SMART HOME SECURITY SYSTEM: APPLICATION OF BOOLEAN ALGEBRA AND LOGIC GATES

Question 1: Boolean Expressions and Simplification

a. Defining Boolean Expressions

- **Motion Sensor in Living Room**:

  Let **M** represent the motion sensor status.

  Expression: **M**

  If motion is detected, **M = 1**; if not detected, **M = 0** (Mano & Kime, 2017).

- **Front Door Opened**:

  Let **D** represent the door sensor status.

  Expression: **D**

  If the door is open, **D = 1**; if closed, **D = 0** (Mano & Kime, 2017).

b. Simplification Using Boolean Laws

Both **M** and **D** are atomic expressions and already in their simplest forms. No further simplification is needed because Boolean variables representing single conditions cannot be reduced further (Mano & Kime, 2017).

Question 2: De Morgan's Theorems Application

a. Applying De Morgan's Theorems

- **Smoke Detected in the Kitchen**:

  Let **S** represent the smoke detector.

  Negation: $\neg \mathbf{S}$ (Floyd, 2013).

- **Windows are Closed**:

  Let **W** represent the window status.

  Negation: $\neg \mathbf{W}$ (Floyd, 2013).

b. Relationship Between De Morgan's Theorems and Boolean Laws

De Morgan's Theorems state that the complement of a conjunction is the disjunction of the complements and vice versa. They relate directly to the laws of complementarity and double negation. According to the complementarity law, $\mathbf{A + \neg A = 1}$ and $\mathbf{A \cdot \neg A = 0}$, while double negation ensures that $\mathbf{\neg(\neg A) = A}$. De Morgan's Theorems thus work with these principles to accurately transform and simplify logical expressions (Floyd, 2013).

Question 3: Truth Tables for Logic Gates

a. Truth Table for AND Gate (Motion Sensor AND Front Door)

M (Motion)	D (Door)	M · D (AND Output)
0	0	0
0	1	0
1	0	0
1	1	1

This table demonstrates that both the motion sensor and door must be triggered for the output to be **1** (Mano & Kime, 2017).

b. Truth Table for NOR Gate (Security Alarm Armed NOR Windows

A (Alarm)	W (Windows)	A + W (OR Output)	NOR Output (¬(A + W))
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

The NOR gate outputs **1** only when both inputs are **0**, confirming standard NOR gate behavior (Floyd, 2013).

Question 4: Equivalence of Boolean Expressions and Truth Tables

a. Equivalence Determination

For the AND condition (**M · D**), the truth table shows that only when both inputs are **1** is the output **1**, which matches the simple Boolean expression **M · D**.

For the NOR condition (¬(A + W)), the truth table verifies that only when both inputs are **0** is the output **1**, matching the Boolean expression for NOR.

b. Algebraic Manipulations and Logical Reasoning

- **AND Operation**:

  By definition, **M · D** outputs **1** only when both **M = 1** and **D = 1**. This corresponds exactly with the truth table, confirming equivalence.

- **NOR Operation**:

  Starting from the OR operation:

  A + W outputs **1** when either **A** or **W** is **1**.

  Taking the complement (**¬(A + W)**) yields **1** only when both are **0**, which matches the NOR truth table outputs (Floyd, 2013).

Thus, the simplified Boolean expressions from Question 1 are fully aligned with the truth tables from Question 3.

Question 5: Integration of System Logic

a. Overall Smart Home Security Logic

The smart home system's logic integrates the following:

- Detection of both motion and door opening:

  $M \cdot D$

- Security alert when neither alarm is armed, nor windows are closed:

  $\neg(A + W)$

The system could logically combine these by using an OR gate to trigger security responses based on either the motion-door combination or alarm-window combination:

Security Trigger = $(M \cdot D) + (\neg(A + W))$

This means the system will activate security measures if either someone is moving and the door is open, or if the alarm is not armed while the windows are not closed. This integrated logic ensures comprehensive coverage of potential security breaches in a smart home environment.

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References:

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Mano, M. M., & Kime, C. R. (2017). *Logic and Computer Design Fundamentals* (5th ed.). Pearson. https://www.pearson.com/en-us/subject-catalog/p/logic--computer-design-fundamentals/P200000003256/9780134080154