Question 1.

The city of Lancaster's water distribution system has 3 wells for water supply. There are 10 pumps at these 3 wells. It is estimated that a pumping rate of 10,000 gallons per minute is needed to satisfy the city's total water demand. There are limits on how much water can be pumped from each well:

- 3000 gal/min from well 1
- 2500 gal/min from well 2
- 7000 gal/min from well 3

There are also different costs of operating each pump and limits on the rate of each pump:

PUMP	MAXIMUM (GAL/MIN)	COST ($/GAL/MIN)	FROM WELL
1	1100	0.05	1
2	1100	0.05	2
3	1100	0.05	3
4	1500	0.07	1
5	1500	0.07	2
6	1500	0.07	3
7	2500	0.13	1
8	2500	0.13	2
9	2500	0.13	3
10	2500	0.13	3

Lancaster wishes to determine the **least cost** way to meet its pumping needs.

1. Formulate the problem as a linear program (LP), using the decision variables

   $$x_j := \text{pump rate per minute of pump } j,\ j = 1, \dots, 10$$

   Hints (steps):

   a. Formulate an objective function to minimize the cost of the pumping plan selected.
   b. Formulate a system of 3 constraints enforcing well capacity.
   c. Formulate a system of 10 constraints enforcing pump capacities.
   d. Formulate a single constraint enforcing the overall pumping requirement.
   e. Complete the model with an appropriate system of variable-type constraints (e.g., nonnegative? Integer? Etc.)

2. Implement the LP in GAMS and solve for the optimal solution.
   a. Copy & paste your GAMS code in Word, or include a screenshot of the GAMS code.
   b. Report the optimal x_j value for $j = 1, \dots, 10$, as well as the optimal objective value (i.e., total cost).

Question 2.

Table 1 Quarterly return rate of stocks

Sample	APPL	GOLD	SBUX	F
1	-0.001	0.01	0.006	0.027
2	0.085	-0.018	0.078	-0.003
3	0.042	-0.04	-0.011	0.02
4	0.111	-0.009	0.03	0.003
5	0.058	-0.029	0.009	0.007
6	0.035	0.026	0.052	-0.005
7	0.06	-0.052	0.029	-0.005
8	0.129	-0.02	0.064	0.008
9	-0.006	0.05	0.056	0.017
10	0.077	-0.066	0.007	0.01
11	-0.05	0.005	-0.008	-0.016
12	-0.006	0.026	0.059	0.016
13	0.093	-0.086	-0.016	-0.018
14	0.083	-0.086	-0.013	0.002
15	0.062	-0.047	-0.01	0.024
16	-0.003	-0.045	0.021	-0.015

The above table shows a random sample (16 quarters) of quarterly return rates of four stocks. Based on this data, you want to determine an investment portfolio (a mix of these four stocks) such that:

- The expected (quarterly) return rate is at least 0.03
- The standard deviation of the portfolio's return rate is minimized

1. Formulate the problem as an optimization problem and solve it. You can directly modify the MeanVar.gms file (shared on Canvas) to build and solve the needed optimization problem. Fill the blanks below with the portfolio you find:

	APPL	GOLD	SBUX	F
Proportion	0.449	0.066	0.445	0.040

(Note: the proportion values must sum up to 1.)
What is the optimal standard deviation value? _____0.173_____

Paste the screenshots of the GAMS model and the part of the solution listing file that shows the solution values.

2. Run the above model 10 times, each time using a different desired return rate as given in the following table; record the corresponding optimal standard deviation values and fill the blanks. (Note: you can either manually run the model 10 times; or create a loop to do it in GAMS; no need to paste screenshots the code for this part).

Mean return rate	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
Risk (SD of return rate)	0.013	0.013	0.014	0.017	0.021	0.025	0.030	0.036	0.044	0.050

Answer these questions:

a. Are all the above 10 instances feasible? Which one(s) is not feasible?

b. Plot the relationship between the expected return and the risk of the optimal investments generated above (paste the plot here), and comment on the relationship (one or two sentences).