Mathematical Induction and Relation Analysis in Real-World Applications

Introduction

Discrete mathematics offers essential tools for modeling and solving real-world problems involving structures, sequences, and relations. In engineering and computing, understanding how patterns grow and how entities relate to one another enables effective system design. This assignment explores two scenarios: one involving a structural pyramid of stacked blocks and another involving a team hierarchy based on experience. Both cases utilize mathematical induction and graphical relation analysis to solve and represent problems effectively.

Scenario 1: Structural Pattern Analysis Using Mathematical Induction

a. Inductive Proof of the Sum of Odd Numbers

The pyramid structure follows the pattern: {1, 3, 5, 7, 9, …}, which is a sequence of the first n odd numbers. The claim is that:

$S_n = 1 + 3 + 5 + \ldots + (2n - 1) = n^2$

Base Case (n = 1):

$S_1 = 1 = 1^2 \rightarrow$ True

Inductive Hypothesis:

Assume for some integer $k \geq 1$, the sum of the first k odd numbers is:

$S_k = 1 + 3 + \ldots + (2k - 1) = k^2$

Inductive Step:

We need to show that:

$S_{k+1} = S_k + (2k + 1) = (k + 1)^2$

Using the hypothesis:

$S_{k+1} = k^2 + (2k + 1) = k^2 + 2k + 1 = (k + 1)^2$

Hence, by mathematical induction, the formula holds for all $n \in \mathbb{N}$.

Number of Blocks in 20 Levels:

$S_{20} = 20^2 = 400$

Thus, the engineer would need **400 blocks** for a 20-level pyramid.

b. Redesigned Tower: Arithmetic Sequence Analysis

The new pattern: {2, 6, 10, 14, ...} is an arithmetic sequence with:

- First term $a = 2$

- Common difference $d = 4$

General formula for the sum of first n terms:

$S_n = n/2 \times [2a + (n - 1)d]$

Substitute values:

$S_n = n/2 \times [4 + 4n - 4] = n/2 \times 4n = 2n^2$

Verification Using Induction

Base Case (n = 1):

$S_1 = 2 = 2(1)^2 = 2 \rightarrow$ True

Inductive Hypothesis:

Assume $S_k = 2k^2$

Inductive Step:

Next term in sequence: $a_{k+1} = 2 + 4k = 4k + 2$

$S_{k+1} = S_k + (4k + 2) = 2k^2 + 4k + 2 = 2(k2+2k+1) = 2(k + 1)^2$

Thus, $S_n = 2n^2$ is proven by induction.

Scenario 2: Relation Analysis Among Developers

Given set:

S = {Alice, Bob, Charlie, David, Emma}

With years of experience:

- Alice (10), Bob (7), Charlie (5), David (3), Emma (1)

a. Is R an Equivalence Relation?

An equivalence relation must be:

- Reflexive: Every person relates to themselves. R is not reflexive since no one is more experienced than themselves..

- Symmetric: If xRy, then yRx. R is not symmetric. If Alice has more experience than Bob, Bob does not have more experience than Alice.

- Transitive: If xRy and yRz, then xRz. R is transitive.

Since **R** is **not reflexive** and **not symmetric**, it is **not** an equivalence relation (Epp, 2011).

b. Directed Graph Representation

The directed graph includes edges from more experienced to less experienced members:

- Alice → Bob, Charlie, David, Emma
- Bob → Charlie, David, Emma

- Charlie → David, Emma
- David → Emma

c. Adjacency Matrix

$$
\begin{array}{c c}
 & \begin{array}{ccccc} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D} & \boldsymbol{E} \end{array} \\
\begin{array}{c} \boldsymbol{A} \\ \boldsymbol{B} \\ \boldsymbol{C} \\ \boldsymbol{D} \\ \boldsymbol{E} \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

d. Hasse Diagram

A Hasse diagram removes redundant paths and transitive edges. The minimal relation connections are:

- Alice → Bob
- Bob → Charlie
- Charlie → David
- David → Emma

This results in a linear diagram showing a strict descending hierarchy (Rosen, 2012).

Conclusion

Through mathematical induction, we have validated two formulas describing block stacking in a pyramid. Additionally, a real-world software development hierarchy was analyzed using relational properties, graphical tools, and adjacency matrices. This assignment illustrates the powerful application of discrete mathematical methods in both engineering and computer science contexts.

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References

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