LOGICAL FOUNDATIONS OF ACCESS CONTROL IN SECURITY SYSTEMS

Introduction

Logic provides the formal backbone of every access-control engine. By mapping real-world credentials—cards, PINs, and biometrics—to propositional variables, engineers can prove that a security policy is both **sound** (never grants access when it should not) and **complete** (never blocks an authorized user). This assignment applies core ideas from discrete mathematics—propositions, truth tables, and equivalent laws—to the design decisions facing a hypothetical security team. The analysis meets three goals: translate English rules into symbolic form, evaluate their behavior exhaustively, and test whether a proposed revision preserves the original intent.

1 Propositions, Quantifiers, and the Primary Rule

Scenario variables

P: “User has a valid access card.”

Q: “User enters the correct PIN.”

R: “User is verified by facial recognition.”

1a. Symbolic form of the original rule

The system grants access *iff* the user both has the card and enters the PIN:

$$\text{Access} \equiv P \wedge Q.$$

The biconditional "**if and only if**" signifies equivalence between the condition and the outcome (Rosen, 2019).

1b. Quantified statement for all authorized users

Let *U(x)* be "*x* is an authorized user." Using the universal quantifier ∀:

$$\forall x\ [\ U(x) \rightarrow (P(x) \land Q(x))].$$

Every authorized individual must simultaneously satisfy card and PIN requirements.

2 Alternate Security Check: (P ∨ Q) ∧ R

2a. Truth table

P	**Q**	**R**	**P∨Q**	**(P∨Q)∧R**
F	F	F	F	F
F	F	T	F	F
F	T	F	T	F
F	T	T	T	**T**
T	F	F	T	F
T	F	T	T	**T**
T	T	F	T	F
T	T	T	T	**T**

2b. Number of granting cases

Access is granted in **three** of the eight possible input combinations: rows 4, 6, 8.

2c. English descriptions of granted cases

1. The user **enters the correct PIN and passes facial recognition** but lacks a valid card.
2. The user **has a valid card and passes facial recognition** without entering a PIN.
3. The user **has a valid card, enters the correct PIN, and passes facial recognition**.

Each granting scenario shares the critical commonality that *R* is true; facial verification is compulsory under this policy.

3 Evaluating a Proposed New Policy

New policy in words

a. If the user has a valid card, no PIN is required.

b. If the user lacks a card, they must supply both a PIN and facial match.

3a. Symbolic translation and simplification

$$Policy = \mathrm{PPP} \lor (\neg P \land Q \land R).$$

Apply absorption and distributive laws (Grimaldi, 2023):

$$P \lor (\neg P \land Q \land R) = (P \lor \neg P) \land (P \lor Q \land R) = T \land (P \lor Q \land R) = P \lor (Q \land R).$$

3b. Equivalence test against $(P \lor Q) \land R$

Equivalence Test Between Proposed Policy and Alternate Security Check

Compare results:

P	Q	R	Proposed Policy P∨(Q∧R)	Alternate Check(P∨Q)∧R
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	**T**	**T**
T	F	F	**T**	F
T	F	T	**T**	**T**
T	T	F	**T**	F
T	T	T	**T**	**T**

When P = T, R = F, the new policy grants access (left column **T**) while the alternate check denies it (right column **F**). Hence the two policies are **not logically equivalent**; the proposed rules would admit a cardholder even if facial recognition failed—an outcome the alternate policy forbids.

Conclusion

Logical modelling exposes subtle gaps long before a line of production code ships. Translating English requirements into symbolic propositions ensures unambiguous intent; constructing truth tables verifies how many—and exactly which—input states produce access; and applying equivalence laws reveals hidden differences between superficially similar policies. For the security system at hand, the colleague’s streamlined idea sacrifices the biometric safeguard under certain conditions. Only by formal reasoning can engineers guarantee that every authorized entry is intentional and every denial justified.

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References

Grimaldi, R. P. (2023). *Discrete and combinatorial mathematics: An applied introduction* (6th ed.). Pearson. https://eu.pearson.com/products/9781292485768

Rosen, K. H. (2019). *Discrete mathematics and its applications* (8th ed.). McGraw-Hill Education. https://www.mheducation.com/highered/product/Discrete-Mathematics-and-Its-Applications-Rosen.html