Q1 $c = a * b$

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$$C(z) = A(z) * B(z)$$

$$\frac{1}{1-7z} = A(z) * B(z)$$

$$A(z) = \sum_{r=0}^{\infty} a_r z^r$$

$$B(z) = \sum_{r=0}^{\infty} b_r z^r$$

$$C(z) = \sum_{r=0}^{\infty} c_r z^r$$

$$C(z) = \frac{1}{1-7z}$$

$$A(z) = 1 + 3z + 5z^2$$

$$B(z) = \sum_{r=0} b_r z^r$$

$$\frac{1}{(1-7z)(5z^2+3z+1)} = \sum_{r=0}^{\infty} b_r z^r$$

let $5z^2+3z+1 = (1-\alpha z)(1-\beta z)$

$$B(z) = \frac{1}{(1-7z)(1-\alpha z)(1-\beta z)}$$

let $\frac{A}{1-7z} + \frac{B}{1-\alpha z} + \frac{C}{1-\beta z} = \frac{1}{(1-7z)(1-\alpha z)(1-\beta z)}$

$$A(1-\alpha z)(1-\beta z) + B(1-7z)(1-\beta z) + C(1-7z)(1-\alpha z) = 1$$

$$\alpha\beta A + 7\beta B + 7\alpha C = 0 \quad -(1)$$

$$A(\alpha+\beta) + \beta + 7B + (\alpha+7)C = 0 \quad -(2)$$

$$A + B + C = 1 \quad -(3)$$

By solving (1), (2), (3)

$$A = \frac{49}{(7-\alpha)(7-\beta)}, \quad B = \frac{\alpha^2}{(\alpha-7)(\alpha-\beta)}, \quad C = \frac{\beta^2}{(\beta-7)(\beta-\alpha)}$$

$$B(z) = \frac{49}{(7-\alpha)(7-\beta)}\frac{1}{(1-7z)} + \frac{\alpha^2}{(\alpha-7)(\alpha-\beta)}\left(\frac{1}{1-\alpha z}\right) + \frac{\beta^2}{(\beta-7)(\beta-\alpha)}\left(\frac{1}{1-\beta z}\right)$$

$$b_r z^r = \frac{7^2}{(7-\alpha)(7-\beta)}\sum_{r=0}^{\infty} 7^r z^r + \frac{\alpha^2}{(\alpha-7)(\alpha-\beta)}\sum_{r=0}^{\infty}(\alpha z)^r + \frac{\beta^2}{(\beta-7)(\beta-\alpha)}\sum_{r=0}^{\infty}(\beta z)^r$$

$$\cancel{\sum_{r=0}^{\infty} b_r z^r =}$$

$$b_r = \frac{7^{r+2}}{(7-\alpha)(7-\beta)} + \frac{\alpha^{r+2}}{(\alpha-7)(\alpha-\beta)} + \frac{\beta^{r+2}}{(\beta-7)(\beta-\alpha)}$$

Q2 : $a_k - 7a_{k-1} + 10a_{k-2} = 3^k$

Multiply z^k on both sides.

$a_k z^k - 7a_{k-1} z^k + 10a_{k-2} z^k = 3^k z^k$

Let $A(z) = \sum_{r=0}^{\infty} a_r z^r$

Take Σ on both sides :→

$$\sum_{k=2}^{\infty} a_k z^k - 7z \sum_{k=2}^{\infty} a_{k-1} z^{k-1} + 10z^2 \sum_{k=2}^{\infty} a_{k-2} z^{k-2} = \sum_{k=2}^{\infty} 3^k z^k$$

$$\sum_{k=0}^{\infty} a_k z^k - a_1 z - a_0 - 7z \left[\sum_{k=0}^{\infty} a_k z^k - a_0\right] + 10z^2 \sum_{k=0}^{\infty} = \sum_{k=0}^{\infty} 3^k z^k - 1 - 3z$$

$$A(z) - z - 0 - 7z\left[A(z) - 0\right] + 10z^2\left[A(z)\right] = \frac{1}{1-3z} - 1 - 3z$$

$$(A(z))\left[1 - 7z + 10z^2\right] = \frac{1 - [(1+2z)(1-3z)]}{(1-3z)}$$

$$A(z)\,[1-2z][1-5z] = \frac{6z^2 + z}{(1-3z)}$$

$$A(z) = \frac{6z^2+z}{(1-2z)(1-3z)(1-5z)}$$

Let $A(z) = \dfrac{\alpha}{1-2z} + \dfrac{\beta}{1-3z} + \dfrac{\gamma}{1-5z}$

$$\alpha(1-3z)(1-5z) + \beta(1-2z)(1-5z) + \gamma(1-2z)(1-3z)$$
$$\parallel$$
$$6z^2+z$$

Putting $\boxed{z=0}$ $\alpha+\beta+\gamma = 0$ —①

$\boxed{z=1}$ $8\alpha+4\beta+2\gamma = 7$ —②

$\boxed{z=-1}$ $24\alpha+18\beta+12C = 5$ —③

Solving ①, ② & ③ we get.

$\cancel{\alpha = -\frac{9}{2}}$ $\cancel{\beta} =$

$\alpha = \dfrac{8}{3}$ $\beta = -\dfrac{9}{2}$ $\gamma = \dfrac{11}{6}$

$$A(z) = \frac{8}{3}\left(\frac{1}{1-2z}\right) - \frac{9}{2}\left(\frac{1}{1-3z}\right) + \frac{11}{6}\left(\frac{1}{1-5z}\right)$$

$$A(z) = \sum_{k=0}^{\infty} a_k z^k = \left[\frac{8}{3}\left(\sum_{r=0}^{\infty} 2^r z^r\right) - \frac{9}{2}\left(\sum_{r=0}^{\infty} 3^r z^r\right) + \frac{11}{6}\sum_{r=0}^{\infty} 5^r z^r\right]$$

$$\sum_{k=0}^{\infty} a_k z^k = \sum_{r=0}^{\infty}\left(\frac{8}{3}\times 2^r - \frac{9}{2}\times 3^r + \frac{11\times 5^r}{6}\right) z^r$$

$$a_k = \frac{8}{3}2^k - \frac{9}{2}3^k + \frac{11}{6}5^k.$$

03 (1)

$$b_r - b_{r-1} = 2\left(\cancel{100}^{25} \times \frac{9}{\cancel{4}_1} - b_{r-1}\right)$$

$$b_r - b_{r-1} = 450 - 2b_{r-1}$$

$$b_r = 450 - b_{r-1}$$

$$b_r + b_{r-1} = 450 \quad - (1)$$

Multiply z^r both sides & take sigma $\sum_{r=1}^{\infty}$

$$\sum_{r=1}^{\infty} b_r z^r + \sum_{r=1}^{\infty} b_{r-1} z^r = 450 \sum_{r=1}^{\infty} z^r$$

$$\sum_{r=0}^{\infty} b_r z^r - \cancel{b_0}^{\,0} + z\sum_{r=0}^{\infty} b_r z^r = 450\left(\sum_{r=0}^{\infty} z^r - 1\right)$$

Let $\sum_{r=0}^{\infty} b_r z^r = B(z)$

$$B(z) = \frac{450\left(\frac{1}{1-z} - 1\right)}{1+z}$$

$$B(z) = \cancel{\frac{450z}{(1-z)^2}} \quad \frac{450z}{(1+z)(1-z)}$$

$$B(z) = \frac{A}{1+z} + \frac{B}{1-z}$$

$$450z = (A+B) + (B-A)z$$

$A+B=0$

$B-A=450$

$$\boxed{\begin{array}{l} B=225 \\ A=-225 \end{array}}$$

$$B(z) = \frac{225}{1-z} - \frac{225}{1+z}$$

$$B(z) = 225\left[\frac{1}{1-z} - \frac{1}{1-(-1)z}\right]$$

$$B(z) = 225\left[\sum_{r=0}^{\infty} z^r - \sum_{r=0}^{\infty} (-1)^r z^r\right]$$

$$B(z) = \sum_{r=0}^{\infty} 225\left[1-(-1)^r\right] z^r = \sum_{r=0}^{\infty} b_r z^r$$

$$\therefore \quad b_n = 225\left[1-(-1)^n\right]$$

(ii) $a_r = \begin{cases} 100\left(\frac{3}{2}\right)^r & 0 \le r \le 9 \\ 100\left(\frac{3}{2}\right)^{10} & r \ge 10 \end{cases}$

$$b_r - b_{r-1} = 2\left[a_r - b_{r-1}\right]$$

$$b_r + b_{r-1} = 2a_r$$

Multiply z^r on both sides & take $\sum_{r=1}^{\infty}$

$$\sum_{r=1}^{\infty} b_r z^r + \sum_{r=1}^{\infty} b_{r-1} z^r = 2 \sum_{r=1}^{\infty} a_r z^r$$

$$\sum_{r=0}^{\infty} b_r z^r - b_0 + z\sum_{r=0}^{\infty} b_r z^r = 2\left[\sum_{r=1}^{9} a_r z^r + \sum_{r=10}^{\infty} a_r z^r\right]$$

$$= 2\left[\sum_{r=1}^{9} 100 \times \left(\frac{3}{2}z\right)^r + \sum_{r=10}^{\infty} a_r z^r\right]$$

Let $\sum_{r=0}^{\infty} b_r z^r = B(z)$

$$(1+z)B(z) = 2\left\{100\left[\frac{\left(\frac{3z}{2}\right)\left[\left(\frac{3z}{2}\right)^9 - 1\right]}{\left(\frac{3z}{2} - 1\right)}\right] + 100 \times \left(\frac{3}{2}\right)^{10}\left[\frac{z^{10}}{1-z}\right]\right\}$$

$$B(z) = \frac{(600z)}{(1+z)}\left[\frac{1 - \left(\frac{3z}{2}\right)^9}{2-3z}\right] + \frac{200 \times \left(\frac{3}{2}\right)^{10} z^{10}}{(1-z)(1+z)}$$

$$B(z) = \frac{120z\left[1-\left(\frac{3z}{2}\right)^9\right]}{(1+z)} + \frac{360z\left[1-\left(\frac{3z}{2}\right)^9\right]}{(2-3z)} + \frac{100\left(\frac{3}{2}\right)^{10} z^{10}}{1+z} + \frac{100\left(\frac{3}{2}\right)^{10} z^{10}}{1-z}$$

$$B(z) = \frac{120z}{1+z} - \frac{120\left(\frac{3}{2}\right)^9 z^{10}}{(1+z)} + \frac{120\left(\frac{3z}{2}\right)\left[1-\left(\frac{3z}{2}\right)^9\right]}{\left(1-\frac{3z}{2}\right)}$$

$$+ \frac{100\left(\frac{3}{2}\right)^{10} z^{10}}{1+z} + \frac{100\left(\frac{3}{2}\right)^{10} z^{10}}{1-z}$$

$$B(z) = 120\left(z - z^2 + z^3 \dots\right) - 80\left(\frac{3}{2}\right)^{10}\left[z^{10} - z^{11} + z^{12} \dots\right]$$

$$+ 120\left[\frac{3z}{2} + \left(\frac{3z}{2}\right)^2 + \dots \left(\frac{3z}{2}\right)^9\right]$$

$$+ \left(\frac{3}{2}\right)^{10} 100\left[z^{10} - z^{11} \dots\right] + \left(\frac{3}{2}\right)^{10} 100 \cdot \left[z^{10} + z^{11} \dots\right]$$

$$b_r = \begin{cases} 0 & r = 0 \\ 120\left[\left(\frac{3}{2}\right)^r - (-1)^{r+1}\right] & 1 \le r \le 9 \\ 120(-1)^{r+1} - 80\left(\frac{3}{2}\right)^{10}(-1)^r + 100 \times \left(\frac{3}{2}\right)^{10}\left[1 + \dots\right] & r \ge 10 \end{cases}$$

$$b_r = \begin{cases} 0 & r = 0 \\ 120\left[\left(\frac{3}{2}\right)^r - (-1)^{r+1}\right] & 1 \le r \le 9 \\ 120(-1)^{r+1} + 100 \times \left(\frac{3}{2}\right)^{10} + 20 \times \left(\frac{3}{2}\right)^{10}(-1)^r & r \ge 10 \end{cases}$$

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Q4

$$\frac{1}{1-z} = 1+z+z^2+z^3 \dots z^r$$

Differentiate both sides

$$\frac{1}{(1-z)^2} = 1+2z+3z^2 \dots + rz^{r-1}$$

Multiply z both sides

$$\frac{z}{(1-z)^2} = 1+2z^2+3z^3 \dots + rz^r$$

Differentiate Again

$$\frac{1+z}{(1-z)^3} = 1+2^2z+3^2z^2 \dots + r^2z^{r-1}$$

Multiply z both sides.

$$\frac{z(1+z)}{(1-z)^3} = z+2^2z^2+3^2z^3 \dots + r^2z^r$$

Differentiate again

$$\frac{z^2+4z+1}{(1-z)^4} = 1+2^3z+3^3z^2 \dots + r^3z^{r-1}$$

Multiply z both sides:

$$z\cancel{(z^2+4z+)}$$

$$A(z) = \frac{z^3+4z^2+z}{(1-z)^4} = z+2^3z^2+3^3z^3 \dots + r^3z^r$$

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$$\frac{A(z)}{1-z} = C(z) = \frac{(z^3+4z^2+z)}{(1-z)^5}$$

$$C(z) \Rightarrow 0(z^0) + (0+1^3)z^1 + (0^3+1^3+2^3)z^2 \dots (0^3+1^3+2^3 \dots n^3)z^n$$

$$C(z) = (z^3+4z^2+z)(1-z)^{-5}$$

$$(1-z)^{-n} = \left(1+nz+\frac{n(n+1)}{\lfloor 2}z^2 \dots\right)$$

$$C(z) = z^3\frac{[5\times 6\times \dots (r+1)]}{\lfloor r-3} + \frac{4z^2(5.6. \dots (r+2))}{\lfloor r-2} + \frac{z(5.6. \dots (r+3))}{\lfloor r-1}$$

$$C_r = \frac{z^3}{\lfloor 4}\frac{\lfloor r+1}{\lfloor r-3} + \frac{4z^2}{\lfloor 4}\frac{\lfloor r+2}{\lfloor r-2} + \frac{z}{\lfloor 4}\frac{\lfloor r+3}{\lfloor r-1}$$

$$\Rightarrow \frac{(r+1)(r)(r-1)(r-2)}{\lfloor 4} + \frac{4(r-1)r(r+1)(r+2)}{\lfloor 4} + \frac{r(r+1)(r+2)(r+3)}{\lfloor 4}$$

$$\Rightarrow \frac{r(r+1)}{\lfloor 4}\left[r^2+2-3r+4r^2-8+4r+r^2+5r+6\right]$$

$$\Rightarrow \frac{r(r+1)}{\lfloor 4}[6r^2+6r] \Rightarrow \frac{\cancel{1\times 2\times 3}}{\cancel{1\times 2\times 3}\times 4}\frac{r^2(r+1)^2}{2^2}$$

$$\Rightarrow \left[\frac{r(r+1)}{2}\right]^2$$

is convergent ... the series

Q5 $a_r \Rightarrow$ Total dollar assets of company in r^{th} year.

$a_r - a_{r-1} \Rightarrow$ ~~Decrease~~ Increase in assets during r^{th} year.

ATQ $\quad a_r - a_{r-1} = 5(a_{r-1} - a_{r-2}) \; \forall r \geq 2$

$$a_r = 6a_{r-1} - 5a_{r-2}$$

Multiply z^r on both sides & take $\sum_{r=2}^{\infty}$,

$$\sum_{r=2}^{\infty} a_r z^r = \sum_{r=2}^{\infty} 6a_{r-1} z^r - \sum_{r=2}^{\infty} 5a_{r-2} z^r$$

$$\sum_{r=0}^{\infty} a_r z^r - a_1 z - a_0 = 6z\left(\sum_{r=0}^{\infty} a_r z^r - a_0\right) - 5z^2 \sum_{r=0}^{\infty} a_r z^r$$

Let $A(z) = \sum_{r=0}^{\infty} a_r z^r$

$$A(z) - 7z - 3 = 6z\,(A(z) - 3) - 5z^2 A(z)$$

$$A(z)\left[1 - 6z + 5z^2\right] = 3 - 11z$$

$$A(z) = \frac{3 - 11z}{1 - 6z + 5z^2} = \frac{3 - 11z}{(5z-1)(z-1)}$$

$$\frac{3-11z}{(5z-1)(z-1)} = \frac{A}{5z-1} + \frac{B}{z-1}$$

$$3-11z = (A+5B)z - (A+B)$$

$z=0$ $\quad \boxed{A+B=-3}$

$z=1$ $\quad 4B=-8$

$\boxed{B=-2}$

$\boxed{A=-1}$

$$\frac{3-11z}{(5z-1)(z-1)} = \frac{-1}{5z-1} - \frac{2}{z-1}$$

$$A(z) = \sum_{k=0}^{\infty} a_k z^k = \frac{1}{1-5z} + \frac{2}{1-z}$$

$$A(z) = \sum_{k=0}^{\infty} a_k z^k = \sum_{k=0}^{\infty} 5^k z^k + 2\sum_{k=0}^{\infty} z^k$$

$$\sum_{k=0}^{\infty} a_k z^k = \sum_{k=0}^{\infty} (5^k+2) z^k$$

$\therefore \quad a_k = 2+5^k$

Expression for total assets in k^{th} year $= 2+5^k$.

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Q6 $\sum u_n = \sum_{n=0}^{\infty} \left(\frac{5n-4n^3}{9n^3+2}\right)^n$

$\Rightarrow u_n = \left(\frac{5n-4n^3}{9n^3+2}\right)^n$

From Cauchy Root test $\lim_{n\to\infty} (u_n)^{1/n} = l$

$l<1$ series is ~~divergent~~ convergent
$l>1$ series is divergent
$l=1$ series may be convergent, divergent

$L = \lim_{n\to\infty} \left(\left|\frac{5n-4n^3}{9n^3+2}\right|^{n}\right)^{\frac{1}{n}}$

$L = \lim_{n\to\infty} \left(\frac{4n^3-5n}{9n^3+2}\right)$

$L = \lim_{n\to\infty} \frac{\cancel{n^3}}{\cancel{n^3}}\left[\frac{4-\frac{5}{n^2}}{9+\frac{2}{n^3}}\right]$

$L = \lim_{n\to\infty} \frac{4-\frac{5}{n^2}}{9+\frac{2}{n^3}}$ as $n\to\infty$ $\frac{1}{n}\to 0^+$

So. $L = \frac{4}{9}$.

As $l < 1$

From Cauchy Root Test we can conclude that the series is convergent.

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Q7 $\frac{1}{n}$ is divergent because.

$S_1 = \frac{1}{1}$

$S_2 = 1 + \frac{1}{2} =$

$S_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

As $\frac{1}{3} > \frac{1}{4}$

so $S_4 > 1 + \frac{1}{2} + \frac{1}{2}$

$S_4 > 1 + \frac{2}{2}$

$S_8 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$\frac{1}{5}, \frac{1}{6}, \frac{1}{7} > \frac{1}{8}$

$S_8 > 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2}$

$S_8 > 1 + \frac{3}{2}$

Similarly.

$S_{16} > 1 + \frac{4}{2}$

$S_{2^n} > 1 + \frac{n}{2}$

as $n \to \infty$

S_∞ becomes infinite and diverges

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As: $\sum_{n\geq 1}^{\infty} \frac{n}{(a+b)n^2} < \sum_{n\geq 1}^{\infty} \frac{n}{an^2+b}$

$\forall\ a, b > 0$ & $n \geq 1$

As $\sum_{n\geq 1}^{\infty} \frac{n}{(a+b)n^2}$ diverges the expression

greater than it should also diverge.

$\sum_{n\geq 1}^{\infty} \frac{n}{(a+b)n^2}$ diverges because $\sum_{n\geq 1}^{\infty} \frac{1}{n}$

diverges.

$\therefore$ $\sum_{n\geq 1}^{\infty} \frac{n}{an^2+b}$ should also diverge

Hence, Proved.

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Q8 (i) Series $= 1 + \sum_{r=1}^{\infty} \frac{\alpha.(\alpha+1)..(\alpha+r-1)\,\beta(\beta+1)..[\beta(t+r-1)]}{\underline{|r}\;\gamma(\gamma+1)..(\gamma+r-1)}$

$= 1 + \sum_{r=1}^{\infty} u_n$

Using D' Alembert's test :→ on $\sum_{n=1}^{\infty} u_n$

$l = \lim_{n\to\infty} \frac{u_{n+1}}{u_n}$

$l < 1$ converging series

$l > 1$ diverging series

$l = 1$ Test fails.

$l = \lim_{n\to\infty} \frac{\alpha.(\alpha+1)....(\alpha+n)\;\beta(\beta+1)..(\beta+n)\;x^{n+1}}{\underline{|n+1}\;\gamma(\gamma+1)..(\gamma+n)} \cdot \frac{\gamma(\gamma+1)..(\gamma+n-1)}{\alpha.(\alpha+1)....(\alpha+n-1)\;\beta(\beta+1)..(\beta+n-1)}$

$l = \lim_{n\to\infty} \frac{(\alpha+n)\;(\beta+n)\;x}{(n+1)\;(\gamma+n)}$

$l = \lim_{n\to\infty} \frac{(n)(n)}{(n+1)\,n} \frac{\left(\frac{\alpha}{n}+1\right)\left(\frac{\beta}{n}+1\right)}{\left(\frac{\gamma}{n}+1\right)} x$

As $n \to \infty$ ~~then~~ $l = x$

$l = x$

$x < 1$ so $l < 1$ series ~~then~~ $\sum u_n$ converges

$1 + \sum u_n$ also conv.

$x > 1$ so $l > 1$ series $\sum u_n$ diverges

series $(1 + \sum u_n)$ also diverges

(ii) Series $= 1 + \sum_{r=1}^{\infty} \frac{\alpha(\alpha+1)\ldots(\alpha+r-1)\beta(\beta+1)\ldots(\beta+r-1)x^r}{\lfloor r \; \gamma(\gamma+1)\ldots(\gamma+r-1)}$

$= 1 + \sum_{r=1}^{\infty} u_r$

By using Raabe's test on $\sum_{r=1}^{\infty} u_r$

$$p = \lim_{n\to\infty}\left[n\left[\frac{u_n}{u_{n+1}} - 1\right]\right]$$

$p > 1$ $\quad \sum u_n$ converges

$p < 1$ $\quad \sum u_n$ diverges

$$p = \lim_{r\to\infty} r\left\{\left[\frac{\alpha(\alpha+1)\ldots(\alpha+r-1)\beta(\beta+1)\ldots(\beta+r-1)x^r \; \gamma(\gamma+1)\ldots(\gamma+r)\lfloor r+1}{\lfloor r \; \gamma(\gamma+1)\ldots(\gamma+r-1) \; \alpha(\alpha+1)\ldots(\alpha+r)\beta(\beta+1)\ldots x^{r+1}} - 1\right]\right\}$$

$$p = \lim_{r\to\infty} r\left[\frac{(r+1)(\gamma+r)}{(\alpha+r)(\beta+r)\,x} - 1\right]$$

$x = 1$

$$p = \lim_{r\to\infty} r\left\{\frac{r^2 + r\gamma + \gamma + r - \alpha\beta - r^2 - (\alpha+\beta)r}{(\alpha+r)(\beta+r)}\right.$$

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$$p = \lim_{n\to\infty} \frac{\cancel{n^2}\left[n(\gamma-\alpha-\beta+1) + \gamma-\alpha\beta\right]}{n\times \cancel{n^2}\left(\frac{\alpha}{n}+1\right)\left(\frac{\beta}{n}+1\right)}$$

$$p = \lim_{n\to\infty} \frac{\left[\gamma+1-\alpha-\beta+\frac{\gamma-\alpha\beta}{n}\right]}{\left(\frac{\alpha}{n}+1\right)\left(\frac{\beta}{n}+1\right)}$$

As $n\to\infty$, $p = (\gamma-\alpha-\beta+1)$

$p = \gamma-\alpha-\beta+1$

If $\gamma-\alpha-\beta>0$

then $\gamma-\alpha-\beta+1>1$

$p>1$

so $\sum u_n$ converges

$\therefore\ 1+\sum_{n=1}^{\infty} u_n$ converges.

If $\gamma-\alpha-\beta\le 0$

$\gamma-\alpha-\beta+1\le 1$

$p\le 1$

so $\sum u_n$ diverges.

$\therefore\ \left(1+\sum_{n=1}^{\infty} u_n\right)$ diverges.