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## Stability Analysis of Luo Converter Using State Space Modeling\*

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In this paper, stability of the Luo converter and the effect of circuit elements on the stability and damping fluctuation frequency is investigated. Along with this, the mathematical modeling of positive output (P/O) Luo converter in continuous conduction mode (CCM) is done by using an average state-space method. By using the obtained mathematical model, the transfer function of the Luo converter is extracted, and by using the location of the roots the mentioned transfer function, stability, and effect of circuit elements on converter behavior are studied. Also, its situations are compared with other converters. The analyses are performed through MATLAB based on simulations, which includes both roots location and bodes diagram.

*Keywords:* Luo converter; mathematical modeling; continues conduction mode; state space modeling; stability.

### 1. Introduction

Dc—dc converters are switching circuits, which convert a determined level of dc voltage to another determined level.<sup>1,2</sup> Among the dc—dc converters, buck, boost, buck-boost, cuk and SEPIC can be mentioned as the most common types. These converters are so applicable in distributed generation,<sup>1</sup> power factor correction,<sup>2,3</sup> aerospace industry,<sup>4</sup> elevators,<sup>5,6</sup> electric motors and renewable resources such as photovoltaic and fuel cells.<sup>7–12</sup> In addition, high dynamic response and the low

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number of switches can be considered as strong points of these converters.<sup>12–15</sup> Luo converter is another type of advanced dc–dc converters.<sup>16</sup> Since output voltage fluctuations of this converter are at a very low level, this converter is very applicable in the industry field. Hence, due to the widespread application of this converter in industry, determining the stability margin of this converter would be indispensable. Extracting transform function is one of the simple methods to analyze the stability of dc–dc converters. Therefore, to realize this, achieving a mathematical model of this converter is necessary. By using the obtained mathematical model, stability analyzing and also the effect of circuit elements on frequency response would be possible.<sup>17</sup> Luo converter is considered a nonlinear converter because of its switching property. There are various kinds of techniques to model nonlinear structures. Methods such as the current injection model, average circuit model and the average state-space model are considered as popular methods in modeling. The Average state space method is suitable for converters that operate in CCM. However, it is not suitable for those which operate in discontinuous conduction mode (DCM).<sup>18</sup> The Current injection technique is suitable for dc–dc converters with basic structures such as buck, boost, buck-boost, in both DCM and CCM.<sup>18,19</sup> Also, the average circuit modeling method has been created before the state space method, and due to its generality for all types of dc–dc converters, it is one of the chief modeling methods.<sup>20</sup>

In this paper, the mathematical modeling of the Luo converter will be done by using average state-space modeling in CCM. The voltage transfer function of this converter will be extracted by using an obtained mathematical model. At the subsequent stage, stability and damping fluctuations frequency of Luo converter will be investigated by root location and bode diagram of an achieved transfer function. With this modeling, the performance of the system can be stabilized at sensitive loads such as medical devices.

## 2. Dynamic Equation of Lou Converter

Figure 1(a) shows the power circuit of the Luo converter. In the following figure, switches  $S_1$  and  $S_2$  are considered as ideal. In this section, the mathematical modeling of the Luo converter in CCM mode will be performed. Assuming switch  $S_1$  is on, and by applying Kirchhoff's voltage law in equivalent circuit shown in Fig. 1(b), the voltage of inductors  $L_1$  and  $L_2$  can be written as (1) and (2), respectively:

$$L_1 \frac{di_{L1}}{dt} = v_i - R_{L1} i_{L1}. \quad (1)$$

$$L_2 \frac{di_{L2}}{dt} = v_i + V_{C1} - v_o - R_{L2} i_{L2}. \quad (2)$$

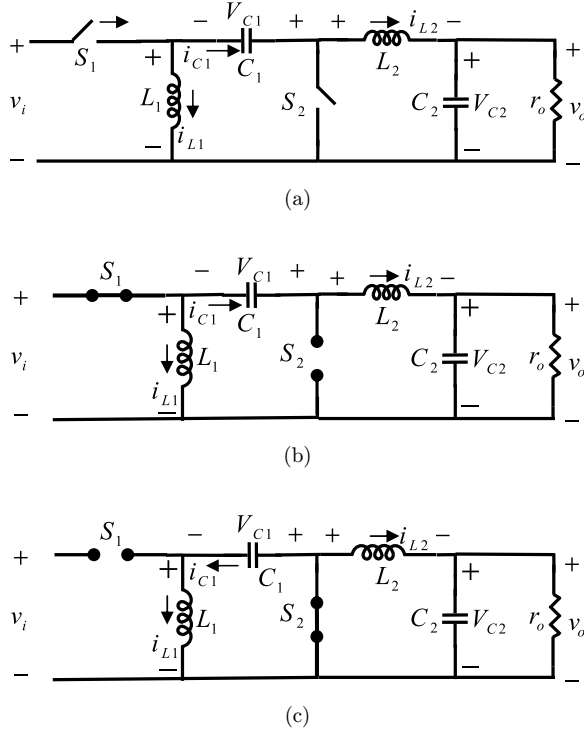


Fig. 1. (a) Schematic diagram for the Luo converter; (b) equivalent circuit of Luo converter when Switch  $S_1$ : on; (c) equivalent circuit of Luo converter when Switch  $S_2$ : on.

The current of capacitor  $C_1$  is also obtained by applying Kirchhoff's current law in the converter as following equation:

$$C_1 \frac{dV_{C1}}{dt} = i_{L2}. \quad (3)$$

Also, converter's output current is represented as a mix of capacitor current  $C_2$  and load current  $i_o$  as follows:

$$i_o = \frac{v_o}{r_o} + i_{C2}. \quad (4)$$

The measure of output current  $i_o$ , when the switch  $S_1$  is on, would be represented as:

$$i_o = i_{L2}. \quad (5)$$

Based on Eqs. (5) and (4), the measure of output voltage changes is obtained as follows:

$$C_2 \frac{dV_{C2}}{dt} = i_{L2} - \frac{v_o}{r_o}, \quad (6)$$

where  $R_{L1}$  and  $R_{L2}$  are equivalent resistance of inductors  $L_1$  and  $L_2$ , respectively,  $v_o$  is the output voltage,  $i_{L1}$  is the current of inductor  $V_{C1}$ ,  $L_2$  is the current of inductor  $i_{L2}$ ,  $L_1$  is the voltage of two ends of the capacitor  $C_1$ ,  $V_{C2}$  is the voltage of two ends of the capacitor  $C_2$ ,  $r_o$  is the output resistance,  $i_o$  is the load current,  $i_{C2}$  is the current of the capacitor  $C_2$ , and  $v_i$  is the input voltage. It should be noted that output voltage  $v_o$  is equal to the voltage of two ends of the capacitor  $C_2$ , so in all obtained equations,  $V_{C2}$  is used instead of  $v_o$ .

According to the obtained equations, when switch  $S_1$  is on, and by considering the voltages of the capacitors  $C_1$  and  $C_2$  and the current of inductors  $L_1$  and  $L_2$  as state variables, state-space equations of Luo converter are obtained as follows:

$$\begin{bmatrix} \frac{di_{L1}}{dt} \\ \frac{di_{L2}}{dt} \\ \frac{dV_{C1}}{dt} \\ \frac{dV_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{R_{L2}}{L_2} & \frac{1}{L_2} & -\frac{1}{L_2} \\ 0 & \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & \frac{-1}{r_o C_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \\ 0 \\ 0 \end{bmatrix} [v_i].$$

$$V_{C2} = [0 \ 0 \ 0 \ 1] \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix}. \quad (7)$$

Figure 1(c) shows equivalent circuit of Luo converter when the switch  $S_2$  is on. According to this figure, dynamic equations related to voltage of inductors  $L_1$  and  $L_2$  are obtained as follows:

$$L_1 \frac{di_{L1}}{dt} = -V_{C1} - R_{L1} i_{L1}, \quad (8)$$

$$L_2 \frac{di_{L2}}{dt} = -R_{L2} i_{L2} - V_{C2}. \quad (9)$$

Also, according to (4), when switch  $S_2$  is on, equations related to the current of capacitors  $C_1$  and  $C_2$  also represented as follows:

$$C_1 \frac{dV_{C1}}{dt} = i_{L1}, \quad (10)$$

$$C_2 \frac{dV_{C2}}{dt} = i_{L2} - \frac{V_{C2}}{r_o}. \quad (11)$$

By using above-achieved equations, state-space equations of Luo converter when the switch  $S_2$  is turned on, are represented as follows:

$$\begin{bmatrix} \frac{di_{L1}}{dt} \\ \frac{di_{L2}}{dt} \\ \frac{dV_{C1}}{dt} \\ \frac{dV_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & -\frac{1}{L_1} & 0 \\ 0 & -\frac{R_{L2}}{L_2} & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{r_o C_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \\ 0 \\ 0 \end{bmatrix} [v_i], \quad (12)$$

$$V_{C2} = [0 \ 0 \ 0 \ 1] \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix}.$$

### 3. Transfer Function of Lou Converter

According to the two state-space equation groups obtained in (7) and (12), transfer function  $\frac{v_o}{v_i}$  of Luo converter is obtained by the following equation<sup>18</sup>:

$$\frac{v_o}{v_i} = [Q'D_1 + Q''D_2][A'D_1 + A''D_2]^{-1}[B'D_1 + B''D_2]. \quad (13)$$

In the above equation, matrixes  $A'$ ,  $A''$ ,  $B'$ ,  $B''$ ,  $Q'$  and  $Q''$  are represented as follows:

$$A' = \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{R_{L2}}{L_2} & \frac{1}{L_2} & -\frac{1}{L_2} \\ 0 & \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{r_o C_2} \end{bmatrix}, \quad (14)$$

$$A'' = \begin{bmatrix} -\frac{R_{L1}}{L_1} & 0 & -\frac{1}{L_1} & 0 \\ 0 & -\frac{R_{L2}}{L_2} & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{r_o C_2} \end{bmatrix}, \quad (15)$$

$$B' = \begin{bmatrix} \frac{1}{L_1} & \frac{1}{L_2} & 0 & 0 \end{bmatrix}^T, \quad (16)$$

$$B'' = [0 \ 0 \ 0 \ 0]^T, \quad (17)$$

$$Q' = Q'' = [0 \ 0 \ 0 \ 1]. \quad (18)$$

According to (13), and also the values of  $A'$ ,  $A''$ ,  $B'$ ,  $B''$ ,  $Q'$  and  $Q''$ , transfer function  $\frac{v_o}{v_i}$  of Luo converter is obtained as follows:

$$H(s) = \frac{v_o}{v_i} = \frac{A_1 s^2 + A_2 s + A_3}{A_4 s^4 + A_5 s^3 + A_6 s^2 + A_7 s + A_8}. \quad (19)$$

In the above-obtained transfer function, values  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $A_5$ ,  $A_6$ ,  $A_7$  and  $A_8$  are defined as follows:

$$A_1 = D_1 C_1 L_1 r_o, \quad (20)$$

$$A_2 = D_1 C_1 R_{L1} r_o, \quad (21)$$

$$A_3 = D_1 D_2 r_o, \quad (22)$$

$$A_4 = C_1 C_2 L_1 L_2 r_o, \quad (23)$$

$$A_5 = C_1 C_2 r_o [L_1 R_{L2} + L_2 R_{L1}] + C_1 L_1 L_2, \quad (24)$$

$$A_6 = C_1 C_2 R_{L1} R_{L2} r_o + C_2 D_2^2 L_2 r_o - C_2 D_1^2 L_1 r_o + C_1 L_1 R_{L2} + C_1 L_2 R_{L1} \\ + C_1 L_1 r_o, \quad (25)$$

$$A_7 = -D_1^2 L_1 + D_2^2 L_2 + C_1 R_{L1} R_{L2} + C_1 R_{L1} r_o - C_2 D_1^2 R_{L1} r_o + C_2 D_2^2 R_{L2} r_o, \quad (26)$$

$$A_8 = -D_1^2 R_{L1} + D_2^2 R_{L2} + D_2^2 r_o. \quad (27)$$

In the above equations  $D_1$  is duty cycle of Luo converter and also  $D_2$  is obtained as follows:

$$D_2 = 1 - D_1. \quad (28)$$

#### 4. Investigation of the Effect of Circuit Elements of Lou Converter on System Stability

In order to investigate the influence of circuit elements of Luo converter on system stability and zeros' and poles' location, the location of transfer function roots and bode diagram related to converter are analyzed in different values of circuit parameters. Therefore, values of  $C_1$ ,  $C_2$ ,  $L_1$  and  $L_2$  are considered as Table 1. Another parameters are  $r_o = 20 \Omega$ ,  $v_i = 120 \text{ V}$ ,  $D_1 = 0.4$ ,  $D_2 = 0.6$ ,  $R_{L1} = R_{L2}|_{L=10 \text{ mH}} = 0.45 \Omega$ ,  $R_{L1} = R_{L2}|_{L=20 \text{ mH}} = 0.55 \Omega$  and  $R_{L1} = R_{L2}|_{L=40 \text{ mH}} = 0.62 \Omega$ . It is be noted that the resistances of inductors were increased by increasing inductor values. So, it can be said that the resistances of inductors have the same effect on stability compared to their relative inductors and their effects can be neglected in the analysis.

Table 1. Different values of circuit parameters of Luo converter.

Parameter/Case study	Case study 1	Case study 2	Case study 3	Case study 4
$C_1$	30 $\mu$ F	50 $\mu$ F	20 $\mu$ F	20 $\mu$ F
$C_2$	20 $\mu$ F	20 $\mu$ F	30 $\mu$ F	50 $\mu$ F
$L_1$	10 mH	10 mH	10 mH	10 mH
$L_2$	10 mH	10 mH	10 mH	10 mH
Parameter/Case study	Case study 5	Case study 6	Case study 7	Case study 8
$C_1$	20 $\mu$ F	20 $\mu$ F	20 $\mu$ F	20 $\mu$ F
$C_2$	20 $\mu$ F	20 $\mu$ F	20 $\mu$ F	20 $\mu$ F
$L_1$	20 mH	40 mH	10 mH	10 mH
$L_2$	10 mH	10 mH	20 mH	40 mH

## 5. Investigation of the Influence of Capacitor $C_1$

By considering the value of capacitor  $C_1$  equals to 30  $\mu$ F, transfer function of Luo converter is obtained based on (19) as below:

$$H(s) = \frac{9 \times 10^6 s^2 + 45 \times 10^7 s + 12 \times 10^{12}}{3s^4 + 7800s^3 + 13757500s^2 - 433125 \times 10^4 s + 775 \times 10^{10}}. \quad (29)$$

Also, by considering  $C_1 = 50 \mu$ F, transfer function of Luo converter is obtained as:

$$H(s) = \frac{3 \times 10^6 s^2 + 15 \times 10^7 s + 24 \times 10^{11}}{s^4 + 2600s^3 + 4852s^2 - 76375 \times 10^4 s + 155 \times 10^{10}}. \quad (30)$$

Figure 2 shows the locations of the roots and bode diagrams of achieved transfer functions for  $C_1 = 30 \mu$ F and  $C_1 = 50 \mu$ F. In addition, the related information to zeros and poles of transfer functions of Luo converter for both mentioned values of  $C_1$  is presented in Table 2. According to Fig. 2(b), when phase value is  $-180^\circ$ , magnitude diagram in that frequency shows  $-29.7$  dB; thus, transfer function would be stable in this state. Also, in Fig. 2(d), because in phase  $-180^\circ$  magnitude diagram is smaller than zero (equals to  $-24.1$  dB), transfer function would be stable by considering  $C_1 = 50 \mu$ F. According to the achieved results for  $C_1 = 30 \mu$ F and  $C_1 = 50 \mu$ F, we can see that Luo converter is in stable state and increasing the value of capacitor  $C_1$  has a very little influence on imaginary values of the poles. As a result, by applying changes in the value of capacitor  $C_1$ , converter is still in stable state and the frequency of damping fluctuations of the converter is also unchanged.

According to the achieved results for  $C_1 = 30 \mu$ F and  $C_1 = 50 \mu$ F, we can see that Luo converter is in stable state and increasing the value of capacitor  $C_1$  has a very little influence on imaginary values of the poles. As a result, by applying changes in the value of capacitor  $C_1$ , converter is still in stable state and the frequency of damping fluctuations of the converter is also unchanged.



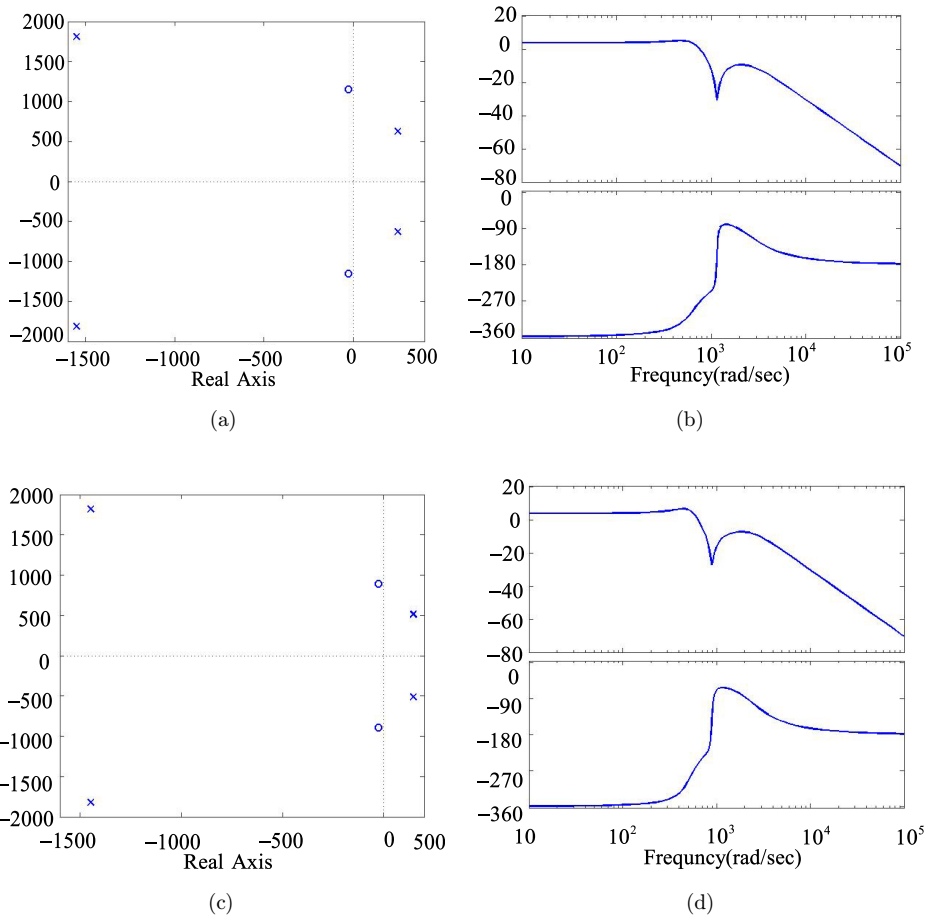


Fig. 2. Roots location and bodes diagram of Luo converter; (a) roots location, considering  $C_1 = 30 \mu\text{F}$ ; (b) bode diagram, considering  $C_1 = 30 \mu\text{F}$ ; (c) roots location, considering  $C_1 = 50 \mu\text{F}$ , (d) bode diagram, considering  $C_1 = 50 \mu\text{F}$ .

Table 2. Location of zero and poles of transfer function, considering  $C_1 = 30 \mu\text{F}$  and  $C_1 = 50 \mu\text{F}$ .

Capacitor value	Poles	Corner frequency of poles (rad/s)	Zeros	Corner frequency of zeros (rad/s)
$C_1 = 30 \mu\text{F}$	$251 \pm 626i$	674	$-25 \pm 1.15 \times 10^3i$	$1.15 \times 10^3$
	$-1.55 \times 10^3 \pm 1.81 \times 10^3i$	$2.38 \times 10^3$		
$C_1 = 30 \mu\text{F}$	$251 \pm 626i$	674	$-25 \pm 894i$	894
	$147 \pm 514i$	535		

## 6. Investigation of the Effect of Capacitor $C_2$

According to (19), transfer function of Luo converter for  $C_2 = 30 \mu\text{F}$  is obtained as follows:

$$H(s) = \frac{6 \times 10^6 s^2 + 3 \times 10^8 s + 12 \times 10^{12}}{3s^4 + 5300s^3 + 7507500s^2 - 46375 \times 10^5 s + 775 \times 10^{10}}. \quad (31)$$

Also, by considering the value of capacitor  $C_2$  as  $C_2 = 50 \mu\text{F}$ , transfer function of Luo converter is achieved as follows:

$$H(s) = \frac{12 \times 10^5 s^2 + 6 \times 10^7 s + 24 \times 10^{11}}{s^4 + 1100s^3 + 1102500s^2 - 9475 \times 10^5 s + 155 \times 10^{10}}. \quad (32)$$

The roots locations and bode diagrams of the above transfer functions are shown in Fig. 3. The information obtained from poles and zeros of these transfer functions is

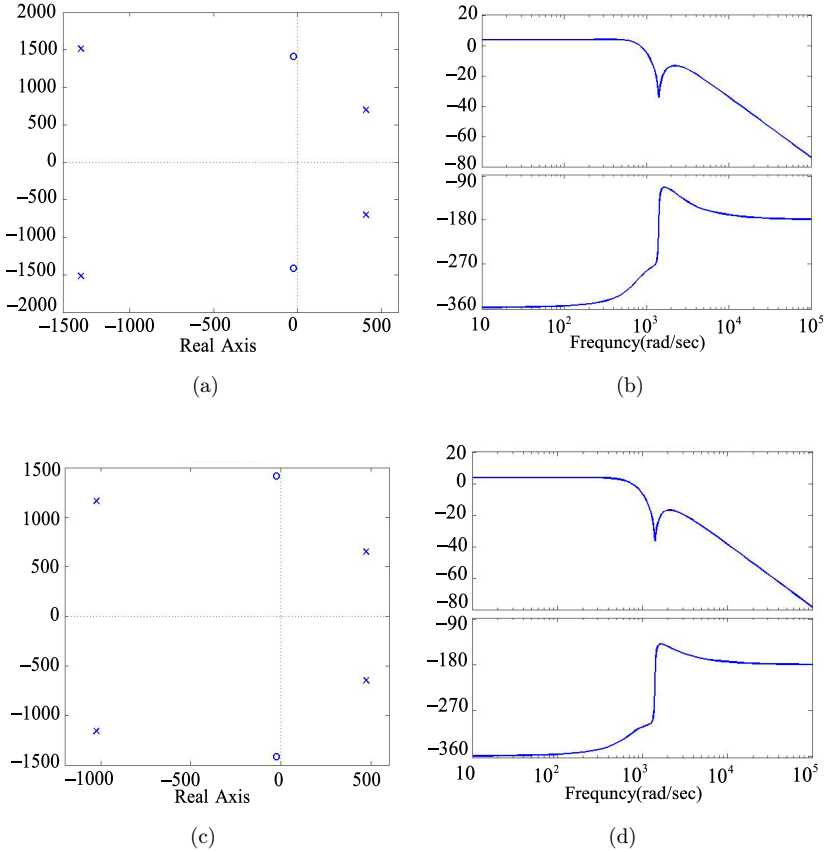


Fig. 3. Roots location and bodes diagram of Luo converter; (a) roots location considering  $C_2 = 30 \mu\text{F}$ ; (b) bode diagram considering  $C_2 = 30 \mu\text{F}$ ; (c) roots location considering  $C_2 = 50 \mu\text{F}$ ; (d) bode diagram considering  $C_2 = 50 \mu\text{F}$ .

Table 3. Location of zero and poles of transfer function, considering  $C_2 = 30 \mu\text{F}$  and  $C_2 = 50 \mu\text{F}$ .

Capacitor value	Poles	Corner frequency of poles (rad/s)	Zeros	Corner frequency of zeros (rad/s)
$C_2 = 30 \mu\text{F}$	$408 \pm 697i$ $-1.29 \times 10^3 \pm 1.51 \times 10^3i$	800 $1.99 \times 10^3$	$-25 \pm 1.41 \times 10^3i$	$1.41 \times 10^3$
$C_2 = 50 \mu\text{F}$	$408 \pm 697i$ $-1.29 \times 10^3 \pm 1.51 \times 10^3i$	800 $1.99 \times 10^3$	$-25 \pm 1.41 \times 10^3i$	$1.41 \times 10^3$

represented in Table 3. In Fig. 3(b), because the magnitude diagram in phase  $-180^\circ$ , is smaller than zero (equals to  $-31 \text{ dB/s}$ ), transfer function is stable by considering  $C_2 = 30 \mu\text{F}$ . Also by considering  $C_2 = 50 \mu\text{F}$ , magnitude diagram which is shown in Fig. 3(d) in the frequency related to phase  $-180^\circ$  equals to  $-32.3 \text{ dB}$ . Since this numeric value is smaller than zero, the transfer function in this state is stable.

From Fig. 3 and the location of the roots of transfer functions obtained in (31) and (32), we can see that by increasing the value of the capacitor  $C_2$ , the converter is still in a stable state. However, system poles are moving to the right side of the axis and also imaginary values of the poles are decreasing. So, by decreasing the imaginary values of the poles, the frequency of damping fluctuations of the converter will be reduced.

## 7. Investigation of the Effect of Inductor $L_1$

According to the fifth case study, transfer function of Luo converter for  $L_1 = 20 \text{ mH}$ , is obtained as follows:

$$H(s) = \frac{3 \times 10^6 s^2 + 75 \times 10^6 s + 3 \times 10^{12}}{s^4 + 2575s^3 + 3788750s^2 - 339687 \times 10^3 s + 19375 \times 10^8}. \quad (33)$$

Considering  $L_1 = 40 \text{ mH}$ , Luo converter transfer function is represented as follows:

$$H(s) = \frac{6 \times 10^6 s^2 + 75 \times 10^6 s + 3 \times 10^{12}}{2s^4 + 5125s^3 + 7113750s^2 - 7896875 \times 10^3 s + 19375 \times 10^8}. \quad (34)$$

Figure 4 shows the roots locations and bode diagrams related to (33) and (34). Also, information about roots of these transfer functions is represented in Table 4. According to bode diagram related to  $L_1 = 20 \text{ mH}$ , in frequency which phase is  $-180^\circ$ , magnitude diagram of transfer function is equals to  $-36.9 \text{ dB}$ . Because the obtained value is a value smaller than zero, this transfer function will be stable. Also, according to bode diagram shown in Fig. 4(d), related to  $L_1 = 40 \text{ mH}$ , magnitude diagram in frequency which the phase is  $-180^\circ$ , equals to  $-42.5 \text{ dB}$  and smaller than zero, then the converter is also stable in this state.

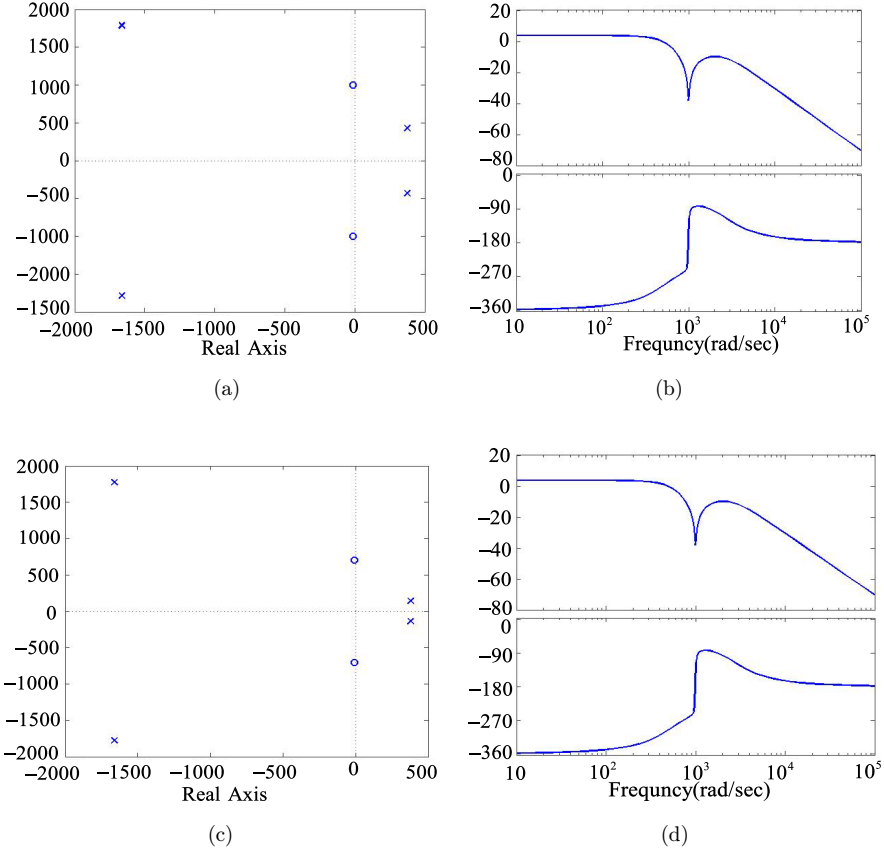


Fig. 4. Roots location and bodes diagram of Luo converter; (a) roots location considering  $L_1 = 20$  mH; (b) bode diagram considering  $L_1 = 20$  mH; (c) roots location considering  $L_1 = 40$  mH; (d) bode diagram considering  $L_1 = 40$  mH.

Table 4. Location of zero and poles of transfer function, considering  $L_1 = 20$  mH and  $L_1 = 40$  mH.

Inductor value	Poles	Corner frequency of poles (rad/s)	Zeros	Corner frequency of zeros (rad/s)
$L_1 = 20$ mH	$376 \pm 429i$	570	$-12.5 \pm 10^3i$	$10^3$
	$-1.66 \times 10^3 \pm 1.79 \times 10^3i$	$2.44 \times 10^3$		
$L_1 = 40$ mH	$380 \pm 140i$	405	$-6.25 \pm 707i$	707
	$-1.66 \times 10^3 \pm 1.78 \times 10^3i$	$2.43 \times 10^3$		

Comparing the location of the roots obtained from changing the value of inductor  $L_1$ , we can see that by increasing the value of the inductor  $L_1$ , the system is still stable and the imaginary values of poles will be decreased. Following reducing the imaginary values of the poles, the frequency of damping fluctuations of the system will be declined.

## 8. Investigation of the Effect of Inductor $L_2$

Transfer function of Luo converter for  $L_2 = 20$  mH is obtained as follows:

$$H(s) = \frac{15 \times 10^5 s^2 + 75 \times 10^6 s + 3 \times 10^{12}}{s^4 + 2575 s^3 + 2588750 s^2 - 146875 \times 10^3 s + 19375 \times 10^8}. \quad (35)$$

By considering the eighth case study, for  $L_2 = 40$  mH, transfer function of Luo converter is obtained a below:

$$H(s) = \frac{15 \times 10^5 s^2 + 75 \times 10^6 s + 3 \times 10^{12}}{2s^4 + 5125s^3 + 3513750s^2 - 1853125 \times 10^3 s + 19375 \times 10^8}. \quad (36)$$

The locations of the roots and bode diagrams of the above transfer functions are represented in Fig. 5. The information related to the roots of obtained the transfer

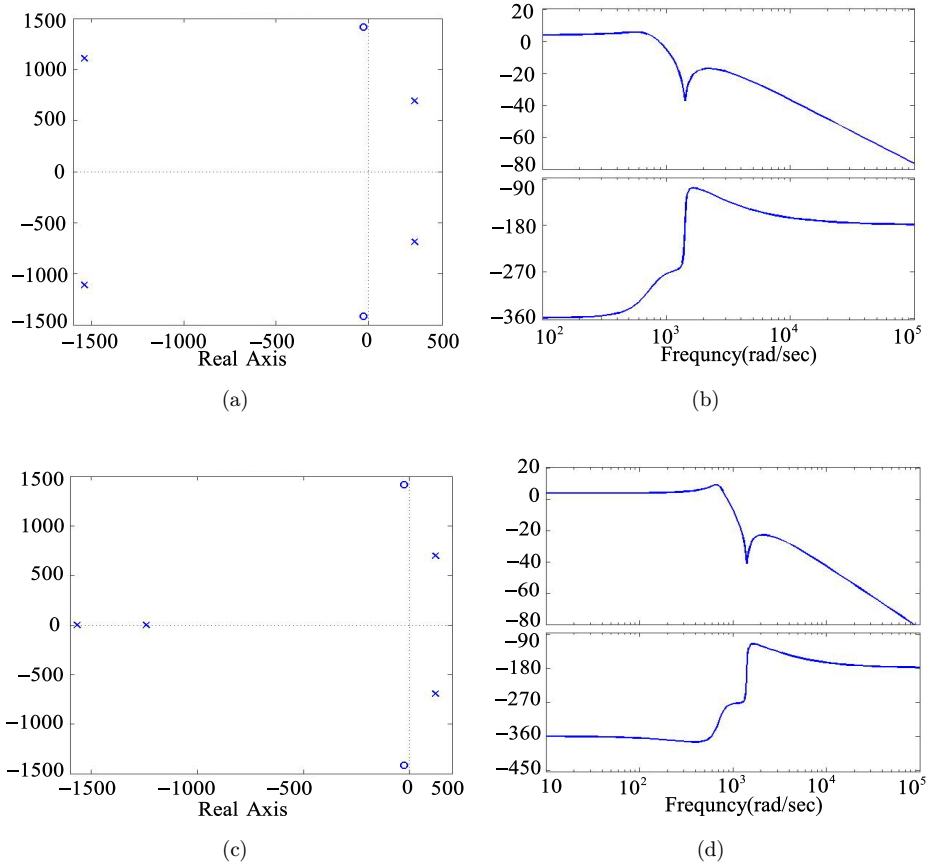


Fig. 5. Roots location and bodes diagram of Luo converter; (a) roots location considering  $L_2 = 20$  mH; (b) bode diagram considering  $L_2 = 20$  mH; (c) roots location considering  $L_2 = 40$  mH; (d) bode diagram considering  $L_2 = 40$  mH.

Table 5. Location of zero and poles of transfer function, considering  $L_2 = 20$  mH and  $L_2 = 40$  mH.

Inductor value	Poles	Corner frequency of poles (rad/s)	Zeros	Corner frequency of zeros (rad/s)
$L_2 = 20$ mH	$251 \pm 690i$ $-1.54 \times 10^3 \pm 1.11 \times 10^3 i$	734 $1.9 \times 10^3$	$-25 \pm 1.41 \times 10^3 i$	$1.41 \times 10^3$
$L_2 = 40$ mH	$122 \pm 969i$ $-1.24 \times 10^3$ $-1.57 \times 10^3$	405 $2.43 \times 10^3$	$-25 \pm 1.41 \times 10^3 i$	$1.41 \times 10^3$

functions are shown in Table 5. According to Fig. 5(b), when transfer function phase is  $-180^\circ$ , magnitude diagram equals to  $-36.6$  dB, so this numeric value is smaller than zero, transfer function would be stable. Also, by considering  $L_2 = 40$  mH, in frequency  $1.48$  rad/s, and when phase diagram equals to  $-180^\circ$ , magnitude diagram shows  $-33.5$  dB. Since this value is smaller than zero, Luo converter is stable, considering  $L_2 = 40$  mH.

By comparing the results obtained for Luo converter for  $L_2 = 20$  mH and  $L_2 = 40$  mH, it can be seen that by increasing the value of inductor  $L_2$ , Luo converter is still in stable state. Also, imaginary values of poles declined, as two real poles were created on the left side of the imaginary axis.

## 9. Comparison

Figure 6 shows comparing the diagram of the studied converter with presented structure<sup>21,22</sup> in term of magnitude and phase of bode diagram based on their poles and zeros. As can be seen, the studied converter has four poles and two zeros. Diagram was related to Ref. 21 has four poles and three zeros that are mixed. In Ref. 22. has five poles and six zeros, some of which are mixed and have one zero at the origin. Therefore, the studied converter has suitable situation in comparison with

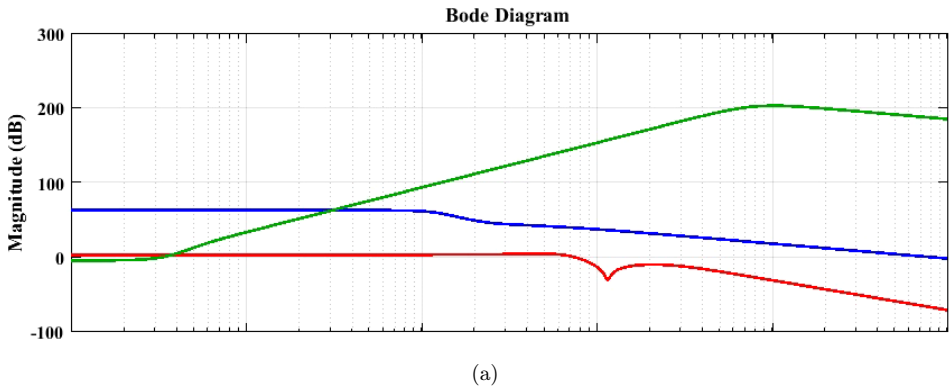


Fig. 6. Bodes diagram comparison; (a) magnitude; (b) phase.

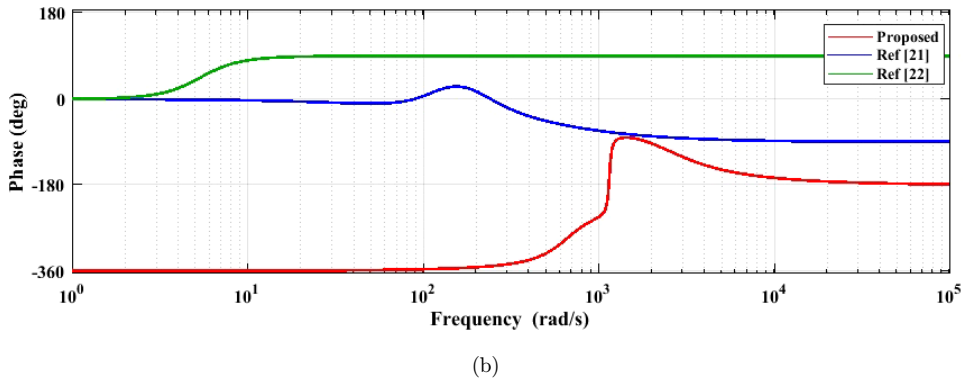


Fig. 6. (Continued)

Refs. 21 and 22. The comparison for magnitude and phase variation is presented in Figs. 6(a) and 6(b), respectively.

## 10. Conclusion

In this paper, the mathematical modeling of the Luo converter is done based on the average state-space model. By using the mentioned model, the voltage transfer function of this converter is extracted. Converter's stability and also the influence of circuit elements on stability and the frequency of damping fluctuations were analyzed by using obtained transfer function and roots location. The results show that Luo converter is stable in normal state and by applying changes in values of circuit elements, it still would be stable. Also, by increasing the values of capacitors  $C_1$  and  $C_2$ , and inductors  $L_1$  and  $L_2$ , the imaginary values of poles will be reduced; following this fall, the frequency of damping fluctuations of the converter will be declined as well. In addition, the comparison results reconfirm the studied converter suitable performance with others. With this modeling, it is possible to optimally design the converter for stable operation at sensitive loads such as medical equipment.

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