Understanding Functions and Sequences in Real-Life Scenarios

Functions and sequences are foundational concepts in discrete mathematics, offering essential tools for solving real-life problems in business, finance, and computing. By analyzing various scenarios, one can grasp how functions work through composition, domain and range analysis, and understand different types of functions such as injective, surjective, and bijective (Rosen, 2012). Furthermore, arithmetic and geometric sequences allow predictions of future values and cumulative outcomes, especially in economic or operational forecasting.

Scenario 1: Composite Functions and Production Costs

We are given two functions:

- Cost function: $C(x) = 0.5x^2 + 10x + 50$

- Widget production function: $x(t) = 5t$

a. Composite Function:

The composite function is:

$C(x(t)) = C(5t) = 0.5(5t)^2 + 10(5t) + 50 = 0.5(25t^2) + 50t + 50 = 12.5t^2 + 50t + 50$

This function $C(x(t))$ gives the total cost of production as a function of time, t, in hours.

b1. Domain of C(x(t)):

Given $t \geq 0$ and maximum widget production is 500,

$x(t) = 5t \leq 500 \Rightarrow t \leq 100$

So, domain: $0 \leq t \leq 100$

b2. Range of C(x(t)):

Evaluate C(x(t)) at endpoints:

- At $t = 0$: $C = 50$

- At $t = 100$: $C = 12.5(100)^2 + 50(100) + 50 = 125000 + 5000 + 50 = 130050$

Range: [50, 130050]

b3. Total Cost for 8 Hours:

$C(5 \cdot 8) = C(40) = 0.5(40)^2 + 10(40) + 50 = 800 + 400 + 50 = 1250$

Scenario 2: Properties of Functions and Inverse

Given:

$h(n) = 500000 + 25n,\ 0 < n < 20000$

a. Nature of Function:

- **Injective:** Yes, since for each unique n, h(n) is unique.

- **Surjective:** No, because the codomain is all natural numbers, but only specific values in steps of 25 are hit.

- **Bijective:** No, not surjective on $\mathbb{N}$, so not bijective.

Inverse Function:

Let $y = h(n) = 500000 + 25n$

Solving for n:

$n = (y - 500000)/25$

Inverse function: $h^{-1}(y) = (y - 500000)/25$, defined for $y \in [500025, 999975]$

b. Guest with Booking Code 500925:

$n = (500925 - 500000)/25 = 925/25 = 37$

Check-in order is 37.

Scenario 3: Salary Sequences

a. Arithmetic Salary Formula:

Starting salary = $30,000

Annual raise = $2,000

Formula:

$a_n = 30000 + (n - 1) \cdot 2000$

5th year salary:

$a_5 = 30000 + 4 \cdot 2000 = 38000$

b. Total Salary Over 10 Years (Arithmetic):

Total = Sum of 10 terms in arithmetic sequence

$S_{10} = 10/2\ (a_1 + a_{10}) = 5\ (30000 + 48000) = 5 \cdot 78000 = 390000$

Geometric Raise (5% increase annually):

Geometric formula:

$a_n = 30000 \cdot (1.05)^{(n-1)}$

10th year salary:

$a_{10} = 30000 \cdot (1.05)^9 \approx 30000 \cdot 1.5513 = 46539$

Conclusion

Understanding the structure and relationships within functions provides clarity for decision-making in production and administration. The ability to analyze injectivity or compose functions directly applies to business operations. Similarly, knowledge of arithmetic and geometric sequences supports financial forecasting and planning (Epp, 2011). These mathematical tools bridge abstract reasoning and practical application, enhancing both theoretical knowledge and real-world problem-solving.

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References

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