TWO-DEGREE-OF-FREEDOM PID CONTROLLERS — THEIR FUNCTIONS AND OPTIMAL TUNING —

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Abstract. "Two-degree-of-freedom (2DOF) PID controller" is a 2DOF controller whose serial compensator is a PID element and whose feedforward compensator is a PD element. In this article, the previous researches on its equivalent transformations and interpretations of its advantages are surveyed first. Then, formulae for optimal tuning is reported. *Copyright ©2000 IFAC*

Key Words. PID control system, two-degree-of-freedom structure, optimal tuning, frequency domain performance index, process control

1. INTRODUCTION

Degree of freedom of a control system is the number of closed-loop transfer functions which can be adjusted independently (Horowitz 1963). The design of control systems is a multi-objective problem and, therefore, the two-degree-of-freedom (abbreviated as "2DOF") control system can naturally attain higher performance than one-degree-of-freedom ("1DOF") system. This fact was already discussed in Horowitz's book, but it took two decades before it was exploited in the framework of PID control (Araki 1984*a*&*b*, 1985). In the above papers, the 2DOF PID was proposed for industrial use together with detailed analysis and a small list of optimal parameters. Consequently, this proposal was adopted by vendors (Hiroi 1986; Namie *et al.* 1988; OMRON Corporation 1988), and further studies followed (see the references in §3). As the result, fairly complete list of optimal parameters are available now. But they are given in table-look-up forms and not necessarily convenient for use in control computers. In the present paper, the previous researches will be surveyed first, and then, simple formulae giving the optimal parameters of the 2DOF PID in terms of the plant parameters will be reported.

2. PRELIMINARIES

A general 2DOF control system is shown in Fig. 1. The main purpose of the 2DOF PID controller was to exploit the advantage of this structure in the framework of PID. So, $C(s)$ (referred to as **serial compensator**) was naturally chosen as

$$C(s) = K_P\left\{1 + \frac{1}{T_I s} + T_D D(s)\right\} \tag{1}$$

where $D(s)$ is the approximate derivative given by $D(s) = s/(1+\tau s)$. As for $C_f(s)$ (referred to as **feedforward compensator**),

$$C_f(s) = -K_P\{\alpha + \beta T_D D(s)\} \tag{2}$$

was suggested (Araki 1984*a*&*b*) for the purpose of keeping the whole controller as simple as the original (i.e. 1DOF) PID, where the integral element is excluded because of the stability requirement and the "-" sign was put because of the reason clarified in §4. The three parameters of $C(s)$, i.e. the proportional gain K_P, the integral time T_I and the derivative time T_D, will be referred to as **basic parameters**, and the two parameters of $C_f(s)$, i.e. α and β, as **2DOF parameters**. These five parameters are treated as tunable. The τ in $D(s)$ is set as $\tau = T_D/\delta$, where δ is the derivative gain. It has been a traditional practice to use a fixed value of δ. We follow this tradition, because our numerical experiments indicated that the change of δ does not influence the optimal values of the other five parameters drastically.

In the following, we consider the next situation, which has been traditionally assumed in literatures (see the references in §4.1) and accepted in practice:

$$H(s) = 1, \quad P_d(s) = P(s). \tag{3}$$

The resulting control system is shown in Fig. 2, where d_m is omitted for simplicity. It can be shown that the 2DOF PID controller makes the steady-state errors to a step reference and a step disturbance robustly 0 if the 1st eq. of (3) hold-

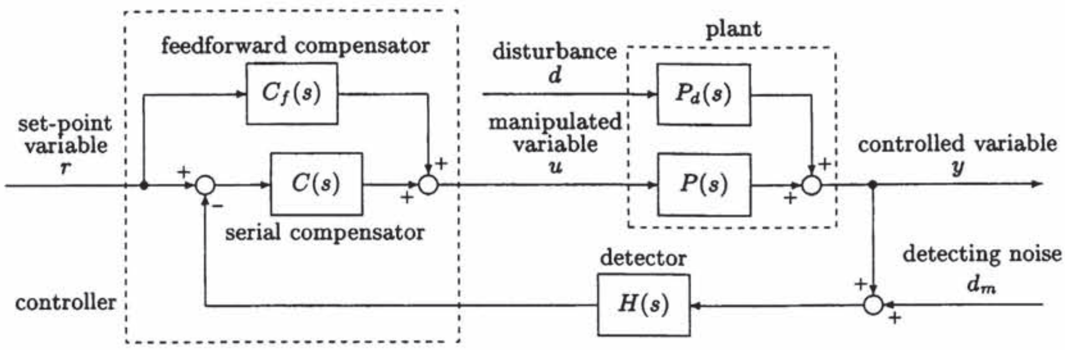
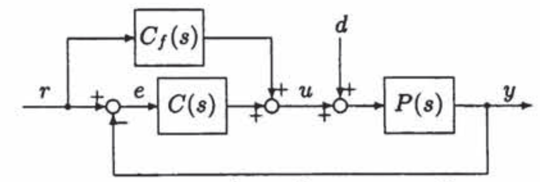
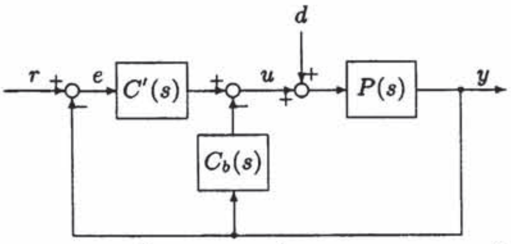


Fig. 1. Two-degree-of-freedom (2DOF) control system



$$C(s) = K_P\left\{1 + \frac{1}{T_I s} + T_D D(s)\right\}$$
$$C_f(s) = -K_P\{\alpha + \beta T_D D(s)\}$$

Fig. 2. Feedforward type (FF type) expression of the 2DOF PID control system



$$C'(s) = K_P\left\{(1-\alpha) + \frac{1}{T_I s} + (1-\beta)T_D D(s)\right\}$$
$$C_b(s) = K_P\{\alpha + \beta T_D D(s)\}$$

Fig. 3. Feedback type (FB type) expression of the 2DOF PID control system

s (Araki and Taguchi 1998). Relaxization of the 2nd eq. of (3) is a topic of study in another article (Taguchi and Araki 1999).

3. EQUIVALENT TRANSFORMATION

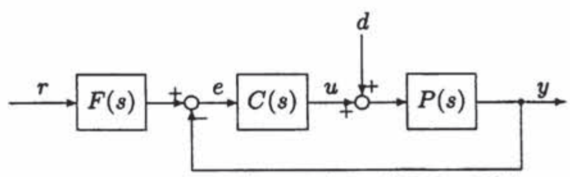
The controller of Fig. 2 is a two-input one-output system in which r and y are the inputs, and u is the output. This controller can be equivalently transformed to Fig. 3 and Fig. 4. These transformations are useful for:

- Understanding the effect of the 2DOF structure from various viewpoints,
- Developing an efficient algorithm in digital implementation (Hiroi and Yamamoto 1985; Hiroi and Nagakawa 1989),
- Introducing nonlinear operations such as magnitude limitation, rate limitation, directional gain adjustment, etc. (Hiroi and Nagakawa 1989; Hiroi *et al.* 1990; Kanda and Hiroi 1991),
- Realizing bumpless switching, implementing an anti-reset-windup mechanism,
- Converting the PID controller already built in to the 2DOF PID (Hiroi and Yonezawa 1985; Hiroi 1986; Yamazaki and Hiroi 1987).

In this paper, we can look at only the first aspect because of the space constraint.

4. THE EFFECT OF THE 2DOF PID

Responses of y to the unit step changes of r and d are called "set-point response (denoted as **r-response**)" and "disturbance response (denoted as **d-response**)," respectively. We focus on these responses, and see how they are improved, as a whole, by the 2DOF PID. Note that these responses are nothing but the indicial responses of the closed-loop transfer functions $G_{yr}(s)$ and $G_{yd}(s)$.



$$F(s) = \frac{1 + (1-\alpha)T_I s + (1-\beta)T_I T_D s D(s)}{1 + T_I s + T_I T_D s D(s)}$$
$$C(s) = K_P\left\{1 + \frac{1}{T_I s} + T_D D(s)\right\}$$

Fig. 4. Reference-filter type (Filter type) expression of the 2DOF PID control system

4.1. *Problem with the conventional PID controller*

Consider the conventional (i.e. 1DOF) PID control system of Fig. 5, whose closed-loop transfer functions are

$$G_{yr1}(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}, \tag{4}$$

$$G_{yd1}(s) = \frac{P(s)}{1 + P(s)C(s)}. \tag{5}$$

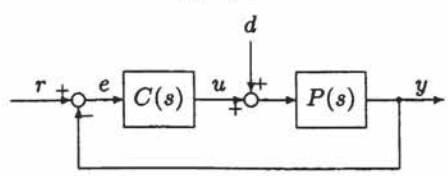
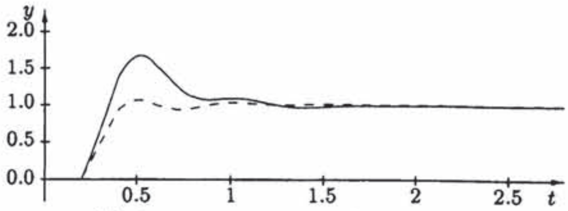
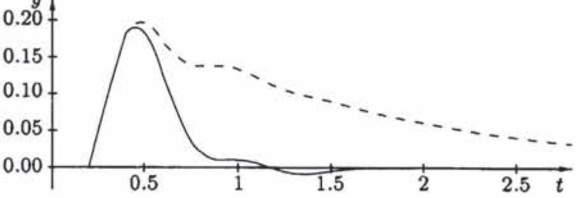


Fig. 5. Conventional 1DOF PID control system

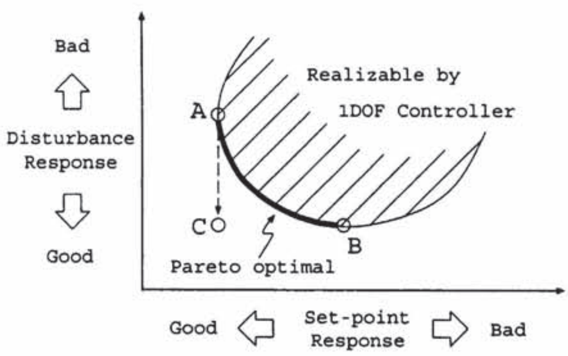


(a) Set-point responses (r-responses)



(b) Disturbance responses (d-responses)

Fig. 6. Responses of conventional PID control system



A: Set-point optimal, B: Disturbance optimal, C: Realizable by 2DOF controller

Fig. 7. Conceptual illustration of performances of 1DOF and 2DOF control systems

These two transfer functions are related as

$$G_{yr1}(s)P(s) + G_{yd1}(s) = P(s). \tag{6}$$

This equation means that, given $P(s)$, $G_{yr1}(s)$ is uniquely determined if $G_{yd1}(s)$ is chosen, and vice versa. This fact causes the following difficulty in the parameter tuning. Namely, if the disturbance response is optimized, the set-point response is often found to be poor, and vice versa. For this reason, some of the classical researches (Chien, Hrones & Reswick 1952; Kuwata 1987) gave two tables: one for the "disturbance optimal (**d-optimal**) parameters", and the other for the "set-point optimal (**r-optimal**) parameters." Fig. 6 shows the example of tuning for the plant $P(s) = e^{-0.2s}/(1+s)$, where the continuous lines are the response for the "d-optimal" tuning ($K_P = 6.0, T_I = 0.40, T_D = 0.084$) and the dotted lines are those for the "r-optimal" tuning ($K_P = 4.75, T_I = 1.35, T_D = 0.094$).

The above situation is conceptually illustrated in Fig. 7. The hatched area is realizable by the conventional (i.e. 1DOF) PID. This means that one cannot optimize the r-response and the d-response at once and are forced to choose one of the next alternatives:

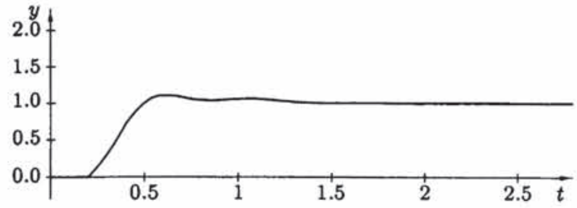


Fig. 8. r-response of 2DOF PID control system

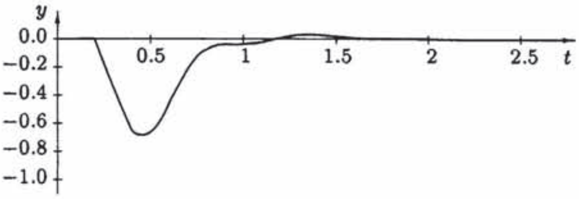


Fig. 9. Response of the second term of $G_{yr2}(s)$

- to choose a Pareto optimal point (on the bold line of Fig. 7), or
- to make the d-optimal tuning and to use rate limiter for r.

Under the classical situation, where the set-point variable is not changed so often, the second alternative is advantageous and, therefore, many of the optimal tuning methods (Takahashi 1949; Hozebroek and van der Waerden 1950*a*&*b*; Wolfe 1951; Cohen and Coon 1953; Lopez *et al.* 1967; Miller *et al.* 1967) only give the d-optimal parameters. However, the situation has changed and the modern control systems are often required to change the set-point variable fairly frequently. To cope with such a situation, the 2DOF PID controller offers a powerful solution. Namely, it enables us to make both the r-response and the d-response practically optimal at once in the linear framework, as explained in the following.

4.2. *Interpretations of the 2DOF effect*

First, we compare responses of the 2DOF PID control system with those of 1DOF system based on Fig. 2. The closed-loop transfer function $G_{yr2}(s)$ of the 2DOF system is

$$G_{yr2}(s) = \frac{P(s)C(s)}{1+P(s)C(s)} + \frac{P(s)C_f(s)}{1+P(s)C(s)} \tag{7}$$

where $G_{yd2}(s)$ remains same with $G_{yd1}(s)$. This indicates, assuming that $C(s)$ is set the same,

- the d-responses of the 2DOF and the 1DOF control systems are same, and
- the r-responses differ by the amount of the second term of $G_{yr2}(s)$ in (7), which can be adjusted by $C_f(s)$.

Thus, appropriate choice of $C_f(s)$ is expected to improve the r-response of the 2DOF system without deteriorating the d-response. The improvement in the case of the plant $P(s) = e^{-0.2s}/(1+s)$

Table 1 Formulae of the optimal parameters for 2DOF PID controller (1)

$P(s) = \frac{e^{-Ls}}{1+Ts}$	K_P	$0.1415 + \frac{1.224}{(L/T) - 0.001582}$
	T_I/T	$0.01353 + 2.200(L/T) - 1.452(L/T)^2 + 0.4824(L/T)^3$
	T_D/T	$0.0002783 + 0.4119(L/T) - 0.04943(L/T)^2$
	α	$0.6656 - 0.2786(L/T) + 0.03966(L/T)^2$
	β	$0.6816 - 0.2054(L/T) + 0.03936(L/T)^2$
$P(s) = \frac{e^{-Ls}}{(1+Ts)^3}$	K_P	$0.4020 + \frac{1.275}{(L/T) + 0.003273}$
	T_I/T	$0.3572 + 7.467(L/T) - 12.86(L/T)^2 + 11.77(L/T)^3 - 4.146(L/T)^4$
	T_D/T	$0.8335 + 0.2910(L/T) - 0.04000(L/T)^2$
	α	$0.6661 - 0.2509(L/T) + 0.04773(L/T)^2$
	β	$0.8131 - 0.2303(L/T) + 0.03621(L/T)^2$
$P(s) = \frac{e^{-Ls}}{s}$	K_P	$\frac{1.253}{L}$
	T_I	$2.388L$
	T_D	$0.4137L$
	α	0.6642
	β	0.6797

is shown in Fig. 8, where the basic parameters are chosen same with the d-optimal tuning for the 1DOF PID and the 2DOF parameters are $\alpha = 0.60$ and $\beta = 0.63$. Here, we find that the overshoot in the r-response is completely suppressed. This improvement is the effect of the second term of $G_{yr2}(s)$ in (7). Actually, the indicial response of the second term is as shown in Fig. 9. This waveform almost exactly matches the overshoot part of the r-response of the 1DOF system, and by superposing it, the overshoot was planed. Namely, the effect of the 2DOF structure appears as the "superposition of the second term of $G_{yr2}(s)$." We assured the same phenomena are observed in most test batches, and so, we determined to include "$-$" sign in the standard form of $C_f(s)$.

Some other remarks are to be made. From the FB-type expression, we can see that the preceded derivative PID (Miyazaki 1981) is obtained as a special case by setting $\alpha = 0$, $\beta = 1$; and the I-PD controller (Kitamori 1979) by setting $\alpha = \beta = 1$. From the filter-type expression, we can see that the 2DOF PID realizes the same function with the rate-limiting operation in the linear framework. From the modern theoretic point, the effect of the 2DOF structure can be interpreted as reallocation of zeros of the set-point transfer function.

5. OPTIMAL TUNING

The observations made in §4.2 suggest the next method of parameter tuning.

Two-step Tuning Method:

Step 1 Optimize the d-response by tuning $C(s)$ (i.e. the basic parameters K_P, T_I, T_D).

Step 2 Let $C(s)$ be fixed and optimize the r-response by tuning $C_f(s)$ (i.e. the 2DOF parameters α, β).

The above method has advantages that the classical result on PID tuning can be utilized in Step 1, that the number of parameters to be optimized at once is not large (i.e. 3 and 2), and that we can maintain intuitive understanding about what are going on in each step. On the other hand, this method does not necessarily guarantee the "overall optimal". To be concrete, the major characteristics (i.e. poles) of the system is basically determined at the first step, and, if that is chosen too extremely, the adjustment in the second step becomes difficult so that we can only attain a very poor set-point response. This phenomena is actually observed if we remove the 2nd assumption of (3) (Taguchi and Araki 1999). Here, we employ the two-step method and construct optimal parameter formulae.

As the basis of our optimal tuning, we consider the next performance criterion.

$$J[\lambda, p;\ H(s)] = \int_0^\infty \left| \lambda(\omega) \left\{ \frac{d^p H(s)}{ds^p} \right\}_{s=j\omega} \right|^2 d\omega \quad (8)$$

where $H(s)$ is the s-function corresponding to a step response ($G_{ed}(s)/s = -G_{yd}(s)/s$ or $G_{er}(s)/s = \{1 - G_{yr}(s)\}/s$). By applying the above performance criterion to representative test batches, we found that

$$\lambda(\omega) = \omega^{1/4}, \quad p = 2 \quad (9)$$

makes the conventional PID control systems "optimal" in the classical sense, i.e.

- the overshoot is less than 20 %, and

Table 2 Formulae of the optimal parameters for 2DOF PID controller (2)

$P(s) = \dfrac{e^{-Ls}}{s(1+Ts)}$	K_P	$0.7608 + \dfrac{0.5184}{\{(L/T) + 0.01308\}^2}$
	T_I/T	$0.03330 + 3.997(L/T) - 0.5517(L/T)^2$
	T_D/T	$0.03432 + 2.058(L/T) - 1.774(L/T)^2 + 0.6878(L/T)^3$
	α	0.6647
	β	$0.8653 - 0.1277(L/T) + 0.03330(L/T)^2$
$P(s) = \dfrac{e^{-Ls}}{s(1+Ts)^2}$	K_P	$0.1778 + \dfrac{0.5667}{(L/T) + 0.002325}$
	T_I/T	$0.2011 + 11.16(L/T) - 14.98(L/T)^2 + 13.70(L/T)^3 - 4.835(L/T)^4$
	T_D/T	$1.262 + 0.3620(L/T)$
	α	0.6666
	β	$0.8206 - 0.09750(L/T) + 0.03845(L/T)^2$
$P(s) = \dfrac{e^{-Ls}}{1+Ts+T^2s^2}$	K_P	$0.3363 + \dfrac{0.5013}{\{(L/T) + 0.01147\}^2}$
	T_I/T	$-0.02337 + 4.858(L/T) - 5.522(L/T)^2 + 2.054(L/T)^3$
	T_D/T	$0.03392 + 2.023(L/T) - 1.161(L/T)^2 + 0.2826(L/T)^3$
	α	$0.6678 - 0.05413(L/T) - 0.5680(L/T)^2 + 0.1699(L/T)^3$
	β	$0.8646 - 0.1205(L/T) - 0.1212(L/T)^2$
$P(s) = \dfrac{e^{-Ls}}{1+2Ts+T^2s^2}$	K_P	$1.389 + \dfrac{0.6978}{\{(L/T) + 0.02295\}^2}$
	T_I/T	$0.02453 + 4.104(L/T) - 3.434(L/T)^2 + 1.231(L/T)^3$
	T_D/T	$0.03459 + 1.852(L/T) - 2.741(L/T)^2 + 2.359(L/T)^3 - 0.7962(L/T)^4$
	α	$0.6726 - 0.1285(L/T) - 0.1371(L/T)^2 + 0.07345(L/T)^3$
	β	$0.8665 - 0.2679(L/T) + 0.02724(L/T)^2$

- the settling time is almost same with or less than that of the "optimal" system tuned by the CHR method.

Taguchi *et al.*(1987), Araki and Taguchi(1998) calculated the optimal parameters of the 2DOF PID and the 2DOF PI for 7 kinds of test batches applying the above criterion. Here, we construct formulae which give the optimal parameters in terms of the plant parameters. Considering easiness to be used in small control computers, we sought solutions in the class of simple rational functions. The results are as given in Tables 1 ~ 3. It must be noted that

- These formulae are effective only in the range $(L/T) \leq 1.0$.
- Generally, change of the 2DOF parameters α and β are not large in the case of PID controller except the 6-th plant.
- Except the 6-th plant, sensitivity of the response to the change of the controller parameters is not very high at the optimal point.
- The 6-th plant $e^{-Ls}/(1 + Ts + T^2s^2)$ is the 2nd order oscillatory system with the damping coefficient being 0.5. The change of the optimal parameters is rather complicated in this case. In addition, sensitivity of the responses to the change of the controller parameters is very high. So, it is recommended not to rely upon the formula, but to carry out deliberate tuning.
- The 7-th plant $e^{-Ls}/(1+2Ts+T^2s^2)$ can be expressed as $e^{-Ls}/(1 + Ts)^2$. So, this plant could be located between the first and the second.

6. CONCLUSION

In this paper, some of the previous researches on the two-degree-of-freedom PID controller were surveyed, and formulae giving their optimal parameters were constructed under the assumptions that the detector has enough accuracy and speed over the range of control purpose and that the major disturbance is the step input entering the plant at the manipulating point. The situation where the major disturbance enters the plant in a different fashion is studied elsewhere (Taguchi and Araki 1999).

7. REFERENCES

Araki, M. (1984*a*). On Two-degree-of-freedom PID control systems. SICE Research Committee on Modeling and Control Design of Real Systems.

Araki, M. (1984*b*). PID Control System with Reference Feedforward (PID-FF Control System). Proc. of 23rd SICE Annual Conference, 31–32.

Araki, M. (1985). Two-Degree-of-Freedom Control System - I. *System and Control* **29**(10), 649–656.

Araki, M. and H. Taguchi (1998). Two-Degree-of-Freedom PID Controller. *Systems, Control and Information* **42**(1), 18–25.

Table 3 Formulae of the optimal parameters for 2DOF PI controller

$P(s) = \dfrac{e^{-Ls}}{1+Ts}$	K_P	$0.1098 + \dfrac{0.7382}{(L/T) - 0.002434}$
	T_I/T	$0.06216 + 3.171(L/T) - 3.058(L/T)^2 + 1.205(L/T)^3$
	α	$0.6830 - 0.4242(L/T) + 0.06568(L/T)^2$
$P(s) = \dfrac{e^{-Ls}}{(1+Ts)^3}$	K_P	$0.2713 + \dfrac{0.7399}{(L/T) + 0.5009}$
	T_I/T	$2.759 - 0.003899(L/T) + 0.1354(L/T)^2$
	α	$0.4908 - 0.2648(L/T) + 0.05159(L/T)^2$
$P(s) = \dfrac{e^{-Ls}}{s}$	K_P	$\dfrac{0.7662}{L}$
	T_I	$4.091L$
	α	0.6810
$P(s) = \dfrac{e^{-Ls}}{s(1+Ts)}$	K_P	$0.1787 + \dfrac{0.2839}{(L/T) + 0.001723}$
	T_I/T	$4.296 + 3.794(L/T) + 0.2591(L/T)^2$
	α	$0.6551 + 0.01877(L/T)$
$P(s) = \dfrac{e^{-Ls}}{s(1+Ts)^2}$	K_P	$0.07368 + \dfrac{0.3840}{(L/T) + 0.7640}$
	T_I/T	$8.549 + 4.029(L/T)$
	α	$0.6691 + 0.006606(L/T)$
$P(s) = \dfrac{e^{-Ls}}{1+Ts+T^2s^2}$	K_P	$0.1000 + \dfrac{0.05627}{\{(L/T) + 0.06041\}^2}$
	T_I/T	$4.340 - 16.39(L/T) + 30.04(L/T)^2 - 25.85(L/T)^3 + 8.567(L/T)^4$
	α	$0.6178 - 0.4439(L/T) - 7.575(L/T)^2 + 9.317(L/T)^3 - 3.182(L/T)^4$
$P(s) = \dfrac{e^{-Ls}}{1+2Ts+T^2s^2}$	K_P	$0.3717 + \dfrac{0.5316}{(L/T) + 0.0003414}$
	T_I/T	$2.069 - 0.3692(L/T) + 1.081(L/T)^2 - 0.5524(L/T)^3$
	α	$0.6438 - 0.5056(L/T) + 0.3087(L/T)^2 - 0.1201(L/T)^3$

Chien, K.L., J. A. Hrones and J. B. Reswick (1952). On the Automatic Control of Generalized Passive Systems. *Trans. of ASME* **74**, 175–185.

Cohen, G. H. and G. A. Coon (1953). Theoretical Consideration of Retarded Control. *Trans. of ASME* **75**, 827–834.

Hiroi, K. (1986). Two-Degree-of-Freedom PID Control System and its Application. *Instrumentation* **29**(2), 39–43.

Hiroi, K., A. Nomura, A. Yoneya and Y. Togari (1990). Advanced Two Degrees of Freedom PID Algorithm. Proc. of 29th SICE Annual Conference, 49–50.

Hiroi, K. and K. Nagakawa (1989). Improvement of a Digital PID Algorithm with Rate Limitation on MV Change. Proc. of 28th SICE Annual Conference, 243–244.

Hiroi, K. and K. Yonezawa (1985). Reference-Filter Type Two-Degree-of-Freedom PID Control System. Proc. of 24th SICE Annual Conference, 217–218.

Hiroi, K. and Y. Yamamoto (1985). Component-Separated Type Two-Degree-of-Freedom PID Control System. Proc. of 24th SICE Annual Conference, 219–220.

Horowitz, I. M. (1963). *Synthesis of Feedback Systems*. Academic Press.

Hozebroek, P. and B. L. van der Waerden (1950*a*). The Optimal Adjustment of Regulators. *Trans. of ASME* **72**, 317–322.

Hozebroek, P. and B. L. van der Waerden (1950*b*). Theoretical Considerations on the Optimal Adjustment of Regulators. *Trans. of ASME* **72**, 309–315.

Kanda, M. and K. Hiroi (1991). Super Two Degrees of Freedom PID Algorithm. Proc. of 30th SICE Annual Conference, 465–466.

Kitamori, T. (1979). A Method of Control System Design Based upon Partial Knowledge about Controlled Processes. *Trans. of SICE* **15**(4), 549–555.

Kuwata, R. (1987). An Improved Ultimate Sensitivity Method and PID; I-PD Control Characteristics. *Trans. of SICE* **23**(3), 232–239.

Lopez, A. M., J. A. Miller, C. L. Smith and P. W. Murrill (1967). Tuning Controllers with Error-Integral Criteria. *Instrumentation Technology* **14**, 57–62.

Miller, J. A., A. M. Lopez, C. L. Smith and P. W. Murrill (1967). A Comparison of Controller Tuning Techniques. *Control Engineering* **14**, 72–75.

Miyazaki, S. (1981). The Practice of Process Control (19th) —Direct Digital Control and its Merits. *Automation* **26**(1), 102–107.

Namie, M., T. Ueda, T. Tsukabe and H. Taguchi (1988). Two-Degrees-of-Freedom PID Controller used for Temperature Control—Simultaneity of Reference Response and Disturbance Response—. *OMRON Technics* **28**(4), 285–291.

OMRON Corporation (1988). Electrical Temperature Controller. Catalog No. SCSS-045.

Taguchi, H. and M. Araki (1999). Effects of Disturbance Input Point in Two-Degree-of-Freedom PID Control System. Preprints of the 42nd Japan Joint Automatic Control Conference, 239–242.

Taguchi, H., M. Doi and M. Araki (1987). Optimal parameters of Two-Degrees-of-Freedom PID Control Systems. *Trans. of SICE* **23**(9), 889–895.

Takahashi, Y. (1949). *Automatic Control*. Kagaku-Gijutsu-Sha.

Wolfe, W. A. (1951). Controller Settings for Optimum Control. *Trans. of ASME* **73**, 413–418.

Yamazaki, S. and K. Hiroi (1987). Application of Reference-Filter Type 2DOF PID to Boiler Control. *Instrumentation* **30**(10), 114–119.

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