AUTOMATA FORMAL LANGUAGES AND LOGIC

Lecture Notes
on
Sets

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1.What is a Set?

Why Sets :

Sets are the fundamental property of mathematics. Now as a word of warning, sets, by themselves, seem pretty pointless. But it's only when we apply sets in different situations do they become the powerful building block of mathematics that they are.

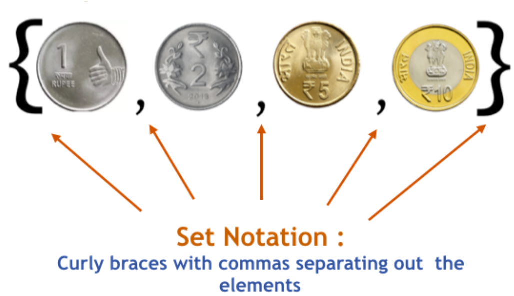
Math can get amazingly complicated quite fast. Graph Theory, Abstract Algebra, Real Analysis, Complex Analysis, Linear Algebra, Number Theory, and the list goes on. But there is one thing that all of these share in common: Sets.

Definition :

A set is an unordered collection of distinct objects, which may be anything (including other sets).

For Example : Here, we have a set of Indian Coins.

We represent Sets in Curly braces with commas separating out the elements



2. Representing Sets

We can represent Sets in 3 different ways namely,

1. Descriptive Form
2. Set-builder Notation
3. Roster Form

1) Descriptive form:

In descriptive form we provide an english description of what is contained in the set. For example : "The set of all even numbers " or "The set of all real numbers less than 150 and so on"

2) Set builder notation :

Set-builder notation is a mathematical notation for describing a set by enumerating its elements or stating the properties that its members must satisfy.

Set-builder notation has three parts: a variable, a colon or vertical bar separator, and a logical rule. These three parts are contained in curly brackets.

Here if we are talking about set of even natural numbers, in set builder notation we can specify it as:
The set of all n, where or we can say such that

The set of all n such that n is a natural number and n is even.

3) Roster form:

In the roster form, we list all elements of a set inside a pair of braces

For example,

Let say we define A as set of all x where x is an integer and its values is greater than and equal to -1 but less than 5 }

then In roster form we can say A = { -1, 0,1, 2, 3, 4 }

3. Set Membership

Given a set S and an object x, we say x is contained in S, if x is one of the elements of S and is denoted by,

$$x \in S$$

x is not a member of S if x is not an element of S and is denoted by,

$$x \notin S$$

4. Order of Sets / Cardinality :

The order of a set or cardinality defines the number of elements a set is having. It describes the size of a set.

For Examples:

1) if Set A = {a, b, c, d, e}

then order of A is 5

2) Here Set B is a set of Sets {{a, b}, {c, d, e, f, g}, {h}} and the size of B = 3

3) if Set C = $\{ n \in N \mid n < 137\}$

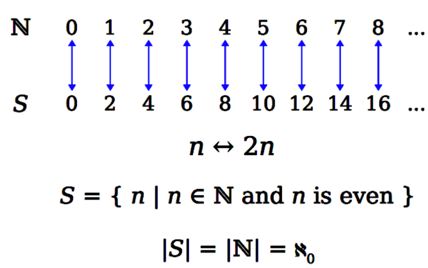
then $|C| = 137$

For finite sets the order (or cardinality) is the number of elements

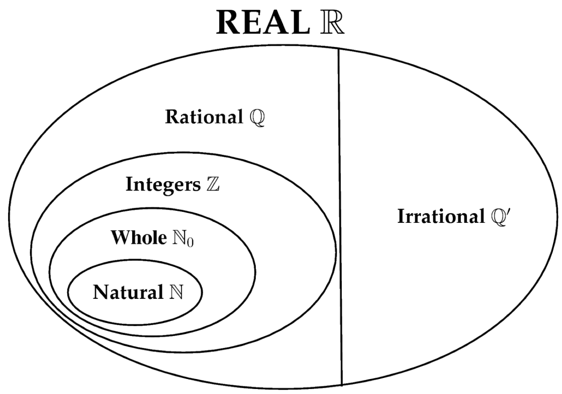
For infinite sets, all we can say is that the order is infinite.. We can ask What is |N|? where N denotes a set of natural number? There are infinitely many natural numbers. Hence order of N can't be a natural number, since it's infinitely large.

Consider the set S, where S is the set of all n such that n is a natural number and is even then, What is order of S? Is it less than or equal or greater than the order of N? Basically we are boiling down to the question , Which infinity is larger?

if we see for every natural number n we can have a corresponding even number 2n, this means the order of S and N is same but can't be a natural number, since it's infinitely large.



Also, There are infinitely many whole numbers {0,1,2,3,4,…}, But there are more real numbers (such as 12.308 or 1.1111115) because there are infinitely many possible variations after the decimal place as well. But both are infinite, so the question is Do all infinite sets have the same cardinality?



Oddly enough, we can say with sets that some infinities are larger than others, but this is a more advanced topic in sets, we will talk about this in detail in Unit 4.

5.Types of Sets :

In this section we discuss about the following types of Sets:

13. Empty Set or Null Set
14. Singleton Set
15. Finite Set
16. Infinite Set
17. Equivalent Set
18. Equal Sets
19. Disjoint Sets
20. Subsets
21. Proper Subset
22. Super Set
23. Universal Set
24. Power Set

1) Empty Set : A null set or an empty set is a valid set with no member. We can use empty braces({ }) or ϕ to denote an empty set.

Does $\phi = \{\phi\}$?
ϕ indicates an empty set , this set contains no elements. Whereas $\{\phi\}$ indicates a set that has one element which happens be an empty set.

Hence they are not equal.

Does 2 = {2}?
2 indicates that it is a number, whereas {2} indicates It is a set that contains a number. Hence they are not equal.

2) Singleton Set : If a set contains only one element it is called to be a singleton set.

Example :

Set A = {1}
Set B ={a}
Set C ={4}
Set D ={car}

3) Finite Sets : A set in which the number of elements are finite (can be counted) are called Finite Sets.
Example

Set A = {4, 8, 12, 16}

Set B = {90, 180}

4) Infinite Sets : A set in which the number of elements are infinite are called Infinite Sets.

Example

N = Set of all natural no's

= {1, 2, 3, … }

Z = Set of all integers

= {..,-3,-2,-1,0,1,2,3..}

5) Equivalent Sets If the number of elements are same for two different sets, then they are called equivalent sets.

Example:

If A = {1,2,3,4} and

B = {Red, Blue, Green, Black}

Here, Set A and B are equivalent,

since |A| = |B|= 4

6) Equal Sets : The two sets A and B are said to be equal if they have exactly the same elements, order of elements do not matter.

Example:

if A = {1,2,3,4} and

B = {4,3,2,1}

Then, Set A and B are equal

& can be denoted as :

A = B

7) Disjoint Sets : The two sets A and B are said to be disjoint if the set does not contain any common element.

Example:

Set A = {1,2,3,4} and

Set B = {5,6,7,8} are disjoint sets, because there is no common element between them.

8) Subsets : A set S is a subset of a set T (denoted $S \subseteq T$) if all elements of S are also elements of T.

Examples:

1. $\{ 1, 2, 3 \} \subseteq \{ 1, 2, 3, 4 \}$
2. $N \subseteq Z$ (every natural number is an integer)
3. $Z \subseteq R$ (every integer is a real number)

We can ask ourselves a question whether there exists any Set S where Ø is a subset of S?

Since the empty set has no elements, ϕ is a subset of every set. Infact all the sets are subsets of itself

9) Proper Subset : If B is a proper subset of A (denoted as $B \subset A$), then all elements of B are in A but A contains atleast one element that is not in B.

Example :
A = {1, 3, 5}
B = { 1, 5}
C = { 1, 3, 5}

then,
B ⊂ A
But, C ⊆ A

10) Superset
if B ⊆ A (B is a subset of A) then,
A ⊇ B (A is a superset of B)

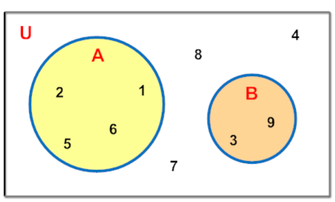
Here,
if B ⊂ A (B is a proper subset of A) then,
A ⊃ B (A is a proper superset of B)

Consider Example :
A = {1, 3, 5}
B = { 1, 5}
C = { 1, 3, 5}

Here, A ⊇ C or C ⊇ A
and A ⊃ B

So we can say every set is a subset and superset of itself.

11) Universal Set : A universal set is a set of all the elements, or members, of any group under consideration, including its own elements. In the given example U contains elements that belong to Set A and B, plus the elements 7, 8 and 4.



12) Power Set : Power set of any set S is the set of all subsets of S, including the empty set and S itself.

For example,

Consider a Set S = {a, b, c, d} then Power Set of S would be

P(S) = {Ø,{a}, {b}, {c}, {d},{a, b}, {a, c}, {a, d}, {b, c},

{b, d}, {c, d},

{a, b, c}, {a, b, d},

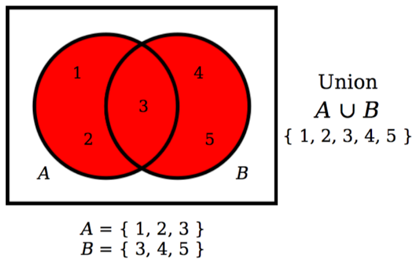
{a, c, d}, {b, c, d},

{a, b, c, d}}

|S| < |P(S)|

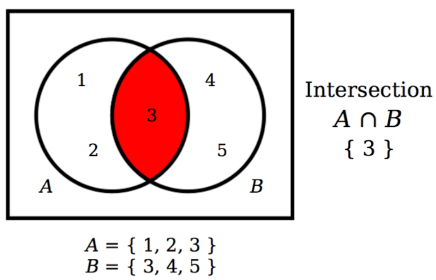
6. Operations on Sets

1)Union : Union of two sets which will contain elements from both the sets.



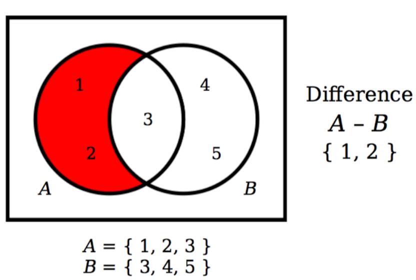
$$A \cup B = \{ x : x \in A \text{ or } x \in B \}$$

2) Intersection : Intersection of two sets will only contain elements that belong to both the sets.



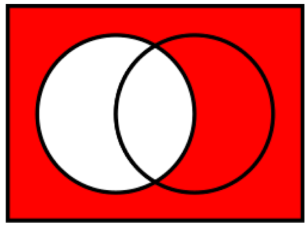
$$A \cap B = \{ x : x \in A \text{ and } x \in B \}$$

3) Difference : Difference between Set A and Set B will only contain those elements that belong to Set A, any element in common will be discarded.



$$A - B = \{ x : x \in A \text{ and } x \notin B \}$$

4) Complement : Complement of a Set A contains all the elements which do not belong to A.



A' = { x : x ∈ U and x ∉ A}

7. List of Set Identities:

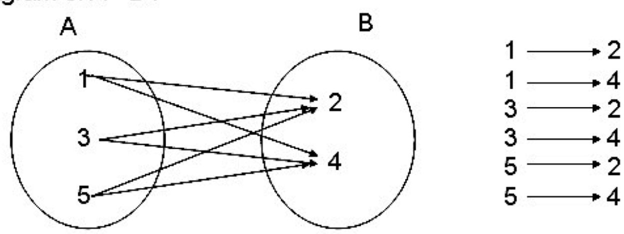
Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

8. Cartesian Product of two sets

The Cartesian product of two sets A and B, denoted A × B, is the set of all possible ordered pairs where ,

the elements of A are first and
the elements of B are second.
A X B = { (a, b) : a ∈ A and b ∈ B}
A = {1, 3, 5} and B ={2, 4} then,
A X B = { (a, b) : a ∈ A and b ∈ B}
A X B = { (1,2), (1,4), (3,2), (3, 4), (5,2), (5,4) }

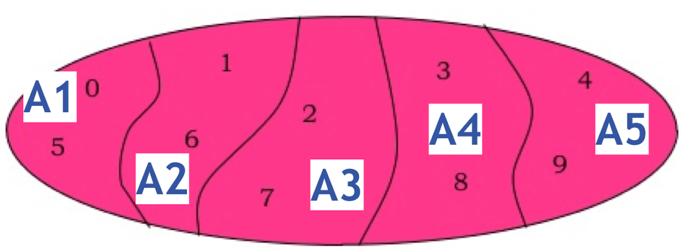
Arrow Diagram of A ×B :



9. Partition of a Set :

A partition of a set is a grouping of its elements into non-empty subsets, in such a way that every element is included in exactly one subset.

For example the regions A1, A2, A3, A4, A5 form partitions of Set A as shown below



A1, A2, A3, A4, A5 are disjoint subsets of A.

A = A1 U A2 U A3 U A4 U A5