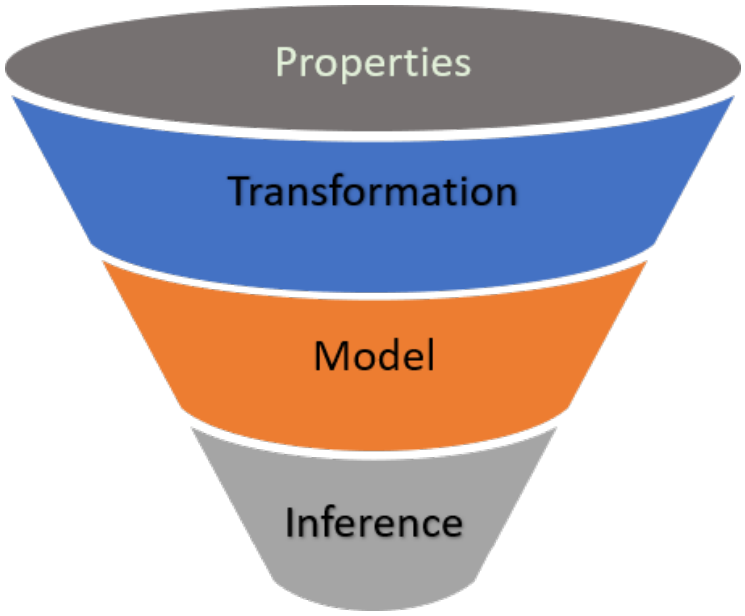
Is there an Economic Case for Energy-Efficient Dwellings in the UK Private Rental Market?

A Project Report Submitted
for the course

Statistics I
in
Bachelor of Mathematics



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ABSTRACT

This is an attempt to replicate and enhance the analyses presented in the paper 'Is there an Economic Case for Energy-Efficient Dwellings in the UK Private Rental Market?' by Franz Fuerst, Michel Haddad, Hassan Adad. This report provides a summary of the methods we used and the sources we used to complete this project.

Keywords - Energy efficiency; Hedonic model; Regression; Split incentive problem; Repeat sales index.

Contents

1 Introduction

The split incentive problem, or the lack of proper incentives to undertake energy efficiency measures, is one of the most pressing issues in the private rental sector.The landlord, makes capital expenditures in energy efficiency and does not profit immediately, but the tenant, reaps the benefits of decreased utility bills and increased thermal comfort. This has an impact on landlords' investment decisions and constitutes a roadblock to greater energy efficiency. The key point of contention is whether the landlord can charge a higher rent for a dwelling that is more energy-efficient. "Is it true that increased home energy efficiency leads to greater home sales prices?"

We address the topic in section 3. The hedonic regression model is used, in which the price of a property is separated into several components connected with its related attributes, and obtains estimates of the contributory value of each component. Implicit prices are estimated by linear regression.

In section 4, we relate the housing prices to the socio-economic environment of the area. In section 5, we address another important question 'how does the sale prices of houses fluctuate with time in years?' We use the repeat sales method introduced by Bailey, Muth, and Nourse (1963). Lastly we put forward our entire summary in section 6.

2 Data and descriptive statistics

The data set contains information on dwellings from a large sample of real estates in the United Kingdom (n=4, 201). There are 43 variables altogether and can be categorised into four classes: Repeat sales transactions variables, Energy Performance Certificate (EPC) variables, Index of Multiple Deprivation (IMD) variables, Geographical location variables. The authors of the original publication also wrote the data article on this dataset. The data was gathered by them from a number of publically available sources and filtered to make it easier to analyse.

- **Step 1**: The information on residences that have been sold at least twice were obtained from Her Majesty's Land Registry's online database, which had residential transaction values from 1995 to 2012.
- **Step 2**: The EPC rating data was acquired from the Domestic Energy Performance Certificate Register online database and then combined with the dataset.
- **Step 3**: The socioeconomic data was then added to the dataset established in the previous stage, and a random draw was performed to collect observations from hundreds of different neighbourhoods across England and Wales to provide a representative sample.

We proceed to explain the variables and highlight the key features of data. In the dataset the 4201 houses are indexed by id running from 1 to 4201.

2.1 Repeat sales transactions variables

The variables price _1 and price_2 represent the first and second sales prices, respectively which were brought on date_1 and date_2 respectively. From these raw variables, two derived variables are formed ln_price_1, ln_price_2 are formed by taking their respective logarithms of price_1 and price_2; days_between_sale represents the number of days between date_1 and date_2. Another variable denoted by perc_change_p2_to_p1, notes the percentage change of price_1 and price_2. The Table 1 show a summary of the transactional variables.[1]

Variable	Mean	Median	Std.Dev	Skewnwss	Kurtosis	Smallest	Largest	Obs	Normal
price_1	120190.8638	100000	189750.5168	23.9595	689.7001	6000	5660000	4201	1.3e-80
price_2	154575.3033	120000	263755.4659	24.4936	706.5711	25000	7900000	4201	8.317e-82
ln_price_1	11.4558	11.5129	0.6549	-0.0051	1.645	8.6995	15.5489	4201	1.6227e-21
ln_price_2	11.7524	11.6952	0.5217	1.1602	5.0146	10.1266	15.8824	4201	1.7283e-35
perc_change_p2_to_p1	0.4986	0.2	0.8323	3.1254	19.9221	-0.622	10.4167	4201	8.4553e-59
days_between_sale	2400.0838	2196	1236.9192	0.5589	-0.3239	187	6156	4201	2.245e-28

Table 1: Descriptive statistics of the transactional variables

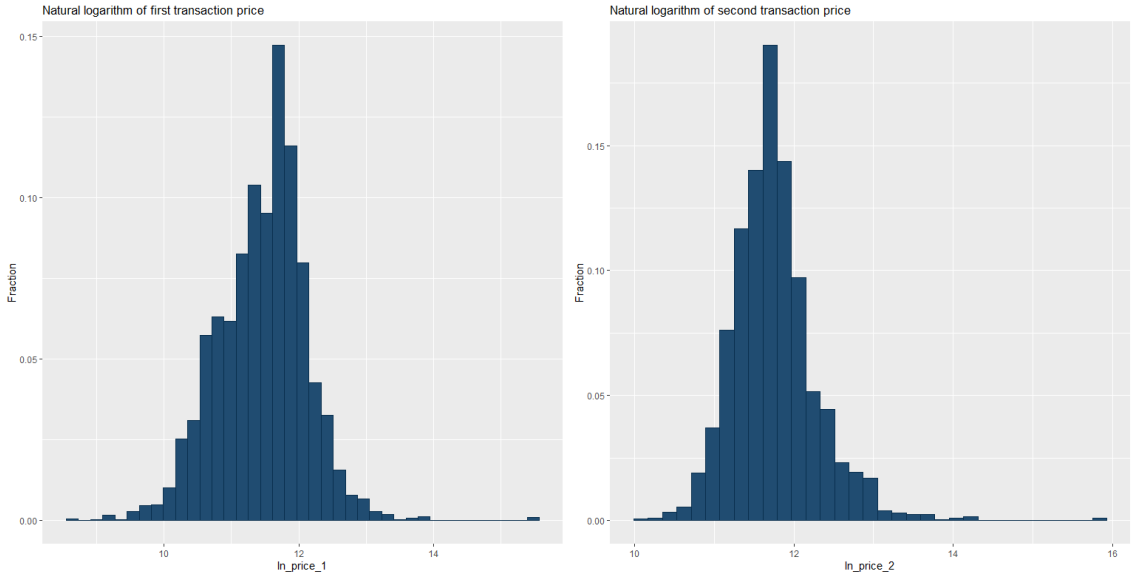

Figure 1: Distributions of the log price paid in the first (upper left hand side) and second (upper right hand side) property sale transactions

[1]The header described as **Normal** refers to the Shapiro-Francia normality test. The null hypothesis is that the data is normally distributed.The R function ShapiroFranciaTest() is being used to calculate the p-values.

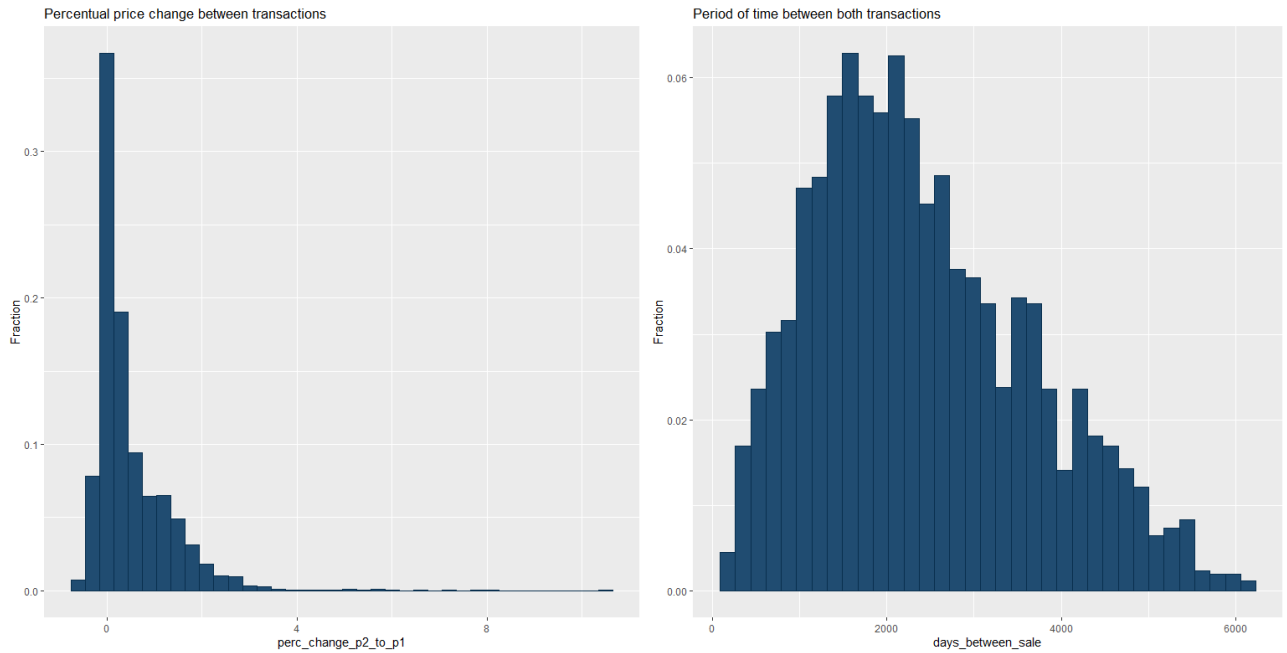

Figure 2: Price variation between the first and second sale transaction (left hand side), and period of time from the first to the second transaction (right hand side)

2.2 Energy performance certificate(EPC) variables

An EPC is a rating system that describes the energy efficiency of real estate properties in the European Union. A score out of hundred denoted by epc_100 is given to each house, where 1 is the least least efficient, and is based on Standard Assessment Procedure (SAP) points. Each house was given an EPC band according to its EPC rating. A house belongs to a particular band in A, B, C, D, E, F, G if the SAP points is in the range of 92-100, 81-91, 69-80, 55-68, 39-54, 21-38, 1-20 respectively. The varaibles epc_rating_a, epc_rating_b, epc_rating_c, epc_rating_d, epc_rating_e, epc_rating_f, epc_rating_g are boolean variables which takes 1 if and only if the house is assigned to band A, B, C, D, E, F, G respectively. Another variable ln_epc_10 is obtained by taking logarithm of epc_100. The Table 2 show the descriptive statistics of these variables.

EPC band	Frequency	Fraction
EPC A	0	0
EPC B	379	0.0902
EPC C	1442	0.3433
EPC D	1480	0.3523
EPC E	699	0.1664
EPC F	162	0.0386
EPC G	39	0.0093

Table 2: Frequency and fraction of the seven EPC bands

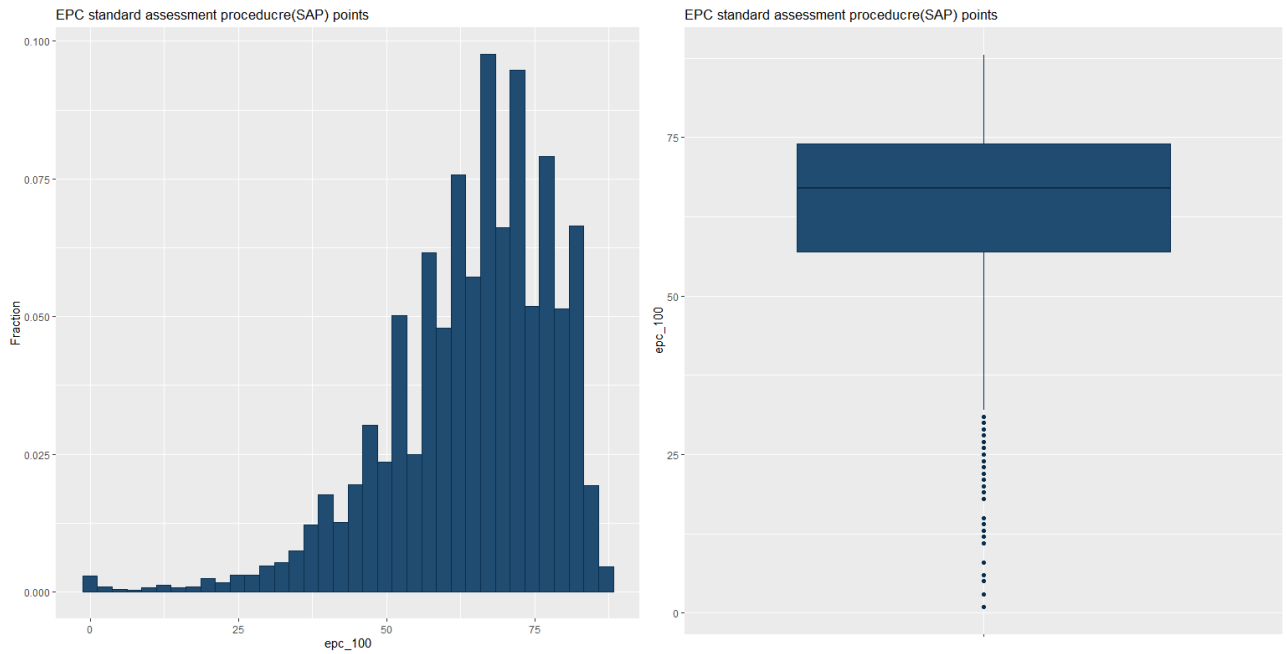

Figure 3: Histogram (left hand side) and box plot (right hand side) of the distribution of the SAP points

In this dataset, no property is assigned to band A. Most houses, around 70%, have the rating C and D. Also, few data is available on EPC F and G.

2.3 Socio-Economic variables

A rank is assigned to each of the Lower Layer Super Output Areas (LSOA) in the United Kingdom in terms of crime (crime_score), education (educ_score), income(income_score) and employment (emp_score), health (health_score) and disability, barrier(barrier_score), and living environment

(living_score). There is also an imd_score, which indicates the amount of impoverishment. In each of these, the subject with the lowest rank is the most disadvantaged. For all the categories, each LSOA is assigned a score from 1 to 10, with 1 representing the most deprived 10% of LSOAs. As a result, we have categorical variables such as imd level, income level, employment level, education level, health level, crime level, barrier level, and living level.

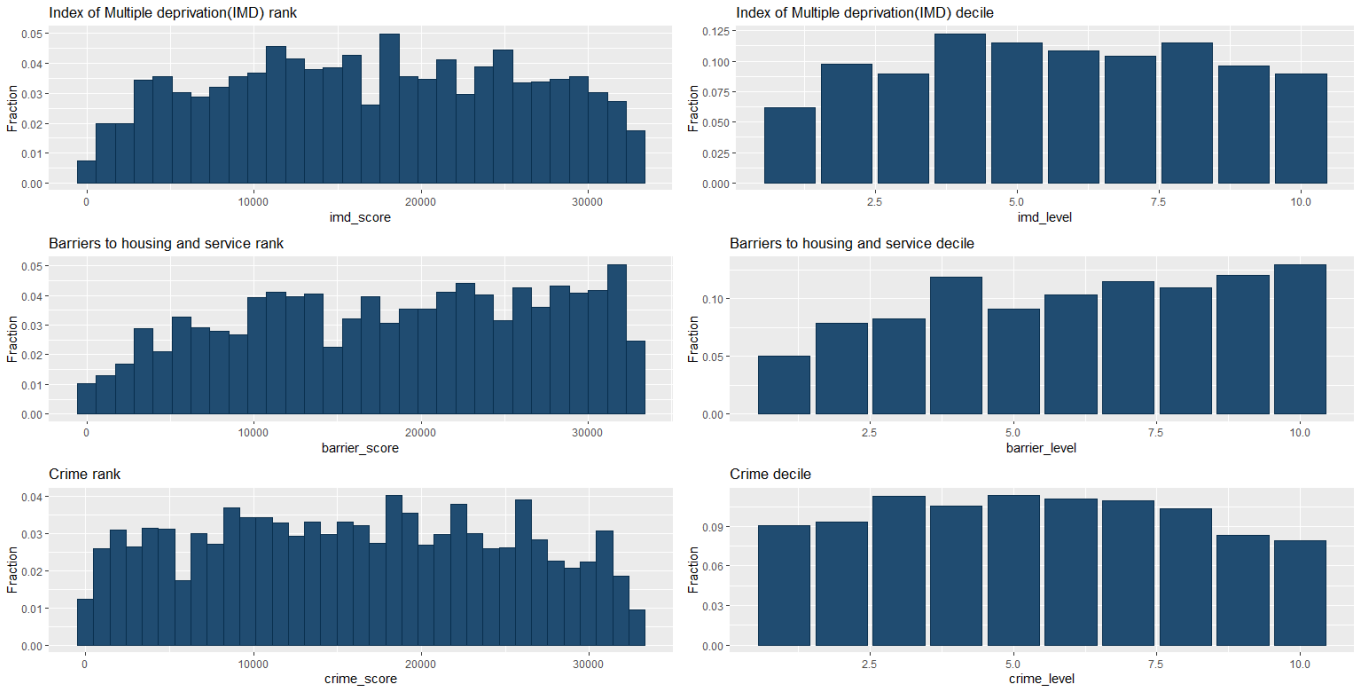

Figure 4: Histograms (left hand side) and bar charts (right hand side) of the IMD and its seven domains, considering their ranks and deciles, respectively

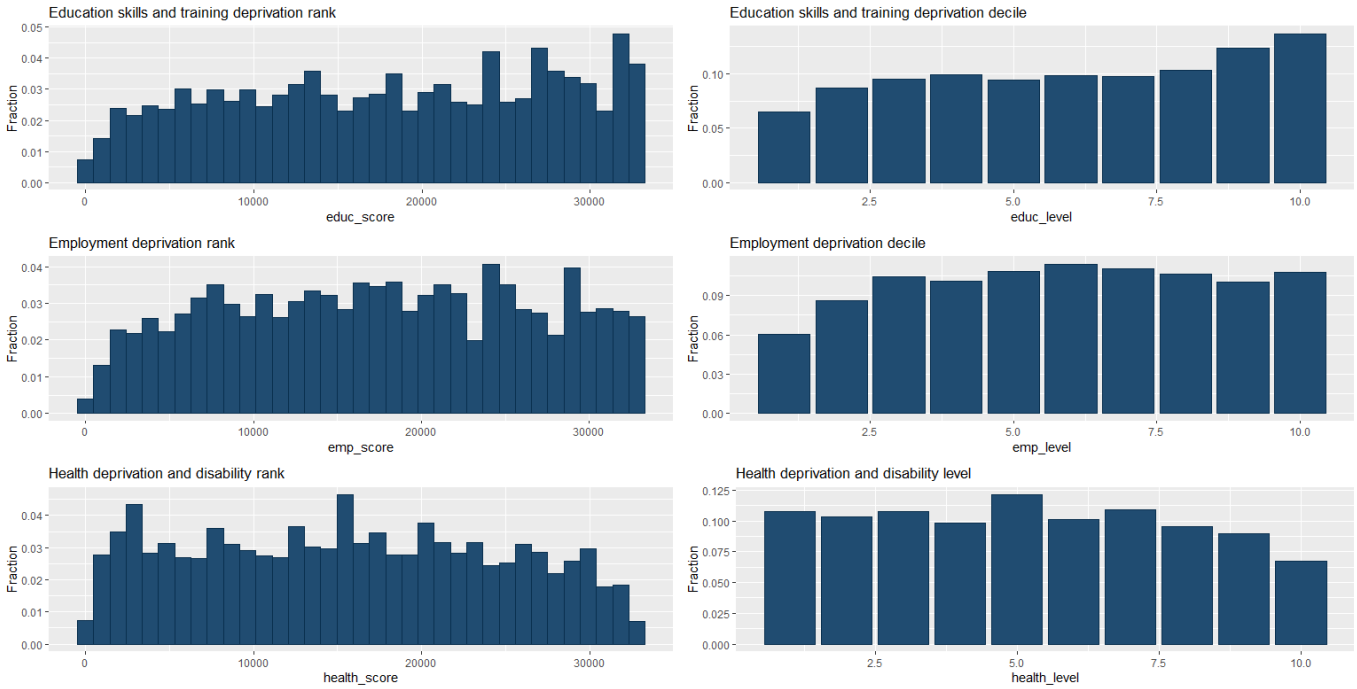

Figure 5: Continued

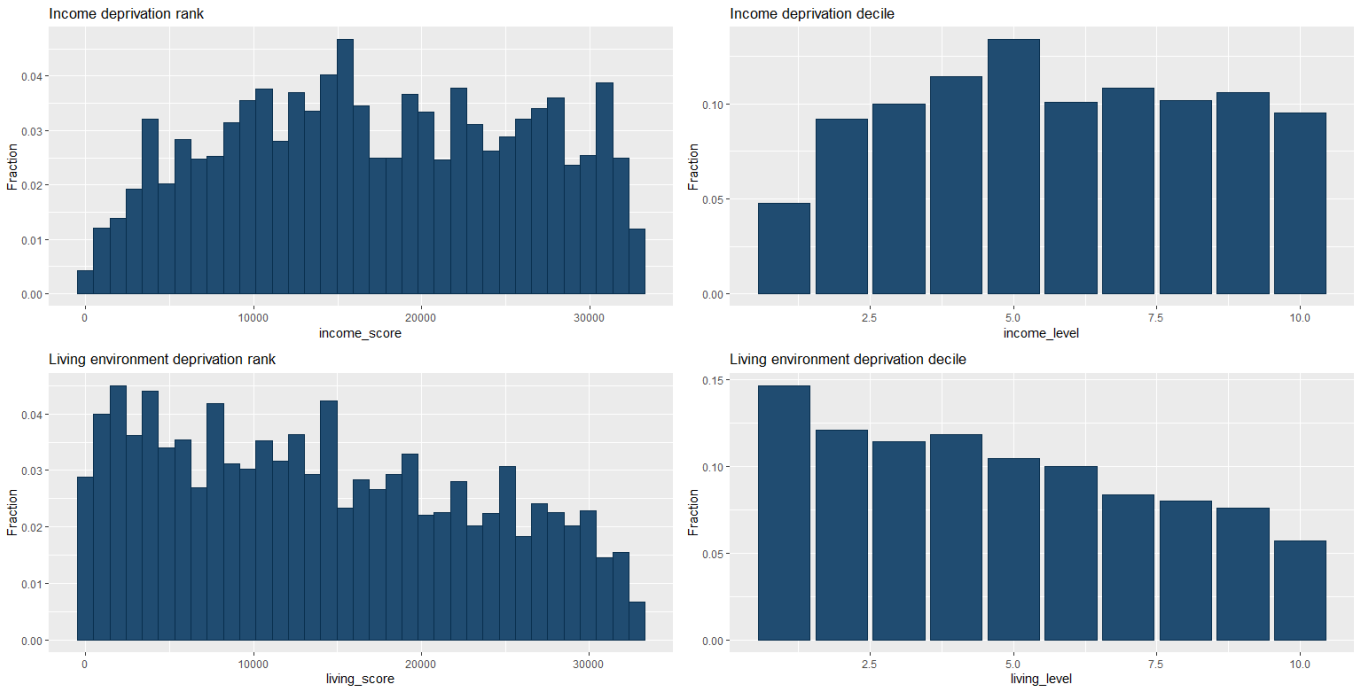

Figure 6: Continued

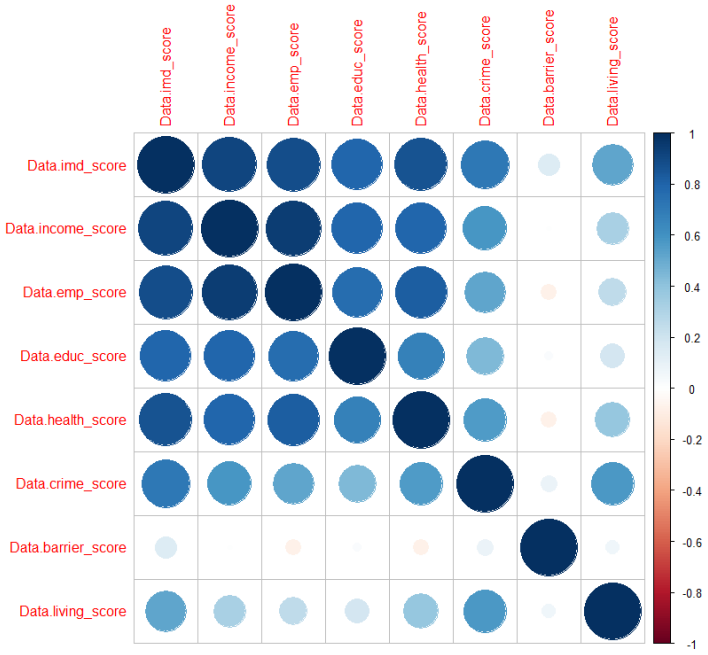

Figure 7: Correlation matrix plot of socio-economic variables labelled according to the colors

2.4 Geographical location variables

The properties in dataset were distributed geographically according to the Office for National Statistics (ONS) categorization, which includes nine regions (previously known as GOR). The variables reg_north_east, reg_north_west, reg_yorkshire_and_the_humber, reg_east_midlands, reg_west_midlands, reg_east_of_england, reg_london, reg_south_east, reg_south_west are the boolean variables indicating the region in which property belongs to.

Geography	Transaction Frequency	Transaction fraction
North West	840	0.2
Yorkshire and the Humber	839	0.1997
West Midlands	599	0.1426
East Midlands	435	0.1035
South East	407	0.0969
London	361	0.0859
South West	287	0.0683
East of England	279	0.0664
North East	154	0.0367

Table 3: Geographical distribution of the transactions included in the dataset

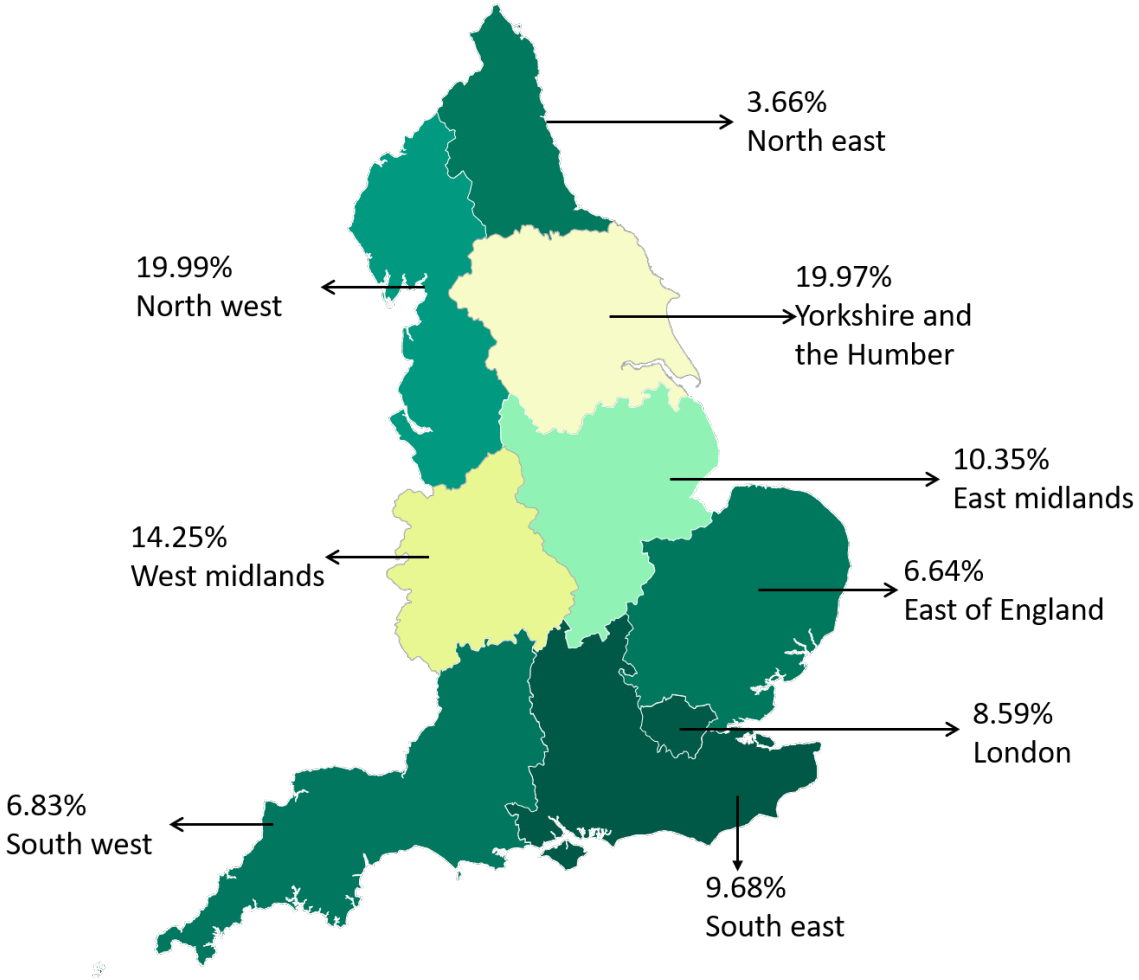

Figure 8: Geographical distribution of the transactions plotted across different regions of UK

3 Hedonic pricing model and results

Let $x_{i1t}, ..., x_{iKt}$ denote locational and physical characteristics, including categorical variables related ith property. Let P_{it} denote the price of ith property. As in the sample data we let $t = 1, 2$. We make a probabilistic model as follows. The dependent variable $\ln(P_{it})$ is written as a linear function of x_i and an error term ε_i is added.

$$\ln(P_{it}) = \beta_{0t} + \sum_{j=1}^{K} \beta_{jt} x_{ijt} + \varepsilon_i$$

The coefficients β_{jt} are called hedonic price indices and are to be estimated.

The random errors capture additional information on the housing prices. We make four assumptions on the distribution of residuals ε in the linear model:

- The mean of ε is 0
- The variance of ε is σ^2
- $\varepsilon \sim N(0, \sigma^2)$
- The random errors are independent of each other.

The method of least square(OLS) is applied to find estimators $\hat{\beta}_{jt}$ of β_{jt} and the estimated model is

$$\ln(\hat{P}_t) = \hat{\beta}_{0t} + \sum_{j=1}^{K} \hat{\beta}_{jt} x_{jt}$$

where $\hat{P}_t$ is the estimated sales price per square meter. The coefficients $\hat{\beta}_{jt}$ can be interpreted as follows. Consider x_{jt} for a particular j. If x_{jt} is a numerical variable, then the above equation can be differentiated to yield

$$\frac{\partial(\ln \hat{P}_t)}{\partial x_{jt}} = \hat{\beta}_{jt}$$

$$\Rightarrow \frac{\partial \hat{P}_t}{\partial x_{jt}} = \hat{\beta}_{jt} \hat{P}_t$$

On the other hand if x_{jt} is a categorical variable, hold all other variables constant and let x_{jt} change to $x_{jt} + \Delta x_{ijt}$ and suppose $\hat{P}_t$ changes to $\hat{P}'_t$. Then,

$$\ln(\hat{P}'_t) - \ln(\hat{P}_t) = \hat{\beta}_{jt} \Delta x_{jt}$$

$$\Rightarrow \hat{P}'_t = \hat{P}_t e^{\beta_{jt} \Delta x_{jt}}$$

Thus, for categorical variables, $e^{\hat{\beta}_{jt}}$ measures the prportion of change broght in the price for unit change in x_{jt}.

3.1 Summary of important terms in regression

3.1.1 Multiple Regression Model

Suppose we would like to predict variable y using $x_1, \cdots, x_K$. We have the data:

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_n x_{iK} + \varepsilon_i = \mathbf{x}_i^T \vec{\beta} + \varepsilon_i$$

for $i = 1, 2, ..., n$. Write

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} & \dots & x_{1K} \\ 1 & x_{21} & x_{22} & x_{23} & \dots & x_{2K} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \\ 1 & x_{n1} & x_{n2} & x_{n3} & \dots & x_{nK} \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \mathbf{e} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

A model of the form $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_n x_K$ is to be fit to data, i.e. $\hat{Y} = X\hat{\beta}$. We need to find $\hat{\beta}$ that minimizes the Standard Square Error (SSE): $S(\hat{\beta}) = ||Y - \hat{Y}||^2$.

- Equate $\partial S/(\partial \hat{\beta}_i) = 0$ and we obtain: $(X^T X)\hat{\beta} = X^T \mathbf{Y}$.
- A formal solution is $\hat{\beta} = (X^T X)^{-1} X^T \mathbf{Y}$.
- The fitted values are $\hat{Y} = X\hat{\beta} = X(X^T X)^{-1} XY = H\mathbf{Y}$ where H is called the projection matrix.
- We make usual four assumptions on $\mathbf{e}$: $E[\mathbf{e}] = 0, \mathbf{e}\ N_n(\vec{0}, \sigma^2 I_K)$
- The covariance matrix of random vector $\hat{\beta}$ is $\Sigma_{\hat{\beta}\hat{\beta}} = \sigma^2 (X^T X)^{-1}$

The following is a summary of the important terms in the hedonic regression that will be used.

3.1.2 Estimated coefficients

The *Estimate* is the estimated coefficient by OLS regression. If the estimated coefficient is positive, then there is positive associativity between the corresponding variable and the price.

3.1.3 Standard Error

Std.Error: The standard error for each coefficient is given by

$$\sigma_{\hat{\beta}_{jt}} = \sigma^2[(X^T X)^{-1}]_{jj}$$

3.1.4 Testing if a coefficient is zero

The testing of hypothesis whether a coefficient might be zero, using t-distribution is as follows. We take the level of significance to be 0.01.

$$\begin{gathered}\text{Null Hypothesis: } \beta_{jt} = 0 \\ \text{Alternate Hypothesis: } \beta_{jt} \neq 0 \\ \text{test statistic: } t_c = \frac{\hat{\beta}_{jt}}{s_{\hat{\beta}_{jt}}} \\ \text{Rejection region: } |t_c| > t_{\frac{0.01}{2}} \\ \text{p-value: } 2P(t > t_c) \text{ if } t_c \text{ is positive} \\ \text{and } 2P(t < t_c) \text{ if } t_c \text{ is negative.}\end{gathered}$$

where the t-distribution has $n - (K + 1)$ degrees of freedom.
If we are 99% confident that $\beta_{jt} \neq 0$, then we consider x_{jt} to be a statistically significant in our model. The p-value for this test is given in the last column. The significance code is given to each coefficient: , **, * indicate that the p-value is less than 0.001, 0.01, 0.05 respectively. The variables whose coefficients have , ** are statistically significant.

3.1.5 Residual standard error

If y is the dependent variable estimated by $\hat{y}$, the residual standard error s is given by

$$s = \sqrt{\frac{SSE}{n - (K + 1)}} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n - (K + 1)}}$$

s is an unbiased estimator of σ.

3.1.6 Multiple R-squared

If y is the dependent variable estimated by $\hat{y}$, then the Multiple coefficient of determination R is given by

$$R^2 = 1 - \frac{SSE}{SS_{yy}} = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \overline{y}_i)^2}$$

represents the proportion of the total sample variation in y that can be "explained" by the multiple-regression model.

3.1.7 Adjust R-squared

Adjust R-squared is defined as

$$\overline{R}^2 = 1 - \frac{n-1}{n-(K+1)}(1-R^2)$$

The value of R may be large due to the excess number of regressors, which may not add to the regression's explanatory power. This is penalised by adjusted R-squared.

3.1.8 Confidence intervals for β_{jt}

To each parameter β_{jt}, the 99% ($\alpha = 0.01$) confidence interval using t-distribution is given by

$$(\hat{\beta}_{jt} - t_{\alpha/2} s_{\hat{\beta}_{jt}}, \hat{\beta}_{jt} + t_{\alpha/2} s_{\hat{\beta}_{jt}})$$

where $t_{\alpha/2}$ is the value such that, if t has t-distribution of $n-(K+1)$ degrees of freedom,

$$P(t > t_{\alpha/2}) = \frac{\alpha}{2}$$

3.2 Results of hedonic regression of prices

Here are some preliminary plots before moving to regression. Figure 9 is the plot of mean of logarithm of prices plotted against the various EPC variables. As you can see here there are no EPC A because our dataset does not contain those variables.

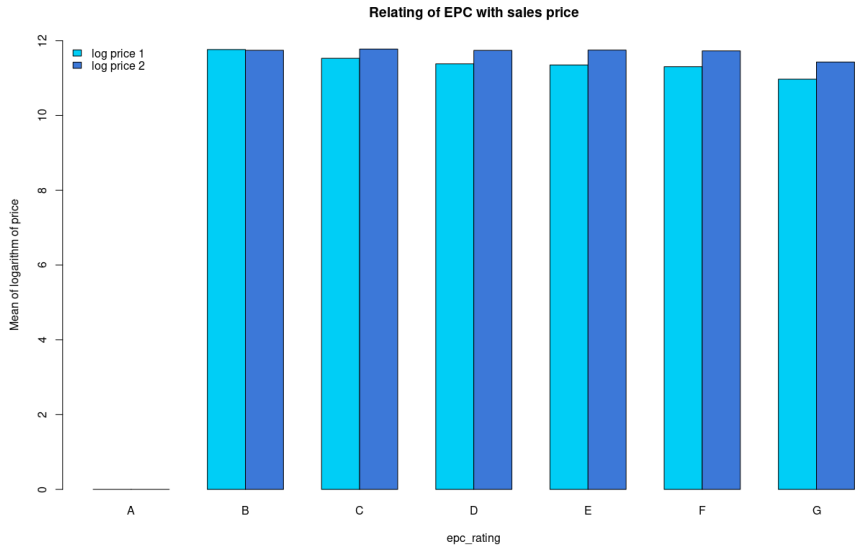

Figure 9: Plot of mean of ln_price_1 and ln_price_2 across different EPCs

In the Figure 10, the boxplot of ln prices again plotted against the different EPCs. This shows that there is a slight but some trend. In the x-axis 2 refers to EPC B, 3 to EPC C and so on. We can observe that there are many outliers captured by the boxplot. EPC C and D are neither the best nor the worst. Hence, there is a wide range of prices due to many investments on these categories. That is why there are so many outliers. On the other hand we can locate an outlier in B which is too far from the rest. That observation might be unusual. Furthermore, there is too less data on EPC G leading to less number of outliers.

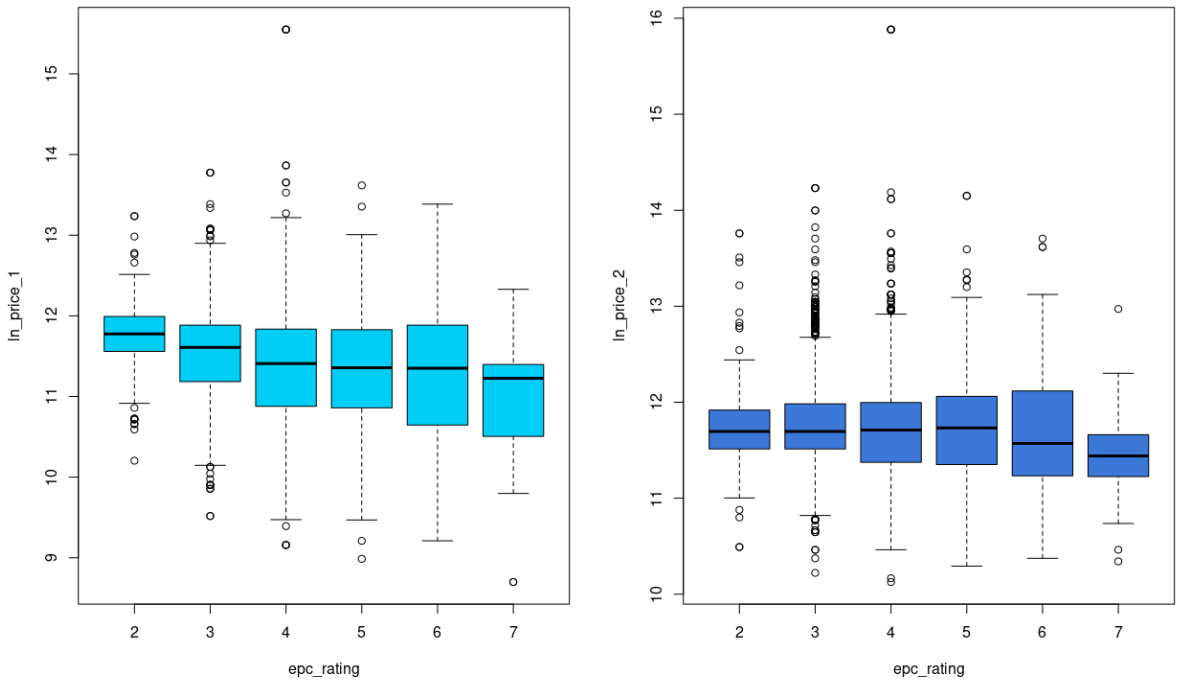

Figure 10: Box Plot for ln_price_1 and ln_price_2 respectively categorised by EPCs where 2:B, 3:C, 4:D, 5:E, 6:F, 7:G

We have used R to estimate the hedonic pricing indices. The logarithm of prices was regressed on categorical variables epc_rating_b, epc_rating_c, epc_rating_e, epc_rating_f, epc_rating_g, and various regional indicators. As most houses have EPC rating D, epc_rating_d was taken as the baseline and is held out of the regression. epc_rating_a is removed as no houses in the sample has EPC rating A. reg_north_west is also taken as reference, as most houses in the sample are from this region. The Table 4 and Table 5 show the summary of this linear model.

Variable	Estimate	Std Error	t-value	Pr(> |t|)	
Intercept	11.201346	0.024276	461.414	0(approx)	***
epc_rating_b	0.412648	0.033911	12.168	0(approx)	***
epc_rating_c	0.139870	0.021929	6.378	$2 \cdot 10^{-9}$	***
epc_rating_e	-0.021471	0.027018	-0.795	0.42684	
epc_rating_f	-0.103260	0.048702	-2.120	0.03405	*
epc_rating_g	-0.371917	0.095488	-3.895	0.00001	***
reg_north_east	-0.002641	0.051658	-0.051	0.95923	
reg_yorkshire_and_the_humber	0.061126	0.028788	2.123	0.03378	*
reg_east_midlands	-0.002814	0.035063	-0.080	0.93605	
reg_west_midlands	0.101695	0.031686	3.209	0.00134	**
reg_east_of_england	0.203688	0.040687	5.006	0.0000006	***
reg_london	0.882822	0.037107	23.791	0(approx)	***
reg_south_east	0.496382	0.035536	13.968	0(approx)	***
reg_south_west	0.239821	0.040296	5.951	$3 \cdot 10^{-8}$	***

Table 4: Hedonic regression of price_1. Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Variable	Estimate	Std Error	t-value	Pr(> |t|)	
(Intercept)	11.56700	0.01797	643.775	0(approx)	***
epc_rating_b	0.03577	0.02510	1.425	0.15415	
epc_rating_c	0.02668	0.01623	1.644	0.10026	
epc_rating_e	0.02064	0.02000	1.032	0.30208	
epc_rating_f	-0.04066	0.03605	-1.128	0.25942	
epc_rating_g	-0.27357	0.07067	-3.871	0.00011	***
reg_north_east	-0.06989	0.03823	-1.828	0.06763	.
reg_yorkshire_and_the_humber	0.05109	0.02131	2.398	0.01653	*
reg_east_midlands	-0.02497	0.02595	-0.962	0.33599	
reg_west_midlands	0.05897	0.02345	2.515	0.01195	*
reg_east_of_england	0.19067	0.03011	6.332	0(approx)	***
reg_london	0.98121	0.02746	35.727	0(approx)	***
reg_south_east	0.47361	0.02630	18.007	0(approx)	***
reg_south_west	0.25482	0.02982	8.544	0(approx)	***

Table 5: Hedonic regression of price_2. Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

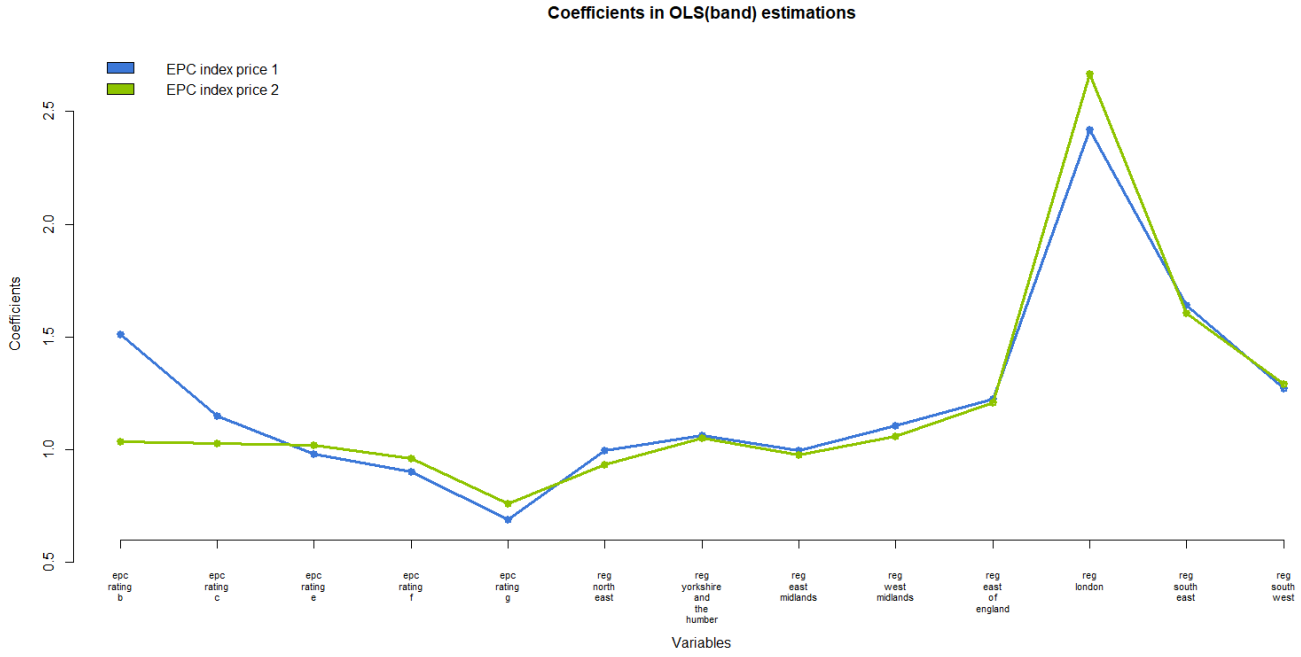

Figure 11: Hedonic Index of variables indicating the exponents of the regression coefficients

The exponent of coefficients can be plotted on a line graph and is shown in above Figure 11. This shows the relation between EPC rating and prices. From the Table 4, we see that the epc_rating_b, epc_rating_c are statistically significant variables in this model. For price_1, epc_rating_b has a hedonic index around 1.5, indicating that houses with EPC rating B experience a price premium. The hedonic index of epc_rating_g is less than 1, indicating that lower energy efficient homes tend to have lower prices. Region London has the highest hedonic index indicating that, with respect to houses in the northwest region, houses in London are expensive.

3.3 Diagnostic plots

Residual is nothing but the difference between fitted values and true values.

$$\varepsilon_i = y_i - \hat{y}_i$$

We made four assumptions on the distribution of these residuals in the beginning and now we diagnose if these assumptions hold, and assess if the model explains the relationship well.

3.3.1 Residuals vs. Fitted

This is a scatter plot of fitted values on the x-axis and residual=true value−fitted values. These graphs indicate if the price and energy performance have a non-linear connection. If the model does not capture the non-linear connection between predictor factors and outcome variables, the pattern may appear non-linear

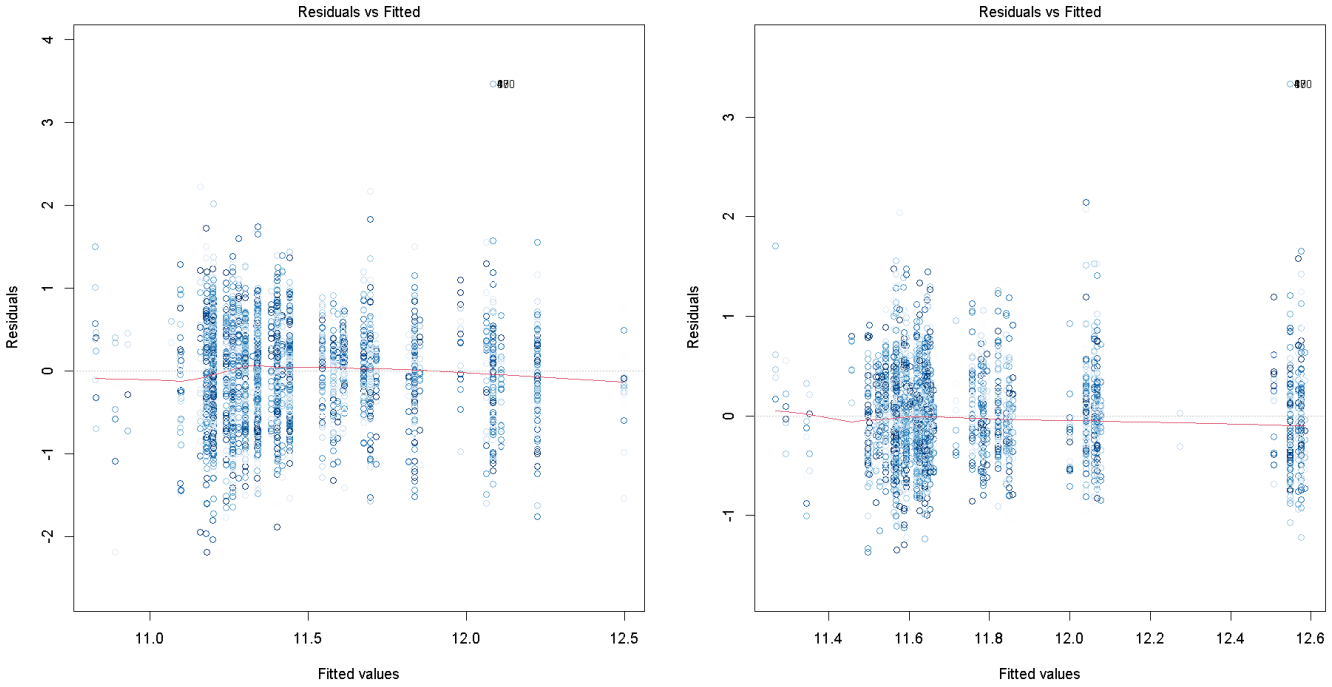

Figure 12: Plot of Residuals vs Fitted

Fig shows the residuals versus fitted plot for the two models. Both the graphs show that the relationship between prices and locational factors, energy performance is nearly linear and our model explains this relationship quite well.

3.3.2 Normal Q-Q plots of residuals

This is a scatter plot with theoretical normal quantiles on the x-axis and quantiles of residues in y-axis. This plot diagnoses if the residuals are normally distributed. The figures below show that the residuals in this model are almost normally distributed, following our assumptions.

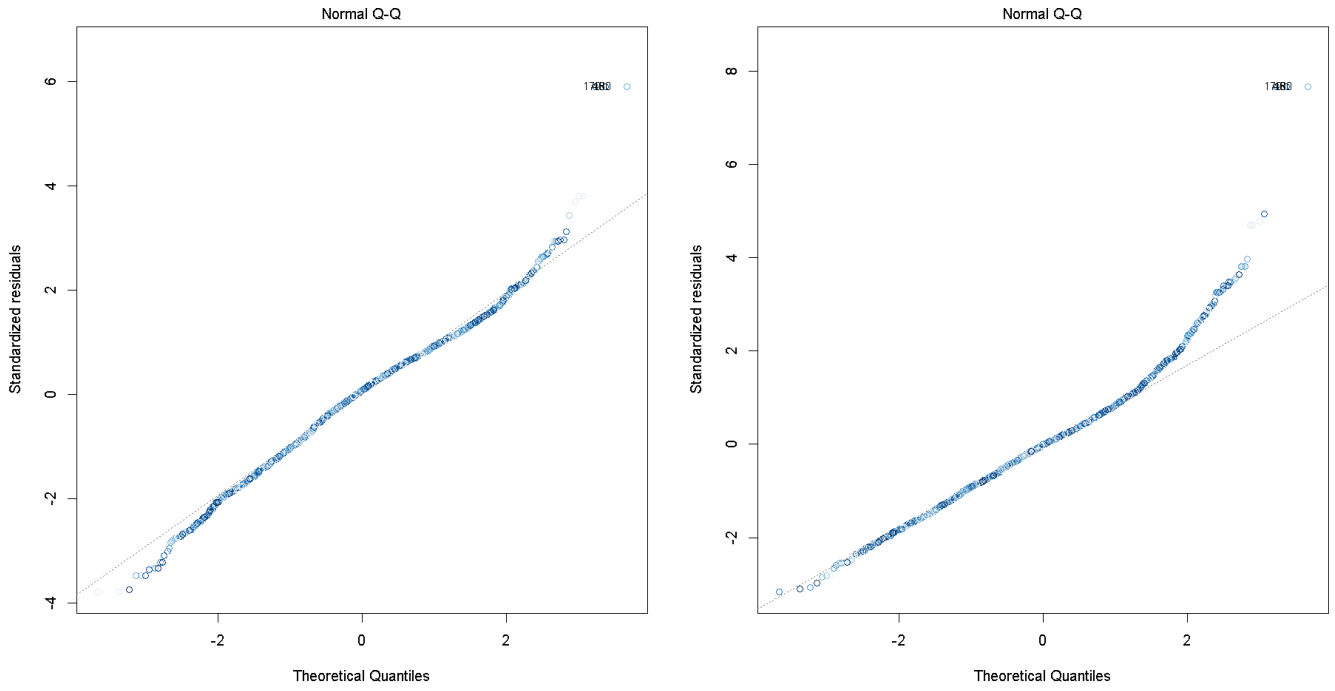

Figure 13: Normal Q-Q plots of residuals

3.3.3 Scale-location plot and assumption of equal variance

The standardized residual is given by $\frac{y_i - \hat{y}_i}{\sqrt{\hat{y}_i}}$ The intensity of the discrepancy between actual and predicted values is measured by the standardised residual.

A scale-location plot is a scatter plot that shows the square root of the absolute value of standardised residuals against fitted values. If residuals are spread uniformly across predictor ranges, then our assumption of homoscedasticity is valid. This is how we may test the equal variance assumption graphically. The assumption is valid if the graph is evenly distributed throughout the horizontal line, as it is in our linear model:

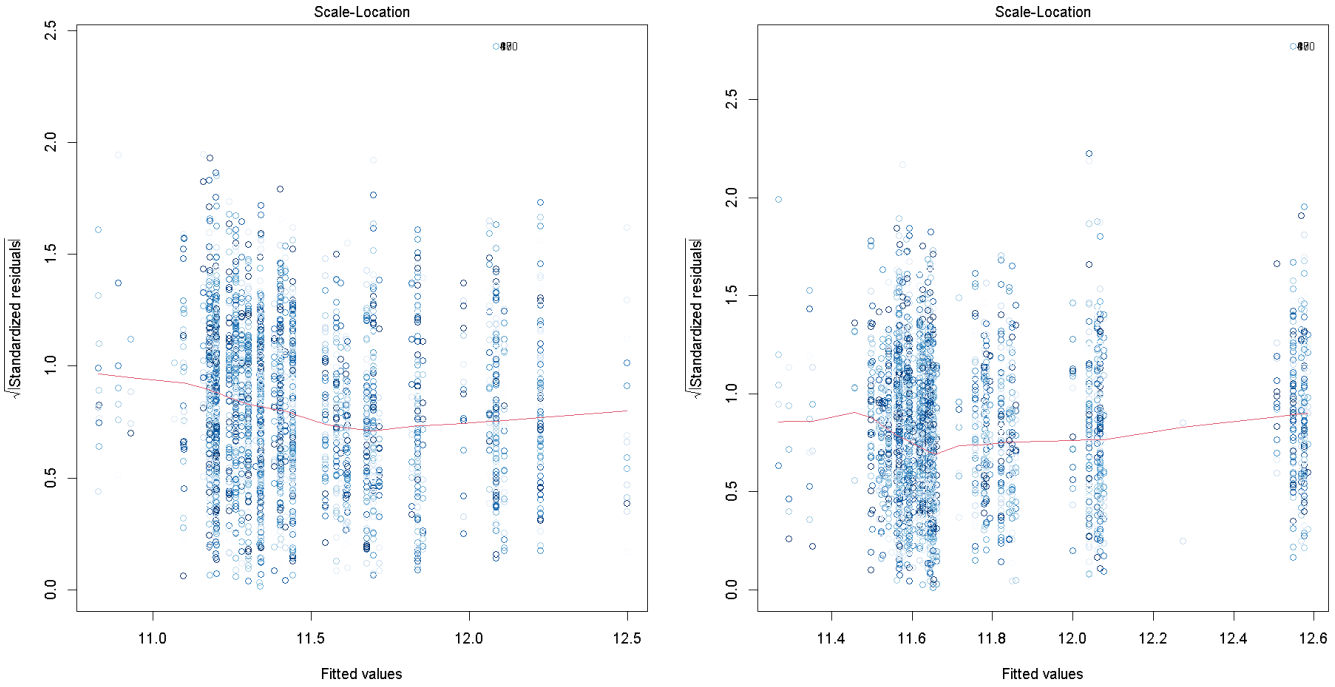

Figure 14: Scale Location plot

All the plots above show that the hedonic pricing model with OLS is indeed a good way to model the dependence of prices and locational, energy factors.

3.3.4 Leverage

The leverage score for the i^{th} independent observation x_i is given as:

$$h_{ii} = [H]_{ii} = \mathbf{x}_i^T (X^T X)^{-1} \mathbf{x}_i = h_{ii} = \frac{\partial \hat{y}_i}{\partial y_i}$$

It is the degree by which ith measured value influences the ith fitted value (i.e., $\hat{y}_i$) Points with extreme values of variables x_i are said to have high leverage. High leverage points have a stronger capacity to shift the regression line and can be influential, causing the outcome and accuracy to be distorted.

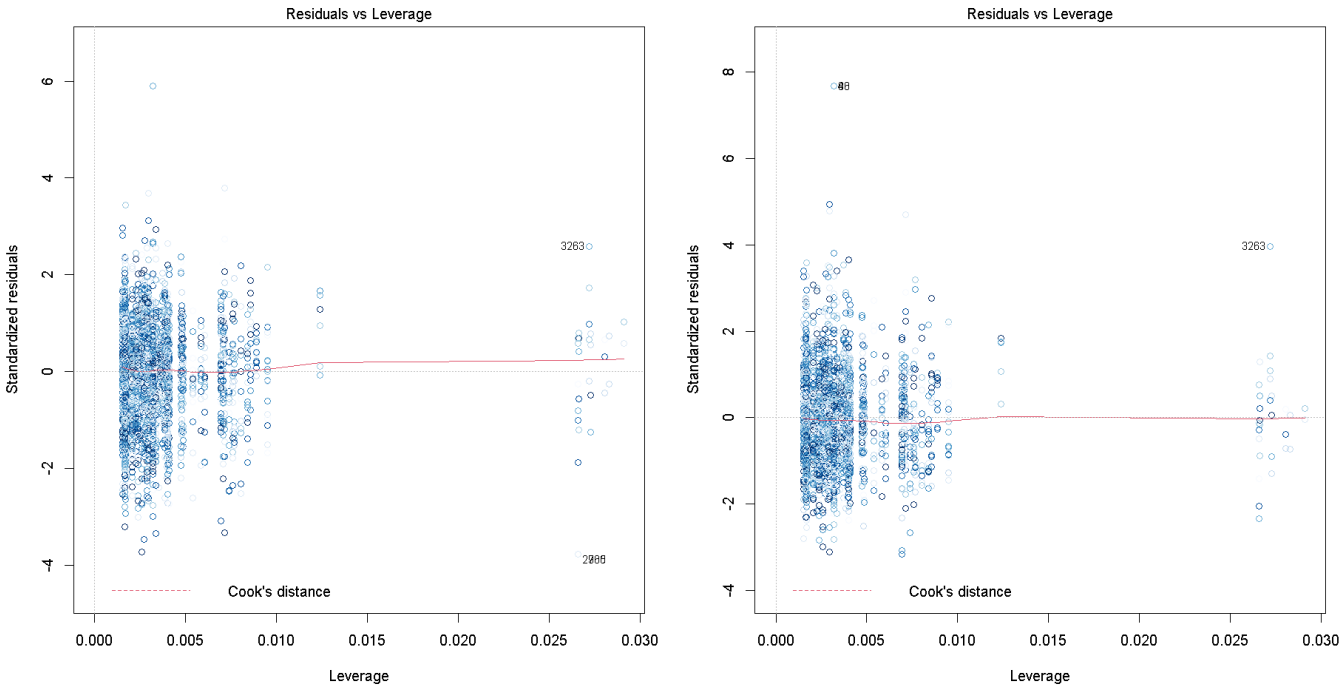

Figure 15: Standarized Residuals vs Leverage

Figure 15 below shows the standarized residuals plotted against the leverage. The contour lines of Cook's Distance are not visible in the graph. Hence, it is evident that the number of outliers are too less. So, we do not need to rerun the model by removing outliers.

3.3.5 Cook's Distance

Large residuals (outliers) and/or excessive leverage in data points can skew the conclusion and accuracy of a regression. The Cook's distance statistic for each observation determines how much the model estimates change when that observation is removed. Cook's distance D_i of observation i is defined as the sum of all the changes in the regression model when observation i is removed from it

$$D_i = \frac{1}{(K+1)s^2} \sum_{j=1}^{n} (\hat{y}_j - \hat{y}_{j(i)}) = \frac{e_i^2}{(K+1)s^2} \left[\frac{h_{ii}}{(1-h_{ii})^2} \right].$$

where $\widehat{y}_{j(i)}$ is the fitted response value obtained when excluding i, and s is the standard error. Figure 16 shows the Cook's Distance of each response value.Hence, we can notice that the Cook's Distance is mostly small for each value.

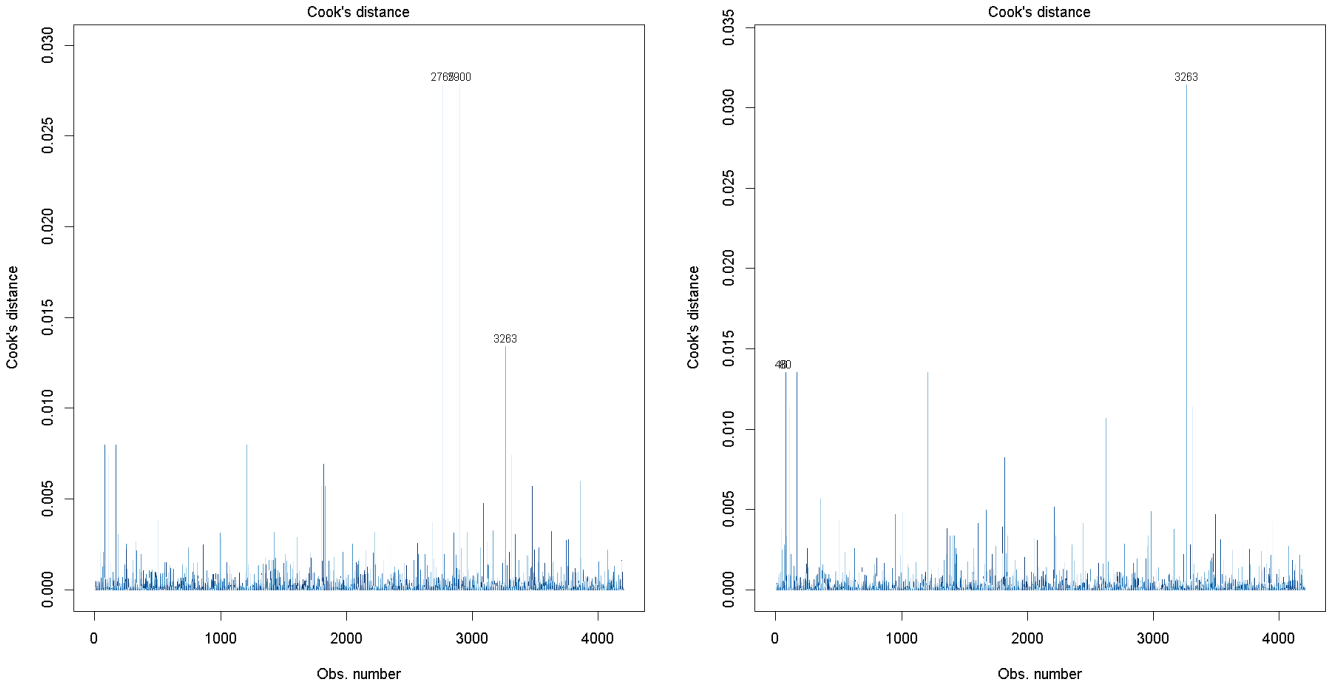

Figure 16: Cook's Distance plotted against row number

As a result, our fitted model is not extremely distorted. We can see there are too less points with high Cook's Distance. This shows that although the data had many outliers pointed out by the boxplot but they mostly have low influence.

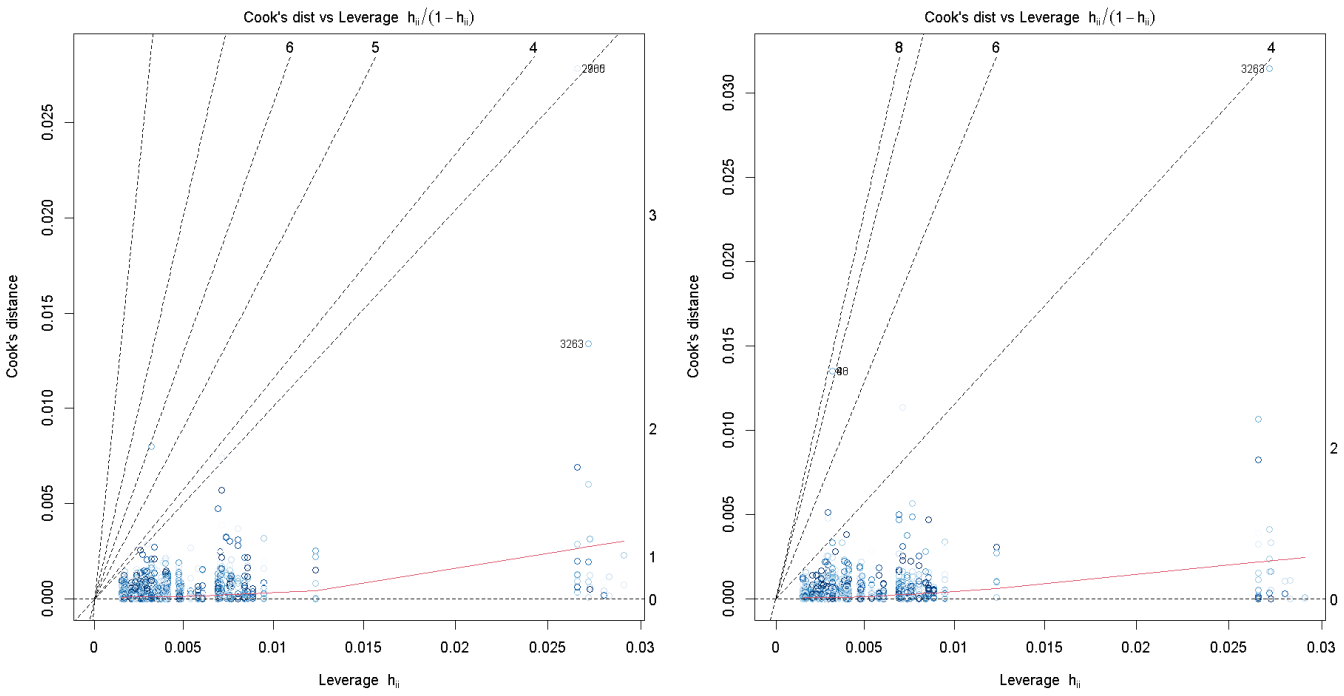

Figure 17: Cook's Distance plotted against Leverage.The contours in the scatterplot are standardized residuals labelled with their magnitudes

On the other hand Figure 17 shows the plot of leverage and cooks distance.

4 Price vs socio-economic variables

We would like to relate price_1 and price_2 with the socio-economic variables imd_score, income_score, emp_score, educ_score, health_score, crime_score, barrier_score, living_score. This would reveal the dependency of housing prices on socio-economic environment. We assume that errors are normally distributed in a number of statistical procedures, including regression. The plots given below clearly show that the price_1 and price_2 variables are not normal.

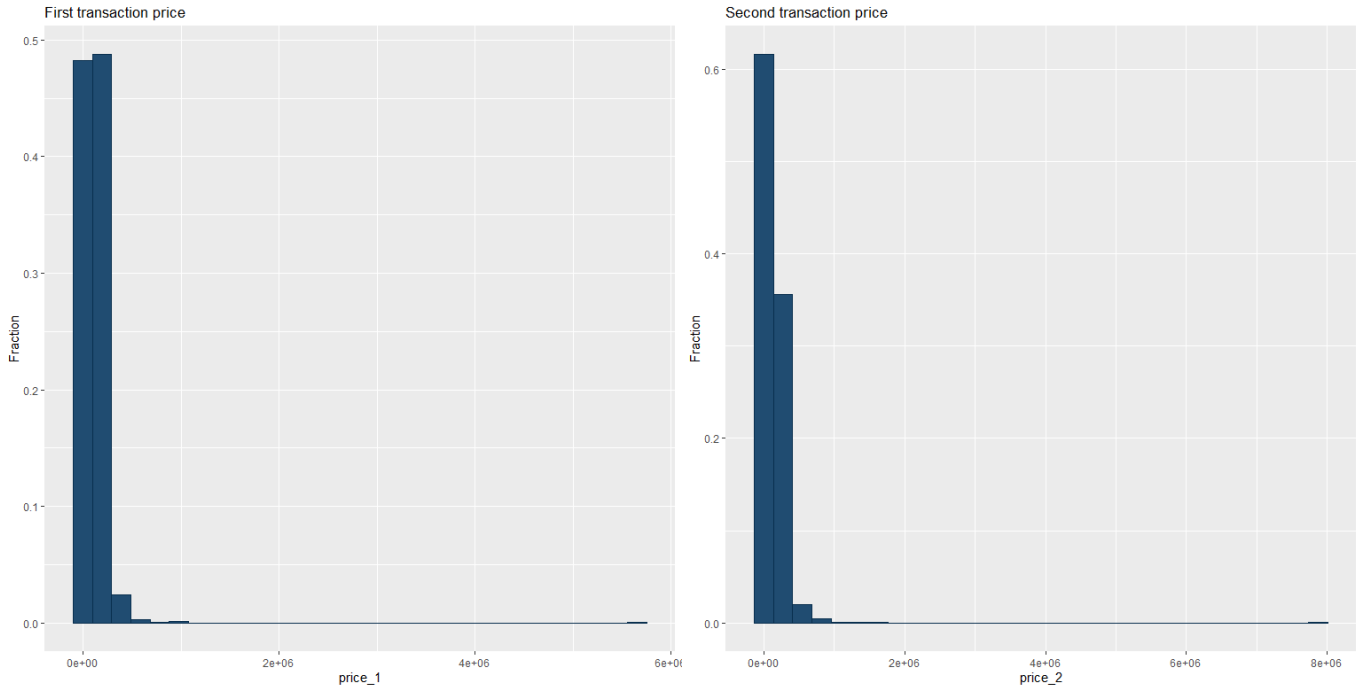

Figure 18: Distributions of the price paid in the first (left hand side) and second (right hand side) property sale transactions

Instead of directly applying linear regression, we start by applying box-cox transformation on the dependent variable, developed by Box and Cox(1964).

4.1 Box-cox transformation

Our data is transformed via the Box-Cox transformation to mimic a normal distribution as nearly as possible. Consider the case when variable y is regressed against variable x. If the data is normally distributed, the linear model provides a good match. One reason is that this assumption underpins all confidence intervals and hypothesis testing. The goal of the box-cox transformation is to convert y to $y^{(\lambda)}$ in such a way that transformed y is close to normal. The family of transformations considered are:

$$y^{(\lambda)} = \begin{cases} \frac{y^{\lambda}-1}{\lambda} & \lambda \neq 0 \\ \ln(y) & \lambda = 0 \end{cases}$$

By using maximum likelihood estimator, a value of λ is found which makes $y^{(\lambda)}$ approximately normal.

4.2 Results

R already has a function in its MASS library, **boxcox(object, ...)** where object refers to the linear model we want to fit. After running the code, it is noticed that,

$$\lambda_{\text{price\_1}} = 0.02$$

$$\lambda_{\text{price\_2}} = -0.3$$

We take $0.02 \approx 0$, and we transform price_1 variable to $\ln(\text{price\_1})$ and price_2 variable to $\frac{(\text{price\_2})^{-0.03}-1}{-0.03}$.

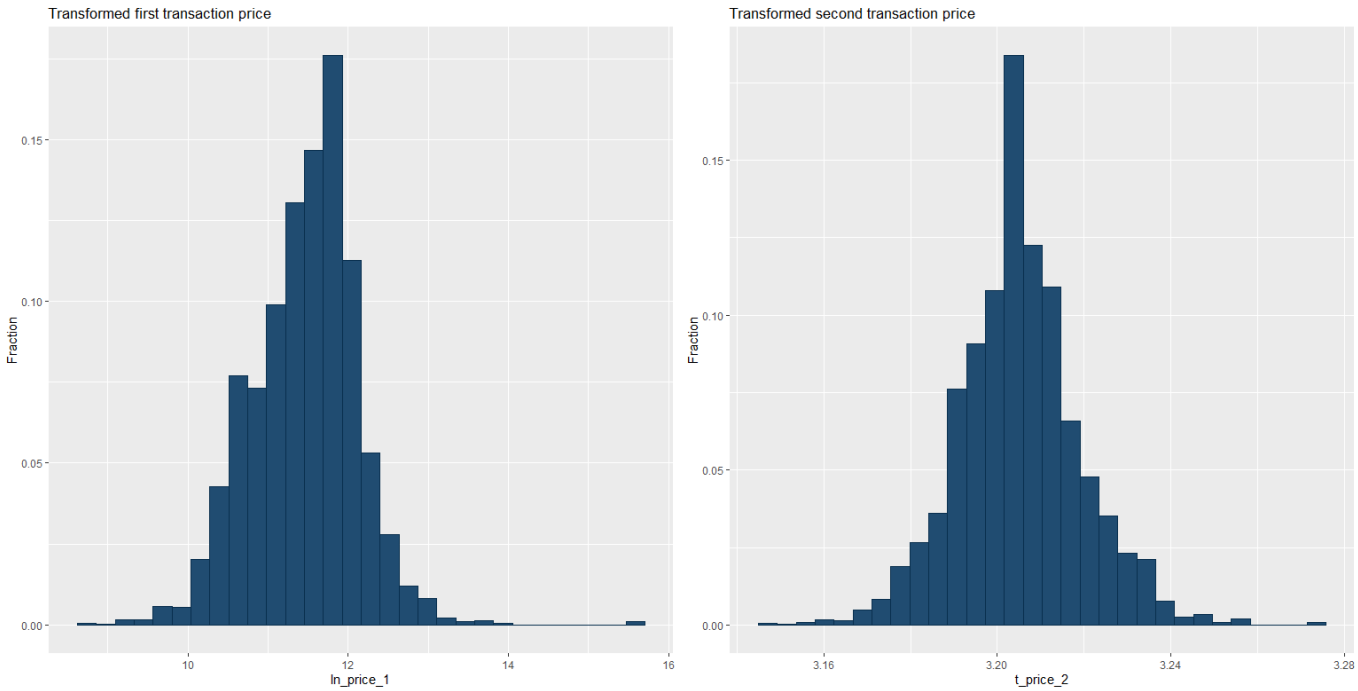

Figure 19: Distributions of the transformed price paid in the first (left hand side) and second (right hand side) property sale transactions

The plots above are symmetric and look like normal distribution. We can now move to regression analysis. [2]

[2] **Note**: The Figure 7 shows the pairwise correlation in the various socio-economic variables. This may result into multicollinearity in our model. Although, the coefficient s and p-values are affected by multicollinearity, but the forecasts, precision of the predictions, and goodness-of-fit statistics are unaffected. Our primary purpose is to generate predictions and hence we are not addressing multicollinearity.

4.3 Summary

Variable	Estimate	Std Error	t-value	Pr(> |t|)	
(Intercept)	10.82817670995	0.04523532930	239.374	0(approx)	***
imd_score	-0.00006253022	0.00000696566	-8.977	0(approx)	***
income_score	-0.00001487051	0.00000390421	-3.809	0.000142	***
emp_score	0.00003609509	0.00000368207	9.803	0(approx)	***
educ_score	0.00003438949	0.00000196613	17.491	0(approx)	***
health_score	0.00003242077	0.00000250168	12.960	0(approx)	***
crime_score	0.00000174045	0.00000167774	1.037	0.299622	
barrier_score	-0.00000001551	0.00000146222	-0.011	0.991539	
living_score	0.00001157480	0.00000167902	6.894	0(approx)	***

Table 6: Trasformed price_1 vs socio-economic variables.
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Variable	Estimate	Std Error	t-value	Pr(> |t|)	
(Intercept)	3.18665988102	0.00087845134	3627.588	0(approx)	***
imd_score	-0.00000193203	0.00000013527	-14.283	0(approx)	***
income_score	-0.00000017160	0.00000007582	-2.263	0.0237	*
emp_score	0.00000103479	0.00000007150	14.472	0(approx)	***
educ_score	0.00000088244	0.00000003818	23.112	0(approx)	***
health_score	0.00000085907	0.00000004858	17.683	0(approx)	***
crime_score	0.00000014584	0.00000003258	4.476	0.000008	***
barrier_score	0.00000004633	0.00000002840	1.632	0.1028	
living_score	0.00000026311	0.00000003261	8.069	0(approx)	***

Table 7: Trasformed price_2 vs socio-economic variables.
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

4.4 Partial residue plots

A partial residual plot is a scatterplot that depicts the connection between a specific independent variable (x_i) and the response variable (y) in the presence of other independent variables. A partial residual plot is a significant tool for determining if a given variable has a linear connection with

the response variable. Partial residuals are computed as:

$$\text{Residuals} + \hat{\beta}_i x_i$$ [3]

In a partial residual plot, these partial residuals are plotted against x_i. The termplot function of R helps us to create such plots. This function additionally takes the regression hyperplane's projection to the x_i versus y plane. The slope of the line is the coefficient of x_i in the original linear model. Moreover, it further adds a smooth curve to the plot. This allows us to determine whether the x_i has any non-linear relationships.

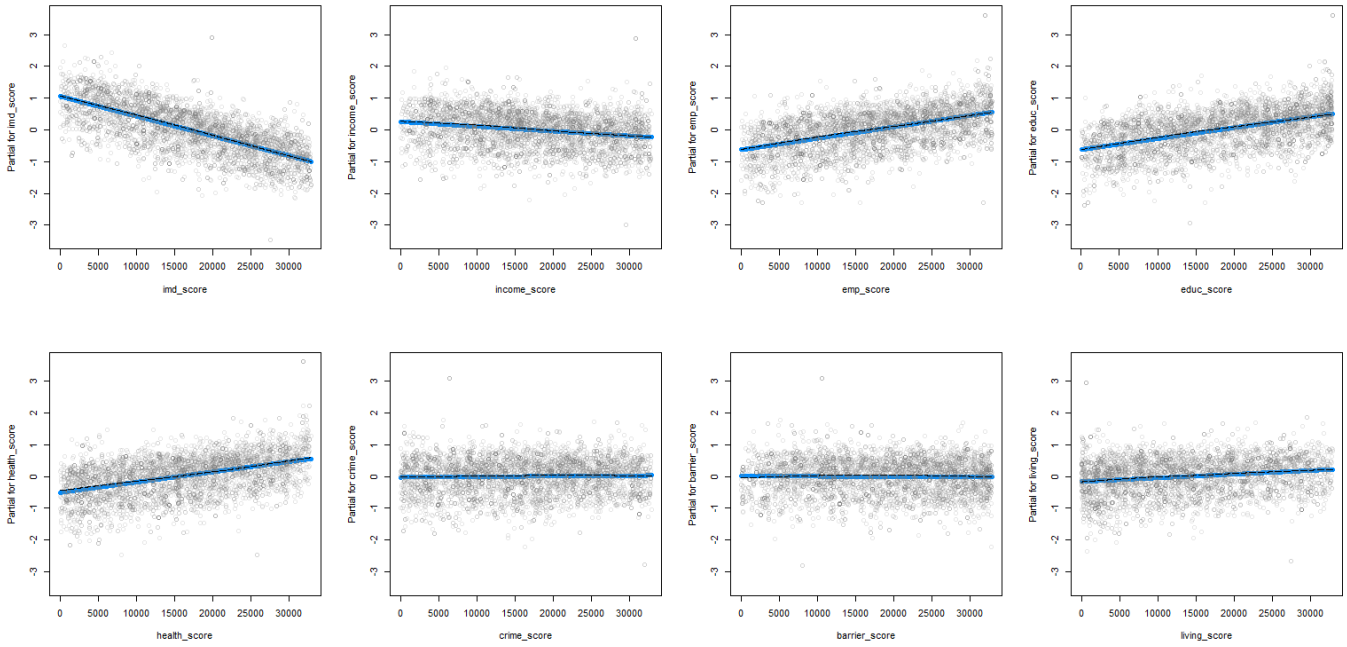

Figure 20: Plot of partial residuals(gray dots) and regression terms(the blue solid line) of transformed price_1 across the predictor variables along with smooth fitted curve

[3] **Note**: The Residuals are the residuals from the full model.

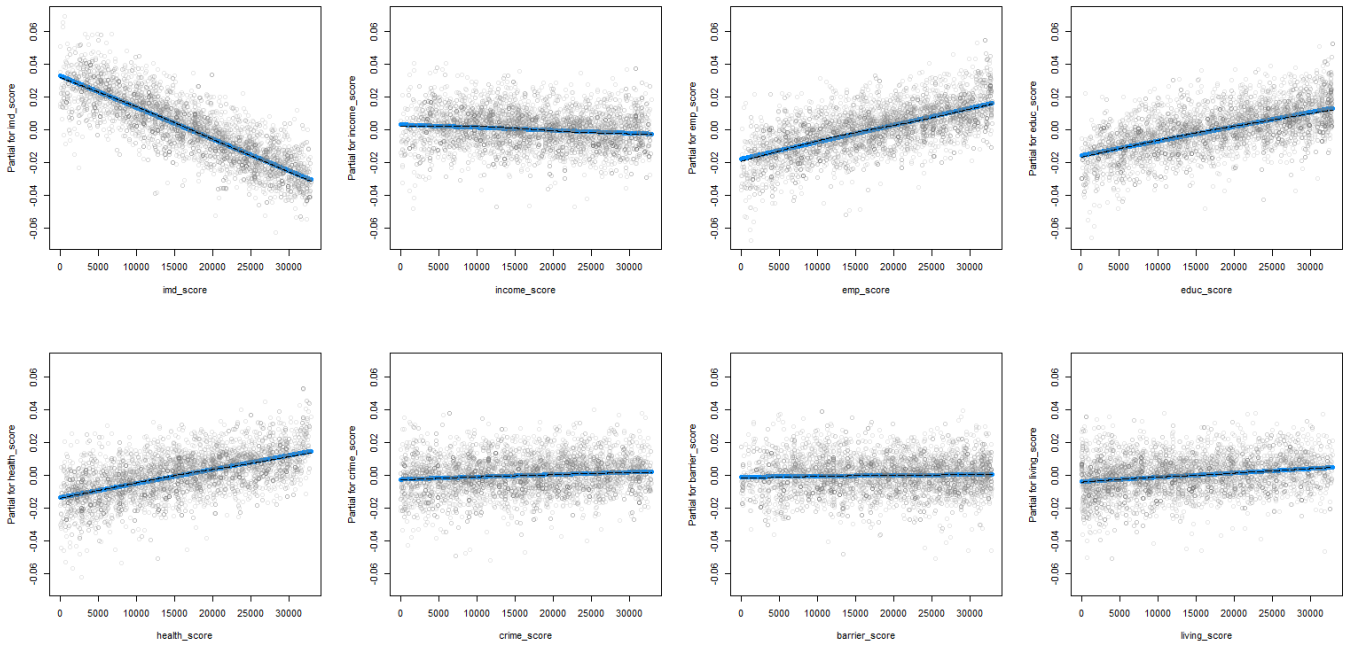

Figure 21: Plot of partial residuals(gray dots) and regression terms(the blue solid line) of transformed price_2 across the predictor variables along with smooth fitted curve(black dashed)

The above plots suggests that there is no non-linear relationship and our model quite efficiently measures the relationship.

5 Repeat sales index

In this section, we utilise the repeat-sales approach to compare the selling prices of the same piece of property over time to see how home values vary over time. If a property does not change, this approach can be used to examine how prices vary over time by comparing the difference in sale prices of the same house.

5.1 Preparing the data for repeat sales index

The original dataset had the repeat sales prices of houses. The repeat sales transactions took place from 1995 to 2012, for this data set. We made a new dataset that consisted of variables Y1995, Y1996, ... and so on upto Y2012.These variables take the value -1 if the first sale transaction occurred in that year, 1 if the second sale transaction occurred in that year, and 0 otherwise, for each house. The variable log_change each derived as:

$$\text{log\_change} = \text{ln\_price\_2} - \text{ln\_price\_1}$$

5.2 Regression

Let there be T time periods where sales can occur from $y_1, ..., y_T$. Each y_t is a categorical variable: $y_t = -1 (y_t = 1$ resp.) if first (second resp.) sale takes place in year y_t and $y_t = 0$ otherwise. Let B'_t denote the (true, but unknown) repeat sales index for year y_t. Let P_{it} and $P_{it'}$ denote the sales prices of ith house at the tth period. A probabilistic model is made as follows:

$$\frac{P_{it'}}{P_{it}} = \frac{B_{t'}}{B_t} U_{itt'}$$

Taking logarithm,

$$\ln(P_{it'}) - \ln(P_{it}) = \ln(B_{t'}) - \ln(B_t) + u_{itt'}$$

where $u_{itt'} = \ln(U_{itt'})$ is the residual. Again, four assumptions on the distribution of the residuals: They have mean 0, constant variance, $u_{itt'} \sim N(0, \sigma^2)$, uncorrelated with each other. The above equation can be written as:

$$\ln(P_{it'}) - \ln(P_{it}) = \sum_{j=1}^{T} y_j \ln(B_j) + u_{itt'}$$

The model in the above equation is fit using linear regression and the estimated log indices are transformed to repeat sales indices with the exponential function. The estimated linear equation will be:

$$\ln(\hat{P}_{t'}) - \ln(\hat{P}_t) = \hat{B}_0 + \sum_{j=1}^{l} \ln(\hat{B}_j) y_j$$

We run OLS regression using R where the dependent variable is log_change and the regressors are Y1995, Y1996, ..., Y2012 (called $y_1, ..., y_T$, $T = 18$). The Y1995 is taken as base year and is held out of the regression.

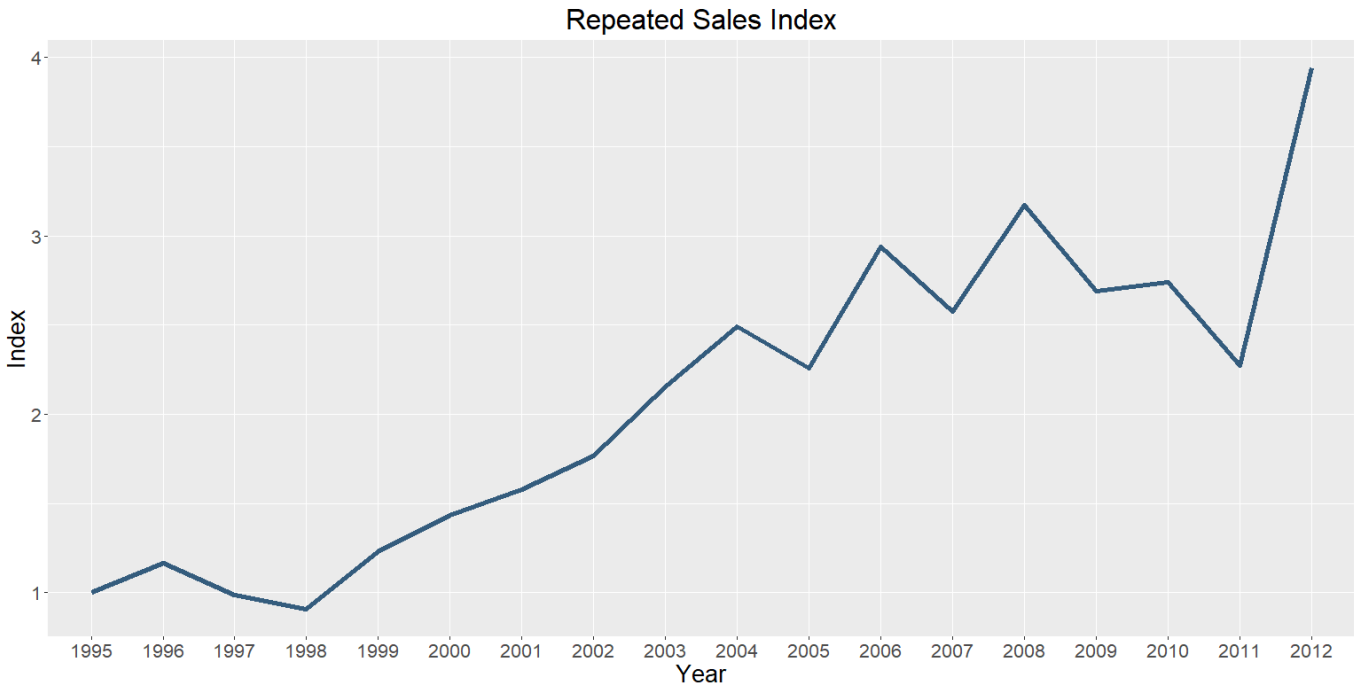

Figure 22: Repeat Sales Index of each year with base year being 1995

The Figure 22 shows the line graph of the estimated coefficients. There is increase of change in prices over years. The inflation is prices is evident the figure. There are major retractions in 1998, 2011. The index reaches its maximum at 2012.

6 Conclusions

Landlords can make informed investments when there is a clear link between price and energy efficiency. In addition, we have taken regional aspects into account. We linked the pricing to socio-economic parameters in the second model. Energy efficiency features have a moderate but considerable influence on both transaction prices, according to the empirical data. Furthermore, the current study was unable to regulate the correlation between factors. Moreover, our model was only focused on pricing, and we were unable to establish a link between EPCs and Regions. In order to produce a comprehensive idea on this divided incentive dilemma, the data can also be enlarged to a much larger database.

6.1 Limitations

- For hedonic regression, a large data must be collected and processed with. However, the data that was utilised only had 4201 values. The Hedonic Price Index calculates how much

individuals are prepared to pay for alleged differences in environmental quality and its implications. However, if individuals are ignorant of the link between environmental attributes and property worth, the value will not be reflected in the price.

- The repeat sales technique has a key flaw in that it excludes homes that sell only once during the data collection period. Another issue is that all houses age, and as a result, the houses alter, which might impact the index, which is not taken into account here.
- While using box-cox transformation, the lambda obtained for price_2 was not zero, and is therefore not possible to interpret. Also box-cox transformation does not always guarantee normality.

6.2 Extensions

The chief findings support existing literature on "Is there an Economic Case for Energy-Efficient Dwellings in the UK Private Rental Market?" by contributing to the observational evidence on the link between energy efficiency ratings and pricing decisions in the housing market, and establishing that the aspects covered by EPCs are broadly important to housing price. Here is the list of extension we have attempted, which were not there in the original paper.

1. Plots for visualization viz.
    - Figure 7
    - Figure 9
    - Figure 10
    - Figure 11
2. Diagnostic plots in subsection 3.3
3. Box-Cox transformation in subsection 4.1
4. Partial Residue Plots, subsection 4.4
5. Repeat sales index of the the price variables across years, section 5

6.3 Future Scope

More complex regressions can be applied in terms of statistical approaches. The correlation between components could not be controlled in the current investigation. The Figure 7 clearly

demonstrates that several factors that we expected to be independent have a significant degree of correlation. Case and Quigley's (1991) hybrid index can also be used. The Case-Shiller(1987, 1989) regression method can be used to make a development on the repeat sales index. A recent method called autoregressive index has been developed by Nagaraja, Brown, and Zhao (2011). On a bigger dataset, the analysis may be run with additional variables, taking into account, housing attributes, etc.

7 References

1. Franz Fuerst, Michel Ferreira Cardia Haddad(2020). "Real estate data to analyse the relationship between property prices, sustainability levels and socio-economic indicators." Data in Brief. 33 106359.

2. F. Fuerst, M.F.C. Haddad, H. Adan(2015). "Is there an economic case for energy-efficient dwellings in the UK private rental market?" Journal for Cleaner Prod. 245 (2020) 118642.

3. F. Fuerst , P. McAllister , A. Nanda , P. Wyatt(2015). "Does energy efficiency matter to home-buyers? an investigation of EPC ratings and transaction prices in England?" Energy Economy. 48 145–156 .

4. Bailey, M.J., Muth, R.F., Nourse, H.O. (1963). "A regression method for real estate price index construction. Journal of the American Statistical Association." 58 933-942

5. Ryan, Thomas P (2015). Modern Regression Methods. Wiley-Interscience.

6. G. E. P. Box and D. R. Cox(1964). "An Analysis of Transformations." Journal of the Royal Statistical Society. Series B Vol. 26. No. 2.

7. Case, K.E., Shiller, R.J. (1987). Prices of single-family homes since 1970: new indexes for four cities. New England Economic Review. Sept./Oct. 45-56.

8. Case, K.E., Shiller, R.J. (1989). The efficiency of the market for single family homes. The American Economic Review. 79 125-137

9. Case, B., Quigley, J.M. (1991). The dynamics of real estate prices. The Review of Economics and Statistics. 73 50-58

10. Chaitra H. Nagaraja. Lawrence D. Brown. Linda H. Zhao(2011). "An autoregressive approach to house price modeling." Ann. Appl. Stat. 5 (1) 124 - 149, March 2011.

11. Repeat Sales House Price Index Methodology. Journal of Real Estate Literature. Vol. 22, No. 1 (2014), pp. 23-46.

Acronyms

EPC Energy Performance Certificate. 4, 6, 7, 15, 16, 18, 31, 33

GOR Government Offices for the Regions. 10

IMD Index of Multiple Deprivation. 4, 8

LSOA Lower Layer Super Output Areas. 7, 8

OLS Ordinary Least Squares. 12, 13, 21, 30

ONS Office for National Statistics. 10

SAP Standard Assessment Procedure. 6, 7

SSE Standard Square Error. 13

Glossary

barrier_score Barriers to housing and services rank (where 1 is most deprived) assigned to the property. 7, 24

crime_score Crime rank (where 1 is most deprived) assigned to the property. 7, 24

educ_score Education skills and training deprivation rank (where 1 is most deprived) assigned to the property. 7, 24

emp_score Employment deprivation rank (where 1 is most deprived) assigned to the property. 7, 24

health_score Health deprivation and disability rank (where 1 is most deprived) assigned to the property. 7, 24

imd_score Index of multiple deprivation (IMD) rank (where 1 is most deprived) assigned to the property. 8, 24

income_score Income deprivation rank (where 1 is most deprived) assigned to the property. 7, 24

living_score Living environment deprivation rank (where 1 is most deprived) assigned to the property. 8, 24

price_1 The first sales transaction Price of the house. 5, 17, 18, 24, 26–28

price_2 The second sales transaction Price of the house. 5, 17, 24, 26, 27, 29, 32