The Number Glossary

1] Factors:
A positive integer ‘f’ is said to be a factor of a given positive integer ‘n’ if f divides n without leaving a remainder.
e.g. 1, 2, 3, 4, 6 and 12 are the factors of 12.

2] Prime Numbers:
A prime number is a positive number which has no factors besides itself and unity.

3] Composite Numbers:
A composite number is a number which has other factors besides itself and unity.

4] Factorial:
For a natural number ‘n’, its factorial is defined as: $n! = 1 \times 2 \times 3 \times 4 \times .... \times n$
(Note: $0! = 1$)

5] Absolute value:
Absolute value of x (written as |x|) is the distance of ‘x’ from 0 on the number line. |x| is always positive.
$|x| = x$ for $x \geq 0$
$= -x$ for $x \leq 0$

Factors, Multiples HCF and LCM

1] Factorization:
Any natural number N can be expressed as $a^m \times b^n \times c^p....$ where a, b, c, ... are all prime numbers.
e.g. $72 = 2^3 \times 3^2$
The number of divisors of N is equal to
$(m + 1) \times (n + 1) \times (p + 1)$ ....

The sum of divisors of N is

$$\frac{a^{m+1} - 1}{a - 1} \cdot \frac{b^{n+1} - 1}{b - 1} \cdot \frac{c^{p+1} - 1}{c - 1} \; .....$$

2] HCF and LCM:
Product of numbers = HCF × LCM

3] HCF and LCM of fractions:

$$\text{HCF of fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

$$\text{LCM of fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

4] Relatively Prime Numbers: Two positive integers are said to be relatively prime to each other if their highest common factor is 1

Properties of numbers

1] For any positive integer n:
the product of any ‘n’ consecutive positive integers is divisible by n!.
Hence, $n(n^2 - 1)$ is divisible by 6.

2] For a prime number ‘p’ and whole number ‘a’, $a^p - a$ is divisible by p.

3] Square of a number can neither end with an odd number of zeroes nor with 2, 3, 7 or 8.

4] Product of two even numbers is even
Product of two odd numbers is odd
Product of odd and even numbers is even

Algebraic Formulae

1] $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
Hence, $a^3 \pm b^3$ *is divisible by* $(a \pm b)$ *and* $(a^2 \mp ab + b^2)$.

2] $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + ... + b^{n-1})$ [for all n]

Hence, $a^n - b^n$ *is divisible by* $a - b$ *for all n.*

3] $a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 ... - b^{n-1})$ [n-even]

Hence, $a^n - b^n$ *is divisible by* $a + b$ *for even n.*

4] $a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 + ... + b^{n-1})$ [n-odd]

Hence, $a^n + b^n$ *is divisible by* $a + b$ *for odd n.*

5] $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)$
Hence, $a^3 + b^3 + c^3 = 3abc$ *if* $a + b + c = 0$

Divisibility Tests

A number is divisible by:

1] **2, 4 & 8** when the number formed by the last, last two, last three digits are divisible by 2, 4 & 8 respectively.

2] **3 & 9** when the sum of the digits of the number is divisible by 3 & 9 respectively.

3] **11** when the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.

4] **6, 12 & 15** when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.

5] **7**, if the number of tens added to five times the number of units is divisible by 7.

6] **13**, if the number of tens added to four times the number of units is divisible by 13.

7] **19**, if the number of tens added to twice the number of units is divisible by 19.

Laws of Indices

1] $a^m \times a^n = a^{m+n}$

2] $a^m \times a^n \times a^p \times ..... = a^{m+n+p+...}$

3] $\left(a^m\right)^n = a^{mn}$

4] $a^m \div a^n = a^{m-n}$

5] $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ where a and p are real numbers and $q \neq 0$

.6] $a^{\frac{1}{n}} = \sqrt[n]{a}$

7] $a^{-n} = \dfrac{1}{a^n}$

8] $a^0 = 1$ (where $a \neq 0$)

9] $(a \times b)^m = a^m \times b^m$ and $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

10] If $a^m = a^n$ and $a \neq -1, 0, 1$ then $m = n$.

11] If $a^m = b^m$ and $m \neq 0$ then
i. $a = \pm b$ for even m
ii. $a = b$ for odd m

6] Binomial theorem:
$(x + y)^n = x^n + n.x^{n-1}y + {}^nC_2x^{n-2}y^2 + {}^nC_3x^{n-3}y^3 ... ... + n.x\,y^{n-1} + y^n$
Hence, to find the remainder when $(x + y)^n$ *is divided by x, find the remainder when* y^n *is divided by x*

$(x + 1)^n = x^n + n.x^{n-1} + {}^nC_2x^{n-2} + ... + n.x + 1^n$
Hence, $(x + 1)^n$ *leaves remainder 1 when divided by x*

Numeric inequalities

1. If $a, b \geq 0$, then $\dfrac{a + b}{2} \geq \sqrt{ab}$.

2. If a, b c, ... k are n positive quantities, then
$$\left(\frac{a^m + b^m + c^m + ... + k^m}{n}\right) > \left(\frac{a + b + c + ... + k}{n}\right)^m$$
unless m is a positive proper fraction.

3. $\dfrac{a}{b} + \dfrac{b}{a} \geq 2$, where a and b are positive numbers.

4. $\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{d} + \dfrac{d}{a} \geq 4$, where a, b, c and d are positive numbers.

5. $a^4 + b^4 + c^4 + d^4 \geq 4abcd$, where a, b, c and d are positive numbers.

Base system

The 'Decimal Number System' or the 'Base 10 system' uses the 10 digits from 0 to 9 to represent any number.

Similarly, the 'Binary System' or the 'Base 2 system' uses 0 and 1 while the 'Octal System' or the 'Base 8 system' uses digits from 0 to 7.

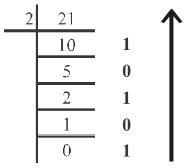
Conventionally, $(..XYZ)_n$ is the representation of a number in Base 'n' notation. (where X,Y,Z are the last 3 digits of the number) and $(..XYZ)_n = \ldots + X \times (n)^2 + Y \times (n)^1 + Z \times (n)^0$

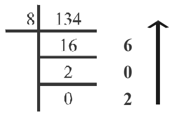
For eg. $(9276)_{10} = 9(10)^3 + 2(10)^2 + 7(10)^1 + 6(10)^0$

$(10110)_2 = 1(2)^4 + 0(2)^3 + 1(2)^2 + 1(2)^1 + 0(2)^0$
$= 16 + 4 + 2 = (22)_{10}$

$(712)_8 = 7(8)^2 + 1(8)^1 + 2(8)^0 = 448 + 8 + 2$
$= (458)_{10}$

Conversion from Decimal to any base 'X' number system:
e.g. Convert $(21)_{10}$ into binary system and
convert $(134)_{10}$ into octal system





$\therefore (21)_{10} = (10101)_2$ $\quad\quad$ $\therefore (134)_{10} = (206)_8$

Comparison between numbers

1] If a > b and c > 0, then a + c > b + c,

$a - c > b - c$, $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

2] If a, b $\geq$ 0, then $a^n > b^n$ and $\frac{1}{a^n} < \frac{1}{b^n}$, where is positive.

3] a < b and x > 0, then $\frac{a+x}{b+x} > \frac{a}{b}$

4] a > b and x > 0, then $\frac{a+x}{b+x} < \frac{a}{b}$

5] If a, b $\geq$ 0, then $\frac{a+b}{2} \geq \sqrt{ab}$

6] $|x - y| = |y - x|$

$|x \,.\, y| = |x| \,.\, |y|$

$|x + y| \leq |x| + |y|$

$|x + y| \geq |x| - |y|$

Quadratic function

1. A function of the form $f(x) = ax^2 + bx + c$ $(a \neq 0)$ is called a quadratic function.

   The graph of a quadratic function:

   If a>0, the graph is an upward bell-shaped curve (parabola) that cuts the X-axis in a maximum of two points.

   If a<0, then the graph is an inverted bell-shaped curve that cuts the X-axis in a maximum of two points.

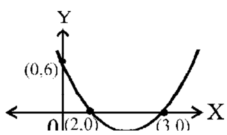
2. An equation of the form:

   $ax^2 + bx + c = 0$ $(a \neq 0)$ is called a quadratic equation.

3. The values of x which satisfy the equation $ax^2 + bx + c = 0$ are called the roots of the quadratic equation .

   The graph of the quadratic function, $f(x) = x^2 - 5x + 6$ will look as follows:

   x = 2 and x = 3 are the roots of the equation. (x – 2) and (x – 3) are the factors of the given quadratic function, i.e. $x^2 - 5x + 6 = (x - 2)(x - 3)$



4. For the quadratic polynomial $ax^2 + bx + c$, let α and roots.

   Then $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$
   $= a(x - \alpha)(x - \beta)$
   $= a[x^2 - (\alpha + \beta)x + \alpha\beta]$

   Hence, we have:

   i) $\alpha + \beta = -\frac{b}{a}$; ii) $\alpha\beta = \frac{c}{a}$

4. The roots α, β are given by:

   $\alpha, \beta = \frac{-b \pm \sqrt{\Delta}}{2a}$; where $\Delta = b^2 - 4ac$ is the disc

   i) If c = a, then $\alpha = \frac{1}{\beta}$

   ii) If b = 0, then $\alpha = -\beta$

   iii) If $\alpha = m + \sqrt{n}$, then $\beta = m - \sqrt{n}$

   iv) If $\alpha = m + in$, then $\beta = m - in$

5. Properties of the discriminant Δ:

   i) If $\Delta \geq 0$, then α, β are real.

   ii) If $\Delta = 0$, then $\alpha = \beta$.

   iii) If $\Delta < 0$, then α, β are complex conjugates.

6. Quadratic inequalities:

   $(x - \alpha)(x - \beta) > 0$ if $x < \alpha$ or $x > \beta$ $(\alpha \leq \beta)$

Ratios

1] If $\frac{a}{b} = \frac{c}{b}$ then

(i) $\frac{a \pm b}{b} = \frac{c \pm d}{d}$ (ii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

2] If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$

then (i) $\frac{a}{b} = \frac{a+c+e+\ldots}{b+d+f+\ldots}$

(ii) $\frac{a}{b} = \left(\frac{a^n + c^n + e^n + \ldots}{b^n + d^n + f^n + \ldots}\right)^{\frac{1}{n}}$

(iii) $\frac{a}{b} = \left(\frac{pa^n + qc^n + re^n + \ldots}{pb^n + qd^n + rf^n + \ldots}\right)^{\frac{1}{n}}$

3] If A $\propto$ B and B $\propto$ C then A $\propto$ C
(A = kB, B = mC $\Rightarrow$ A = (km)C)

4] If A $\propto$ C and B $\propto$ C, then (A $\pm$ B) $\propto$ C and $\sqrt{AB}$
(A = kC; B = mC $\Rightarrow$ A $\pm$ B = (k $\pm$ m)C)

5] If a number is increased by x%, multiply the number by $\frac{100+x}{100}$. If a number is decreased by x%, multiply the number by $\frac{100-x}{100}$

6] % increase/decrease in a no. = $\frac{\text{increase/decrease}}{\text{original number}} \times 1($

Percentages

1. A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called the rate percent. It is denoted by the symbol %.
2. To find the % equivalent of a fraction or a decimal, express the fraction with the denominator 100. The numerator is the required answer.
3. To find the fraction or decimal equivalent of a percentage divide the given percentage by 100.
4. To increase a number by given %, multiply the number by the factor $\frac{100 + \text{Rate}}{100}$.
5. To decrease a number by given % multiply the number by the factor $\frac{100 - \text{Rate}}{100}$.
6. To find the % increase of a number:
   % increase = $\frac{\text{Total increase}}{\text{Initial value}} \times 100$ = $\frac{\text{Final value - Initial value}}{\text{Initial value}} \times 100$
7. To find the % decrease of a number:
   % decrease = $\frac{\text{Total decrease}}{\text{Initial value}} \times 100$ = $\frac{\text{Initial value - Final}}{\text{Initial value}} \times 100$
8. Absolute percent change = $\frac{|\text{New value - Original value}|}{\text{Original value}} \times 100$
9. If the price of a commodity increases by r%, then reduction in consumption, so as not to increase the expenditure is $\left(\frac{r}{100 + r} \times 100\right)\%$
10. If the price of a commodity decreases by r%, then the increase in consumption so as not to decrease the expenditure is $\left(\frac{r}{100 - r} \times 100\right)\%$
11. If A's income is r% more than that of B, then B's income is less than that of A by $\left(\frac{r}{100 + r} \times 100\right)\%$.
12. If A's income is r% less than that of B, then B's income is more than that of A by $\left(\frac{r}{100 - r} \times 100\right)\%$.

Simple & Compound interest

1. When a sum of money is lent by A to B, A is called the lender (creditor) and B is called the borrower (debtor). The sum lent is called the Principle (P).
2. Interest (I) is the extra money paid by the borrower to the lender for the use of the money for a specified time.
3. The time for which the money is borrowed is called period (N). The interest paid per 100 rupees in a year is called rate percent per annum(R). The sum of interest and principal is callede the amount (A). A = P + I
4. Simple interest is always calculated only or original principal. For simple interest, I = $\frac{P \times N \times R}{100}$
5. For calculating compound interest, the interest is added back to the principal and the interest is calculated on the sum of the interest and the principal.

   For compound interest, amount A = $P\left(1 + \frac{R}{100}\right)^N$

Profit and Loss

1. Profit = SP − CP ... (SP > CP)
2. Loss = CP − SP ... (CP > SP)
3. % Profit = $\frac{\text{Profit}}{\text{CP}} \times 100 = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$
4. % Loss = $\frac{\text{Loss}}{\text{CP}} \times 100 = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$
5. Discount% = $\frac{\text{Discount}}{\text{Marked Price}} \times 100$
6. If two items are sold, each at Rs.x, one at a gain of p% and the other at a loss of p%, there is an overall loss given by $\frac{p^2}{100}$%.

   The absolute value of the loss is given by $\frac{2p^2x}{100^2 - p^2}$.
7. If CP of two items is same and % Loss and % Gain on the two items are equal, then net loss or net profit is zero.
8. Buy x get y free i.e., if x + y articles are sold at cost price of x articles, then the percentage discount = $\frac{y}{x + y} \times 100$
9. By using false weight, if a substance is sold at cost price, the overall gain % is given by

   $\frac{100 + \text{Gain}\%}{100} = \frac{\text{True Scale or Weight}}{\text{False Scale or Weight}}$.
10. In case of successive discounts a% and b%, the effective discount is $\left(a + b - \frac{ab}{100}\right)\%$

Mixtures & Alligations

1. If P_1 and P_2 are the prices of two quantities Q_1 and Q_2, then the average price of the mixture, given by P_m is: $P_m = \frac{P_1Q_1 + P_2Q_2}{Q_1 + Q_2}$
2. When two mixtures M_1 and M_2, each containing ingredient A and B in the ratio a : b and x : y, respectively, are mixed, the proportion of the ingredients A and B i.e., $q_A : q_B$, in the compound mixture is given by:

$$\frac{q_A}{q_B} = \frac{M_1 \times \left(\frac{a}{a+b}\right) + M_2 \times \left(\frac{x}{x+y}\right)}{M_1 \times \left(\frac{b}{a+b}\right) + M_2 \times \left(\frac{y}{x+y}\right)}$$

3. If a vessel contains 'a' litres of liquid A, then if 'b' litres of mixture be withdrawn and replaced by liquid B, and the operation repeated 'n' times in all, then:

$$\frac{\text{Liquid A left in vessel after } n^{th} \text{operation}}{\text{Initial quantiry of liquid A in vessel}} = \left(\frac{a-b}{a}\right)^n$$

$$\frac{\text{Liquid A left after } n^{th} \text{operation}}{\text{Liquid B left after } n^{th} \text{operation}} = \frac{\left(\frac{a-b}{a}\right)^n}{1 - \left(\frac{a-b}{a}\right)^n}$$

Time, Speed, Distance

1. Distance covered by a body is given by
   Distance = Speed × Time

2. Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

   → If a part of a journey is travelled at speed s_1 km/hr in t_1 hours and remaining part at speed s_2 km/hr in t_2 hours then, Total distance travelled = $s_1t_1 + s_2t_2$

   Average speed = $\dfrac{s_1t_1 + s_2t_2}{t_1 + t_2}$

   → If equal time intervals are travelled with different speeds, say 'a' and 'b' km/hr, then the average speed is equal to their arithmetic mean.

   Average speed = $\dfrac{a+b}{2}$ km/hr

   → If equal distances are travelled with different speeds, say 'a' and 'b' km/hr, then the average speed is equal to their harmonic mean.

   Average speed = $\left(\dfrac{2ab}{a+b}\right)$ km/hr

3. If the ratio of the speeds of A and B is x : y, then :
   → the ratio of time taken to travel equal distances is y : x
   → the ratio of distance travelled in equal time intervals is x : y

4. Relative speed
   If A and B are travelling at 'a' and 'b' km/hr, (a > b)
   Case (i): In the same direction
   → A gains (a - b) km over B in 1hour.
   → If they are 'x' km apart, A will overtake B in $\dfrac{x}{a-b}$ hrs
   Case (ii): In the opposite direction
   → A and B together cover (a + b) km in 1hour.
   → If they are 'x' km apart, A will meet B in $\dfrac{x}{a+b}$ hrs

5. Conversion units
   1 hour = 60 minutes = 60 × 60 seconds
   1 kilometer = 1000 meters
   1 km/hr = $\dfrac{5}{18}$ m/sec;

2-D Mensuration

For any triangle (Δ ABC):

Area = $\dfrac{1}{2}$ × (base) × (height)

= $\sqrt{s(s-a)(s-b)(s-c)}$

(where a, b, c are its sides; s = $\dfrac{a+b+c}{2}$)

= r × s (r = radius of incircle)

= $\dfrac{abc}{4R}$ (R = radius of circumcircle).

For any right angled triangle (Δ ABC):

Area = $\dfrac{1}{2}$ × Product of perpendicular sides.

For any equilateral triangle (Δ ABC):

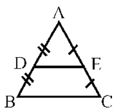
Area = $\dfrac{\sqrt{3}}{4}$ × (side)^2.

Lines, Angles and Triangles

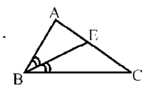
1. Sum of the angles in a straight line = 180° (Angles in a linear pair).
2. Vertically opposite angles are congruent (equal).
3. If any point is equidistant from the endpoints of a segment, then it must lie on the perpendicular bisector of the segment.
4. When two parallel lines are intersected by a transversal, corresponding angles are equal, alternate angles are equal and co-interior angles are supplementary. (All acute angles formed are equal to each other and all obtuse angles are equal to each other).
5. The ratio of intercepts formed by a transversal intersecting three parallel lines is equal to the ratio of corresponding intercepts formed by any other transversal.

For any triangle (Δ ABC):

6. Sum of interior angles = 180°.
7. Measure of exterior angle = Sum of remote interior angles.
8. Area = $\dfrac{1}{2}$ × (base) × (height) = $\sqrt{s(s-a)(s-b)(s-c)}$
   (where a, b, c are its sides; s = $\dfrac{a+b+c}{2}$)
   = r × s (r = radius of incircle)
   = $\dfrac{abc}{4R}$ (R = radius of circumcircle).
9. The line segment joining the midpoint of any two sides is parallel to the third side and has half the length of the third side. (Midpoint theorem)
10. If DE ∥ BC, then $\dfrac{AD}{DB} = \dfrac{AE}{EC}$.
    (Basic proportionality theorem - BPT)



11. If AD is the median i.e., BD = DC, then $AB^2 + AC^2 = 2(AD^2 + DC^2)$.
    (Appollonius theorem)
12. If ∠ABE = ∠EBC, then $\dfrac{BA}{BC} = \dfrac{AE}{EC}$.
    (Interior angle bisector theorem)



For any right angled triangle (Δ ABC):

13. If ∠ABC = 90°, then $AB^2 + BC^2 = AC^2$.
    (Pythagoras theorem)

Special Triangles

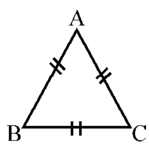
Equilateral triangle:

Each angle = 60°.

Height = $\dfrac{\sqrt{3}}{2}$ × side.

Area = $\dfrac{\sqrt{3}}{4}$ × (side)^2.

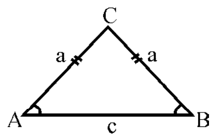
inradius = $\dfrac{\text{height}}{3}$, circumradius = $\dfrac{2}{3}$ × height.



Isosceles Triangle:

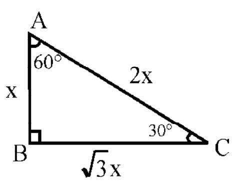
Angles opposite to equal sides are equal.

Area = $\dfrac{a}{4}\sqrt{4c^2 - a^2}$



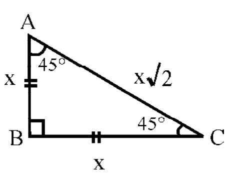
30°–60°–90° triangle:

Area = $\dfrac{\sqrt{3}}{2}$ × x^2



45°– 45°–90° triangle:

Area = $\dfrac{x^2}{2}$



Congruency & Similarity of Triangles

Congruency

Two triangles are congruent if their corresponding sides and angles are congruent.

Tests of congruence: (SSS / SAS / AAS / ASA)

Similarity

Two triangles are similar if their corresponding angles are congruent and corresponding sides are in proportion.

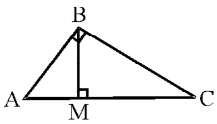
Tests of similarity: (AA / SSS / SAS)

For two similar triangles:

Ratio of the sides = Ratio of heights = Ratio of Medians = Ratio of Angle bisectors = Ratio of circumradii.

Also, Ratio of the areas = Ratio of square of the sides

For any right angled triangle (Δ ABC):



$\Delta ABC \sim \Delta AMB \sim \Delta BMC$

$BM^2 = AM \times MC$

$AB \times BC = BM \times AC$

Polygons and their properties

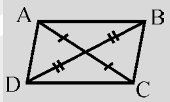
For any regular polygon:

(A polygon which has all its sides and angles equal)

1. Sum of internal angles = 180°(n-2).
2. Measure of an internal angle = $\dfrac{180°(n-2)}{n}$.
   (where n is the number of sides)

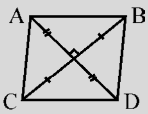
Properties of some special polygons:

Parallelogram:



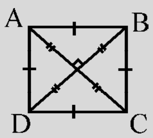
3. Opposite sides are parallel and congruent.
4. Opposite angles are congruent.
5. Diagonals bisect each other.
6. Area of parallelogram = Base × height .

Rhombus:



7. Opposite sides are parallel and all sides are equal.
8. Opposite angles are congruent.
9. Diagonals bisect each other at 90°.
10. Area = $\dfrac{1}{2}$ × Product of diagonals.

Square:

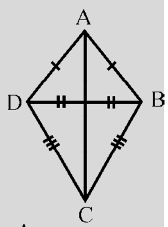


11. All sides are congruent and opposite sides are parallel. All angles are right angles.
12. Diagonals are congruent and bisect each other at 90°.
    ℓ (diagonal) = $\sqrt{2}$ × (side).
13. Area = (side)^2 = $\dfrac{(\text{diagonal})^2}{2}$

Rectangle :

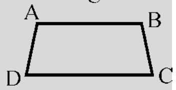
14. Area = Length × Breadth

Kite:



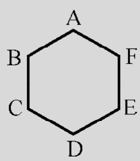
15. Two pairs of adjacent sides are congruent.
16. Diagonals intersect each other at 90° and longer diagonal bisects shorter diagonal.
17. Area = $\dfrac{1}{2}$ × product of diagonals.

Isosceles Trapezium:



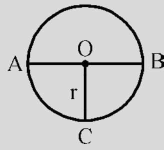
18. One pair of opposite sides is parallel.
19. Non-parallel sides are congruent.
20. Area = $\dfrac{1}{2}$ × sum of parallel sides × height.

Regular Hexagon:



21. Area = $\dfrac{3\sqrt{3}}{2}$ (side)^2.
22. Six equilateral triangles are formed by joining the opposite vertices of the hexagon.

Circle and its properties

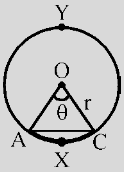


For the given circle,

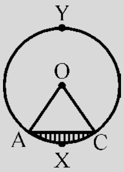
1. Radius of the circle (OA, OB, OC) = r
2. Diameter of the circle (AB) = 2r
3. Area = πr^2
4. Circumference = $2\pi r$

Chord Properties:

5. Chords equidistant from the the centre of a circle are equal.
6. A line from the centre, perpendicular to a chord, bisects the chord.
7. Equal chords subtend equal angles at the centre.
8. The diameter is the longest chord of a circle.
9. Chord AC divides the length of the circle into two parts:
   minor arc – AXC
   major arc – AYC
   The area bound by OAXC is a 'sector'.
10. Measure of arc AXC = m$\angle$AOC = θ.

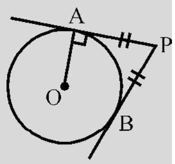


11. ℓ (arc AXC) = $\dfrac{\theta}{360°} \times 2\pi r$.
12. Area (sector OAXC) = $\dfrac{\theta}{360°} \times \pi r^2$



13. Chord AC divides the area of the circle into two parts:
    The shaded area is the minor segment, the rest is the major segment.
14. Area of minor segment
    = Area of sector OAXC– area of ΔOAC
    $= r^2\left[\dfrac{\pi\theta}{360} - \dfrac{\sin\theta}{2}\right]$;
    (where θ = m$\angle$AOC (in degrees))

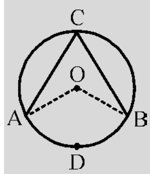
Tangent Properties:



PA and PB are tangents to the given circle, OA is the radius.

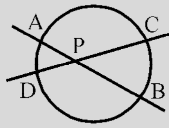
15. PA = PB.
16. OA $\perp$ PA.

Inscribed angle / Subtended Angle:



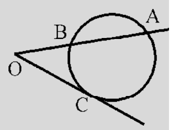
$\angle$ACB is inscribed in the arc ACB & subtended by arc ADB / chord AB.

17. m$\angle$AOB = 2 × m$\angle$ACB.
    (Inscribed Angle Theorem)
18. Angle inscribed in a semicircle or that subtended by a diameter is a right angle.
19. For a cyclic quadrilateral, opposite angles are supplementary.

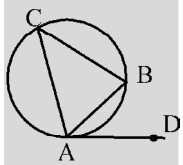


Secant, Tangent and Chord Properties.

20. If two secants viz. AB and CD, intersect at P, then AP × BP = CP × DP.
    Note: This equation will hold even if the secants meet outside the circle.



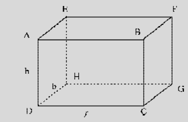
21. If a tangent (OC) and a secant (AB) meet externally at O, then $OC^2 = OA \times OB$.
    (Tangent – Secant theorem)
22. The angle made by chord (AB) with the tangent at A (AD) is equal to the angle subtended by it on the opposite side.
    m$\angle$BAD = m$\angle$ACB.
    (Tangent Chord Property)



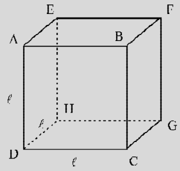
3D Mensuration

Solids and their associated formulae:

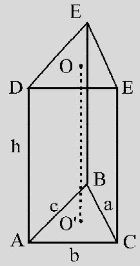
1. **Cuboid:**
Total surface area
$= 2(\ell \times b + b \times h + h \times \ell)$
Volume $= \ell \times b \times h$
Length of body diagonal
$= \sqrt{\ell^2 \times b^2 + h^2}$



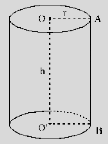
2. **Cube;**
Total surface area $= 6\,\ell^2$
Volume $= \ell^3$
Body diagonal $= \sqrt{3} \times \ell$



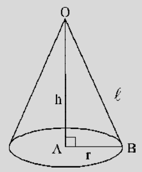
3. **Right Prism** (Solid with rectangular vertical faces and bases as congruent polygons):
Total surface area = Perimeter of base × height + 2 × area of base
Volume = Area of base × height



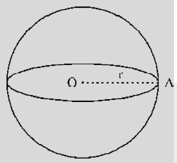
4. **Right circular cylinder:**
curved surface area $= 2\pi rh$
Total surface area $= 2\pi r(r + h)$



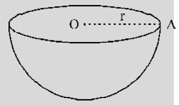
5. **Right circular cone:**
Slant height, $\ell = \sqrt{r^2 + h^2}$
Curved surface area $= \pi r \ell$
Total surface area $= \pi r \ell + \pi r^2$
Volume $= \frac{1}{3}\pi r^2 h$



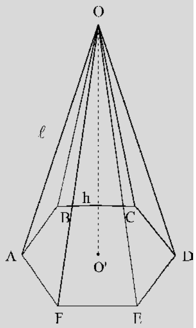
6. **Sphere:**
Surface area $= 4\pi r^2$
Volume $= \frac{4}{3}\pi r^2$



7. **Hemisphere:**
Curved surface area $= 2\pi r^2$
Total surface area =
$3\pi r^2$
Volume $= \frac{2}{3}\pi r^3$

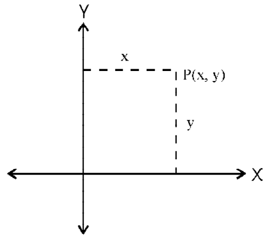


8. **Pyramid:**
Total surface area = Area of base
$+ \frac{1}{2} \times$ Perimeter of base ×
Slant height
Volume $= \frac{1}{3} \times$ Base area ×
height.



Coordinate Geometry, Functions and Graphs

1. Any point P on the coordinate plane can be represented by (x, y) where:



x = the x coordinate = distance of P from the Y–axis
y = the y coordinate = distance of P from the X–axis

2. If the value of y can be expressed in terms of x, then y is said to be a function of x. This is expressed as y = f(x). E.g. $y = 3x^2 + 4x + 5$. In this case, if we know the relation, y = f(x), then the value of y can be determined for any given value of y.

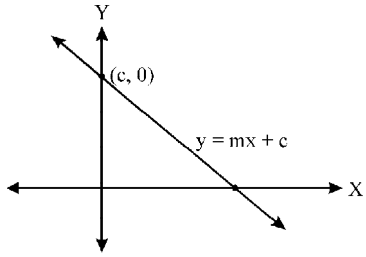
3. The graph of a function f(x) is the set of points of the form (x, y) which satisfy the equation y = f(x).

Linear Function

1. If the value of y changes linearly with x, i.e the change in y is directly proportional to the change in x then y is said to be a linear function of x.

Thus, for any two points (x_1, y_1) and (x_2, y_2), we have $\frac{y_1 - y_2}{x_1 - x_2}$ = constant, say 'm'.

This constant value is called the slope of the line.



2. The graph of a linear function is a straight line.

3. Equation of a line may be written in the form: y = mx + c, where m is the slope of line and 'c' is the y intercept i.e., the line meets y axis at the point (c, 0).

Graph of the line
y = mx + c
looks as follows:

4. Case (i) m > 0: A positive slope indicates that the line makes an acute angle with the positive direction of the X-axis.

Case (ii) m < 0: A negative slope indicates that the line makes an obtuse angle with the positive direction of the X-axis.

Case (iii) m = 0: A zero slope indicates that the line is parallel to the X-axis.

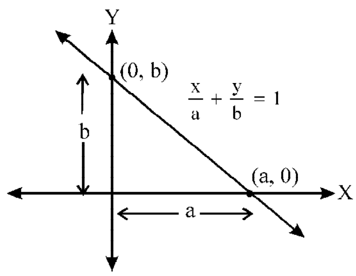
Case (iv) $m = \infty$: An infinite slope indicates that the line is perpendicular to the X-axis.

5. Equation of the X – axis is 'y = 0'.
Equation of the Y – axis is 'x = 0'.

6. Equation of a line parallel to the X – axis is of the form 'y = a' {a is the distance of the line from the X-axis}
Equation of a line parallel to Y – axis is of the form 'x = b'{b is the distance of the line from the Y-axis}

7. Equation of a line with x and y intercepts as 'a' and 'b' respectively (i.e. which meets the X-axis and Y- axis at points (a, 0) and (0, b) respectively) is

$\frac{x}{a} + \frac{y}{b} = 1$ (or bx + ay = ab).



8. A given pair of values for x and y, say (x_1,y_1) is said to be the solution of two simultaneous linear equations, if it satisfies both the equations.
If we draw the graphs (straight lines) of these two linear equations, then the solution (x_1,y_1) is the point of intersection of the two lines.
Note: Solving simultaneous linear equations in two variables involves elimination of one of the variables to obtain the value of the other variable.

Sequence, Series & Progression

1] For an Arithmetic Progression
$a_n = a_1 + (n - 1)d$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d] = \frac{n}{2}(a_1 + a_n)$$

(a_1 is the first term, d is the common difference)

2] If x, y and z are in A.P. then, $y - x = z - y$ and y is the Arithmetic Mean of x and z.

3] For a Geometric Progression
$a_n = a_1 r^{n-1}$

$$S_n = a_1\left[\frac{1 - r^n}{1 - r}\right]$$

$$S_\infty = \frac{a_1}{1 - r} \text{ if } |r| < 1$$

(a_1 is the first term, r is the common ratio)

4] If x, y and z are in G.P. then, $xz = y^2$ and y is the Geometric Mean of x and z.

5] For the first 'n' natural numbers

$\rightarrow$ Their sum $= \dfrac{n(n + 1)}{2}$

$\rightarrow$ Sum of their squares $= \dfrac{n(n + 1)(2n + 1)}{6}$

$\rightarrow$ Sum of their cubes $= \left[\dfrac{n(n + 1)}{2}\right]^2$

Set Theory

1] A set is a collection of objects. A set is represented in one of the following ways:
The set of prime numbers less than 10 is written as {2,3,5,7} or {x/ x is a prime number less than 10}

2] The number of elements in a set is called its **cardinal number** and is written as n(A). A set with cardinal number 0 is called a null set while that with cardinalnumber ∞ is called an infinite set.

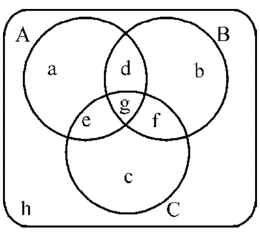
3] Set A is said to be a subset of Set B if each and every element of Set A is also contained in Set B. Set A is said to be a **proper subset** of Set B if Set B has at least one element that is not contained in Set A.

4] The **Universal set** is defined as the set of all possible objects under consideration. Every other set is then a subset of the universal set.

5] **Union of two sets** is represented as $A \cup B$ and consists of elements that are present in either Set A or Set B or both.

6] **Intersection of two sets** is represented as $A \cap B$ and consists of elements that are present in both Set A and Set B.

7] **Venn Diagram:** A venn diagram is used to visually represent the relationship between various sets.
What do each of the areas in the figure represent?
a- only A
b- only B
c- only C
d- A and B, not C
e- A and C, not B
f- B and C, not A
g- A, B and C
h- None of A, B or C



8] **Some important properties:**
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Logarithms

If $a^n = b$, then $\log_a b = n$

Conventionally, $\log b$ represents $\log_{10} b$

1] $\log_b 1 = 0$

2] $\log_a a = 1$

3] $\log_a b = \dfrac{1}{\log_b a}$ OR $\log_a b \times \log_b a = 1$

4] $\log_b(m \times n) = \log_b m + \log_b n$

5] $\log_b\left(\dfrac{m}{n}\right) = \log_b m - \log_b n$

6] $\log_b(m)^n = n \log_b(m)$

7] $\log_b m = \dfrac{\log_a m}{\log_a b} = \log_a m \times \log_b a$

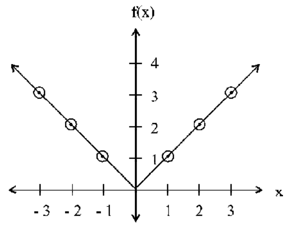
8] $\log_{b^\beta}(a^\alpha) = \dfrac{\alpha}{\beta}\log_b a$

9] $\log_x a = \log_x b$ if and only if $a = b$

Other types of functions

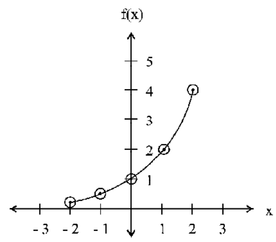
Modulus function:

1. A function f(x) is called a modulus function if $f(x) = P|Qx + R| + S$.
2. A special application is $f(x) = |x|$, i.e., f(x) takes only the magnitude of x.
$f(x) = -x$ if $x < 0$
$= x$ if $x \geq 0$



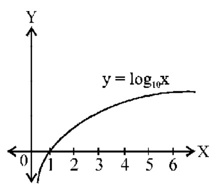
Exponential function:

1. A function $f(x) = a^x$ is called an exponential function. ($a > 0$ and $x \in R$)
2. The graph of the function $f(x) = 2^x$ is as shown alongside.
3. The exponential functions a^x will never be negative for any value of x. The value of f(x) approaches 0 when x tends to negative infinity.



Logarithmic function:

1. A function $f(x) = \log x$ is called a logarithmic function.
2. The graph of the function $f(x) = \log_{10} x$ is as shown along-side.



Even and Odd Functions:

1. A function f(x) is said to be an even function of x if $f(-x) = f(x)$, for all values of x. The graph of an even function is symmetrical about the Y-axis.
2. A function f(x) is said to be an odd function of x if $f(-x) = -f(x)$, for all values of x.

Counting Principles

PRINCIPLE OF COUNTING:

When to MULTIPLY?

When two tasks are performed in succession, i.e., they are connected by an 'AND', to find the total number of ways of performing the two tasks, you have to MULTIPLY the individual number of ways.
eg: If there are 3 boys and 4 girls, then to select a boy AND a girl you have to select 1 boy out of 3 and 1 girl out of 4. So, the number of ways of selecting $= 3 \times 4 = 12$.

When to ADD?

When only one of the two tasks is performed, i.e. the tasks are connected by an 'OR', to find the total number of ways of performing the two tasks you have to ADD the individual number of ways.
eg: If there are 3 boys and 4 girls, then to select a child you either select a boy OR select a girl. So, the number of ways of selecting a child $= 3 + 4 = 7$.

SPECIAL CASES:

1] **Linear arrangement of r out of 'n' distinct items(^nP_r):** The first item in the line can be selected in 'n' ways AND the second in (n – 1) ways AND the third in (n – 2) ways AND so on. So, the total number of ways of arranging 'r' items out of 'n'is **(n)(n – 1)(n – 2)...(n – r + 1).**

2] **Linear arrangement of 'n' items out of which 'p' are alike, 'q' are alike, 'r' are alike:**

Number of ways = $\dfrac{n!}{p!\,q!\,r!...}$

3] **Circular arrangement of 'n' distinct items:**
Fix the first item AND then arrange all the other items linearly w.r.t. the first item. This can be done in $1 \times (n-1)! = (n-1)!$ **ways.**

4] **Selection of r items out of 'n' distinct items(^nC_r):**
Arrange of r items out of n = Select r items out of n AND arrange r items out of r.

$$^nP_r = {}^nC_r \times r! \Rightarrow {}^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Permutations & Combinations - Summary Table

Sr. No	Situation		Number of ways
1	**Linear Arrangement of:**		
	'r' out of 'n' distinct items	*Without repetition*	$^nP_r = \dfrac{n!}{(n-r)!}$
		Repetition allowed	n^r
	'n' out of 'n' distinct items	*Without repetition*	$^nP_n = n!$
		Repetition allowed	n^n
	'n' out of 'n' items not all distinct	*Without repetition*	$\dfrac{n!}{p!\,q!\,r!}$
	Circular Arrangement of:		
	'n' out of 'n' distinct items	*Without repetition*	$(n-1)!$
	'n distinct beads of a necklace	*Without repetition*	$\dfrac{(n-1)!}{2}$
2	**Selection of:**		
	'r' out of 'n' distinct items	$^nC_r = \dfrac{^nP_r}{r!} = \dfrac{n!}{r!\,(n-1)!}$	
	Total selections from:		
	'n' distinct items	$^nC_0 + {}^nC_1 + C^n_2 + C^n_3 + C^n_4 + ... + {}^nC_n = 2^n$	
3	**Partitioning**		
	Similar Items - Distinct Groups	*no groups empty*	$^{n-1}C_{r-1}$
		some groups may be empty	$^{n+r-1}C_{r-1}$
	Distinct Items - Distinct Groups	*some groups may be empty*	r^n
		arrangement within a group matters	$\dfrac{(n+r-1)!}{(r-1)!}$ (E.g. 6 rings - 4 fingers)
	Similar Items - Similar Groups	List the various possibilities and find number of ways for each possibility	
	Distinct Items - Similar Groups	List the various possibilities and find number of ways for each possibility	
4	Dearrangement of 'n' items	$n!\left[1 - \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} - ... \dfrac{1}{n!}\right]$	