classmate

Date

Page

D.S. Assignment

Q1 It did not rain yesterday → True
The door is closed → True

(a) Door is open → False
It didn't rain yesterday → True
False or True
⇒ True.

(b) If it rained yesterday → p
Its sunday today → q
$p \to q \Rightarrow \sim p \vee q$
p is False ; $\sim p$ is True
∴ whole statement is True.

Q2 Oranges are ripe along the path $= p$
It is safe to walk along the path $= q$
Dogs have been seen in this area $= r$

3 atomic propositions.

(i) $p \wedge q$

(ii) $(\sim r \wedge q) \wedge p$.

(iii) $(p \rightarrow q) \Leftrightarrow \sim r$

(iv) $(r \rightarrow \sim q) \wedge p$

(v) It is necessary q is similar to $p \rightarrow q$.
for p, it is sufficient that q is $q \rightarrow p$,

It is necessary but not sufficient is
$(p \rightarrow q) \wedge \sim (q \rightarrow p)$

$\Rightarrow (\sim p \vee q) \wedge \sim (\sim q \vee p) \Rightarrow (\sim p \vee q) \wedge (q \wedge \sim p)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

So answer of (v) is

$$[\sim q \vee (\sim p \wedge \sim r)] \wedge [\sim q \wedge (\sim p \wedge \sim r)]$$

$\underline{\underline{o}}$ Contrapositive of $p \to q$ is $\sim q \to \sim p$.

ⓐ If it rains today $\to p$
field is crowded $\to q$
we can't play football $\to r$
Contra positive of $(p \vee q) \to r \Rightarrow \sim r \to \sim(p \vee q)$

we can play football, $\leftarrow \sim r \to \sim p \wedge \sim q$
if it doesn't rain today and the field is not crowded.

(ii) Contrapositive of $A \Leftrightarrow B$ is $\sim B \Leftrightarrow \sim A$.

A → Lakers advance to final

B → They win their remaining matches.

Contrapositive is :→

If Lakers do not win their remaining matches if and only if they will not advance to finals.

Q4 (a)

A	B	$A \rightarrow B$	$\sim B$	$(A \rightarrow B) \Longrightarrow \sim B$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	1	0	0

(b)

X	Y	Z	$X \oplus Y$	$(X \oplus Y) \rightarrow Z$
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	1
1	1	1	0	1

(c)

P	Q	R	$(P \rightarrow Q)$	$Q \rightarrow R$	$(P \rightarrow Q) \wedge (Q \rightarrow R)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	1

Q5 (a)

X	Y	X ∧ Y	~X	~Y	~X ∧ ~Y	(X ∧ Y) ∨ (~X ∧ ~Y)
0	0	0	1	1	1	1
0	1	0	1	0	0	0
1	0	0	0	1	0	0
1	1	1	0	0	0	1

(b)

X	Y	~X	~X ∨ Y
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

(c)

X	Y	X ∧ Y	~X	(X ∧ Y) ∨ ~X
0	0	0		1
0	1	0	1	1
1	0	0		0
1	1	1	0	1

From truth table we can conclude that statements 2 and 3 are logically equivalent.

Q.6.

X	Y	Z	$X \wedge Y$	$Y \oplus Z$	$X \oplus Y \oplus Z$	$(X \wedge Y) \vee Y$
0	0	0	0	0	0	0
0	0	1	0	1	1	0
0	1	0	0	1	1	1
0	1	1	0	0	0	1
1	0	0	0	0	1	0
1	0	1	0	1	0	0
1	1	0	1	1	0	1
1	1	1	1	0	1	1

A	B	$A + B$	$A \to (A+B)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	0	0

Contd.

$(X \wedge Y) \vee Y \vee Z$	$(X \wedge Y) \vee Y \vee Z \to X \oplus Y \oplus Z$
0	1
1	1
1	1
1	0
0	1
1	0
1	0
1	1

If I set the true value of X as 0 then have 4 cases of Y and Z then 1st formula is logically equivalent to the 2nd formula.

Q7 $(p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee \sim r)$

$p \vee \sim q \Rightarrow A$

$\Rightarrow (A \vee \sim r) \wedge (A \vee r)$ {Reverse Distributive law}

$\Rightarrow A \vee (r \wedge \sim r)$

$\Rightarrow A \vee (0)$

$\downarrow A$

$\Rightarrow (p \vee \sim q) \wedge (p \vee q \vee \sim r)$ (Reverse Distributive)

$\Rightarrow p \vee [\sim q \wedge (q \vee \sim r)]$ Distributive law)

$\Rightarrow p \vee [(\sim q \wedge q) \vee (\sim q \wedge \sim r)]$

$\cancel{p \vee \sim q} \vee \sim r$

$\Rightarrow p \vee [0 \vee (\sim q \wedge \sim r)]$

$\Rightarrow p \vee (\sim q \wedge \sim r)$

Q8 $a \rightarrow b$ can be written as $\sim a \vee b$.

$\sim(\sim a \rightarrow \sim b)$ is same $\sim(\sim a) \vee \sim b \Rightarrow a \vee \sim b$

So it simplifies to :-

$$\sim((a \vee \sim b) \wedge \sim c) \Rightarrow \sim(a \vee \sim b) \vee c$$

(Demorgan's law)

$$(\sim a \wedge b) \vee c$$

(DeMorgan's Law Again).