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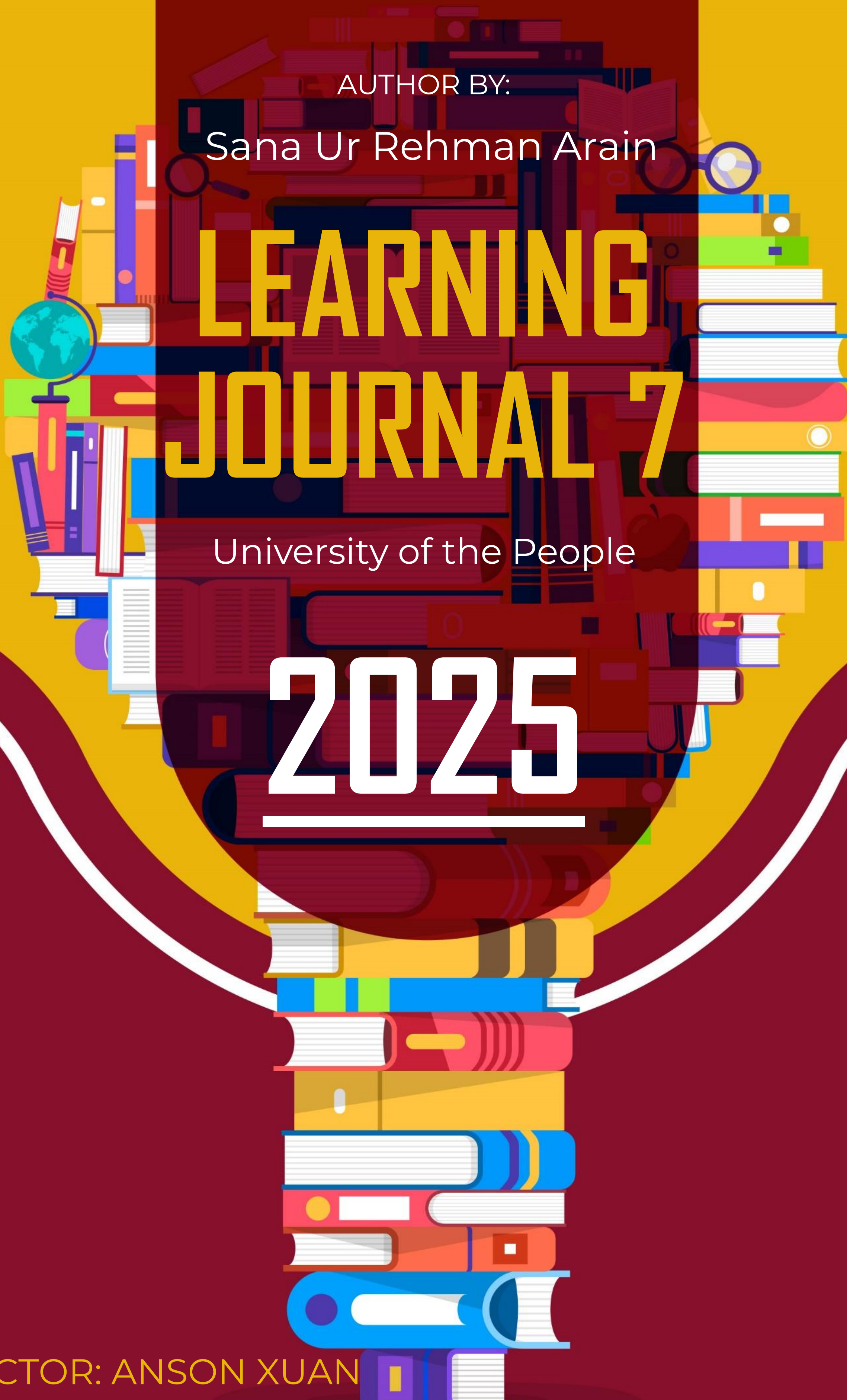
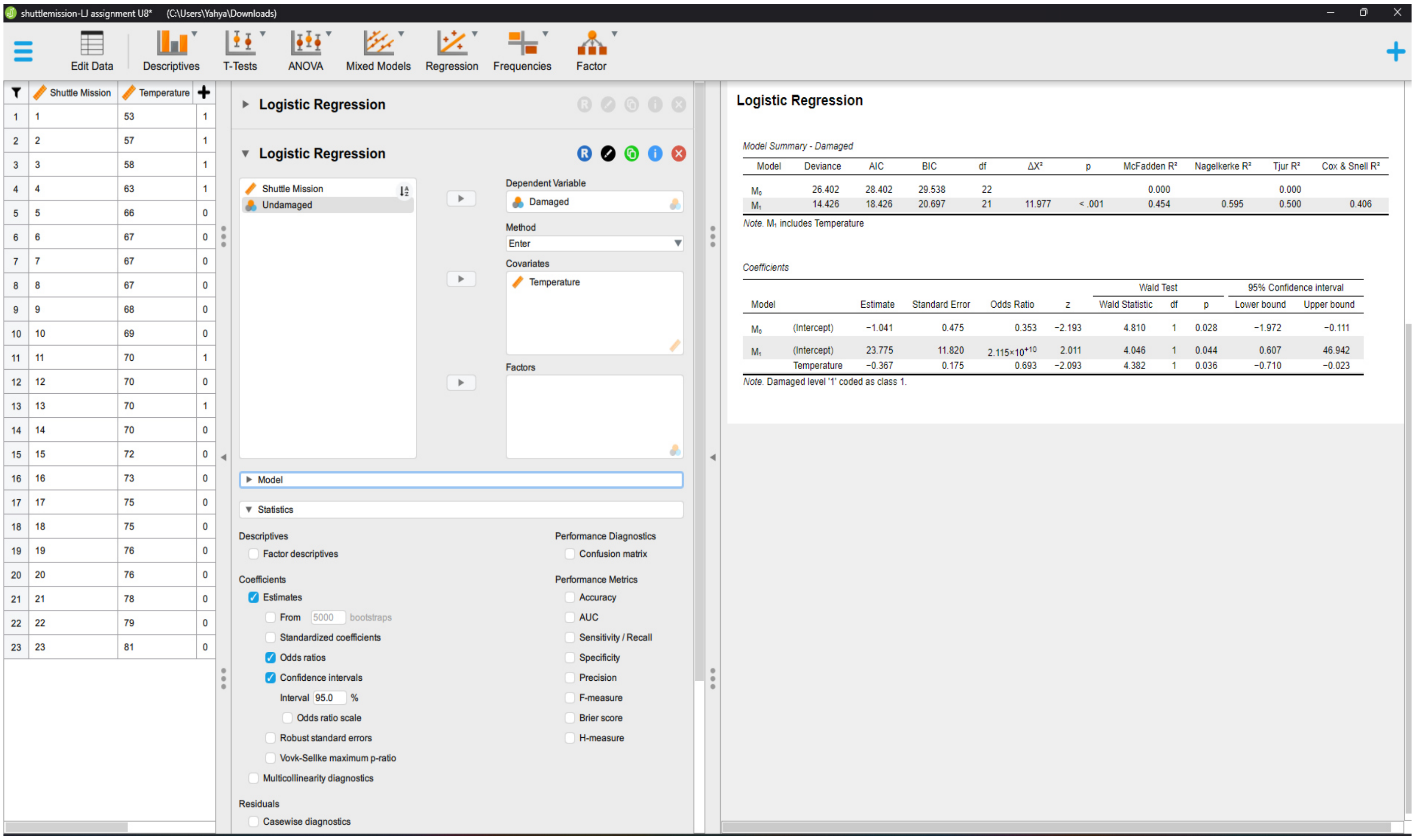


INSTRUCTOR: ANSON XUAN

MATH 1281-01 Statistical Inference – AY2025-T3

a: Run Logistic Regression in JASP



Logistic Regression ▼

Model Summary - Damaged

Model	Deviance	AIC	BIC	df	ΔX^2	p	McFadden R^2	Nagelkerke R^2	Tjur R^2	Cox & Snell R^2
M_0	26.402	28.402	29.538	22			0.000		0.000	
M_1	14.426	18.426	20.697	21	11.977	< .001	0.454	0.595	0.500	0.406

Note. M_1 includes Temperature

Coefficients

						Wald Test			95% Confidence interval	
Model		Estimate	Standard Error	Odds Ratio	z	Wald Statistic	df	p	Lower bound	Upper bound
M_0	(Intercept)	−1.041	0.475	0.353	−2.193	4.810	1	0.028	−1.972	−0.111
M_1	(Intercept)	23.775	11.820	$2.115\times10^{+10}$	2.011	4.046	1	0.044	0.607	46.942
	Temperature	−0.367	0.175	0.693	−2.093	4.382	1	0.036	−0.710	−0.023

Note. Damaged level '1' coded as class 1.

b. Logistic Regression Equation

From the output:

- Intercept (β_0) = 23.775
- Temperature coefficient (β_1) = –0.367

Equation:

p^ = 1 / (1 + e^ (-(23.775 - 0.367·Temperature)))

c. Statistical Significance of β_1

- **p-value for Temperature = 0.036**

Since $0.036 < 0.05$, the coefficient is **statistically significant at the 5% level**.

Interpretation: Temperature has a significant effect on the probability of O-ring damage.

d. Is Temperature a Justified Cause?

Yes. The model with Temperature (M_1) significantly improves fit over the model without it ($p < 0.001$ for ΔX^2).

Also, Temperature is statistically significant ($p = 0.036$), and model fit metrics (e.g., Nagelkerke $R^2 = 0.595$) show decent explanatory power.

Interpretation: There is sufficient statistical evidence to justify that temperature contributed to O-ring damage.

e. Estimated Probabilities

Using:

$$\widehat{}\{p\} = \backslash frac\{1\}\{1 + e^{\{-(23.775 - 0.367 \cdot \backslash text\{Temperature\})\}}\}$$

Let's calculate:

1. **T = 51**

$$\widehat{}\{p\} = \backslash frac\{1\}\{1 + e^{\{-(23.775 - 0.367 \cdot 51)\}}\} = \backslash frac\{1\}\{1 + e^{\{-4.038\}}\}$$

$$= \backslash frac\{1\}\{1 + 0.0176\} = 0.9827$$

2. T = 53°F:

$$\widehat{}\{p\} = \backslash frac\{1\}\{1 + e^{\{-(23.775 - 0.367 \cdot 53)\}}\} = \backslash frac\{1\}\{1 + e^{\{-3.304\}}\}$$

$$= \backslash frac\{1\}\{1 + 0.0368\} = 0.9644$$

3. T = 55°F:

$$\widehat{}\{p\} = \backslash frac\{1\}\{1 + e^{\{-(23.775 - 0.367 \cdot 55)\}}\} = \backslash frac\{1\}\{1 + e^{\{-2.570\}}\}$$

$$= \backslash frac\{1\}\{1 + 0.0765\} = 0.9289$$

4. T = 57°F:

$$\widehat{}\{p\} = \backslash frac\{1\}\{1 + e^{\{-(23.775 - 0.367 \cdot 57)\}}\} = \backslash frac\{1\}\{1 + e^{\{-1.836\}}\}$$

$$= \backslash frac\{1\}\{1 + 0.1594\} = 0.8626$$

Temperature (°F)	$\hat{p}$ (Estimated Probability)
51	0.9827
53	0.9644
55	0.9289
57	0.8626

These calculations show that as temperature decreases, the probability of O-ring damage increases, which aligns with engineering expectations for material behavior at lower temperatures.