Question 1:

Choice 1:

I wrote my code in the following link and shared with you:

https://colab.research.google.com/drive/1GteEDtyWHrRucZrc3hAnod6f7CoxU5gH#scrollTo=WUikjYDrYkH4

Question 2:

If we consider a linearly separable problem for a single-layer perceptron when the bias terms are both zero (b1=0 and b2=0) and the output of the linear combiner is processed by either the sigmoid function or a soft nonlinearity based on the error function, the weights grow without bound. However, when both bias terms are nonzero, regardless of which nonlinearity is used on the output, the stationary points become finite. Thus, when the optimal decision boundary is shifted away from the origin of the signal space, the algorithm converges to a unique finite point.

Reference: (Shynk and Bershad, 1991, 1992; Shynk, 1990).

Question 3:

Logical functions performance:

A	B	A AND B	A OR B	NOT A
False	False	False	False	True
False	True	False	True	True
True	False	False	True	False
True	True	True	True	False

Not logical function:

We have only one input variable at a time and one output which has only two possible states: 0 and 1 or False and True. If we pick two parameters as w=-1 and b=0.5, then we get:

$\hat{y} = w*x + b$

if $x = 0$ then $w*x + b = 0.5 >= 0 \rightarrow y = 1$

if $x = 1$ then $w*x + b = -0.5 < 0 \rightarrow y = 0$

So: $x=0 \rightarrow y =1$

$x = 1 \rightarrow y = 0$

Therefore, a perceptron can implement the NOT logical function.

AND logical function:

$\hat{y} = w1*x1 + w2*x2 + b$

We have three parameters: w1, w2 and b. if we consider w1= w2 = 1 and b = -1.5 then:

if $x1 = x2 = 0$ then $w1*x1 + w2*x2 + b = -1.5 < 0 \rightarrow y = 0$

if $x1 = 0, x2 = 1$ then $w1*x1 + w2*x2 + b = -0.5 < 0 \rightarrow y = 0$

if $x1 = 1, x2 = 0$ then $w1*x1 + w2*x2 + b = -0.5 < 0 \rightarrow y = 0$

if $x1 = x2 = 1$ then $w1*x1 + w2*x2 + b = 0.5 < 0 \rightarrow y = 1$

Therefore, a perceptron can implement the AND logical function.

OR logical function:

$\hat{y} = w1*x1 + w2*x2 + b$

We have three parameters: w1, w2 and b. if we consider w1= w2 = 1 and b = -0.5 then:

if $x1 = x2 = 0$ then $w1*x1 + w2*x2 + b = -0.5 < 0 \rightarrow y = 0$

if $x1 = 0, x2 = 1$ then $w1*x1 + w2*x2 + b = 0.5 < 0 \rightarrow y = 1$

if $x1 = 1, x2 = 0$ then $w1*x1 + w2*x2 + b = 0.5 < 0 \rightarrow y = 1$

if x1 = x2 = 1 then w1*x1 + w2*x2 + b = 1.5 >= 0 → y = 1

Therefore, a perceptron can implement the OR logical function.

XOR logical function:

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

The XOR gate function is:

$$x1x`2 + x`1x2$$

After some calculation it can concluded that the above term is equal to:

$$(x1 + x2)(x1x2)`$$

From this expression, we see that an XOR gate consists of an OR and AND gate. So, the XOR function is not linearly separable and a single-layer perceptron cannot perform XOR logical function.