Stellar Evolution and Nucleosynthesis:

Main Sequence and Post Main Sequence

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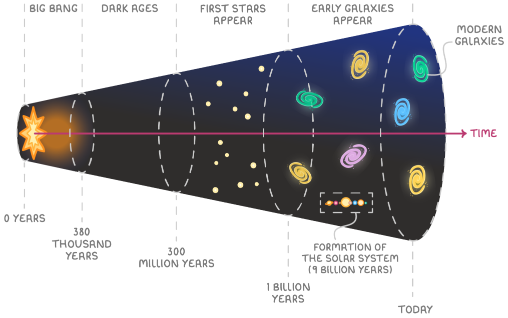
MOTIVATION

Why the study of stars?

- Stars – Only available natural labs of elemental synthesis
- Nucleosynthesis → Production of elements → chemical enrichment of the universe.



Timeline of the Universe [1]

OBSERVATIONS

- Evolution of stars $\rightarrow$ Photometrically Observed

OBSERVATIONS

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$$\downarrow$$

- luminosity & effective temperature ($E = \sigma T^4$)
- color ($\lambda_{\mathrm{max}} = \frac{b}{T}$) & magnitude

OBSERVATIONS

luminosity & effective temperature

$\downarrow$

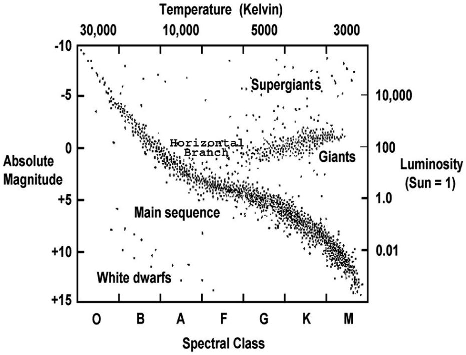
Hertzsprung–Russell (HR) diagram

OBSERVATIONS

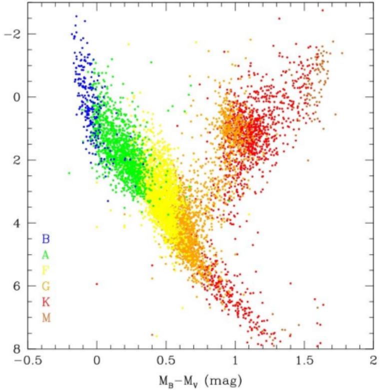
luminosity & effective temperature
$\downarrow$
Hertzsprung–Russell (HR) diagram

color & magnitude
$\downarrow$
Color Magnitude diagram (CMD)

OBSERVATIONS - HR & CMD

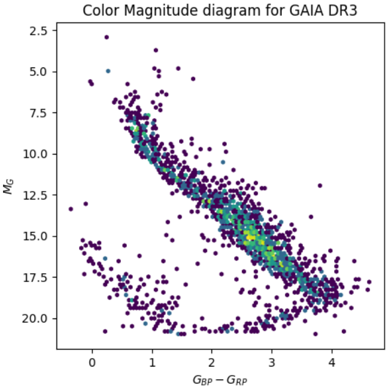


HR diagram [2]



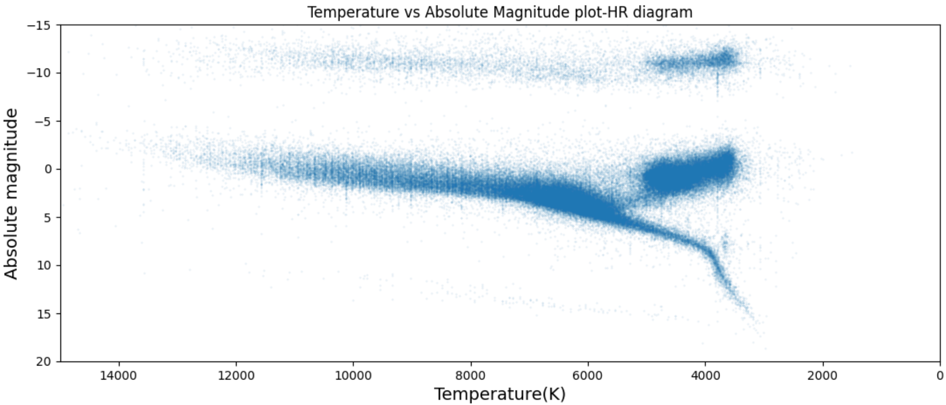
CMD [3]

OBSERVATIONS - From GAIA DR3

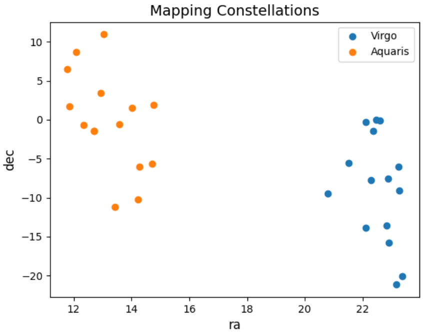


CMD from GAIA DR3

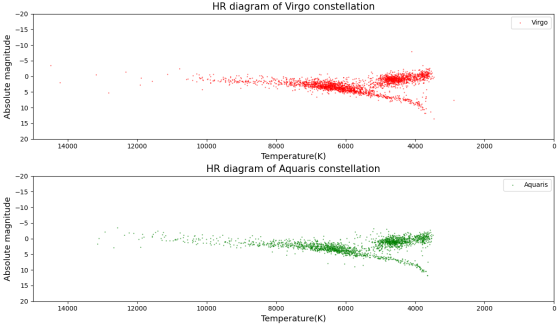
OBSERVATIONS - From the HIPPARCOUS catalogue



HR diagram from HIPPARCOUS



Physical position of clusters in the sky



HR diagram of clusters

OBSERVATIONS - Virgo & Aquarius

One key implication of HR diagram

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One key implication of HR diagram $\rightarrow$ Age (rather how far a star has evolved!)

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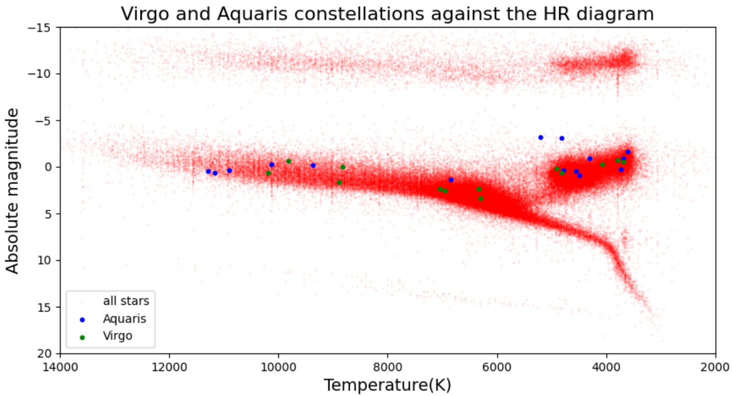
One key implication of HR diagram → Age (rather how far a star has evolved!)



Position of cluster stars in the HR Diagram

OBSERVATIONS - Isochrones

Isochrones $\rightarrow$ Same age, different mass

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- Messier 2 cluster
- SDSS data
- MIST

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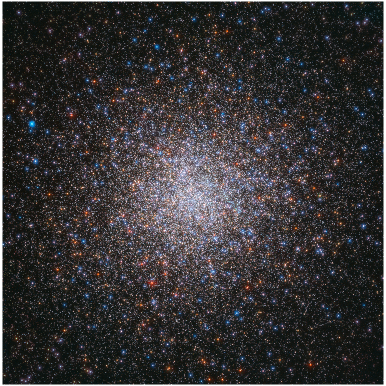
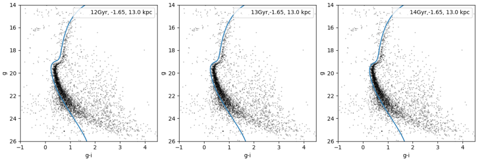


Messier 2 [4]

OBSERVATIONS - Isochrones

In our code, we generate isochrones by:

- **Step 1: Varying the age**

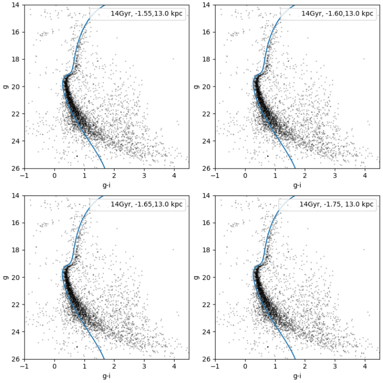


CMD with variation of age

In our code, we generate isochrones by:

- Step 1: Varying the age
- **Step 2: Varying the Metallicity**

Age= fixed at 14 Gyrs

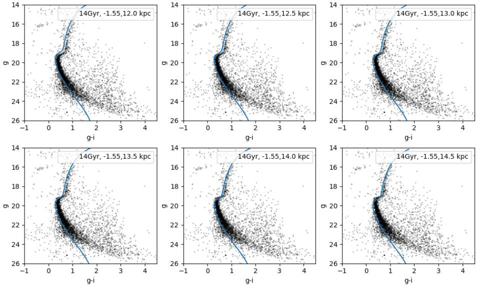


CMD with variation of metallicity

In our code, we generate isochrones by:

- Step 1: Varying the age
- Step 2: Varying the Metallicity
- **Step 3: Varying the Distance**

Age= fixed at 14 Gyrs
Metallicity = fixed at -1.55

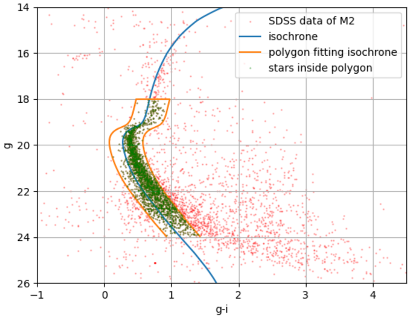


CMD with variation of distance

OBSERVATIONS - Isochrones

Final fit of CMD:

- Age = **14 Gyrs**
- Metallicity = **-1.55**
- Distance = **13.5 kpc**



Best fit CMD for M2 cluster

STRUCTURE - Dynamics

Stars are basically:

- Static (i.e. Time Independent)
- Gaseous
- Spherical
- Isolated

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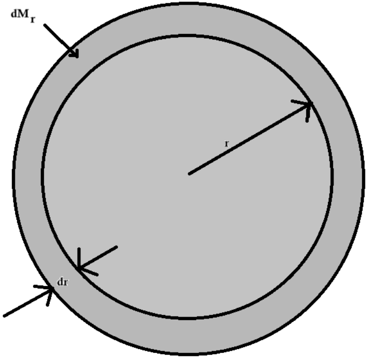
Structure explained by $\rightarrow$ Basic laws of terrestrial physics!

Basic laws of Physics

1. **Mass Conservation**

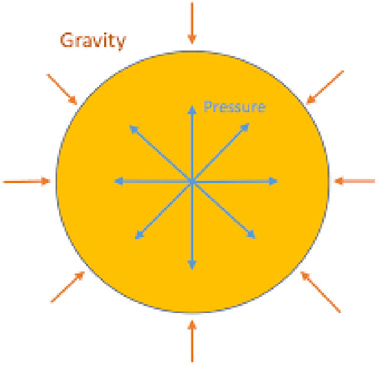


Mass Conservation in stars

STRUCTURE - Dynamics

Basic laws of Physics

1. Mass Conservation
2. **Hydrostatic Equilibrium**



Gravity and Pressure must balance each other [5]

STRUCTURE - Dynamics

Basic laws of Physics

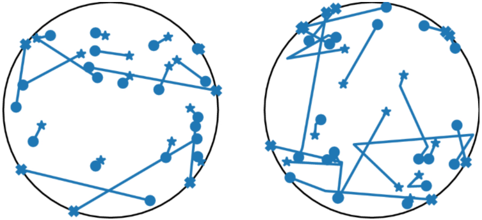
1. Mass Conservation
2. Hydrostatic Equilibrium
3. **Power Conservation**

Power conservation [6]

STRUCTURE - Dynamics

Basic laws of Physics

1. Mass Conservation
2. Hydrostatic Equilibrium
3. Power Conservation
4. **Energy Transport**



Radiative Transport MCMC simulation [7]

STRUCTURE - Dynamics

Equations of Stellar Structure:

1. Mass Conservation $\rightarrow \dfrac{dM_r}{dr} = 4\pi r^2 \rho$
2. Hydrostatic Equilibrium $\rightarrow \dfrac{dP}{dr} = -\dfrac{GM_r \rho}{r^2}$
3. Power Conservation $\rightarrow \dfrac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$
4. Energy Transport $\rightarrow \dfrac{dT}{dr} = -\dfrac{3\kappa\rho}{4acT^3}\dfrac{L}{4\pi r^2}$

STRUCTURE - Dynamics

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Complementary equations:

$\mathrm{P} = \mathrm{P}\ (\rho,\ \mathrm{T},$ composition$)$
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Boundary Conditions:

$$\left.\begin{matrix} M_r \to 0, \\ L_r \to 0 \end{matrix}\right\} \quad \text{as } r \to 0$$

$$\left.\begin{matrix} T \to 0, \\ p \to 0, \\ \rho \to 0 \end{matrix}\right\} \quad \text{as } R \to R_\star$$

STRUCTURE - Dynamics

Another important idea: **Virial Theorem** (=Ideal gas equation + Hydrostatic equilibrium)

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$$\text{Gravitational Potential} \xleftrightarrow{\textbf{Virial Theorem}} \text{Total Internal Energy}$$

$$-\frac{1}{2}\Omega = U$$

$$\implies \overline{T} = \frac{\alpha}{3}\frac{MG\, m_g}{k}\left(\frac{3M}{4\pi\overline{\rho}}\right)^{-1/3}$$

STRUCTURE - Fusion

- Source of energy $\rightarrow$ Nuclear fusion

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Not enough energy to fuse!

Quantum Tunneling to the Rescue

Solution: "Quantum Tunneling"

Even if $V = \infty \rightarrow$ "finite penetration probability"

STRUCTURE - StatStar

Input

M_0

L_0

T_0

X

Z

$\rightarrow$

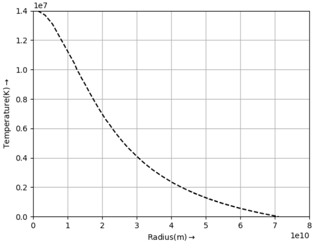
Structure equations

$$
\begin{aligned}
\frac{dM_r}{dr} &= 4\pi r^2 \rho, \\
\frac{dP}{dr} &= -\frac{GM_r \rho}{r^2}, \\
\frac{dL_r}{dr} &= 4\pi r^2 \rho \epsilon, \\
\frac{dT}{dr} &= -\frac{3\kappa\rho}{4acT^3}\frac{L}{4\pi r^2}
\end{aligned}
$$

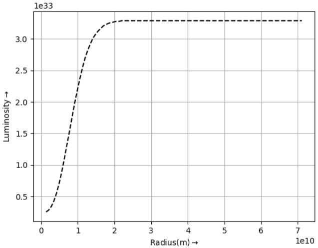
$\xrightarrow{\textbf{StatStar}}$

```
A Homogeneous Main-Sequence Model
 The surface conditions are:              The central conditions are:
 Mtot =  1.000000E+00 Msun                 Mc/Mtot     =  4.00418E-04
 Rtot =  1.020998E+00 Rsun                 Rc/Rtot     =  1.90000E-02
 Ltot =  8.607100E-01 Lsun                 Lc/Ltot     =  7.67225E-02
 Teff =  5.500200E+03 K                    Density     =  7.72529E+01
 X    =  7.000000E-01                      Temperature =  1.41421E+07
 Y    =  2.920000E-01                      Pressure    =  1.46284E+17 dynes/cm**2
 Z    =  8.000000E-03                      epsilon     =  3.17232E+02 ergs/s/g
                  dlnP/dlnT  =  2.49808E+00
```

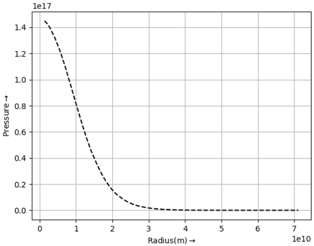
Output



Temperature vs Radius profile



Luminosity vs Radius profile

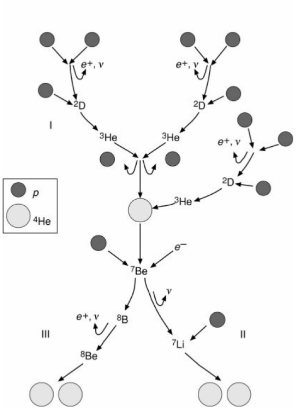


Pressure vs Radius profile

MODELS: Nucleosynthesis in Evolution

Main Sequence

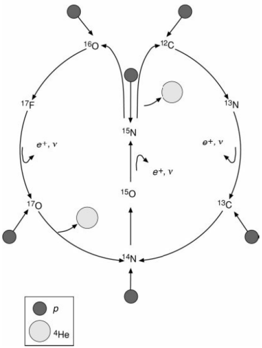
1. **pp chain** $\rightarrow$ Direct



pp chain reaction [8]

Main Sequence

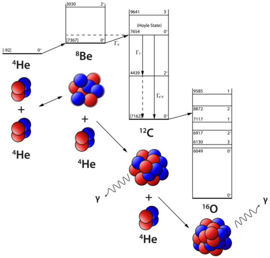
1. pp chain → Direct
2. **CNO cycle** → Carbon, Nitrogen, Oxygen as Intermediates



CNO cycle reaction [8]

Post Main Sequence

1. **Triple α process** $\rightarrow$ He burning



Helium Burning [9]

Main Sequence

1. pp chain $\rightarrow$ Direct
2. CNO cycle $\rightarrow$ Carbon, Nitrogen, Oxygen as Intermediates

MODELS: Nucleosynthesis in Evolution

Main Sequence

1. pp chain $\rightarrow$ Direct
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Post Main Sequence

1. Triple α process $\rightarrow$ He burning
2. **"Holy grail" reaction**

$$\downarrow$$

$$^{4}_{2}He + ^{12}_{6}C \rightarrow ^{16}_{8}O + \gamma$$

MODELS: Nucleosynthesis in Evolution

Evolution $\rightarrow$ Mass $\rightarrow$ temperature, luminosity changes
Emperically, $\log L = \alpha \log T_{\mathrm{eff}} + \text{constant}$
$\rightarrow$ Also found from solving the stellar structure equations

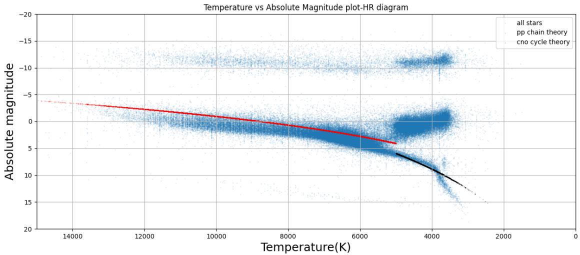
Main Sequence Models!

$$\text{pp chain} : \log L = 5.6 \log T_{eff} + \text{constant}$$
$$\text{CNO cycle} : \log L = 8.4 \log T_{eff} + \text{constant}$$

MODELS - Fitting with data

Equations of MS

- pp chain:
  $\log L = 5.6 \log T_{eff} + \text{constant}$
- CNO cycle:
  $\log L = 8.4 \log T_{eff} + \text{constant}$



pp chain and CNO cycle models with HIPPARCOS data

RESULTS - Resonance in ^{16}O

^{16}O in stellar nucleosynthesis

- $^{4}_{2}He + ^{12}_{6}C \rightarrow ^{16}_{8}O + \gamma$
- C/O ratio

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R-Matrix

Mathematical framework $\rightarrow$ resonances in nuclear reactions.

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The code!

- AZURE2 $\rightarrow$ R-matrix code (C++)
- Calculation of differential and total cross-sections
- Nuclear astrophysics

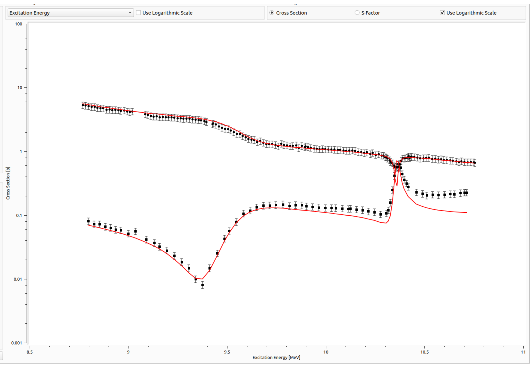
RESULTS - Resonance in ^{16}O

Our task:

- Elastic Scattering $^{12}C(\alpha, \alpha)^{12}C$
- R-Matrix fit $\rightarrow$ cross-section of ^{16}O

Results:

Spin	Fitting	Lit.
1	9.5 MeV	9.580 MeV
4^+	10.4 MeV	10.35 MeV.



Best Fit for σ

FUTURE PROSPECTS

1. MESA, POET
2. StatStar- for higher mass
3. In general, high mass stars
4. Generalise isochrone fitting

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Thank you