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An Inventory Model for Time Varying Deterioration and Price Dependent Quadratic Demand with Salvage Value

R. Venkateswarlu[1] and R. Mohan[2]

GITAM School of International Business, GITAM University,
Visakhapatnam – 530 045, India.
E-mail: rangavajhala_v@yahoo.co.in
[2]F-Civil, Dept of Mathematics, College of Military Engineering
Pune – 400 011, India.
E-mail: mohan_rayappan@yahoo.com

Abstract

A deterministic inventory model is developed for deteriorating items when the demand rate is assumed to be a function of price which is quadratic in nature and the deterioration rate is proportional to time. The model is solved when shortages occur in inventory. Later, the case of salvage is discussed. The sensitivity of the model is discussed with a numerical example.

Keywords: Quadratic demand, deterioration rate, price dependent models, shortages.

1. Introduction

It is well known that the demand rate for physical goods may be depending on time, stock and price. It is also true that the selling price is an important component in inventory system. An economic lot size model for price dependent demand under quantity and freight discounts was developed by Burwell (1997). An inventory system of ameliorating items for price dependent demand rate was considered by Mondal et al (2003). You (2005) developed an inventory model with price and time dependent demand. Ajanta Roy (2008) has developed an inventory model for deteriorating items with price dependent demand and time varying holding cost. Inventory modelers so far have considered two types of price dependent demand scenarios, linear and exponential. The linear price dependent demand implies a uniform change in the demand rate of the product per unit price where as exponential price dependent demand implies a very high change in demand rate of the product per unit price. These two scenarios are quite unusual in realistic situations. Thus *quadratic price dependent demand* may be an alternative approach to the existing two scenarios. The following functional form denotes the price dependent quadratic demand:

$$D(p) = ap^2 + bp + c;\ a \neq 0, b \neq 0, c \geq 0 . \quad (1)$$

The functional form given above explains the accelerated growth/decline in the demand patterns which may arise due to seasonal demand rate (Khanra and Chaudhuri, 2003). We may explain different types of realistic demand patterns depending on the signs of a and b. Bhandari and sharma (2000) have studied a Single Period Inventory Problem with Quadratic Demand Distribution under the Influence of Marketing Policies. Khanra and Chaudhuri (2003) have discussed an order-level inventory problem with the demand rate represented by a continuous quadratic function of time. Sana and Chaudhuri (2004) have developed a stock-review inventory model for perishable items with uniform replenishment rate and stock-dependent demand. Recently, Ghosh and Chaudhuri (2004) have developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. They have used a two-parameter Weibull distribution to represent the time to deterioration. Kalam et al (2010) have studied the problem of production lot-size inventory model for Weibull deteriorating item with quadratic demand, quadratic production and shortages. An order level EOQ model for deteriorating items in a single warehouse system with price depended demand in non-linear (quadratic) form has been studied by Patra et al (2010). Recently Venkateswarlu and Mohan (2011) have developed inventory management models for deteriorating items with constant deterioration and time dependent quadratic demand rate. Most of the authors have assumed that the deterioration of units is the complete loss to the inventory system.

Since the literature on price dependent quadratic demand is seldom, we would like to develop inventory models with time varying deterioration rate in a cycle time when demand rate follows price dependent quadratic demand. The sensitivity analysis is carried out with suitable illustrations at the end.

2. Assumptions and notations

The mathematical model is developed on the following assumptions and notations:

(i) The selling rate $D(p)$ at time t is assumed to be $D(p) = a\,p^2 + bp + c$, $a \neq 0, b \neq 0, c \geq 0$. Here, a is the rate with which the change in the rate demand rate itself increases b is the rate with which the demand rate increases and c is the initial rate of demand.

(ii) Replenishment rate is infinite and lead time is zero.

(iii) p is the selling price per unit,
(iv) Inventory holding cost per unit per unit time is h (>0)
(v) A is the cost of placing an order
(vi) Shortages are allowed and are fully backlogged
(vii) $\theta(t) = \theta t$ is the deterioration rate, $0 < \theta < 1$.
(viii) C is the unit cost of an item
(ix) $I(t)$ is the inventory level at time t
(x) T is the total time of the cycle
(xi) During time t_1, inventory is depleted due to deterioration and demand of the item. At time t_1 the inventory becomes zero and shortages start occurring.
(xii) C_1 is the shortage cost per unit per unit time.
(xiii) The order quantity in one cycle is q.
(xiv) The salvage value γ*C, $0 \le \gamma < 1$ is associated with deteriorated units during a cycle time.

3. Formulation and solution of the model

The objective of the model is to determine the optimum profit for items having price dependent quadratic demand and time varying deterioration rate without shortages.

The inventory level I(t) be the inventory level at time t ($0 \le t \le T$) and depletes as the time passes due to selling rate and deterioration. The differential equation which describes the inventory level at time t can be written as

$$\begin{aligned} &\frac{dI(t)}{dt} + \theta\, t\, I(t) = -(ap^2 + bp + c) \qquad 0 \le t \le t_1 \\ &\frac{dI(t)}{dt} = -(ap^2 + bp + c) \qquad t_1 \le t \le T \end{aligned} \tag{2}$$

with $I(t) = 0$ at $t = t_1$

The solution of the above differential equation (2) using the boundary conditions is given by

$$= (ap^2+bp+c)\left[(t_1-t)+\theta(\frac{t_1^3}{6}+\frac{t^3}{3}-\frac{t_1t^2}{2})+\theta^2(\frac{t_1^5}{40}-\frac{t^5}{15}-\frac{t_1^3t^2}{12}++\frac{t_1t^4}{8}\right], \quad 0<t<t_1$$
$$= (ap^2+bp+c)\left(t-t_1\right), \qquad t_1<t<T$$

$$I(0)=(ap^2+bp+c)\left[t_1+\frac{\theta t_1^3}{6}+\frac{\theta^2 t_1^5}{40}\right] \tag{3}$$

Now stock loss due to deterioration

$$D=(ap^2+bp+c)\left[\theta\frac{t_1^3}{6}+\theta^2\frac{t_1^5}{40}\right] \tag{4}$$

$$q=D+\int_0^T (ap^2+bp+c)dt$$
$$=(ap^2+bp+c)\left[\frac{\theta t_1^3}{6}+\frac{\theta^2 t_1^5}{40}\right]+(ap^2+bp+c)T \tag{5}$$
$$q=(ap^2+bp+c)\left[\frac{\theta t_1^3}{6}+\frac{\theta^2 t_1^5}{40}+T\right]$$

Holding Cost $HC=h(ap^2+bp+c)\left[\frac{t_1^2}{2}-\theta\frac{t_1^4}{12}+\theta^2\frac{t_1^6}{90}\right]$ (6)

Now shortages during the cycle, let S $=(ap^2+bp+c)\frac{1}{2}\left[T-t_1\right]^2$ (7)

The total profit per unit time is given by

$$P(T,t_1\ p)=p(ap^2+bp+c)-\frac{1}{T}(A+Cq+HC+C_1S) \tag{8}$$

$$=p(ap^2+bp+c)-\frac{1}{T}\left[\begin{array}{l} A+C(ap^2+bp+c)\left[T+\theta\frac{t_1^3}{6}+\theta^2\frac{t_1^5}{40}\right] \\ +h(ap^2+bp+c)\left[\frac{t_1^2}{2}-\theta\frac{t_1^4}{12}+\theta^2\frac{t_1^6}{90}\right]+\frac{C_1}{2}(ap^2+bp+c)\left[T-t_1\right]^2 \end{array}\right]$$

Let $t_1=\beta T$, $0<\beta<1$, then the profit function becomes

$$P(T,p)=p(ap^2+bp+c)-\frac{A}{T}-(ap^2+bp+c)\left[\begin{array}{l} C\left[1+\theta\frac{\beta^3T^2}{6}+\theta^2\frac{\beta^5T^4}{40}\right] \\ +h\left[\frac{\beta^2T}{2}-\theta\frac{\beta^4T^3}{12}+\theta^2\frac{\beta^6T^5}{90}\right]+\frac{C_1}{2}*T*(1-\beta)^2 \end{array}\right] \quad (9)$$

To maximize the objective function P(T,p), the necessary condition for maximizing the profit are

$$\frac{\partial P(T,p)}{\partial T}=0 \text{ and } \frac{\partial P(T,p)}{\partial p}=0 \quad \text{also to satisfy the condition}$$
$$\frac{\partial^2 P(T,p)}{\partial T^2}*\frac{\partial^2 P(T,p)}{\partial p^2}-\frac{\partial^2 P(T,p)}{\partial T\partial p}>0 \text{ provided } \frac{\partial^2 P(T,p)}{\partial T^2}<0, \frac{\partial^2 P(T,p)}{\partial p^2}<0 \quad (10)$$

Depending on the signs of '*a*' and '*b*', one may have the following different types of relative demand patterns:

(i) $a > 0$ and $b > 0$ gives accelerated growth in demand
(ii) $a > 0$ and $b < 0$ gives retarded growth in demand
(iii) $a > 0$ and $b < 0$, gives retarded decline in demand
(iv) $a < 0$ and $b < 0$ gives accelerated decline in demand.

The above optimality conditions are satisfied only for retarded decline model (i.e., $a > 0$, $b < 0$ and $c > 0$).

4.1 Numerical example

To illustrate the model developed here, we consider the following hypothetical values:

A= 300, a = 1, b = -180, c = 1500,

C1=1.2, h = 0.7, C= 1, θ = 0.1, β = 0.9

Hence we obtain

	f(T, p)	T	p	q	t1
Quadratic	2026.462	1.262	5.028	798.184	1.236
Linear	1939.541	1.256	4.855	801.743	1.13

We have compared the quadratic demand model with linear demand model and found that the model has shown nearly 5 % improvement in the total profit *f(p,T)*, and there is a marginal gain in optimal price. It is also observed that the re-order time is slightly increased and lot size is decreased in case of quadratic price dependent model.

With the numerical example given above, the sensitivity analysis is performed by changing the parameters (increasing/decreasing by 20% and 50%) one at a time keeping other parameters unchanged. The results are shown in the following table:

	%	f(p, T)	T	p	q	t1
a	-50%	1981.719	1.259	4.938	800.205	1.133
	-20%	2008.238	1.261	4.991	799.207	1.135
	20%	2045.15	1.264	5.066	797.792	1.138
	50%	2074.102	1.266	5.125	796.498	1.139
b	-50%	5836.483	1.238	10.797	813.461	1.114
	-20%	2876.17	1.242	6.24	810.535	1.118
	20%	1486.055	1.286	4.264	783.686	1.157
	50%	966.907	1.328	3.527	760.16	1.195
c	-50%	157.571	2.092	2.943	505.723	1.883
	-20%	1069.984	1.454	4.162	698.642	1.309
	20%	3260.549	1.133	5.917	886.173	1.02
	50%	5637.726	0.999	7.289	1002.27	0.899
C	-50%	2354.054	1.252	4.765	848.745	1.127
	-20%	2154.782	1.258	4.923	818.456	1.132
	20%	1901.762	1.267	5.133	778.428	1.14
	50%	1721.49	1.277	5.29	750.133	1.149
C1	-50%	2028.817	1.268	5.027	802.349	1.141
	-20%	2027.403	1.265	5.027	800.375	1.138
	20%	2025.524	1.26	5.028	796.869	1.134
	50%	2024.12	1.256	5.029	794.023	1.13
A	-50%	2166.178	0.887	4.969	564.446	0.798
	-20%	2076.719	1.126	5.006	713.592	1.013
	20%	1981.154	1.387	5.047	876.267	1.248
	50%	1919.998	1.557	5.073	982.733	1.401

Observations

(i) f(p, T) is highly sensitive to the changes in *b*, *c* and C and it is moderately sensitive to the changes in *a* and A; and it is slightly sensitive to the changes in C_1.

(ii) p is highly sensitive to the changes in the parameters b and c, moderately sensitive to the changes in a and C; and less sensitive to the changes in C_1 and A.

(iii) It is found that, q is highly sensitive to the changes in the parameters c and A; moderately sensitive to the changes in b and C; and somewhat sensitive to the changes in C_1 and a.

(iv) It can also be seen from the above table, T is highly sensitive to the changes in the parameters c and A; moderately sensitive to the changes in b; and slightly sensitive to the changes in the parameters a, C and C_1.

(v) Though the parameter C has huge impact on the profit, it maintains inverse relationship with the parameter C. This may be due to the behaviour of the model (retarded decline).

Conclusion

In this paper, a deterministic inventory model is developed for time dependent deteriorating items. It is assumed that the demand rate is a quadratic function of price. Shortages are allowed and completely backlogged in this model. Stock out is a most common phenomenon in real situations due to uncertainties prevailing in the market. Thus the total profit of the stored items sometimes more than its backorder cost. Hence shortages are economically viable in such situations. Another important component is the salvage value which is also a feasible in these situations.

The deterioration factor, which is time dependent, has been considered in this model as most items undergo either direct spoilage (ex. vegetables, blood, fruits etc.) or physical damage (ex. photo films, unstable liquids etc.) as time progresses.

It is usual to consider the demand rate to be either linear function or exponential function of time/price. Many real market situations need not follow this trend. Thus it is believed that these situations can well be explained through quadratic price dependent demand rate. Under these assumptions, the model is developed and solved for optimal profit. It is interesting to note that only retarded decline model (i.e., $a > 0$, $b < 0$ and $c > 0$) is viable under these situations. It is found that the total profit in this model is more than the total profit obtained in linear demand model. It is also noted that the profit is highly sensitive to the changes in parameters *b, c* and C. Thus the unit cost of the item is significant to determine the profit of the inventory system when shortages are allowed.

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