Homework 9:

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Q1:

Well

1 2 3

w_1, w_2, w_3

Pump: P

1 2 3 ... 10

$P_1 \quad P_2 \quad \cdots \quad P_{10}$

$\dot{P}_i$: pumping rate: Gal/min , $i = 1, \ldots, 10$

Constraints:

$$\begin{cases} 0 \le \dot{P}_1 \le 1100 \\ \vdots \\ 0 \le \dot{P}_{10} \le 2500 \end{cases}, \quad \underbrace{\sum_{i=1}^{10} \dot{P}_i = 10^4 \ \frac{Gal}{Min}}_{\text{overall pumping requirement}}$$

$$CP_1 = \dot{P}_1 * 0.05$$

$$\vdots$$

$$CP_{10} = \dot{P}_{10} * 0.13$$

$0 \le \dot{P}_1 + \dot{P}_4 + \dot{P}_7 \le 3000$ ← well 1

$0 \le \dot{P}_2 + \dot{P}_5 + \dot{P}_8 \le 2500$ ← well 2

$0 \le \dot{P}_3 + \dot{P}_6 + \dot{P}_9 + \dot{P}_{10} \le 7000$ ← well 3

objective function: $Cost = \sum_{i=1}^{10} CP_i * \dot{P}_i$

$$\Rightarrow Cost = 0.05 * \dot{P}_1 + 0.05 * \dot{P}_2 + \cdots + 0.13 * \dot{P}_{10}$$

GAMS Code:

1:

* Lancaster's water distribution system: This is a LP

```
Set pump / 1*10 /;

alias(pump,j);

parameter c(j) 'costs of operating each pump'

	/1 0.05, 2 0.05, 3 0.05, 4 0.07, 5 0.07, 6 0.07, 7 0.13, 8 0.13, 9 0.13, 10 0.13/;

*positive variables x(j) 'x(j)= pump rate per minute of pump j, j=1,…,10' ;

positive variables x(j) 'x(j) is pump rate of pump j';

variable cost;


equations

	Con1,

	Con2,

	Con3,

	Con4,

	Con5,

	Con6,

	Con7,

	Con8,

	Con9,

	Con10,

	Con11,

	Con12,

	Con13,

	Con14,

	Obj

;


Con1 .. x('1') + x('4') + x('7') =l= 3000;

Con2 .. x('2') + x('5') + x('8') =l=2500;

Con3 .. x('3') + x('6') + x('10') =l=7000;
```

```
Con4 .. x('1') =l=1100;

Con5 .. x('2') =l=1100;

Con6 .. x('3') =l=1100;

Con7 .. x('4') =l=1500;

Con8 .. x('5') =l=1500;

Con9 .. x('6') =l=1500;

Con10 .. x('7') =l=2500;

Con11 .. x('8') =l=2500;

Con12 .. x('9') =l=2500;

Con13 .. x('10') =l=2500;

Con14 .. sum(j, x(j)) =e= 10000;

Obj ..  cost =e= sum(j, c(j) * x(j));


model m1 /all/ ;


solve  m1 minimizing cost using LP;
```

2:

xj (j=1,2,…,10) values are as follows:

```
---- VAR x  x(j) is pump rate of pump j

      LOWER     LEVEL     UPPER    MARGINAL

1       .     1100.000    +INF       .
2       .     1100.000    +INF       .
3       .     1100.000    +INF       .
4       .     1500.000    +INF       .
5       .     1400.000    +INF       .
6       .     1500.000    +INF       .
7       .      400.000    +INF       .
8       .         .       +INF      0.060
9       .     1900.000    +INF       .
10      .         .       +INF      EPS
                       LOWER     LEVEL     UPPER    MARGINAL
```

And cost value is:

```
                         LOWER     LEVEL     UPPER    MARGINAL

---- VAR cost             -INF    772.000    +INF       .
```

Q2:

1.

	APPL	GOLD	SBUX	F
Proportion	0.42	0	0.41	0.17

(Note: the proportion values must sum up to 1.)

What is the optimal standard deviation value? _____0.025_____

the screenshots of the GAMS model:

```
$TITLE Mean-variance model.

* MeanVar.gms:  Mean-Variance model.
* Consiglio, Nielsen and Zenios.
* PRACTICAL FINANCIAL OPTIMIZATION: A Library of GAMS Models, Section 3.2
* Last modified: Apr 2008.

SET Assets /APPL, GOLD, SBUX, F/;

ALIAS(Assets,i,j);

PARAMETERS
         ExpectedReturns(i)  Expected returns  /APPL 0.0480625, GOLD -0.0238125, SBUX  0.0220625, F  0.0045000 /;

table
         VarCov(i,j)
              APPL          GOLD          SBUX            F
APPL  2.471529e-03 -0.0010721458 1.241958e-04 -1.396667e-05
GOLD -1.072146e-03  0.0016157625 7.590542e-04  1.618333e-04
SBUX  1.241958e-04  0.0007590542 9.847292e-04  1.203333e-05
F    -1.396667e-05  0.0001618333 1.203333e-05  2.010667e-04
;



POSITIVE VARIABLES
    x(i) Holdings of assets;

VARIABLES
    z           Objective function value

*x(i) is a portfolio vector
```

```
*x(i) is a portfolio vector
EQUATIONS
     ReturnDef    Equation defining the portfolio return,
     NormalCon    Equation defining the normalization contraint,
     objDef       Equation defining the portfolio standard deviation
;

ReturnDef ..   SUM(i, ExpectedReturns(i)*x(i)) =g= 0.03;

NormalCon ..   SUM(i, x(i))   =e= 1;

ObjDef    ..   z              =e= sqrt(SUM((i,j), x(i)*VarCov(i,j)*x(j)));


MODEL MeanVar 'PFO Model 3.2.3' /ReturnDef,NormalCon,ObjDef/;

solve MeanVar min z using NLP;
```

the solution listing file:

xj and z values are as follows:

```
Compilation
Equation Listing   SOLVE MeanVar Using NLP From line 48
Equation
Column Listing     SOLVE MeanVar Using NLP From line 48
Column
Model Statistics   SOLVE MeanVar Using NLP From line 48
Solution Report    SOLVE MeanVar Using NLP From line 48
SolEQU
SolVAR
  x
  z
```

```
  ReturnDef  Equation defining the portfolio return
  NormalCon  Equation defining the normalization contraint
  objDef  Equation defining the portfolio standard deviation

---- VAR x  Holdings of assets

         LOWER     LEVEL     UPPER    MARGINAL

APPL       .       0.420     +INF       .
GOLD       .        .        +INF     0.021
SBUX       .       0.410     +INF       .
F          .       0.170     +INF      EPS

                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR z             -INF      0.025     +INF       .

  z  Objective function value
```

2.

Mean return rate	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
Risk (SD of return rate)	0.013	0.013	0.014	0.017	0.021	0.025	0.030	0.036	0.044	0.050

a. All instances are feasible except the last one. This is because by putting xj values in return definition formula, the result is 0.0480625 which should be 0.05:

ReturnDef .. SUM(i, ExpectedReturns(i)*x(i)) =e= 0.05;

b. As we see in the following plot, for larger values of mean return rate, the risk of the optimal investments generated become higher.