Q1 Given:→ f^n $f: X \to Y$

To Prove:→ f is onto if and only if $f(f^{-1}(B)) = B$ for all $B \subseteq Y$.

Proof:→ ① If f is onto. prove that $f(f^{-1}(B)) = B$.

$f: X \to Y$ is onto.

so every element of Y has a pre image in X.

Since the f^n is not bijective there can be multiple pre images of Y in X. ~~Map them~~

$f^{-1}(y) = x$

Take f both side.

$f(f^{-1}(y)) = f(x) = y$.

Since it is true for y it will be true to $\forall B \subseteq Y$.

classmate

Date

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(2) If $f(f^{-1}(B)) = B \ \forall \ B \subseteq Y$ then f is onto.

$\Rightarrow$ $f^{-1}(B)$ exists $\forall \ B \subseteq Y$.

$f^n : f^{-1} \quad Y \rightarrow X$

Considering the domain of f^{-1} as Y.

$y \in Y$, let image of y under f^{-1} be x.

i.e. $x \in X$ s.t. $f^{-1}(y) = x$

Take f both sides.

$f(f^{-1}(y)) = f(x)$

But

$f(f^{-1}(y)) = y$

So $f(x) = y$. Since we assumed y to be in Y, Then $f(x)$ is a onto function.

Q2 $~~X \to Y~~$ $f(x) = y$ $g(y) = z$ $h(z) = v$

LHS :- $(h o g) o f(x) = hog(f(x)) = h(g(f(x)))$

RHS :- $h o (g o f) = h(g o f(x)) = h(g(f(x)))$

LHS = RHS

Hence, Proved.

Q3 For $f : X \to Y$ $y = f(x)$ to be an equivalence relation it should satisfy following properties :→

① Symmetric :→ If $f(x_1) = f(x_2)$ $x_1 R x_2$
$f(x_2) = f(x_1)$ $x_2 R x_1$

② Reflexive :→ $f(x_1) = f(x_1)$

③ Transitive :→ $f(x_1) = f(x_2)$ $x_1 R x_2$
$f(x_2) = f(x_3)$ $x_2 R x_3$
$\therefore$ $f(x_1) = f(x_3)$ $x_1 R x_3$

$\therefore$ Relation ~~Function~~ is equivalence relation.

Q4 Power set of set A is defined as the ~~subset~~ set of all subsets of set A.

Let set A = $\{a_1, a_2, a_3 \ldots\ldots\}$

No. of subsets = $2 \times 2 \times 2 \ldots\ldots$ (no. of elements of set)

$= 2^n$ where $n = |A|$

So $|P(A)|$ = No. of subsets of set A = 2^n