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LEARNING JOURNAL 4

University of the People

2025

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MATH 1281-01 Statistical Inference – AY2025-T3

PART 1: Reading and Writing Scores Analysis

(a) Hypotheses:

H_0: μd = 0 (There is no difference between reading and writing scores)

H_1: μd ≠ 0 (There is a difference between reading and writing scores)

(b) Checking conditions:

1. Random sample: ✓ Stated that random sample of 250 students was taken
2. Independent observations: ✓ Each student's scores are independent
3. Normal/Large sample: ✓ Histogram appears roughly symmetric, n=250 > 30
4. Related pairs: ✓ Reading and writing scores are from same students

(c) T-test Analysis: Given:

- Mean difference (x̄d) = -0.545
- Standard deviation of differences (sd) = 8.887
- n = 250

t-statistic calculation:

$t = (\bar{x}d - \mu_0)/(sd/\sqrt{n})$

$t = (-0.545 - 0)/(8.887/\sqrt{250})$

$t = -0.545/(8.887/15.811)$

$t = -0.545/0.562$ $t = -0.969$

Degrees of freedom = n - 1 = 250 - 1 = 249

With p-value = 0.39 (given) > $\alpha = 0.05$, we fail to reject H_0.

Conclusion: There is not sufficient evidence to conclude a difference between average reading and writing scores.

(d) Potential errors:

Type II error is most likely here - failing to detect a real difference between reading and writing scores if one exists. This could occur due to:

- High variability in scores (SD = 8.887)
- Sample size might not be large enough to detect small differences
- Natural variation in student performance between subjects

(e) Confidence Interval:

Since p > 0.05 and we failed to reject H_0, 0 would be included in the confidence interval.

95% CI = $\bar{x}d \pm (t_{0.025,249})(sd/\sqrt{n})$

$= -0.545 \pm (1.969)(8.887/\sqrt{250})$

$= -0.545 \pm (1.969)(0.562)$

$= -0.545 \pm 1.106$

$= (-1.651, 0.561)$

PART 2: Fuel Efficiency Analysis

(1) Hypotheses:

H_0: μmanual - μautomatic = 0 (No difference in average MPG)

H_1: μmanual - μautomatic ≠ 0 (There is a difference in average MPG)

(2) T-statistic calculation:

From data:

Manual: $\bar{x}_1 = 19.85$, $s_1 = 4.51$, $n_1 = 26$

Automatic: $\bar{x}_2 = 16.12$, $s_2 = 3.58$, $n_2 = 26$

Pooled standard deviation:

$sp = \sqrt{[((n_1-1)s_1^2 + (n_2-1)s_2^2)/(n_1+n_2-2)]}$

$sp = \sqrt{[((25)(4.51^2 + (25)(3.58^2))/(50)]}$

$sp = \sqrt{[(25(20.34) + 25(12.82))/50]}$

$sp = \sqrt{(828.95/50)}$

$sp = \sqrt{16.579}$

$sp = 4.072$

$t = (\bar{x}_1 - \bar{x}_2)/(sp\sqrt{(1/n_1 + 1/n_2)})$

$t = (19.85 - 16.12)/(4.072\sqrt{(2/26)})$

$t = 3.73/(4.072(0.277))$

$t = 3.73/1.128$

$t = 3.308$

(3) Degrees of freedom:

$df = n_1 + n_2 - 2 = 26 + 26 - 2 = 50$

(4) Conclusion:

Given p-value = 0.0029 < α = 0.05, we reject H_0. There is strong evidence to conclude that there is a significant difference in the average fuel efficiency between cars with manual and automatic transmissions. The data suggests that manual transmission vehicles have higher average MPG (mean = 19.85) compared to automatic transmission vehicles (mean = 16.12).

The analysis shows statistical significance and practical significance with a difference of approximately 3.73 MPG in favor of manual transmissions.