

EAFIT UNIVERSITY

IT AND SYSTEMS DEPARTMENT

CHOICE OF THE PROJECT

PSEUDOCODES

Course: Numerical analysis.

Teacher: Edwar Samir Posada Murillo.

Semester: 2020-2.

Project name: Numerical views.

Project Repository: [Link Repo1](#) [Link Repo2](#)

Members:

Mariana Ramírez Duque (marami21@eafit.edu.co)

Nicolás Roldán Ramírez (nroldanr@eafit.edu.co) Mateo

Sánchez Toro (msanchezt@eafit.edu.co) Maria Cristina

Castrillon (Mcastri6@eafit.edu.co)

Report description: We will write the pseudocode of each method. The code can be found on the repository link cited above

1 Methods:

Algorithm 1: Incremental search

```
input : f: function to find root of, X0:  
        first root approximation,  
        Delta: delta(x),  
        N: maximum number of iterations,  
  
    if (y=0) then  
    |   x0 is root;  
    else  
    |   x1 ← [x0  
    |   delta] y1 ← [f  
    |   (x1)] counter  
    |   ← [1]  
    while y1 * y0 > 0 and counter < n dodo  
    |   x0 ← [x1]  
    |   y0 ← [y1]  
    |   x1 ← [x0 + delta]  
    |   y1 ← [f(x1)]  
    |   counter ← counter + 1  
    if (y1==0) then  
    |   x1 is root;  
    else  
    |   if (y1 * y0 < 0) then  
    |   |   [x0,x1] define an interval;  
    |   else  
    |   |   print("didn't find interval in N iterations")
```

Algorithm 2: Bisection

```
input :  $x_i$ : lower limit of the interval,  $x_s$ :  
         the upper range,  
          $f$ : function to find,  
          $N$ : maximum number of iterations,  $tol$ :  
         tolerance,  
  
          $y_i \leftarrow f(x_i)$   
          $y_s \leftarrow f(x_s)$   
         if ( $y_i = 0$ ) then  
         |   print("xi is root")  
         else  
         |   if ( $y_s = 0$ ) then  
         |   |   print("xs is root")  
         else  
         |   if ( $y_i \neq y_s > 0$ ) then  
         |   |   print("Inappropriate range")  
         else  
         |    $x_m \leftarrow (x_i + x_s)/2$   
         |    $y_m \leftarrow f(x_m)$   
         |    $error \leftarrow tol + 1$   
         |    $counter \leftarrow [1]$   
         while  $y_m \neq 0$  and ( $error > tol$ ) and ( $counter < n$ ) do do  
         ( $y_i \neq y_m > 0$ )  $x_i \leftarrow x_m$ ,  $y_i \leftarrow y_m$   
         else  
         |    $x_s \leftarrow x_m$ ,  $y_s \leftarrow y_m$   
         |    $x_{aux} \leftarrow x_m$   
         |    $x_m \leftarrow x_i + x_s/2$   
         |    $y_m \leftarrow f(x_m)$   
         |    $error \leftarrow x_m - x_{aux}$   
         |    $counter \leftarrow +1$   
         if ( $y_m == 0$ ) then  
         |   print("xm is root")  
         else  
         |   if ( $error < tol$ ) then  
         |   |   print("Xm is root with an error equal to error")  
         else  
         |   print("The method failed")
```

Algorithm 3: False Rule

input : a: lower limit of the interval, b:
the upper range,
f: function to find,
N: maximum number of iterations, tol:
tolerance,

$ya \leftarrow f(a)$
 $yb \leftarrow f(b)$
if ($ya == 0$) **then**
| print("a is root")
else
| **if** ($yb == 0$) **then**
| | print("b is root")
else
| **if** ($ya * yb > 0$) **then**
| | print("Inappropriate range")
else
| $p \leftarrow a - f(a) * (b - a) / f(b) - f(a)$
| $yp \leftarrow f(p)$
| $error \leftarrow tol + 1$
| $counter \leftarrow [1]$
while $ypf = 0$ and ($error > tol$) and ($contador < n$) **do do**
($ya * yp > 0$) $a \leftarrow p$, $ya \leftarrow yp$
else
| $b \leftarrow p$, $yb \leftarrow yp$
| $p_{aux} \leftarrow p$
| $p \leftarrow a - f(a) * (b - a) / f(b) - f(a)$
| $yp \leftarrow f(p)$
| $error \leftarrow p - p_{aux}$
| $counter \leftarrow +1$
if ($yp == 0$) **then**
| print("p is root")
else
| **if** ($error < tol$) **then**
| | print("p is root with an error equal to error")
else
| print("The method failed")

Algorithm 4: Multiple roots

input : f: function to find root of, fprime: derivative of f, f2prime: second derivative of f, tol: error tolerance,
N: maximum number of iterations,
X0: first root approximation

output: root approximation x

```
x ← x0
fun ← (x)
funprime ← fprime(x)
fun2prime ← f2prime(x)
error ← infinity
for i ← 0 to N do
    if error ≤ tol(i,i) = 0 then
        | break;
    error ← xx ← x0
    fun ← f(x)
    funprime ← fprime(x)
    fun2prime ← f2prime(x)
    error ← infinity
    error ← abs(error - x)
```

Algorithm 5: Gaussian elimination

input : nxn matrix A, column vector b

output: solution vector x

```
if (A is not square) or (size of A and size of b are not computable) then
    | break ;
if det(A) = 0 then
    | break ;
A ← [A b]
for i ← 1 to n - 1 do
    if  $\bar{A}(i,i) = 0$  then
        | fund l such that  $\bar{A}(l,i) \neq 0$  ;
        | switch  $\bar{A}(i)$  and  $\bar{A}(l)$ 
        for j ← i + 1 to n do
            | multiplier  $M_{j,i} \leftarrow \frac{\bar{A}(j,i)}{\bar{A}(i,i)}$ 
            |  $\bar{A}_j \leftarrow \bar{A}_j - M_{j,i} \bar{A}_i$ 
x ← susreg( $\bar{A}$ )
```

Algorithm 6: Gaussian elimination with partial pivot

input : square $n \times n$ matrix A , n vector b
output: solution vector x
if $\det(A) == 0$ **then**
 break ;
for $i \leftarrow 1$ **to** $n - 1$ **do**
 champion $\leftarrow i$
 for $j \leftarrow \text{champion} + 1$ **to** $n - 1$ **do**
 if $\text{abs}(A(j,i)) == \text{abs}(A(\text{champion},i))$ **then**
 champion $\leftarrow i$
 Swap row in $A(\text{champion})$ with $A(i)$
 Swap value in $b(\text{champion})$ with $b(i)$
 for $j \leftarrow i + 1$ **to** $n - 1$ **do**
 multiplicand $\leftarrow \frac{A(j,i)}{A(i,i)}$
 $A_j \leftarrow A_j - M_{j,i} * A_i$
 $x \leftarrow \text{susreg}(A)$

Algorithm 7: Gaussian elimination with total pivot

input : square $n \times n$ matrix A , n vector b
output: solution vector x
if $\det(A) == 0$ **then**
 break ;
posStamp $\leftarrow [from 0 \text{ to } n - 1]$
for $i \leftarrow 1$ **to** $n - 1$ **do**
 row $\leftarrow i$
 col $\leftarrow i$
 for $j \leftarrow \text{row}$ **to** $n - 1$ **do**
 for $k \leftarrow \text{col}$ **to** $n - 1$ **do**
 if $\text{abs}(A(i,j)) == \text{abs}(A(\text{row},\text{col}))$ **then**
 row $\leftarrow i$ col $\leftarrow j$
 if col $\neq i$ **then**
 Swap column $A(\text{col})$ with $A(i)$ Swap value in posStamp(col) with posStamp(i)
 if row $\neq i$ **then**
 Swap row $A(\text{row})$ with $A(i)$ Swap value in $b(\text{row})$ with $b(i)$
 for $j \leftarrow i + 1$ **to** $n - 1$ **do**
 multiplicand $\leftarrow \frac{A(j,i)}{A(i,i)}$
 $A_j \leftarrow A_j - M_{j,i} * A_i$
 $x \leftarrow \text{susreg}(A)$
Sort x using posStamp

Algorithm 8: Fixed Point

input : $f(x)$, $g(x)$, x_0 , tol , N
output: The approximate root
Define variables
 $\text{int } i = 0$; double
 $x = x_0$;
Start While ($\text{AbsoluteValue}(f(x)) \leq \text{tol}$ $\vee i = N$) $x = g(x)$;
 $i++$;
End While
Start if ($\text{AbsoluteValue}(f(x)) \leq 1E-8$)
print "Root:" + x ;
else
print "Not possible to obtain root"
End if

Algorithm 9: Secant

input : iter , x_i , $f(x_i)$, E
output: The approximate root
 $I = f(x_0)$
If $I = 0$ Then Show:
" x_0 is Root" $Y_1 = f(x_1)$
 (X_1)
Counter = 0
Error = Tolerance + 1
While
 $X_2 = X_1 - ((Y_1 * (X_1 - x_0)) / \text{Den})$
Error = $\text{Abs}((X_2 - X_1) / X_2)$ $x_0 = X_1$
 $I = Y_1$ $X_1 = X_2$
 $Y_1 = f(X_1)$
Counter = Counter + 1 End
while
If $Y_1 = 0$ Then
Show: " x_1 is Root"
Otherwise If Error \leq Tolerance Then
Show: " x_1 'is an approximate root with a tolerance' Tolerance " Otherwise If
Den = 0 Then
Show: 'There is possibly a multiple root'

Secant Newton

Algorithm 10: Newton

```
input : f(x), g(x), x0, tol, N
output: The approximate root
if fx == 0:
    return the root
end if
if dfun == 0:
    return Error, derivative cannot be 0. end
if
    counter = 0
    error = tolerance + 1
    while error > tol and fx != 0 and dfx != 0 and counter < iterations:
        xn = xi - (fx / dfx)
        fx = fun.evaluate2 (xn)
        dfx = dfun.evaluate2 (xn)
    while
        if type(error) == 0 :
            error = — xn-xi —
        else:
            error = — (xn-xi) / xn —
        end if
        xi = xn
        counter +1
    if
        fx == 0:
            return xi is root
        elif error < tol:
            return xn is an approximation to a root with a tolerance ”
        elif dfx
            == 0:
            return xn is a possible multiple root
        else:
            return The method failed in n iterations
    end if
```

Extra method Aikten Extra

method Steffensen

Algorithm 11: Extra method Aikten

input : function f , float tolerance, integer maxIterations
output: solution vector x

$x \leftarrow f(1)$
 $x1 \leftarrow f(2)$
 $x2 \leftarrow f(3)$
 $aikten1 \leftarrow 1$ $aikten2 \leftarrow 0$
for $i \leftarrow 1$ **to** $maxIterations$ **do**
 if $abs(aikten1 - aikten0) \neq tolerance$ **then**
 break ;
 $aikten1 \leftarrow aikten2$ $aikten2 \leftarrow aiktkenEcuation(x, x1, x2)$
 $x \leftarrow x1$ $x1 \leftarrow x2$ $x2 \leftarrow f(i + 3)$

Algorithm 12: Extra method Steffensen

input : function f , float tolerance, integer maxIterations, float approximation
output: root of f

$x0 \leftarrow approximation$
 $x1 \leftarrow f(x0)$
 $x2 \leftarrow f(x1)$
 $x3 \leftarrow aiktkenEcuation(x0, x1, x2)$
for $i \leftarrow 1$ **to** $maxIterations$ **do**
 if $abs(x0 - x3) \neq tolerance$ **then**
 break ;
 $x0 \leftarrow x1$
 $x1 \leftarrow x2$
 $x2 \leftarrow f(i + 3)$
 $x3 \leftarrow aiktkenEcuation(x0, x1, x2)$
 $x \leftarrow x3$

Algorithm 13: Extra method Muller

input : function f , root approximation 1 x_0 , root approximation 2 x_1 , root approximation 3 x_2 , tolerance tol , Maximum number of iterations N

output: root approximation

$h_1 = x_1 - x_0$

$h_2 = x_2 - x_1$

$t_1 = (f(x_1) - f(x_0))/h_1$

$t_2 = (f(x_2) - f(x_1))/h_2$

$d = (t_2 - t_1)/(h_2 + h_1)$

for $i = 3$ **to** N **do**

$b \leftarrow t_2 + h_2 * d$

$D = (b^2 - 4 * f(x_2) * d)^{1/2}$

if $abs(b - D) < abs(b + D)$ **then**

$E = b + D;$

else

$E = b - D$

$h = (-2 * f(x_2))/E$

$e = x$

$x = x_2 + h$

$e = abs(e - x)$

if $abs(e) \leq tol$ **then**

break;

$x_0 = x_1$

$x_1 = x_2$

$x_2 = x$

$h_1 = x_1 - x_0$

$h_2 = x_2 - x_1$

$t_1 = (f(x_1) - f(x_0))/h_1$

$t_2 = (f(x_2) - f(x_1))/h_2$

$d = (t_2 - t_1)/(h_2 + h_1)$

Vandermonde

Algorithm 14: Vandermonde

input : x, y

output: Polynomial coefficients, Vandermonde polynom

Repeat

Repeat

$A(j,i) = X(j)^{(n-i)}$

until $(j < n)$ **until** $(i$

$< n)$

print: $(\text{Reverse } A) * y$

Newton Divided difference

Algorithm 15: Newton Divided difference

input : vector x_0, x_1, \dots, x_n , vector values $f(x_0)$; a point to evaluate

output: Divided differences $F_{0,0} \dots F_{n,n}$

Step 1

for $i=0, \dots, n$ **do**

 Set $F_{i,0} = f(x_i)$

Step 2

for $i=1, \dots, n$ **do**

for $j=1, 2, \dots, i$ **do**

 Set $F_{i,j} = (F_{i,j-1} - F_{i-1,j-1}) / (x_i - x_{i-j})$

 End ;

Output $(F_{0,0}, \dots, F_{i,i}, \dots, F_{n,n})$

STOP

Algorithm 16: Lagrange

input : vector x_0, x_1, \dots, x_n , vector values $f(x_0)$; a point to evaluate

output: P Lagrange polynomial $P(x)$ evaluated at z

Step 1

Initialize variables. Set P_z equal zero. Set n to the number of pairs of points (x, y) .

Set L to be the all ones vector of length n

Step 2

for $i=1, 2, \dots, n$ **do**

 Step 3

for $j=1, 2, \dots, i$ **do**

 Step 4

if $i = j$ **then** $L_i = (z - x_j) / (x_i - x_j) * L_i$ **then**

 break ;

 Step 5

$P_z = (L_i * y_i + P_z)$

Step 6 Output P_z . STOP

Lagrange Neville

Algorithm 17: Neville

input : Numbers x_0, x_1, \dots, x_n , values $f(x_0), f(x_1), \dots, f(x_n)$

output: the table Q with $p(x) = Q_{n,n}$

Step 1

for $i=1, 2, \dots, n$ **do**

for $j=1, 2, \dots, i$ **do**

 Set $Q(i,j) = ((x - x_{i-j})Q_{i,j-1} - (x - x_i)Q_{i-1,j-1}) / (x_i - x_{i-j})$

Step 2

Output(Q);

STOP

Simple LU

Algorithm 18: Simple LU

input : $n \times n$ matrix A , column vector b

output: solution vector x

if (A is not square) or (size of A and size of b are not computable) **then**

 break ;

if $\det(A) = 0$ **then**

 break ;

$A \leftarrow LU$

$z \leftarrow \text{progresiveSustitution}([L \ b])$

$x \leftarrow \text{RegressiveSustitution}([U \ z])$

Algorithm 19: Partial Pivoting LU

input : nxn matrix A, column vector b

output: solution vector x

if (*A is not square*) or (*size of A and size of b are not computable*) **then**

 | break ;

if $\det(A) = 0$ **then**

 | break ;

$PA \leftarrow LU$

$z \leftarrow \text{progressiveSustitution}([L P * b])$

$x \leftarrow \text{RegressiveSustitution}([U z])$

Algorithm 20: SOR

input : nxn matrix A, column vector b, initial aproximation X0, weighing factor w, tolerance, maximum iterations Nmax

output: solution vector x, final number of iterations iter, error err

$A \leftarrow D - L - U$

$T \leftarrow (D - w * L)^{-1} * ((1 - w) * D + w * U)$

$C \leftarrow w * (D - w * L)^{-1} * b$

$xant \leftarrow x0$

$count \leftarrow 0$

$error = 100$

while $error > tolerance$ and $counter < nmax$ **do**

 | $xact = T * xant + C$

 | $error = norm(xant - xact)$

 | $xant = xact;$

 | $cont = cont + 1;$

$x = xact$

$iter = cont$

$err = error$

Algorithm 21: Gauss-Seidel

input : nxn matrix A, column vector b, initial approximation X0, weighing factor w, tolerance, maximum iterations Nmax

output: solution vector x, final number of iterations iter, error err

$$A \leftarrow D - L - U$$

$$T \leftarrow (D - L)^{-1} * U$$

$$C \leftarrow (D - L)^{-1} * b$$

$$x_{ant} \leftarrow x_0$$

$$count \leftarrow 0$$

$$error = 100$$

while $error > tolerance$ and $counter < nmax$ **do**

$$x_{act} = T * x_{ant} + C$$

$$error = norm(x_{ant} - x_{act})$$

$$x_{ant} = x_{act};$$

$$cont = cont + 1;$$

$$x = x_{act}$$

$$iter = cont$$

$$err = error$$

Algorithm 22: Jacobi

input : nxn matrix A, column vector b, initial approximation X0, weighing factor w, tolerance, maximum iterations Nmax

output: solution vector x, final number of iterations iter, error err

$$A \leftarrow D - L - U$$

$$T \leftarrow (D)^{-1} * (L + U)$$

$$C \leftarrow (D)^{-1} * b$$

$$x_{ant} \leftarrow x_0$$

$$count \leftarrow 0$$

$$error = 100$$

while $error > tolerance$ and $counter < nmax$ **do**

$$x_{act} = T * x_{ant} + C$$

$$error = norm(x_{ant} - x_{act})$$

$$x_{ant} = x_{act};$$

$$cont = cont + 1;$$

$$x = x_{act}$$

$$iter = cont$$

$$err = error$$
