## Introduction

Competitions force discrete-mathematics ideas out of the textbook and into logistics: committees must be picked, calendars must align, and students must be split into balanced teams. Counting techniques, the Division Algorithm, and the algebra of permutations provide organizers with fast, error-free answers that scale far beyond ten participants.

## Forming the Planning Committee: Combinations versus Permutations

First, the school needs a three-member committee. If **order does not matter**, we apply the basic combination rule

$$\mathbf{(10 / 3) = 10! / 3!\ 7! = 120.}$$

The argument uses *unordered* selections: we divide 10! by the factorial of the chosen group (3!) and the ignored remainder (7!) to avoid over-counting identical line-ups of the same three people (Rosen, 2019).

When order matters—for example, if the first person becomes chair, the second secretary, and the third treasurer use a permutation of length 3:

$$\mathbf{P\ (10, 3) = 10 \cdot 9 \cdot 8 = 720.}$$

Here every factorial term in the denominator except $(10 - 3)!$ disappears, preserving each distinct ordering (Mazur, 2020). The factor-of-six gap $(720 \div 120)$ equals 3!, the number of ways to order the same three nominees.

## Scheduling the Knock-Out Rounds: A Modular-Arithmetic Approach

Each round is 30 days after the previous one. Because the *Division Algorithm* guarantees a unique quotient and remainder, we compute $30 \bmod 7$:

$$\mathbf{30 = 7 \times 4 + 2 \Longrightarrow 30 \bmod 7 = 2.}$$

A remainder of 2 means every 30-day jump lands two weekdays later. If the **final** is on **Monday**, the **semifinal** sits 30 days earlier—**Saturday**. One more 30-day step places the **quarterfinal** on **Thursday**. Modular arithmetic collapses a potentially messy calendar into a simple traversal along the cyclic group Z7 of weekday classes (Rosen, 2025).

## Assigning Teams: Multinomial Coefficients and Order Independence

Finally, the ten competitors must be split into labelled teams:

- Team A: 2 students
- Team B: 3 students
- Team C: 5 students

Because every student appears exactly once, we apply the **multinomial coefficient**

$$\mathbf{10! \,/\, 2!3!5! = 2520}$$

to count distinct assignments. A step-by-step choice achieves the same total:

1. **A first:** (10 / 2) = 45.
2. **B next:** from the remaining eight, (8 / 3) = 56.
3. **C last:** the residual five form Team C automatically ((5 / 5) = 1).

The product 45 × 56 × 1 again equals **2 520**. Reversing the order—choosing Team C first, then A, then B—yields

$$\mathbf{(10 / 5)\ (5 / 2)\ (3 / 3) = 252 \times 10 \times 1 = 2520,}$$

Underscoring that multiplication in N is commutative and that the labelled result depends only on subgroup sizes, not on when they were selected. Algebraically, the symmetric group S10 acts on the participant set; divisors 2!, 3!, and 5! “quotient out” reordering *within* each team, leaving a stable count (Grimaldi, 2023).

## Conclusion

Combinatorial reasoning, modular arithmetic, and group properties jointly transform event management into a sequence of provably correct calculations. Combinations distinguish ordered from unordered committees; the Division Algorithm reduces a 30-day interval to a two-day weekday shift; and multinomial coefficients, immune to selection order, enumerate all possible labelled team splits. Together these results illustrate how discrete mathematics provides crisp solutions to real-world scheduling and assignment problems.

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## References

Mazur, D. R. (2020). Combinatorics: A guided tour. American Mathematical Society. https://www.amazon.com.br/Combinatorics-Guided-David-R-Mazur/dp/1470453002

Rosen, K. (2025). Discrete Mathematics and Its Applications (8th ed.). McGraw Hill. https://www.mheducation.com/highered/product/Discrete-Mathematics-and-Its-Applications-Rosen.html

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