EAFIT UNIVERSITY
IT AND SYSTEMS DEPARTMENT

USER'S MANUAL

Objective: The goals of this assignment are to:

1. *Fully understand what you are trying to test by defining the behavior of different methodos*
2. *Provide end-user documentation.*
3. *Give you the opportunity to experiment dynamically with each of the methods*

Course: Numerical analysis.
Teacher: Edwar Samir Posada Murillo.
Semester: 2020-2.
URL: https://numericalviews.sebasmd.com/
Project name: Numerical views.
Project Repository: Link Repo1 Link Repo2

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Contents

0.1 INTRODUCTION

Below you will find a description of how our application is made up and how you can interact with each of its subsection
Our page consists of the following sessions.

1. HOME
2. METHODS
3. ABOUT
4. HELP

1 HOME

1.1 FUNCTION PLOTTER

The application has a grapher which is included in order to show a graphical solution to the user, so he has an idea where some possible roots exist and improve its understanding.
You can modified the graph scale is the function is evaluated again with the new bounds and also select the color from the right side
Input: Funtions
Output: graph

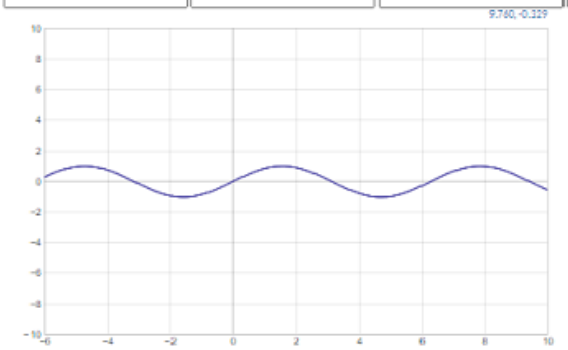
NUMERICAL VIEWS

METHODS ABOUT HELP

FUNCTION PLOTTER

sin(x)

-6 10 -10 10



56 49 155

R G B

Plot

2 METHODS

On this page you can find the consolidated method which is classified as follows:
• FUNCTION ROOTS
• EQUATION SYSTEMS
• FACTORIZATION
• INTERPOLATION
Each of these groups contains their respective methods, which you can access by click on their name.

METHODS

FUNCTION ROOTS

Incremental Search | Bisection | False Position | Newton Method

Fixed Point Method | Secant Method | Multiple Roots

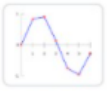
EQUATION SYSTEMS

Doolittle Method | Gauss Seidel Method | Jacobi Method | Gauss Simple Method

Gauss Pivots

FACTORIZATION

LU Simple

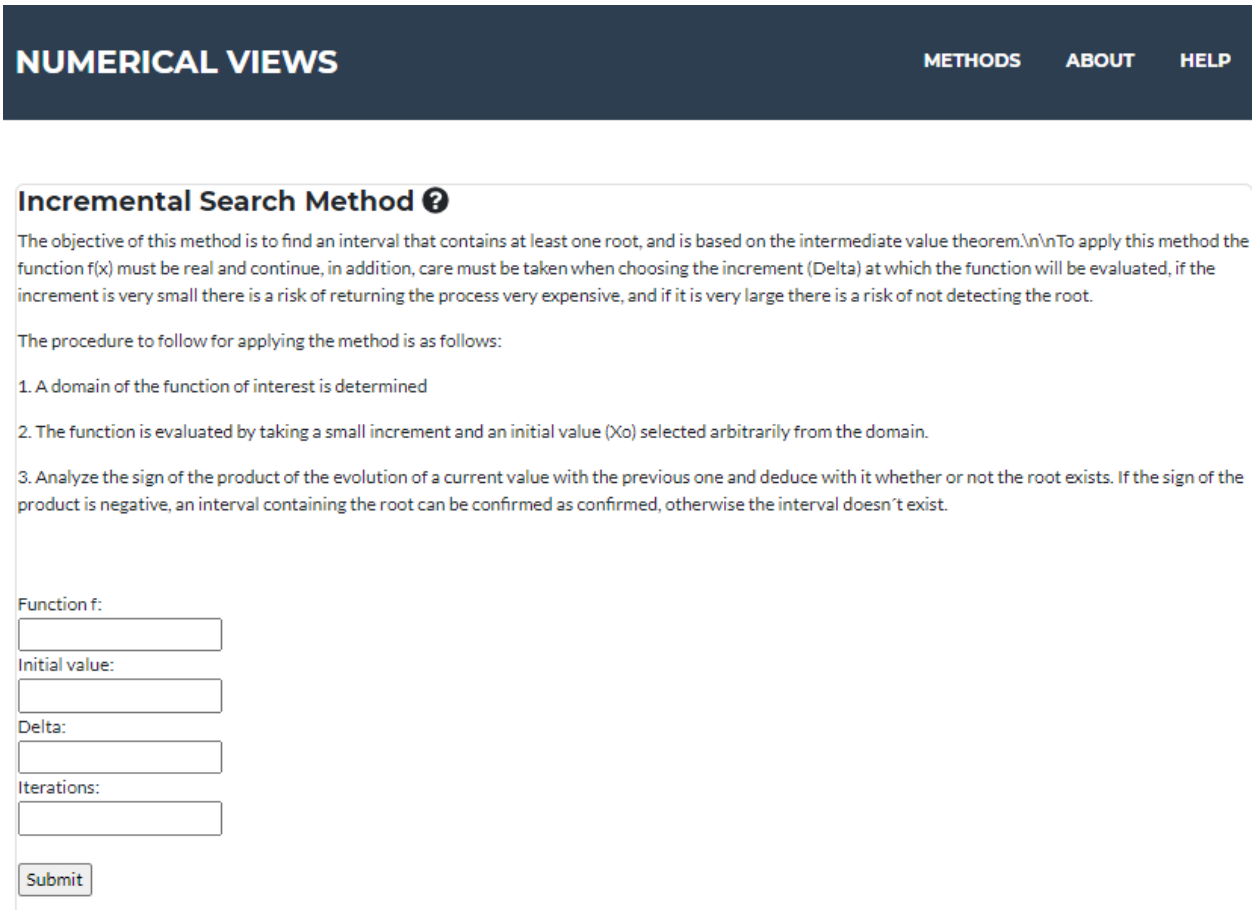


INTERPOLATION

Vandermonde | Newton (Divided difference method) | Lagrange | Splains

Neville

The application has an online help system which aims to guide the user through each method theory, to enter this system it is necessary to find the "?" icon



Next we will describe each group of methods, with their contributions and considerations

2.1 FUNCTION ROOTS

Incremental search The objective of this method is to find an interval that contains at least one root, and is based on the intermediate value theorem. To apply this method the function f(x) must be real and continue, in addition, care must be taken when choosing the increment (Delta) at which the function will be evaluated, if the increment is very small there is a risk of returning the process very expensive, and if it is very large there is a risk of not detecting the root. The procedure to follow for applying the method is as follows:

1. A domain of the function of interest is determined
2. The function is evaluated by taking a small increment and an initial value (Xo) selected arbitrarily from the domain.

3. Analyze the sign of the product of the evolution of a current value with the previous one and deduce with it whether or not the root exists. If the sign of the product is negative, an interval containing the root can be confirmed as confirmed, otherwise the interval doesn´t exist.

Input: Funtion f(x), Initial value, Delta, max Iterations

Considerations:
- The function must be continuous and differentiable.
- The initial value must exist in the function.
- Tolerance must have a positive value.
- The iteration number must be positive.

Bisection The objective of this method is to find the root of a function, taking an initial interval and gradually halving this, until an approximation or the root that satisfies the function is found.

1. Choose an initial interval for function f (x)
2. Afterwards, we seek to find the root with greater accuracy within the interval by dividing in half and observing if the initial conditions are preserved.
3. The Xmed is compared with each of the limits of the interval and it‘s observed that the product changes sign and a new interval is assigned.
4. The process is repeated, and the interval becomes small until it reaches an approximation of the root or the exact root.

Input: Funtion f(x), Lower interval value (a), Higher interval value (b), tolerance, Max Iterations

Considerations:
- The value of a must be minor than b
- Tolerance must have a positive value.
- The iteration number must be positive.
- The quantity of x must be equal to that of y
- The function must be continuous and differentiable.

False positions The objective of this method is to find the intersubsection of a straight formed by points a and b with the x-axis, and to obtain new smaller intervals, which allows an approximation to a root. This method preserves all the characteristics and conditions that the bisubsection method has, except for the way to calculate the intermediate point of the interval The procedure to follow for applying the method is as follows:

1. If you have two points (a, f (a)) and (b, f (b)) and draw the line that joins these two points, you can see that one point is below the x-axis and another above this, and an intermediate point (Xm, 0), with this intermediate point you can compare the limits and obtain a new interval

2. If f (A) and f (B) ¡ 0, then the root is on the left side of the interval.

3. . If f (A) and f (B) ¿ 0, then the root is on the right side of the interval.

4. To find the intersubsection of the line with the X axis we use the following formula: Xm= a - ((f(a)*(b - a))/(f(b) - f(a)))

The False position method converges more quickly than the bisubsection method because when one of its initial values remains fixed, the number of calculations is reduced while the other initial value converges towards the root
Input: Funtion f(x), Lower interval value (a), Higher interval value (b), tolerance, Max Iterations

Considerations:
• The value of a and b must exist in the funtion
• Tolerance must have a positive value.
• The iteration number must be positive.
• The function must be root.
• The function must be continuous and differentiable.

Newton The objective of this method is to find a root of a function from an initial value, a tolerance and a number of iterations, in this case it is not necessary to have an interval. The Newton method due to its speed and effectiveness, it is one of the most used methods; this method is a variable of the fixed point method, so a function g must be calculated, this function g can be calculated in the form: g (X) = X - (f (X) / f '(x)) Once the function g is defined, the following steps must be performed, as in the fixed point method.

1. You must choose an initial approximation Xo

2. Calculates X1 = g(Xo)

3. Calculates X2 = g(X1)

4. Calculates Xn=g(Xn-1)

5. The previous step is repeated until reaching an approximation of the root.

Input: Funtion f(x), Function fprime, Initial x (x0), tolerance, Iterations

Considerations:
• Tolerance must have a positive value.
• The iteration number must be positive.
• The function must be root.
• Newton's method is generally faster than the other methods. If the derivative approaches zero, the method loses its speed because is possible to be a case of multiple root.

Fixed point The objective of this method is to find a root of a function from an initial value, a tolerance and a number of iterations, in this case it is not necessary to have an

interval. From an equation F (X) = 0 an equation X = g (X) is generated, to which a solution is sought, and the following must be taken into account

1. A value of X is searched for, when replacing it in g, the result is X.
2. You must choose an initial approximation Xo
3. X1 = g (Xo) is calculated
4. Calculates Xn=g(Xn-1)
5. And the previous step is repeated until reaching an approximation

Input: Funtion f, Function g, Initial x (x0), tolerance, Iterations

Considerations:
- Tolerance must have a positive value.
- The iteration number must be positive.
- The function must be root.
- The function must be continuous and differentiable.

Secant The objective of this method is to find a root of a function from two initial values, a tolerance and a number of iterations, in this case it is not necessary to have an interval. The secant method is defined as a variant of Newton's method. From the iterative equation that defines Newton's method, the derivative is replaced by an expression that approximates it: X2 = X1 - ((f (X1) * (X1-Xo)) / (f (X1) -f (Xo))

1. You must choose two initial approximations X1 and X0
2. X2 = Expression is calculated, Xn = Expression (n-1)
3. And the previous step is repeated until reaching an approximation.

Input: Funtion f, Initial (x0), Initial (x1), tolerance, Iterations

Considerations:
- Tolerance must have a positive value.
- The iteration number must be positive.
- The function must be root.
- The function must be continuous and differentiable.
- For the method to be faster the initial values must be close to the root

2.2 EQUATION SYSTEMS

Doolittle This method is part of the LU methods for matrix factorization, where a matrix A is factored into L (Upper Triangular) and U (Lower Triangular). Given Ax = b, we would have LUx = b and if Ux = z, then Lz = b. In this specific method, The diagonal of L and U

are equal L [i] [i] = 1. After we have the factorized matrix, progressive substitution is made with the L matrix and the results b to find the values of z, then backward substitution is used to find the values of x, which would be the solution of the system

Input: matrix, b

Considerations:
• The determinant of the matrix cannot be 0.
• The diagonal of the matrix cannot have 0.

Gauss Seidel Gauss-Seidel is an iterative method for determining the solution of equations. It is based on generating approximations to the result based on an initial approximation. The method first tries to convert the given matrix into a diagonally dominant matrix by permutations of rows and columns. Subsequently, each equation is taken and the variable of the diagonal is put in terms of the other variables, after this it is there is the first approximation assigning initial values to the variables using the equations generated previously, the assigned values are obtained through the calculations carried out in the same cycle. This process is repeated until the dispersion value is less than the tolerance value, or until the number of iterations is exceeded

Input: matrix, Xo, B, Tolerance, iterations, Rel

Considerations:
• The determinant of the matrix cannot be 0.
• The diagonal of the matrix cannot have 0.
• Tolerance must have a positive value.
• The iteration number must be positive.

Jacobi Jacobi is an iterative method for determining the solution of equations. It is based on generating approximations to the result based on an initial approximation. The method first tries to convert the given matrix into a diagonally dominant matrix by permutations of rows and columns. Subsequently, each equation is taken and the variable of the diagonal is put in terms of the other variables, after this it is there is the first approximation assigning initial values to the variables using the equations generated previously, the assigned values are obtained through the calculations carried out in the same cycle. This process is repeated until the dispersion value is less than the tolerance value, or until the number of iterations is exceeded.

Input: matrix, Xo, B, Tolerance, iterations, Rel

Considerations:
• The determinant of the matrix cannot be 0.
• The diagonal of the matrix cannot have 0.
• Tolerance must have a positive value.
• The iteration number must be positive.

SOR After calculating the new value X by the Gauss-seidel method, that result is modified for a much more praised result. It´s general formula is:

$$X^{(k+1)} = (1-w)X_i^{(k)} + w\left|-\sum_{j=1}^{i-1}\frac{a_{ij}}{a_{ii}}X_i^{(k+1)} - \sum_{j=i+1}^{n}\frac{a_{ij}}{aii}X_i^{(k)} + \frac{b_i}{a_{ii}}\right|$$

Where: w : Factor of relaxation
w: ¡ 1 Subrelaxation and¿ 1 Overrelaxation
If w = 1 then it´s the general formula of Gauss-Seidel
Input: matrix, Xo, B, Tolerance, iterations, Rel

Considerations:
• The determinant of the matrix cannot be 0.
• The diagonal of the matrix cannot have 0.
• Tolerance must have a positive value.
• The iteration number must be positive.
• It´s to be known that we don't take w = 0 because it would´t find better results on the initial approximation, nor we take w 2 because this causes diversion.

Gauss Simple This method is defined as a set finite of operations whereby is direct. The principal idea of the method is convert a system of equations in other equivalent simpler, by elemental operations of line. Given a system of equations AX=b, where A represent the matrix of coefficients of size nxn, X the vector column of unknown quantity and b the vector of term independent terms; will proceed in the following way:

1. To get the matrix increase Ab of the system
2. To convert all elements under of A11 in zeros using the second operation introduce before. In the process to find a multiplier
3. To convert in zeros all elements under A22.
4. To continue with a similar process to the 3 and 4 processes until to get a triangular matrix, that is, a matrix with all elements under the principal diagonal equal to 0
5. To find the value of the Xn term, with this value to find Xn-1 and like this with regressive substitution until get the X1 term.

Input: matrix, B

Considerations:
• The determinant of the matrix cannot be 0.
• The diagonal of the matrix cannot have 0.

• Propagation and rounding error: Is the error that to expand in the intermediate operations maked and that to have influence in the result.

Gauss Pivot Is a method that to search in each stage of the process of the gaussian elimination that the multiplier (Aik/Akk) be as small as possible, that is, to search to replace the largest element in the principal diagonal. Above mentioned to solve the problem of an element of diagonal near to 0 that to be in the simple gaussian elimination.
Basically, the method woks how the simple gaussian elimination with an operation more in each stage, search the largest element in the principal diagonal. Then, given a system of equations AX=b, where A represent the matrix of coefficients of size nxn, X the vector column of unknown quantity and b the vector of term independent terms; will proceed in the following way:

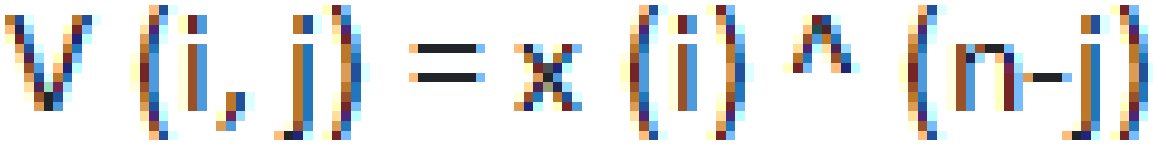
1. To get the matrix increase Ab of the system
2. To search the largest element in absolute value of the first column and to exchange the line that to have it with the first line, otherwise that the first line has it
3. To convert all elements under of A11 in zeros using the second operation introduce before. In the process to find a multiplier.
4. Again, to search the largest element in absolute value of the second column that to be in positions largest or equal to A22. Then, to exchange the line that contain that with the second line, otherwise that the second line has it
5. To convert in zeros all elements under A22
6. To do a similar process of the steps 2,3, 4 and 5 until to find a triangular superior matrix.
7. To find the value of the Xn term, with this value to find Xn-1 and like this with regressive substitution until get the X1 term.

Input: matrix, B

Considerations:
• The determinant of the matrix cannot be 0.
• The diagonal of the matrix cannot have 0.
• This method solves problems that can happen in the execution and the solution of the simple gaussian elimination. That is, this method doesn't have problems in the execution when to find a zero in the principal diagonal, unless the elements below that number are also zeros, which indicates that the system hasn't unique solution. Equally, the method reduces the rounding effect that occurs in the divisions with the numerator close to 0.

2.3 INTERPOLATION

Vandermonde A Vandermonde matrix is one that presents a geometric progression in each row. The indices of the matrix of size n×n
In the first element of each row there are only ones (being the power of zero) and in the second element there are a series of arbitrary numbers. In the third are those same numbers squared. In the fourth are those same numbers cubed and in the following columns raised to the immediately higher power so that in element n of each row those numbers are raised to the power n-1.
The elements of the matrix are given in a general way as:



Where n is a dimension of the matrix, x(i) are the values of the coordinate vector and j are the columns. The objective of this method is to find an interpolating polynomial that represents a function or a problem without exact definition, that passes through a set of given points

Input: Matrix
To establish the size of the matrix, enter the number in the variables field in this way, the application assigns the requested spaces.

Considerations:
• The quantity of x must be equal to that of y
• X vector or Y vector can´t contain a repeat value.

Newton (divided difference) Newton (Divided difference method) takes as parameters a set of points and a value to evaluate and using an approximation of the Taylor series, calculates the corresponding polynomial and evaluates the value.
p(x) = b0 + b1(x-x0)+b2(x-x0)(x-x1)+b3(x-x0)(x-x1)(x-x2)
Where the bi are the diagonal of the aux matrix and are calculated taking into account the previous rows. This b0 = f [x0] nb1 = f [x1, x0] nb2 = f [x2, x1, x0] nbn = f [xn, ..., x0]

Input: Matrix
To establish the size of the matrix, enter the number in the variables field in this way, the application assigns the requested spaces.

Considerations:
• The quantity of x must be equal to that of y
• X vector or Y vector can´t contain a repeat value.

Lagrange The objective of this method is to find an interpolating polynomial that represents a function or a problem without exact definition, that passes through a set of given points.

1. We consider n the given points x of the function, with their respective value of y; given these points there is a single polynomial of degree at most n-1 that passes through them.

2. In order to define the polynomial, find the values of each Li (x) with i from 1 to n

3. Once the values of L have been calculated, we can confirm the polynomial and then some additional calculations.

4. The values of L can be calculated from the following structure, additionally the structure of the Lagrange polynomial can be observed.

Input: Matrix
To establish the size of the matrix, enter the number in the variables field in this way, the application assigns the requested spaces.

Considerations:
• The quantity of x must be equal to that of y
• X vector or Y vector can´t contain a repeat value

Splains (lineal, quadratic, cubic) The Splains seeks to interpolation of a given data set, this set must be composed by x,y pairs for which the method generate a corresponding piecewise function s(x). There are 3 kinds of splain (lineal, cuadratic and cubic) which will determine the degree of the funcions defined in s(x). The set of x values must be unique between them, otherwise the method wont be able to interpolate.

1. Variables: number of x values you want to set (max=15).

2. Splain: splain method you want to use.

3. X: X values of the data set you want to interpolate.

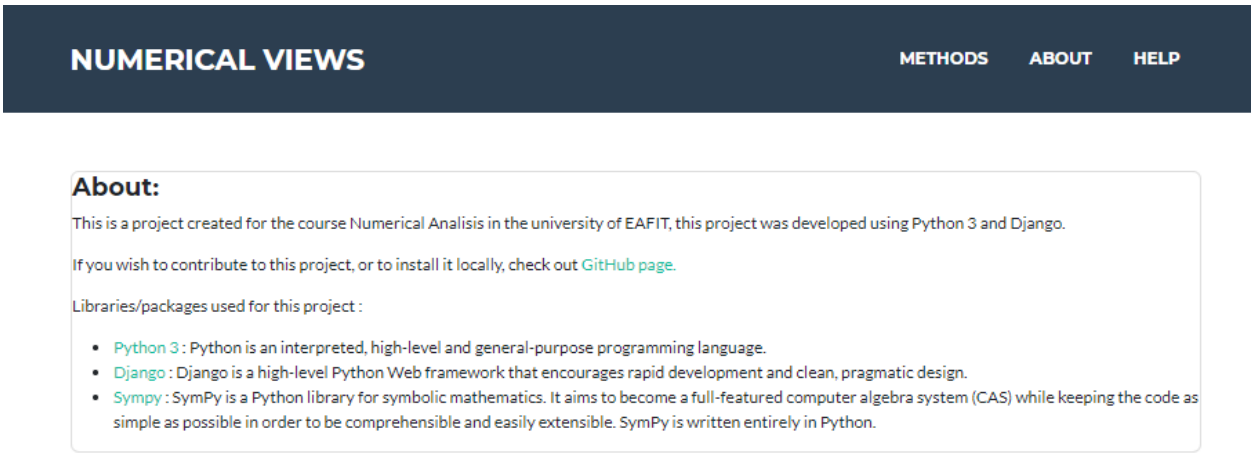
4. Y: Y values of the data set you want to interpolate matching verticaly the corresponding X value.

Input: Matrix
To establish the size of the matrix, enter the number in the variables field in this way, the application assigns the requested spaces.

Considerations:
• The quantity of x must be equal to that of y
• X vector or Y vector can´t contain a repeat value

3 ABOUT

On this page you can find information associated with libraries and packages used.



4 HELP

On this page you can find general information that can help resolve concerns about how to enter the functions.
The application has a description of the method that allows the user to understand what it is about.
The application has an online help system which aims to guide the user through each method theory, to enter this system it is necessary to find the "?" icon.

Help:

The application has an online help system which aims to guide the user in the theory of each method, to enter this system it is necessary display the "?" icon next to each method which you find by doing Click displays a window with the respective help, when finishing the reading the help, it can be closed like any other window by returning the user to the application in a natural way.

- Functions must be written as the function that is taken into account a special notation, some examples are cited in order to clarify how use the different prefixes:
  Sen (x): Sin(x) Ln(x): Log(x) E^(x):E**x X ^ 2:x**2
- EXAMPLE: log(sin(x)**2+1)-1/2

Functions:

- F (x): Simple function
- F '(X): First derivative of this function
- F '' (X): second derivative of the previous function
- G (X): Another different function

If you have identified an error or a bug that is worth reporting, please create an issue on the GitHub project.