Analyzing Student Enrollment Relations Using Reflexivity, Symmetry, and Transitivity

Introduction

In discrete mathematics, understanding the properties of **reflexivity**, **symmetry**, and **transitivity** is essential for analyzing relations, especially when solving real-world problems like grouping students. In this scenario, the university uses a relation R based on **shared course enrollment**, where **xRy** means "students **x** and **y** are enrolled in at least two common courses." The goal is to determine which pairs satisfy this relation, identify groupings based on equivalence, and analyze whether the relation is a **partial order** or an **equivalence relation**.

1. Determining the Set of Related Student Pairs RRR

We begin by examining shared courses among all students. The enrollment sets are:

- a: {Math, Physics, CS}
- b: {Math, CS, English}
- c: {Physics, Chemistry}
- d: {Math, CS}
- e: {Physics, English}

We compare each pair to see if they share **at least two courses**:

- (a, a): 3 common courses → ✓
- (b, b): 3 common courses → ✓

- (c, c): 2 common courses → ✓
- (d, d): 2 common courses → ✓
- (e, e): 2 common courses → ✓
- (a, b): Math & CS → ✓
- (a, d): Math & CS → ✓
- (b, d): Math & CS → ✓
- (c, e): Physics only → ✗
- Others (e.g., a & c, b & e, etc.): either one or no common course → ✗

✅ **Relation R includes these pairs**:

R={(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(b,a),(a,d),(d,a),(b,d),(d,b)}

We included reverse pairs because the relation is undirected (e.g., if **aRb**, then **bRab**).

2. Forming Study Groups Based on Equivalence

To form study groups where each group contains **only students equivalently related**, we must determine **equivalence classes** based on R.

Let's look at the structure:

- **Group 1**: a, b, d (mutually share ≥2 courses)
- **Group 2**: c (not related to anyone else)
- **Group 3**: e (also unrelated to others)

✅ **Groups formed:**

- $\mathbf{G_1}$ = {a, b, d}
- $\mathbf{G_2}$ = {c}
- $\mathbf{G_3}$ = {e}

Total: **3 groups**

3. Partial Order vs. Equivalence Relation

A **partial order** relation must be:

- **Reflexive**: **xRx**
- **Antisymmetric**: If **xRy** and **yRx**, then **x=y**
- **Transitive**: If **xRy** and **yRz**, then **xRz**

An **equivalence relation** must be:

- **Reflexive**
- **Symmetric**: If **xRy**, then **yRx**
- **Transitive**

The key difference is **symmetry vs. antisymmetry**, which plays a central role in classifying the relation (Rosen, 2019).

4. Is R a Partial Order or Equivalence Relation?

Let's check the properties of R:

- **Reflexive**: Every student is related to themselves → ✓

- **Symmetric**: If **aRb**, then **bRa** → ✓
- **Transitive**:
  - **aRb** and **bRd** ⇒ **aRd**?
    - Yes: a and d share Math and CS → ✓
  - Similar for all in group {a, b, d} → ✓
- **Antisymmetric**:
  - **aRb** and **bRa**, but **a ≠ ba** → ✗

✅ **Conclusion**: R is **not a partial order** (fails antisymmetry)

✅ R **is an equivalence relation** (meets reflexivity, symmetry, transitivity), forming equivalence classes or partitions of the set SSS (Grimaldi, 2022).

Conclusion

By analyzing shared enrollments, we identified that relation **R** successfully groups students based on equivalent course overlap. It satisfies all properties of an equivalence relation, leading to three study groups: one shared group and two individual ones. Understanding such properties allows us to model and solve practical problems using mathematical reasoning, a fundamental tool in computing and data-driven decision-making.

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References

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Rosen, K. H. (2019). *Discrete mathematics and its applications* (8th ed.). McGraw-Hill Education. https://www.mheducation.com/highered/product/Discrete-Mathematics-and-Its-Applications-Rosen.html