Sanchit Jalan
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LA Assignment - 3

Q1 Given: V is set of all complex-valued functions f on the real line s.t. $\forall t \in R$

$$f(-t) = \overline{f(t)}$$

The bar denotes complex conjugation.

To show: (i) Show that V, with foll. properties:-

$$(f+g)(t) = f(t) + g(t)$$

$$(cf)(t) = cf(t)$$

is a vector space over the field of R ~~but not a vector space over the field of C~~. but not a vector space over field C.

(ii) Give an example of V which isn't real valued

Proof: let $a, b, c \in V$.

(i) Associativity in addn:-

$$(a+(b+c))(t) = a(t) + (b+c)(t) = a(t) + b(t) + c(t)$$

$$= (a+b)(t) + c(t) = ((a+b)+c)(t).$$

$\therefore \forall t \in R \quad a, b, c \in V \quad a+(b+c) = (a+b)+c.$

(ii) Commutativity in addition:

$$(a+b)(t) = a(t) + b(t) = b(t) + a(t) = (b+a)(t)$$

so, $\forall t \in R$, $a, b \in V \Rightarrow a+b = b+a$

(iii) Additive identity:

Let $x(t)$, $x \in V$ be s.t. $x(t) = 0 \quad \forall t \in R$.

$$(a+x)(t) = a(t) + x(t) = a(t).$$

$\forall t \in R$, $a + x = a \quad \forall a \in V$.

(iv) Additive inverse:

Let $\exists$ a fn $(-a)(t) = -a(t)$

$$\overline{-a(t)} = -a(-t) \quad (a \in V)$$

$\therefore (-a)(t) \in V.$

$$(a+(-a))(t) = a(t) + (-a)(t) = a(t) + (-a(t)) = 0$$

$\therefore \forall a \in V, \exists -a \in V$ s.t. $a + (-a) = 0$, $\forall t \in R$.

Good Write

(v) Associativity in scalar multiplication:

let $a, b \in \mathbb{R}$, $f \in V$

$(a(bf))(t) = a \times (bf)t = ab \times f(t)$

$= ((ab)f)(t)$.

$\forall\, a, b \in \mathbb{R}$, $f \in V$ $\quad a(bf) = (ab)f$, $\forall\, t \in \mathbb{R}$.

(vi) Distribution of scalar over addition

let $a, b \in \mathbb{R}$, $f \in V$.

$((a+b)f)(t) = (af)(t) + (bf)(t) = af(t) + bf(t)$.

$\forall\, a, b \in \mathbb{R}$, $f \in V$

$(a+b)f = af + bf$, $\forall\, t \in \mathbb{R}$.

(vii) Distribution of scalar multiplication over addition of vectors.

let $a \in \mathbb{R}$, $f, g \in V$.

$(a(f+g))(t) = a \times (f+g)(t)$

$= a \times (f(t) + g(t))$

$= af(t) + ag(t) = (af)(t) + (ag)(t)$

$\therefore \forall\, a \in \mathbb{R}$, $f, g \in V$, $a(f+g) = af + ag$, $\forall\, t \in \mathbb{R}$.

(viii) Scalar multiplication by 1, let $f \in V$.

$(1 f)(t) = 1 \cdot f(t) = f(t)$.

$\forall f \in V$, $1 \cdot f = f$, $\forall\, t \in \mathbb{R}$

So, V is satisfying all the condition of a vector subspace.

$\therefore$ V is a vector ~~sub~~space over $\mathbb{R}$.

Hence, Proved.

Now, we need to prove, V is not a vector space over C.

Let $f(t) = it$

$f \notin V$ as $f(-t) = i(-t) = -it = \overline{f(t)}$

$f(t)$ will satisfy other properties as well.

if V is a vector space under C, then.

$m(t) = (x+iy)(f(t)) = (x+iy)(it) = -yt + ixt$

$x, y \in R$ should also be $\in V$. i.e $m \in V$.

$m(-t) = (x+iy) f(-t) = (x+iy)(-it) = yt - ixt$

$m(t) = -yt + ixt$

$\overline{m(t)} = -yt - ixt$

$m(-t) \neq \overline{m(t)}$ $\therefore m \notin V$.

$\therefore$ V is not a vector space over C.

Hence, proved.

Example! $f(t) = it$ is a fn which is not real valued

$f(-t) = -it = \overline{f(t)}$ $\forall t \in R$. $\therefore$, it satisfies the property. But it is not a element of V.

Q2 Thm: The subspace spanned by a non-empty subset S of a vector subspace V is the set of all linear combination of vectors in S.

Given: Subset S of V s.t. S is not empty, i.e.

$S \subseteq V$, $S \neq \phi$.

subspace spanned by $S = A = \{a_1 s_1 + a_2 s_2 \cdots a_k s_k\}$

$s_1, s_2, s_3 \cdots s_k \in S$. $a_1, a_2 \cdots a_k \in R$.

To Prove: A is subspace of V.

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Proof: A subset 'S' of a vector space 'V' is a subspace of the vector space iff. $\forall a, b \in S \Rightarrow a+b \in S$ & $\alpha a \in S \Rightarrow \alpha \in R$

Condition - 1:

Let $x, y \in A$, so they are linear combinations of vectors in S i.e.

$x = \alpha_1 x_1 + \alpha_2 x_2 \dots \alpha_m x_n$ where $\alpha_1, \alpha_2 \dots \alpha_m \in R$ & $x_1, x_2, x_3 \dots x_n \in S$.

$y = \beta_1 y_1 + \beta_2 y_2 \dots \beta_m y_n$, where $\beta_1, \beta_2 \dots \beta_n \in R$ & $y_1, y_2, y_3 \dots y_n \in S$.

$x+y = (\alpha_1 x_1 + \alpha_2 x_2 \dots \alpha_n x_n) + (\beta_1 y_1 + \beta_2 y_2 \dots \beta_n y_n)$

$(\alpha_1, \alpha_2 \dots \alpha_n) \in R$ & $(\beta_1, \beta_2 \dots \beta_n) \in R$.

$(x_1, x_2 \dots x_n) \in S$ $(y_1, y_2 \dots y_n) \in S$

$\therefore$ $(x+y)$ is also a linear combination of S.

$\therefore$ $(x+y) \in A$.

So, condition - 1 satisfied.

Condition - 2:

Let $z \in A$

Let $z = z_1 y_1 + z_2 y_2 \dots z_l y_l$

where $(y_1, y_2 \dots y_l) \in R$ $(z_1, z_2 \dots z_l) \in S$.

Let $r \in R$

$rz = ry_1 z_1 + ry_2 z_2 \dots ry_l z_l$

$ry_1, ry_2 \dots ry_l \in R$

$z_1, z_2 \dots z_l \in S$

$\therefore$ $rz \in A$. So condition 2 satisfied.

$\therefore$ A is subspace of V.

Hence, Proved.