

BQP

1. Functions computable by polynomial sized circuits (uniform families or even just periodic circuits) of TPCPs.
2. Circuit cleanup - Using gate-by-gate Stinespring dilations can reduce to
 - a) Unitary gates
 - b) Ancilla initialization
 - c) Measurement at the end
3. Reasonably intuitive, realistic extension of (2) is a probabilistic classical TM with a quantum tape.

The **B**, by analogy with **BPP** means bounded error probabilistic. Shoenhorns **Prob/QProb** models into framework of deterministic questions. Input is classical deterministic.

$\Pr(\text{Correct answer at the end}) > \frac{2}{3}$ (say)

It's even better to say $> 1 - \epsilon \cdot \text{poly}(|\text{input}|, \log(\epsilon))$.

QFT vs. DFT on $\mathbb{Z}/2^n\mathbb{Z}$

Input: n qubits vs. 2^n floats.

Performance: $\text{poly}(n)$ vs. $\mathcal{O}(N \log N)$ where $N = 2^n$.

$$\mathbb{Z}/2^n\mathbb{Z} \hookleftarrow \mathbb{Z}/2^{n-1}\mathbb{Z} \hookleftarrow \mathbb{Z}/2^{n-2}\mathbb{Z} \hookleftarrow \mathbb{Z}/2^{n-2}\mathbb{Z} \hookleftarrow \dots$$

The basic (classical + quantum) complexity classes:

