1. **Consider a symmetric game where each player must decide whether to hunt or forage for food. If both players decide to hunt, they successfully capture an animal for meat, and they each get a payoff of 10. If one player hunts and one forages, the hunter will be injured by the prey and get payoff 0, while the forager will gather some food and get payoff 5. If both players forage, they will help each other find food faster and each will get payoff 7.**
2. **Draw the matrix representation of this game.**
3. **Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.**
4. **Identify any pure strategy Nash equilibria. Explain your answer.**

**Solution**:

***Player 2***



|  |  |  |
| --- | --- | --- |
|  | **Hunt** | **Forage** |
| **Hunt** |  |  |
| **Forage** |  |  |

***Player 1***

1. No, both the players do not have strictly dominant strategy. If both the players will hunt or forage they will get the better payoff with cooperation. Otherwise if one tries to Hunt and other tries to Forage, one will get nothing and other will get the Forage.
2. There are Two Nash equilibria. (Hunt, Hunt) and (Forage, Forage). As there is no dominant strategy, they should expect to use strategies that are best responses to each other.
3. **You (player 1) are browsing the WWW on a computer and things are frustratingly slow. A popup appears that promises increased speed if you click on it. It will manage this by using an alternative protocol to standard TCP. A classmate (player 2) is having the same experience and has seen the same popup. If you both stay with TCP, you will each experience a 1 ms delay (a payoff of -1). If one player stays with TCP and the other goes with the alternative, the TCP player will experience a 4 ms delay and the alternative player will experience zero delay. If you both switch to the alternative protocol, you will each experience a 3 ms delay.**
4. **Draw the matrix representation of this game.**
5. **Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.**
6. **Identify any pure strategy Nash equilibria. Explain your answer.**

**Solution:**

***Player 2***

|  |  |  |
| --- | --- | --- |
|  | **Hunt** | **Forage** |
| **Hunt** | 10, 10 | 0, 5 |
| **Forage** | 5, 0 | 7, 7 |

|  |  |  |
| --- | --- | --- |
|  | **TCP** | **Alt.** |
| **TCP** | -1, -1 | -4, 0 |
| **Alt.** | 0, -4 | -3, -3 |

***Player 1***

1. The above game is similar to Prisoners Dilemma. Both players have strictly dominant strategy that they should use **Alternative Protocol.**
2. There are no pure strategy Nash equilibria. Because nothing support this condition, (S, T) is a Nash Equilibrium if S is a best response to T, and T is a best response to S
3. **Two drivers are playing chicken, the game where they drive straight toward each other and see if one or both will swerve to avoid a crash. If both drivers swerve, the game is a tie, and each gets payoff 0. If one swerves and one stays straight, the driver who swerves is declared a "chicken" and gets payoff -5, while the driver who stays straight wins and gets payoff +5. If both drivers continue straight, they will crash and both get payoff -50.**
4. **Draw the matrix representation of this game.**
5. **Does either (or both) player(s)have a strictly dominant strategy? Explain your answer.**
6. **Identify any pure strategy Nash equilibria. Explain your answer.**

**Solution:**

***Driver 2***

***Driver 1***

|  |  |  |
| --- | --- | --- |
|  | **Straight** | **Swerves** |
| **Straight** | -50, -50 | +5, -5 |
| **Swerves** | -5, +5 | 0, 0 |

1. This is same as Hawk-Dove Game, one will win, and one will lose. They want to avoid Swerves- Swerves condition. They both don’t have strictly dominant strategy.
2. Two Nash Equilibria, (Straight, Swerves) and (Swerves, Straight). This is the game where one driver swerves while another driver doesn’t.
3. **Draw the matrix representation of the game Rock Paper Scissors Lizard Spock. Use 0 for tie, 1 for win, -1 for lose.**

**Solution:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Rock** | **Paper** | **Scissors** | **Lizard** | **Spock** |
| **Rock** | 0,0 | -1, 1 | 1, -1 | 1, -1 | -1, 1 |
| **Paper** | 1, -1 | 0,0 | -1, 1 | -1, 1 | 1, -1 |
| **Scissors** | -1, 1 | 1, -1 | 0,0 | 1, -1 | -1, 1 |
| **Lizard** | -1, 1 | 1, -1 | -1, 1 | 0,0 | 1, -1 |
| **Spock** | 1, -1 | -1, 1 | 1, -1 | -1, 1 | 0,0 |

1. **There are 80 cars which begin in city A and must travel to city B. There are two routes between city A and city B:**

**Route I: begins with a local street leaving city A, which requires a travel time in minutes equal to 10 plus the number of cars which use this street, and ends with a highway into city B which requires one hour of travel time regardless of the number of cars which use this highway.**

**Route II: begins with a highway leaving city A. This highway takes one hour of travel time regardless of how many cars use it, and ends with a local street leading into city B. This local street near city B requires a travel time in minutes equal to 10 plus the number of cars which use the street.**

1. **Draw the network described above and label the edges with the travel time in minutes needed to move along the edge, in terms of x. Let x be the number of travelers who use Route I. The network should be a directed graph as all roads are one-way.**

**Route I**

60

10+ x

10+(80-x)

60

**Route II**

1. **What would be the travel time per car if all cars chose to use Route I?**

Total Travel Time for Route I = 10 + x + 60

If 80 cars go through Route I = 10 + 80 + 60

Travel Time per car for Route I = 150

1. **Assume that cars simultaneously chose which route to use. Find the Nash equilibrium value of x.**

Nash Equilibrium value of x = 40

Total time to travel 40 cars from Route I and Route II = 110

1. **Explain your answer to part c.**

If x= 40, every car will take the same time to reach the destination. If there was uneven balance, there would be incentive for those in more congested route to switch.

**Now the government builds a new (two-way) road connecting the nodes where local streets and highways meet. The new road is very short and takes no travel time. This adds two new routes.**

**Route III: consists of the local street leaving city A (on Route I), the new road, and the local street into city B (on Route II).**

**Route IV: consists of the highway leaving city A (on Route II), the new road, and the highway leading into city B (on Route I).**

**Route I**

60

10+ x

**Route IV**

**Route III**

10+y

60

**Route II**

1. **What would the travel time be per car if all cars chose Route III?**

Total Travel Time for Route III = 10 + x + 10 + y

If 80 cars go through Route III = 10 + 80 + 10 + 80

Travel Time per car for Route III = 180

1. **What would the travel time be per car if all cars chose Route IV?**

Total Travel Time for Route IV = 60 + 60

If 80 cars go through Route IV = 120

1. **What happens to total travel time as a result of the availability of the new road?**

Because of the availability of new road, increases the travel time for all the cars in Route III whereas decreases the time on Route IV.

1. **If you can assign travelers to routes, it’s possible to reduce total travel time relative to what it was before the new road was built. That is, the total travel time of the 80 cars can be reduced (below that in the original Nash equilibrium from part c) by assigning cars to routes. There are many assignments of routes that will accomplish this. Find one. Explain why your reassignment reduces total travel time by giving the number of cars assigned to each of the Routes I, II, III, IV, the travel time per car on each route, and the total travel time of the 80 cars.**

Yes, total time can be reduced by assigning different route to different cars.

If we assign 30 cars to Route I (A-C-B) the total time taken will be 100 min.

If we assign 30 cars to Route II (A-D-B) the total time taken will be 100 min.

And assigning remaining 20 cars to Route IV(A-D-C-B) the total time taken will be 120 min which is faster from Route I when all cars travelling to it.

1. **Consider an auction in which there is one seller who wants to sell one unit of a good and a group of bidders who are each interested in purchasing the good. The seller will run a sealed-bid, second-price auction. There will be around a dozen other bidders in addition to your firm. All bidders have independent, private values for the good. Your firm’s value for the good is c.**
2. **What bid should your firm submit? Explain your answer.**

Best bid to submit the value for good is c, if my firm bid higher than the value of c then there is a risk of overpaying. If my firm bid the value lower than the value of c then we are underpaying and the risk to lose that bid is more.

1. **How does your bid depend on the number of other bidders who show up?**

Our bid doesn’t depend on the number of other bidders because bidders submit simultaneous sealed bids to the sellers. Bids are independent of the number of bidders in this case.

1. **A seller wants to sell one unit of a good to some bidders, by running a sealed-bid second-price auction. Assume that there are two bidders who have independent, private values v which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. (If there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.)**
2. **Show that the seller’s expected revenue is 6/4.**

For value 1 or 3 there can be 4 combinations for the values i.e.

Case 1: (1,1),

Case 2: (1,3),

Case 3: (3,1),

Case 4: (3,3)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Bidder 2** | |
| **Bidder 1** |  | **1** | **3** |
| **1** | 1 | 1 |
| **3** | 1 | 3 |

Chances of winning the second bid in case 1, case2, case 3 is 1 and Case 4 is 3.

Probability of choosing either one of the cell is ¼.

Expected value of the seller = (1+1+1+3)

=

1. **Suppose that there are three bidders who have independent, private values v which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. What is the seller’s expected revenue in this case? Explain your answer.**

If there are 3 bidders, each will have two option and probability of getting selected is ½. Now there are 8 combinations.

One of the combinations is (3,3,3) choosing the second highest bid is 3. For other 7 cases the second highest value is 1.

The expected value for the seller = (7+3)

=

1. **Briefly explain why changing the number of bidders affects the seller’s expected revenue.**

Yes, the seller’s revenue will be affected as number of bidders changed. With the increase in the number of bidders it lowers the chances of receiving maximum revenue. When all the bidders place the highest bid, the seller receives the maximum revenue.