LiNGAM (Hidden common variables)

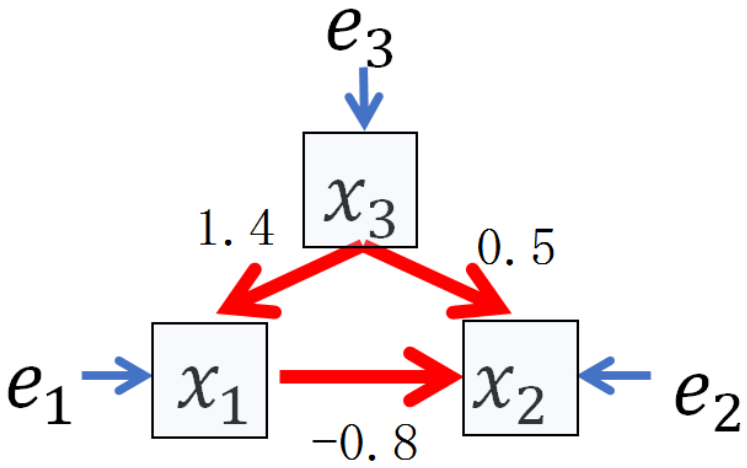
Idea

$$E = \arg\min_{\mu(\nu)} \left[max\left(\max(e_i), \max\left(\iint p(e_i, e_j) log\left(\frac{p(e_i, e_j)}{p(e_i)p(e_j)} \right) de_i de_j \right) \right) + \theta\, penaltyF \right] \quad \theta > 0$$

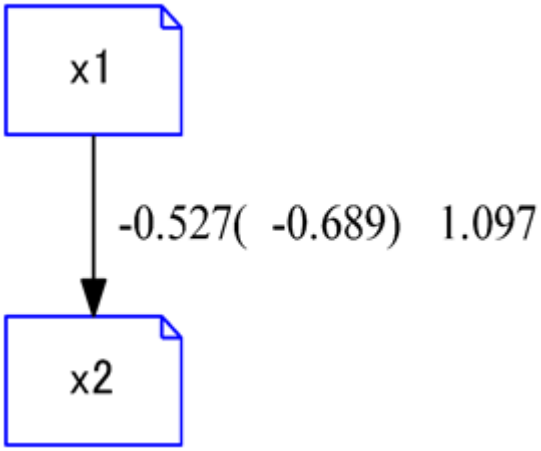
$$\mu \sim t - \text{distribution} \equiv \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

Hidden common variables Test case1

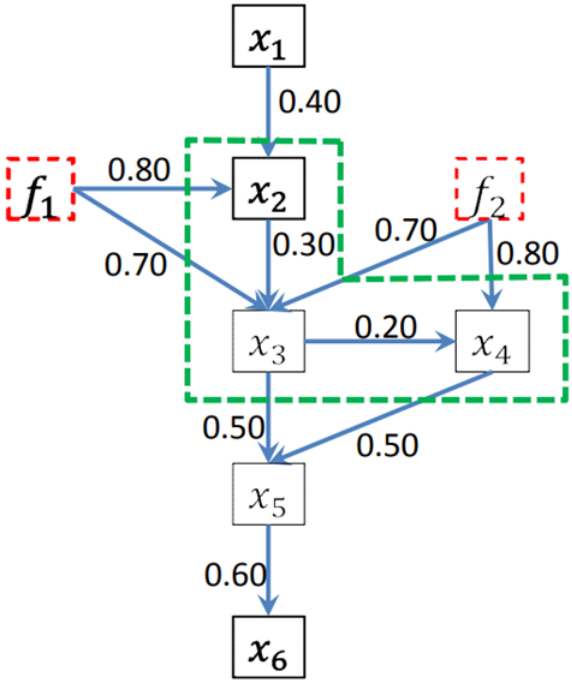
$$x_1 = 1.4x_3 + e_1$$
$$x_2 = -0.8x_1 + 0.5x_3 + e_2$$
$$x_3 = e_3$$



Hidden common variable is x3



Hidden common variables Test case2



$$x_1 = e_1$$

$$x_2 = 0.4x_1 + 0.8f_1 + e_2$$

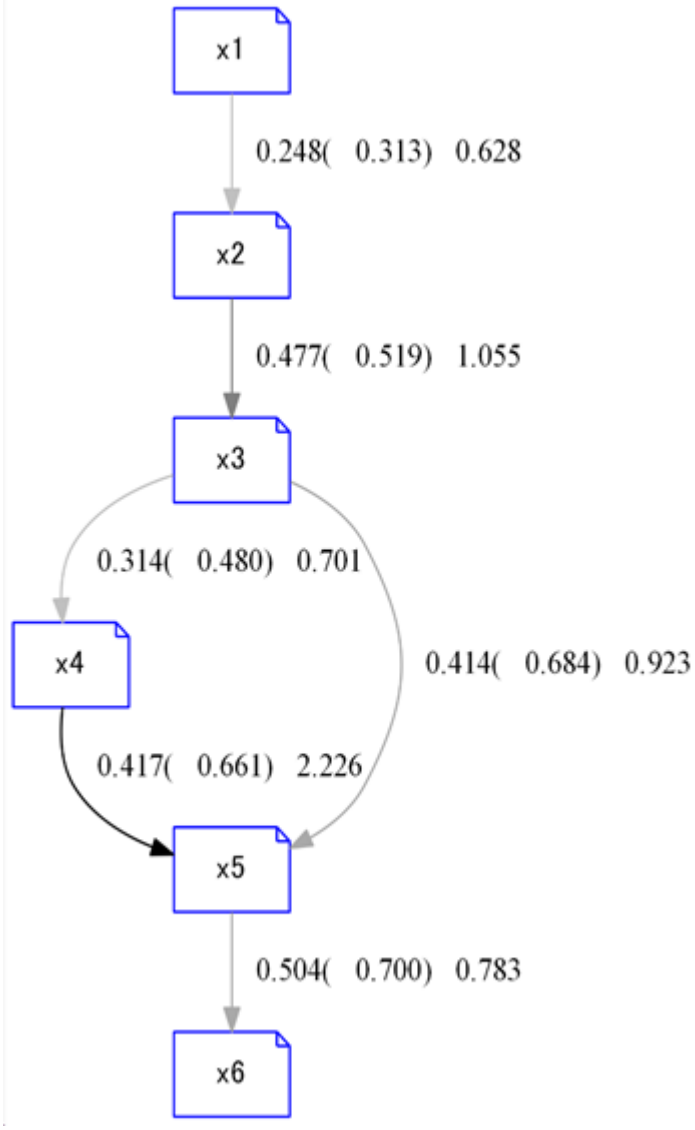
$$x_3 = 0.3x_2 + 0.7f_1 + 0.7f_2 + e_3$$

$$x_4 = 0.2x_3 + 0.8f_2 + e_4$$

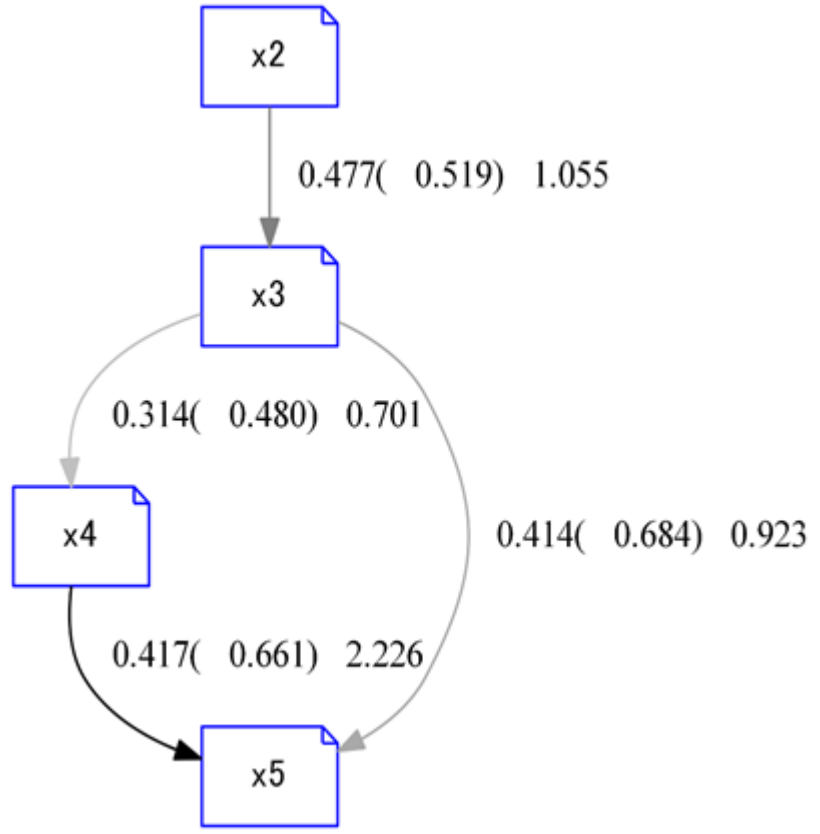
$$x_5 = 0.5x_3 + 0.5x_4 + e_5$$

$$x_6 = 0.5x_5 + e_6$$

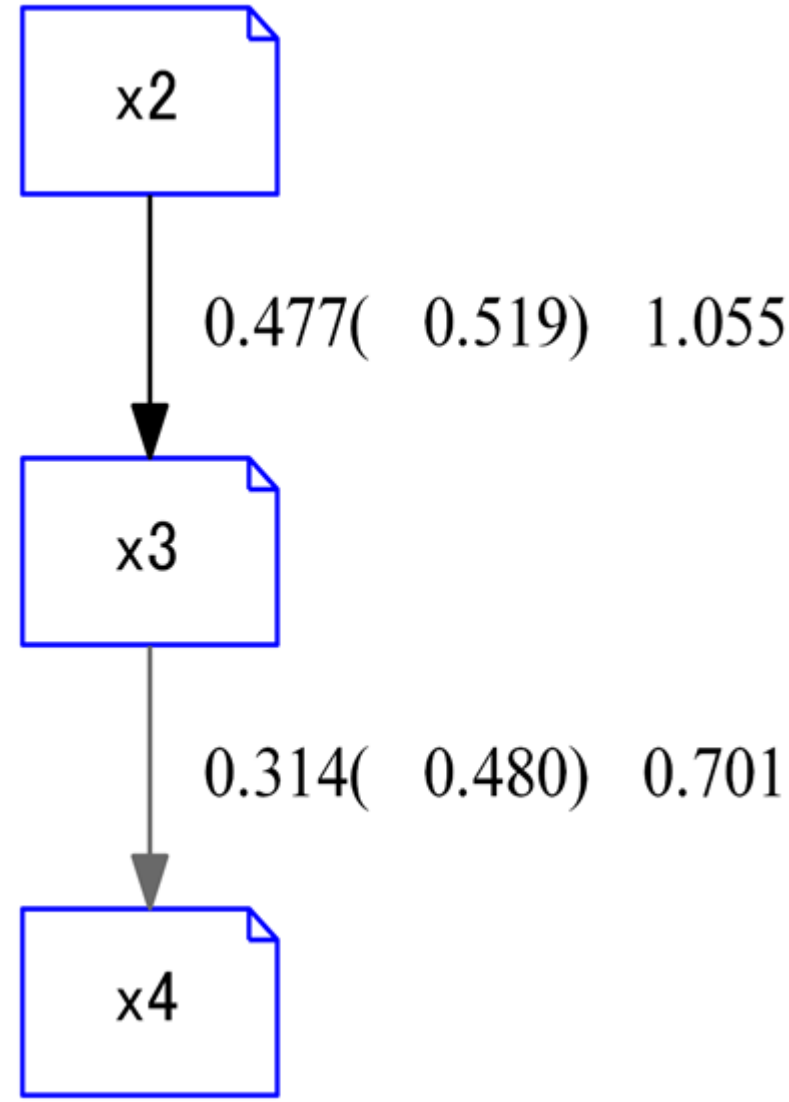
Hidden common variable is f1, f2



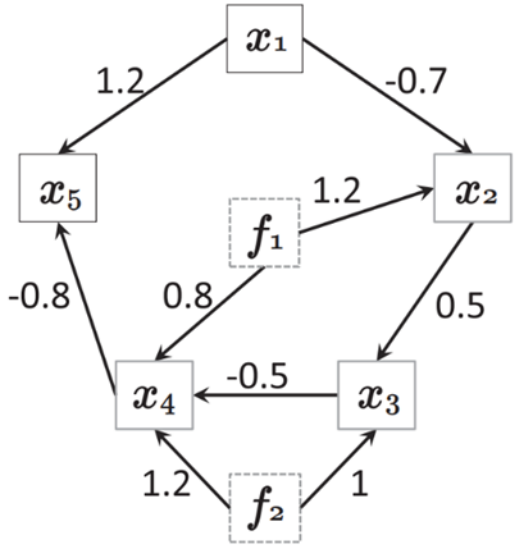
Hidden common variable is x1,x6,f1, f2



Hidden common variable is x1,x5,x6,f1, f2



Hidden common variables Test case3



$$x_1 = e_1$$

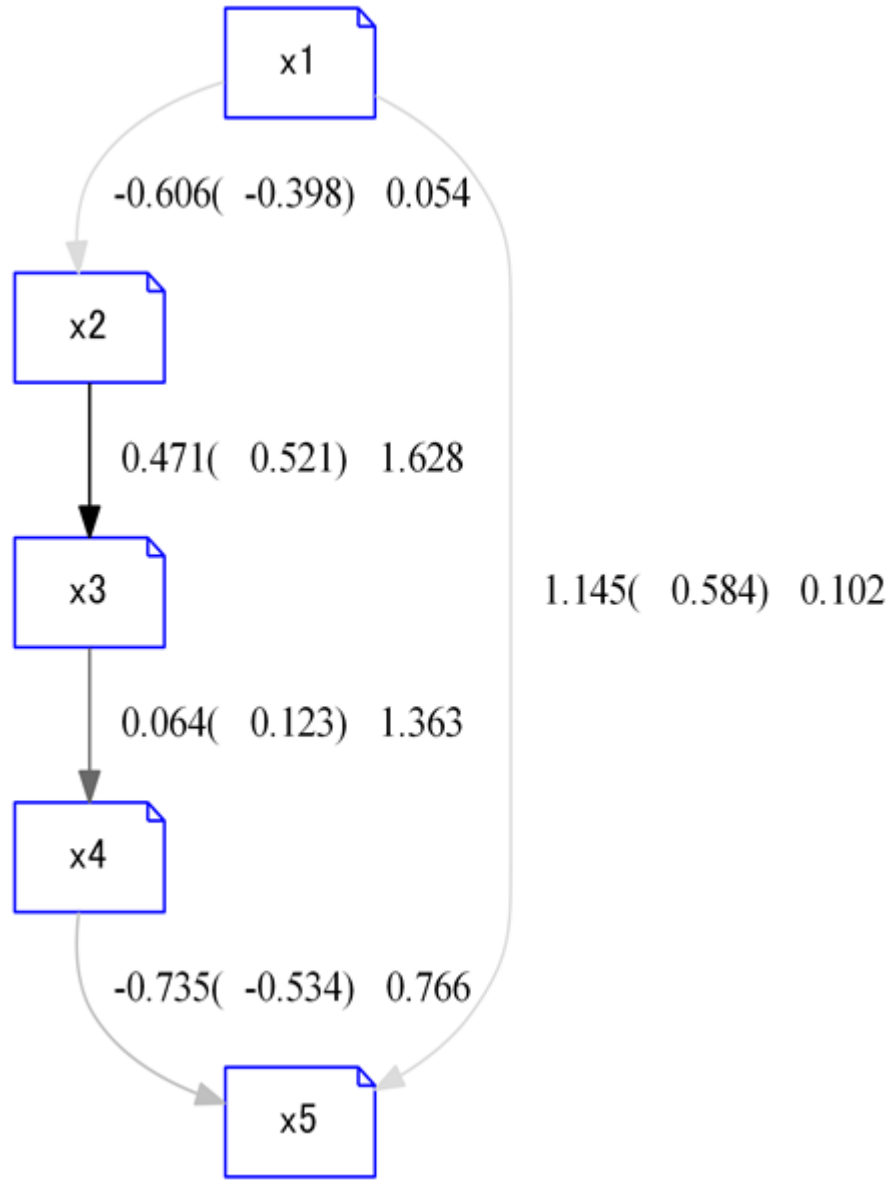
$$x_2 = -0.7x_1 + 1.2f_1 + e_2$$

$$x_3 = 0.5x_1 + f_2 + e_3$$

$$x_4 = -0.5x_3 + 0.8f_1 + 1.2f_2 + e_4$$

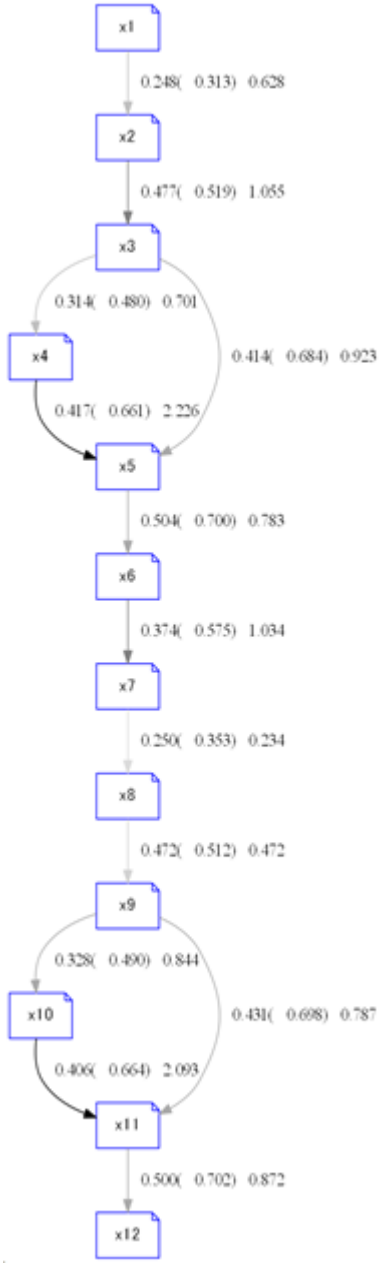
$$x_5 = 1.2x_1 - 0.8x_4 + e_5$$

Hidden common variable is f1, f2

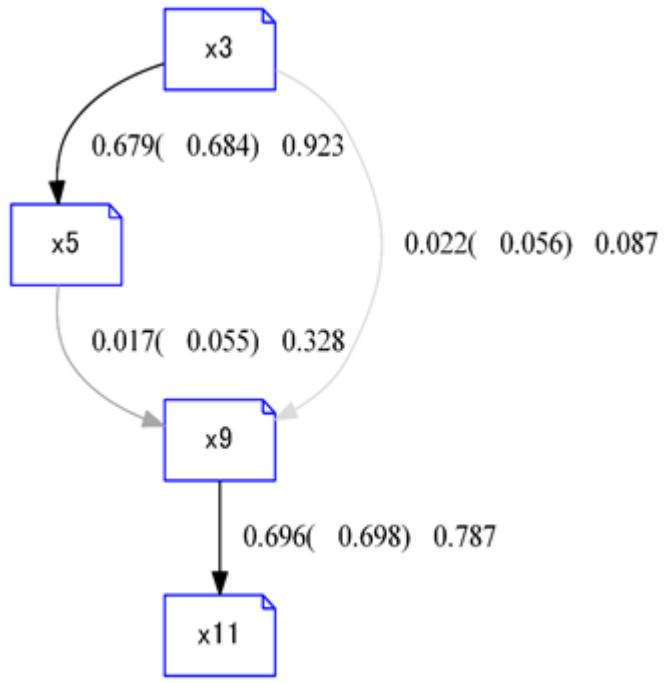


Hidden common variables Test case4 (Combine two "Test case2")

Hidden common variable is f1, f2,f3,f4



Hidden common variable is x1,x2,x6,x7,x8,x12,f1, f2,f3,f4


x3
0.679(0.684) 0.923
x5
0.022(0.056) 0.087
0.017(0.055) 0.328
x9
0.696(0.698) 0.787
x11