Sanchit Jalan

2022101070

__/__/____

Assignment - 1

Q1

Given: Two system of equivalent linear eqn's.

To Prove: These two systems have same solution.

Proof: Let the first system be :

$$a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n = z_1 = A_1 \text{ (let)}$$

$$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = z_2 = A_2 \text{ (let)}$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n = z_m = A_m \text{ (let)}$$

___/___/___

Let system 2 be:

$$B_{11}x_1 + B_{12}x_2 + \dots B_{1n}x_n = z_1$$

$$B_{21}x_1 + B_{22}x_2 + \dots B_{2n}x_n = z_2$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$B_{m1}x_1 + B_{m2}x_2 + \dots B_{mn}x_n = z_n$$

Now since they are equivalent they one can be expressed as linear combination of the other.

Hence, 2 systems can be represented as follows:

$$(C_1 A_{11} + C_2 A_{21} + \dots C_m A_{m1})x_1 +$$
$$(C_1 A_{12} + C_2 A_{22} + \dots C_m A_{m2})x_2 + \dots$$
$$\dots (C_1 A_{1n} + C_2 A_{2n} + \dots C_m A_{mn})x_n$$

Here c_i is a constant value $\forall i \in \{1, 2, \dots m\}$

This is bcoz linear combinations involve elementary operations such as multiplying, adding subtracting which do not change solns.

If we assume $x_1', x_2', \dots x_n'$ to be the set of soln's of system 1, clearly it also is the soln to system 2 as well, which is just a linear combination of system 1.

Hence, Proved.

__/__/____

Given: A $m \times n$ matrix B.

Q) To Prove: By a sequence of elementary operation, we can convert B to a row reduced matrix for any given matrix B.
i.e. any matrix B is row equivalent to a row reduced matrix.

Proof: Row Reduced matrix is as follows:

1. First non zero entry in each non-zero of row is 1.
2. Each column of Row containing the leading non-zero entry of some row has all its other entries zero.

B_{ij} = element of i^{th} row & j^{th} column, B_i = i^{th} row of B.

Follows the below steps to convert to row reduced form.

Step 1: If Row 1 has a non zero entry, divide row 1 by leading non zero entry.

Step 2: Now for the column of first non-zero entry of row 1 to be zero, (say column k), we do $\forall i \geq 2$, $B_i = B_i - B_{ik} \times B_1$

Step 3: Follow the same process for row 2 and all other rows.

These steps will give us an row reduced matrix by doing elementary operations on any B.

Hence, Proved.

_/_/___

Q3

Given: A is square matrix which is $n \times n$ & $AX = 0$ is a homogenous system of linear eqn.

To Prove: A is row equivalent to I_n iff $AX = 0$ has only trivial solns.

Proof: Forward proof:

① Assuming A is equivalent to I_n

we have $AX = 0$

As equivalent system of LE have same soln.

$AX = 0 \Rightarrow IX = 0 \Rightarrow X = 0$ is $x_1 = 0, x_2 = 0 \dots x_n = 0$

$\therefore$ This system have trivial solns.

Backward Proof:

Assuming $AX = 0$ has only trivial solutions

Let A' be row reduced echelon matrix which is row equivalent to A.

$\therefore A'X = 0$ has same as $AX = 0$

$\therefore A'X = 0$ has only trivial solution.

$\therefore$ number of non-zero, r in A' is $\geq n$ i.e. $r \geq n$ — ①

But as A is a square matrix, A' is also square matrix,

$\therefore A'$ has only n rows.

$\therefore r \leq n$ — ②

$\therefore r = n$ (① and ②)

__/__/____

Hence number of non zero rows = n = no. of column.

As this is echelon matrix having n rows, each row shd. have only one '1' & rest of entries as 0 & leading '1' must be on i^{th} column in i^{th} row.

Hence A' is an identity matrix

Hence Proved.

As both forward & backward proof are true.

$A_{n\times n}$ is row equivalent to I_n iff $A_{(n\times n)}$ has trivial solns.

Hence, Proved.