classmate

Date ______

Page ______

# DS Assignment - 3

Q1 (a)

| | | |
|---|---|---|
| 1. | Given | |
| 2. | $(\forall x\, A(x) \wedge \exists x\,(A(x) \to B(x)))$ | [Assm] |
| 3. | $\forall x\, A(x)$ | $\wedge \mathcal{E}(2)$ |
| 4. | $\exists x\,(A(x) \to B(x))$ | $\wedge \mathcal{E}(2)$ |
| 5. | a $A(a) \to B(a)$ | [Assm] |
| 6. | $A(a)$ | $\forall \mathcal{E}(3)$ |
| 7. | $B(a)$ | $\to \mathcal{E}(5)$ |
| 8. | $A(a) \wedge B(a)$ | $\wedge I\ (6,7)$ |
| 9. | $\exists x\ [A(x) \wedge B(x)]$ | $\exists I\ (5,8)$ |
| 10. | $(\forall x\ A(x) \wedge \exists x\,(A(x) \to B(x))) \to [\exists x\,(A(x) \wedge B(x)]$ | $\to I(2\text{-}9)$ |

(b)

| | | |
|---|---|---|
| 1. | | Given |
| 2. | $\forall x\ (A)(x) \to B(x)$ | (Assm) |
| 3. | $\exists x \sim B(x)$ | (Assn) |
| 4. | a $\sim B(a)$ | (Assn) |
| 5. | $A(a) \to B(a)$ | $\forall \mathcal{E}(2)$ |
| 6. | $A(a)$ | [Assn] |
| 7. | $B(a)$ | $\to \mathcal{E}(5)$ |
| 8. | $\sim B(a) \wedge B(a)$ | $\wedge I\ (4,7)$ |
| 9. | $\perp$ | $\perp I\ (8)$ |
| 10. | $\sim A(a)$ | $\sim I(6,\ 7\text{-}9)$ |
| 11. | $\exists x \sim A(x)$ | $\exists I\ (10)$ |
| 12. | $[\exists x \sim B(x) \to \exists x \sim A(x)]$ | $\to I\ (3\text{-}11)$ |
| 13. | $[\forall x\ A(x) \to B(x)] \to [\exists x \sim B(x) \to \exists x \sim A(x)$ | $\to I\ (2\text{-}12)$ |

Q2 For the 2 formulae's to be logically equivalent :->

$\forall x \, \forall y \, (P(x) \to Q(y)) \longleftrightarrow (\exists x \, P(x)) \to (\forall y \, Q(y))$

needs to be proved.

**Forward Proof** ->

| | | |
|---|---|---|
| 1. | Given | Given |
| 2. | $\forall x \, \forall y \, (P(x) \to Q(y))$ | (Assm) |
| 3. | $\exists x \, P(x)$ | (Assm) |
| 4. | $a$ $P(a)$ | (Assm) |
| 5. | $\forall y \, (P(a) \to Q(y))$ | $\forall E$ (2) |
| 6. | $b$ T | T (I) |
| 7. | $P(a) \to Q(b)$ | $\forall E$ (3) |
| 8. | $P(a)$ | Reiteration (4) |
| 9. | $Q(b)$ | $\to E$ (7) |
| 10. | $\forall y \, Q(y)$ | $\forall I$ (6-9) |
| 11. | $\forall y \, Q(y)$ | $\exists E$ (4-9) |
| 12. | $\exists x \, P(x) \to \forall y \, Q(y)$ | $\to I$ (3-11) |
| 13. | $\forall x \, \forall y \, (P(x) \to Q(y)) \to [\exists x \, P(x) \to \forall y \, Q(y)]$ | $\to I$ (2-12) |

## Backward Proof:→

| | | |
|---|---|---|
| 1. | | Given |
| 2. | $\exists x\, P(x) \rightarrow \forall y\, Q(y)$ | [Assm] |
| 3. | $a$ $T$ | $T$ [I] |
| 4. | $b$ $T$ | $T$ [I] |
| 5. | $P(a)$ | [Assm] |
| 6. | $\exists x\, P(x)$ | $\exists I(6)$ |
| 7. | $\forall y\, Q(y)$ | $\rightarrow E(2, 6)$ |
| 8. | $Q(b)$ | $\forall E\ (7)$ |
| 9. | $P(a) \rightarrow Q(b)$ | $\rightarrow I\ (5, 8)$ |
| 10. | $\forall y\ P(a) \rightarrow Q(y)$ | $\forall I\ (4-9)$ |
| 11. | $\forall x\ \forall y\ P(x) \rightarrow Q(y)$ | $\forall I\ (3-10)$ |
| 12. | $[\exists x\, P(x) \rightarrow \forall y\, Q(y)] \rightarrow [\forall x\, \forall y\ P(x) \rightarrow Q(y)]$ | $\rightarrow I\ (2-11)$ |

Thus we can conclude that both statements are logically equivalent as we can derive one statement from another and vice versa.

Q3

| | | |
|---|---|---|
| 1. | | Given |
| 2. | $\exists x \exists y [P(x) \to Q(y)]$ | Assm |
| 3. | $\forall x P(x)$ | Assm |
| 4. | a $\exists y [P(a) \to Q(y)]$ | (Assm) |
| 5. | $P(a)$ | $\forall E(3)$ |
| 6. | b $P(a) \to Q(b)$ | (Assm) |
| 7. | $Q(b)$ | $\to E(5,6)$ |
| 8. | $\exists y Q(y)$ | $\exists I(7)$ |
| 9. | $\exists y Q(y)$ | $\exists I(4-8)$ |
| 10. | $\exists y Q(y)$ | $\exists I(2-9)$ |
| 11. | $\forall x P(x) \to \exists y Q(y)$ | $\to I(3-10)$ |
| 12. | $\exists x \exists y [P(x) \to Q(y)] \to [\forall x P(x) \to \exists y Q(y)]$ | $\to I($ |
| 13. | $\forall x P(x) \to \exists y Q(y)$ | (Assm) |
| 14. | $\sim(\exists x \exists y (P(x) \to Q(y)))$ | (Assm) |
| 15. | a $T$ | (TI) |
| 16. | b $T$ | (TI) |
| 17. | $P(a) \to Q(b)$ | (Assm) |
| 18. | $\exists y\, P(a) \to Q(y)$ | $\exists I(17)$ |
| 19. | $\exists x \exists y [P(x) \to Q(y)]$ | $\exists x (18)$ |
| 20. | $\exists x \exists y (P(x) \to Q(y)) \wedge \sim[\exists x \exists y\, P(x) \to Q(y)]$ | $\wedge I($ |
| 21. | $\perp$ | $\perp I(20)$ |
| 22. | $\sim[P(a) \to Q(b)]$ | $\sim I(17-21)$ |
| 23. | $\sim[P(a) \wedge \sim Q(b)]$ | Assm |
| 24. | $P(a)$ | Assm |
| 25. | $\sim Q(b)$ | Assm |
| 26 | $P(a) \wedge \sim Q(b)$ | $\wedge I(24,25$ |
| 27. | $P(a) \wedge \sim Q(b) \wedge \sim[P(a) \wedge \sim Q(b)]$ | |
| 28. | $\perp$ | $\perp I$ 27 |
| 29. | $Q(b)$ | $\sim I(25-28)$ |
| 30. | $P(a) \to Q(b)$ | $\to I(24-29)$ |
| 31. | $(P(a) \to Q(b)) \wedge [\sim P(a) \to Q(b)]$ | $\wedge I(23,$ |
| 32. | $\perp$ | |

| | | |
|---|---|---|
| 33. | $P(a) \wedge \sim Q(b)$ | Reiteration |
| 34. | $P(a)$ | $\wedge E$ (33) |
| 35. | $\sim Q(b)$ | $\wedge E$ (33) |
| 36. | $\forall x\, P(x)$ | $\forall I$ (34) |
| 37. | $\forall y \sim Q(y)$ | $\forall I$ (35) |
| 38. | $\exists y\, Q(y)$ | |
| 39. | c $Q(c)$ | (Assn) |
| 40. | $\sim Q(c)$ | $\forall E$ (37) |
| 41. | $Q(c) \wedge \sim Q(c)$ | $\wedge I$ (39, 40) |
| 42. | $\perp$ | $\perp I$ (41) |
| 43. | $\perp$ | |
| 44. | $\exists x\, \exists y\, [P(x) \rightarrow Q(y)]$ | $\exists I$ |
| 45. | $\exists x\, \exists y\, [P(x) \rightarrow Q(y)]$ | $\exists I$ |
| 46. | $\forall x\, P(x) \rightarrow \exists y\, Q(y) \longrightarrow \exists x\, \exists y\, P(x) \rightarrow Q(y)$ | $\rightarrow I$ (13-45) |
| 47. | $(\exists x\, \exists y\, P(x) \rightarrow Q(y)) \longleftrightarrow [\forall x\, P(x) \rightarrow \exists y\, Q(y)]$ | $\leftrightarrow I$ (1-46). |

Hence, Proved.

Q4

| | | |
|---|---|---|
| 1. | $\forall x \; f(x) = f(2x)$ | [Given] |
| 2. | $a$ $\quad f(2a) = f(a)$ | $\forall E$ (1) |
| 3. | $b = 2a$ $\quad f(b) = f(2b)$ | $\forall E$ (1) |
| 4. | $f(2a) = f(2 \cdot 2a)$ | (Subs.) |
| 5. | $b = 4a$ $\quad f(b) = f(2b)$ | $\forall E$ (7) |
| 6. | $f(4a) = f(2 \cdot 4a)$ | (Subs.) |
| 7. | $f(8a) = f(2a)$ | Trans (4, 6) |
| 8. | $f(8a) = f(a)$ | Trans (2, 7) |
| 9. | $\forall x \; f(x) = f(8x)$ | |

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