambda}$$

$$= \frac{k}{\lambda^2} \rightarrow std[X] = \frac{\sqrt{k}}{\lambda}$$

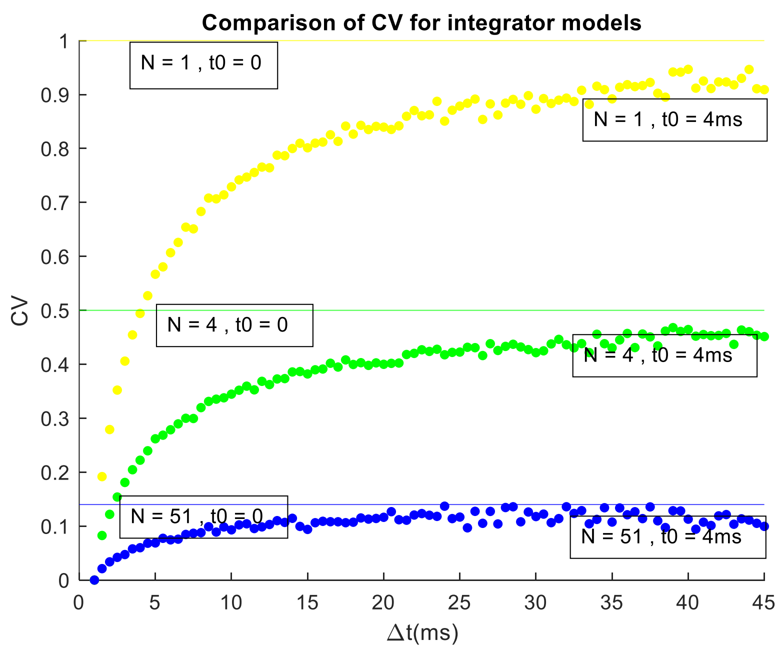
$$\textit{computing CV}:$$

$$CV = \frac{std[X]}{E[X]} = \frac{\frac{\sqrt{k}}{\lambda}}{\frac{k}{\lambda}} = \frac{1}{\sqrt{k}}$$

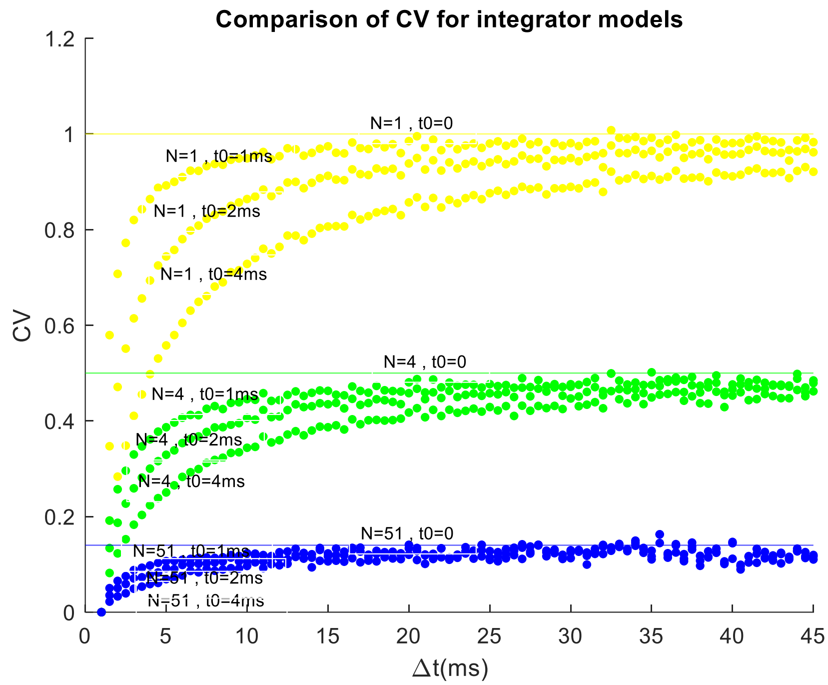
f. The below plot compare the CV computed for the generated spike sets (100 time each one 500 trails for 10 seconds) and the term $\frac{1}{\sqrt{K}}$:



g. In this section we try to simulate the plot of how CV varies across different firing rates (represented as Δt here).

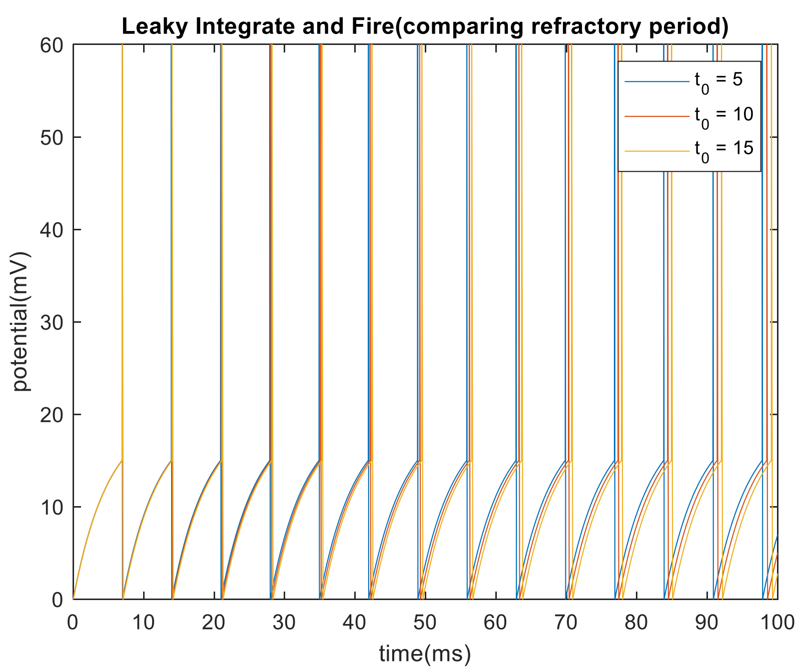


In the below figure we try to compute CV for different refractory periods to show the effect of refractory period on the shape of CV's function:

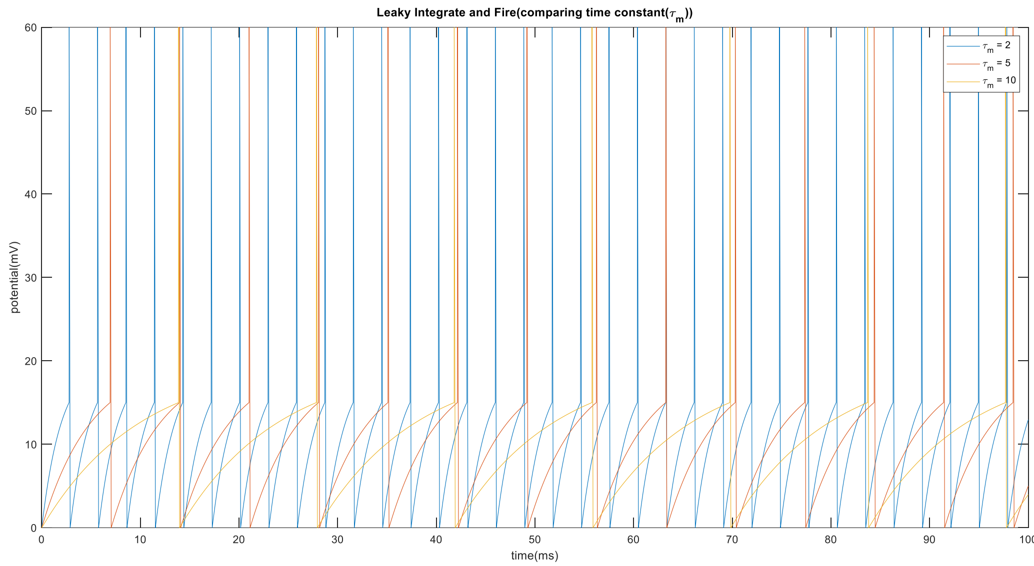


Leaky Integrate and Fire Neuron

a. In this section we try to generate spikes in time, using leaky integrate and fire model. The first figure is indicating the spikes generated by a constant current input with three different refractory periods:



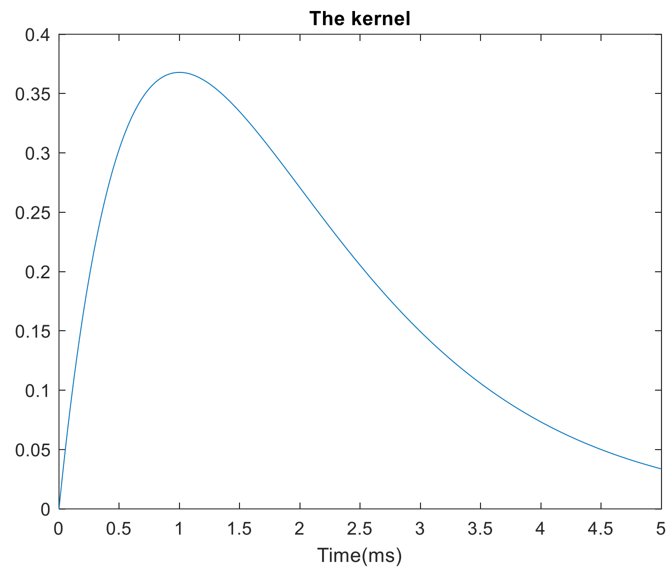
The next figure indicates spikes generated by a constant current input with three different time constants:



b. In this section we find the mean firing rate for a spike train generated by leaky integrate and fire model with a constant input:

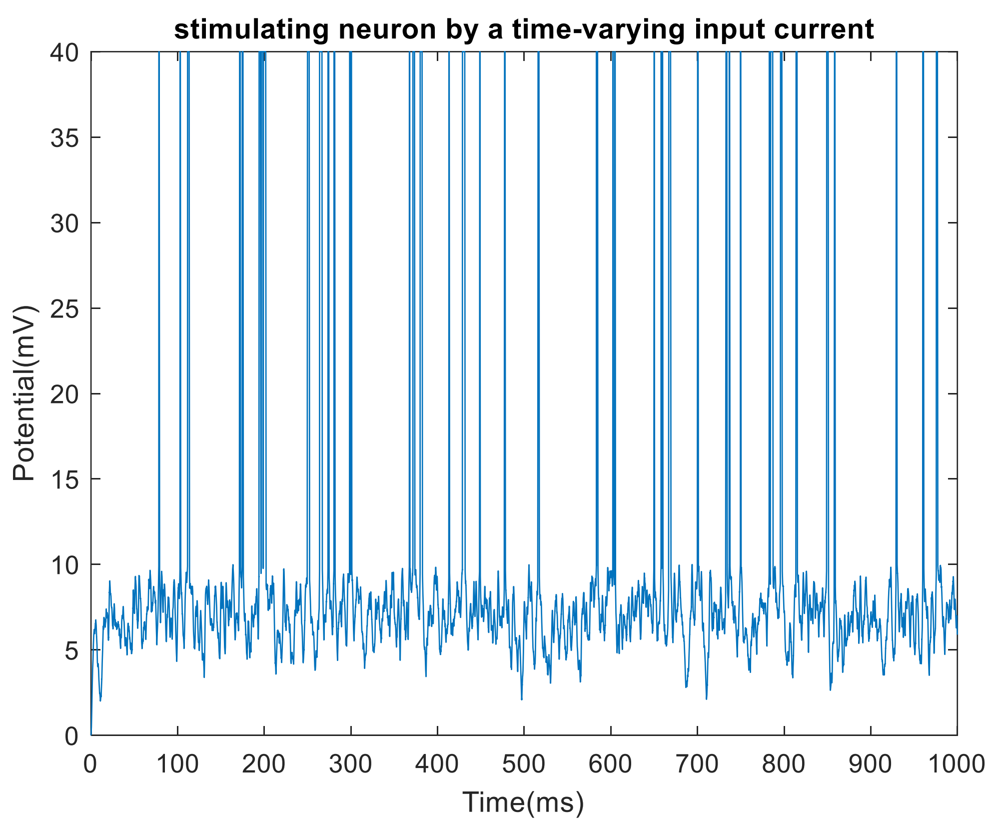
$$V_{th}: the\ threshold\ voltage$$
$$V_{rest}: the\ rest\ potential$$
$$I: the\ constant\ current\ input$$
$$\tau_m: time\ constant$$
$$r: the\ amount\ of\ time\ from\ reaching\ V_{th}\ to\ the\ time\ neuron\ cannot\ generate\ new\ spike$$
$$main\ equation:\ \tau_m \frac{dV_m}{dt} = -V_m + I$$
$$\rightarrow V_m = V_{rest} e^{-t/\tau_m} + I$$
$$\rightarrow if V_m = V_{th} \rightarrow t_{th} = -\tau_m \ln \frac{V_{th} - I}{V_{rest}}$$
$$thus\ a\ single\ spike\ takes\ -\tau_m \ln \frac{V_{th} - I}{V_{rest}}$$
$$+ r\ seconds\ to\ start\ from\ resting\ potential\ and\ finish\ the\ spiking\ process.$$
$$\rightarrow Firing\ Rate = \frac{1}{\tau_m \ln \frac{V_{rest}}{V_{th} - I} + r}\ (if\ all\ the\ values\ are\ in\ standard\ units)$$

c. In this section we try to generate a model for neuron spiking with EPSP kernel. First we declare the kernel: (it can have a non-constant time constant)



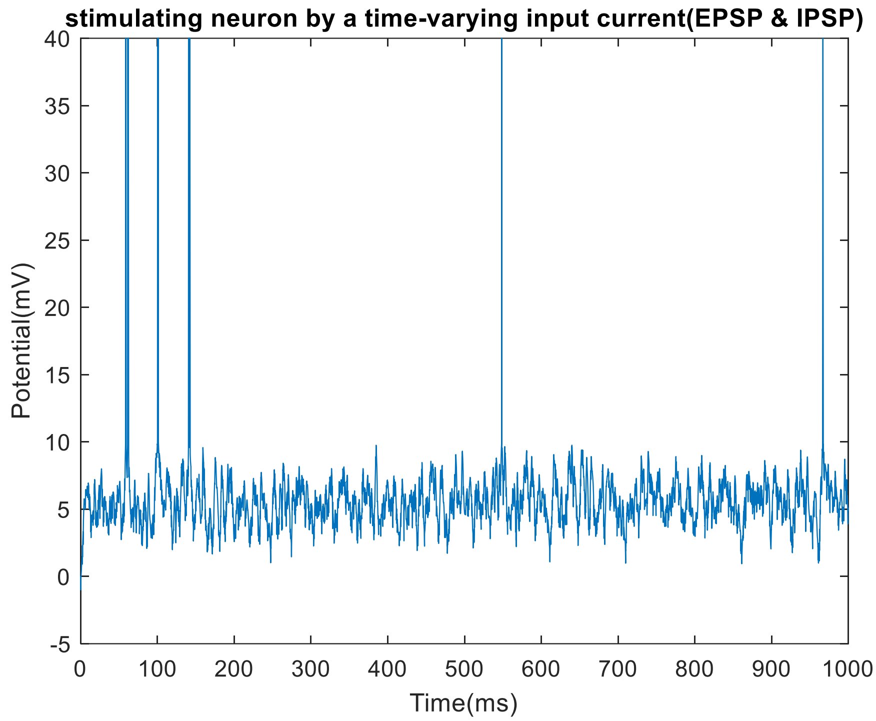
Then we give a specific weight to the kernel and try to generate the final spike of post-synaptic neuron by the integration of the pre-synaptic inputs.

The figure below is generated for weight 1.5, threshold voltage=10Mv, 50 pre-synaptic neurons and firing rate of 100Hz:

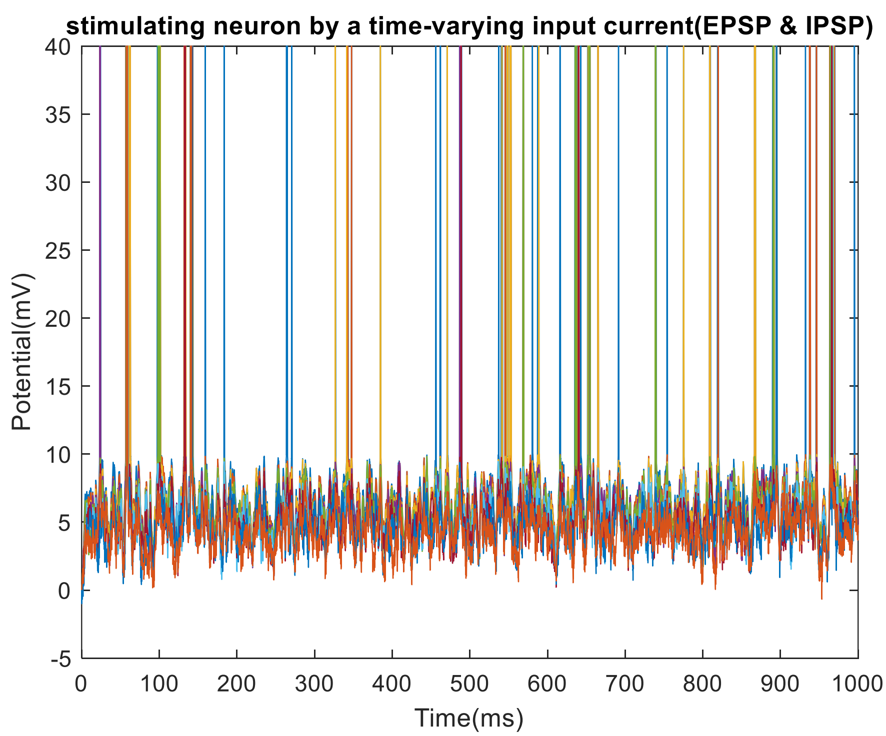


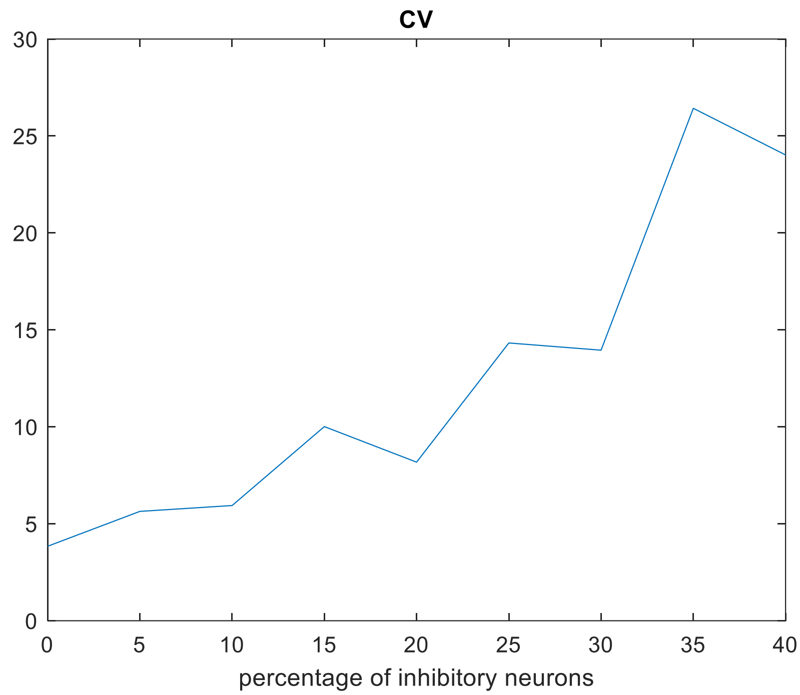
d. In this section we first show how the spikes of post-synaptic neuron will change if we consider the IPSPs too, and then, we try to find the effect of different parameters on CV.

- Now let's see with the previous conditions and adding 20% of inhibitory neurons how the neuron will behave:



- In this part we will vary the percentage of inhibitory neurons and see how will the CV change:

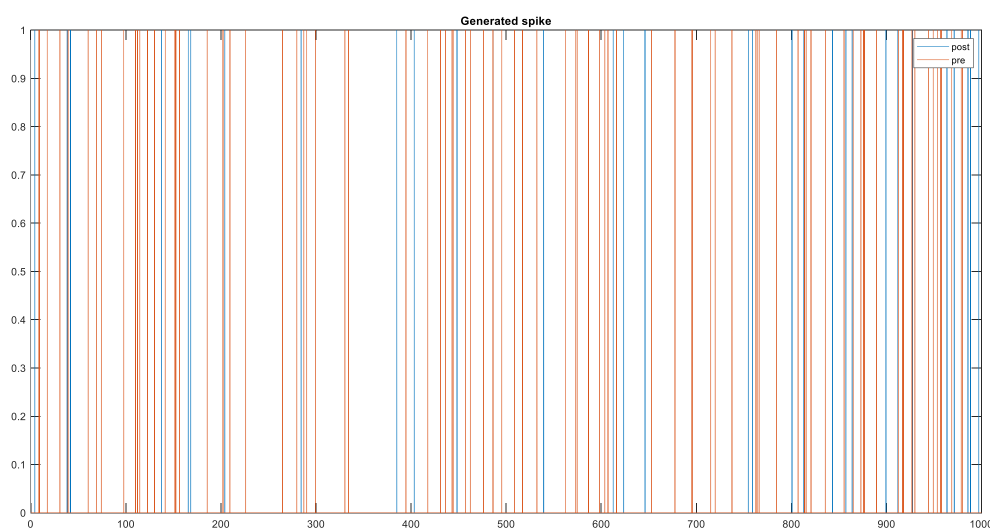


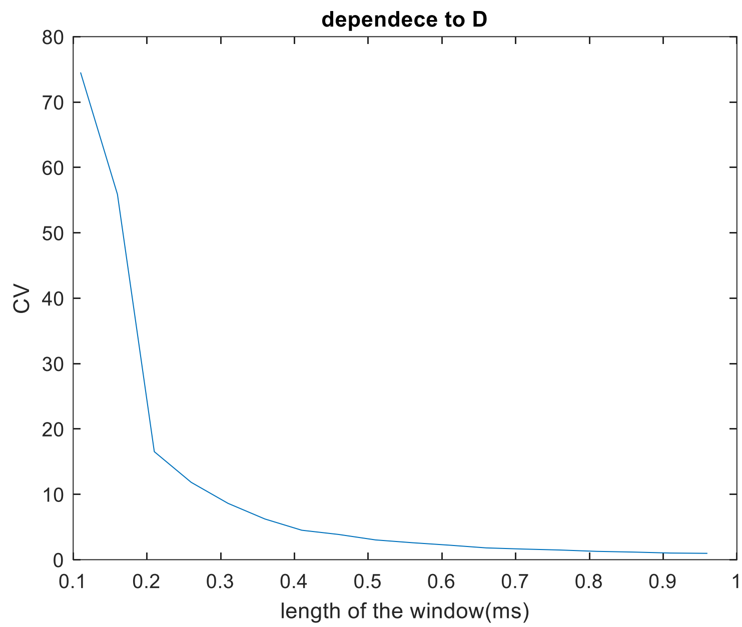


As you can see by increasing the number of inhibitory neurons CV will increase sharply.

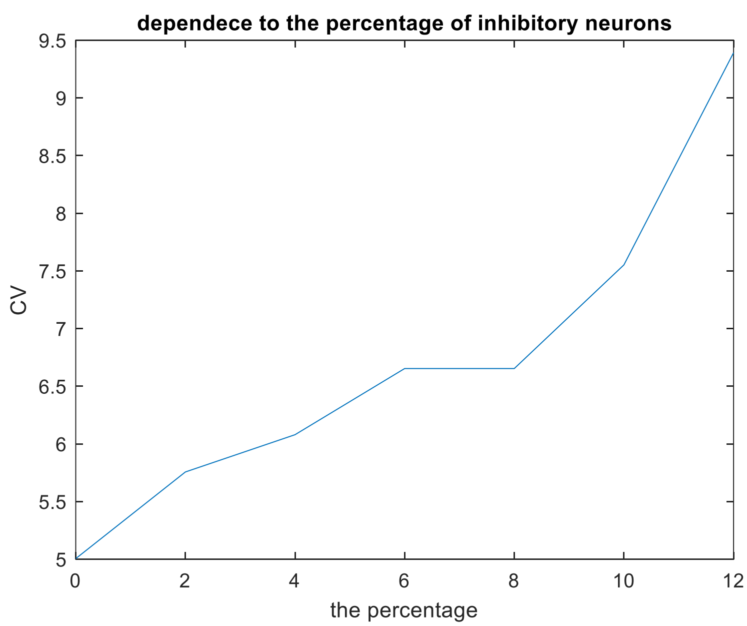
By increasing the firing rate, we will decrease the CV for output. If the weight of kernel decreases the output firing rate will decrease.

e. In this section we try to find the dependency of a post-synaptic neuron to the parameters of integration above the pre-synaptic ones. Thus at first let's generate a spike train from the pre-synaptic Poisson process.





f. In this section we try to add the inhibitory pre-synaptic neurons to the experiment and see how does the CV vary by changing the ratio of inhibitory and excitatory neurons.



As you can see by increasing the number of inhibitors the CV will rise so the randomness will increase. Increasing the number of neurons will help to generate less random data (decreasing CV).