> *Задание* 1

> $FourierSeries := \mathbf{proc}(f, n, a, b)\ \mathbf{local}\ a0, ak, bk, l;$

$l := \dfrac{b-a}{2};$

$a0 := \dfrac{1}{l} int(f, x = a..b);$

$ak := \dfrac{1}{l} int\left(f \cdot \cos\left(\dfrac{k \cdot \mathrm{Pi} \cdot x}{l}\right), x = a..b\right);$

$bk := \dfrac{1}{l} int\left(f \cdot \sin\left(\dfrac{k \cdot \mathrm{Pi} \cdot x}{l}\right), x = a..b\right);$

$\mathbf{return}\ \dfrac{a0}{2} + simplify\left(sum\left(\left(ak \cdot \cos\left(\dfrac{k \cdot \mathrm{Pi} \cdot x}{l}\right) + bk \cdot \sin\left(\dfrac{k \cdot \mathrm{Pi} \cdot x}{l}\right)\right), k = 1..n\right)\right);$

end proc:

> $f := x \rightarrow piecewise(-\mathrm{Pi} \leq x < 0, -\mathrm{Pi} - x, 0 \leq x < \mathrm{Pi}, \mathrm{Pi}):$

$a0 = \dfrac{1}{\mathrm{Pi}} int(f(x), x = -\mathrm{Pi}..\mathrm{Pi});$

$an = simplify\left(\dfrac{1}{\mathrm{Pi}} int(f(x) \cdot \cos(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint;$

$bn = simplify\left(\dfrac{1}{\mathrm{Pi}} int(f(x) \cdot \sin(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint;$

$f = FourierSeries(f(x), \mathrm{infinity}, -\mathrm{Pi}, \mathrm{Pi});$

$$a0 = \frac{\pi}{2}$$

$$an = \frac{(-1)^n - 1}{\pi n^2}$$

$$bn = \frac{-(-1)^n + 2}{n}$$

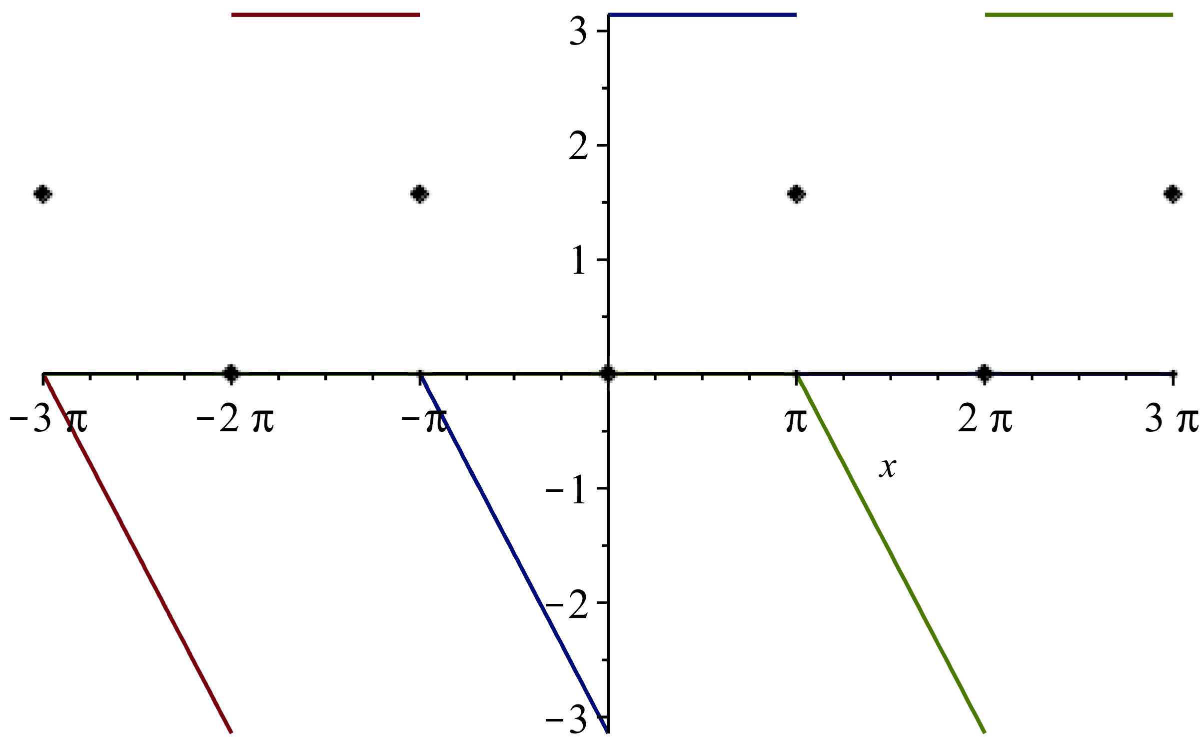
$$f = \frac{\pi}{4} + \left(\sum_{k=1}^{\infty} \frac{((-1)^k - 1)\cos(kx) - \pi \sin(kx)\, k\, ((-1)^k - 2)}{\pi k^2}\right) \tag{1}$$

> $g := (x, n) \rightarrow piecewise(-\mathrm{Pi} + 2\,\mathrm{Pi} \cdot n < x < 2\,\mathrm{Pi} \cdot n, -\mathrm{Pi} - x + 2\,\mathrm{Pi} \cdot n, +2\,\mathrm{Pi} \cdot n < x < \mathrm{Pi} + 2\,\mathrm{Pi} \cdot n, \mathrm{Pi}):$

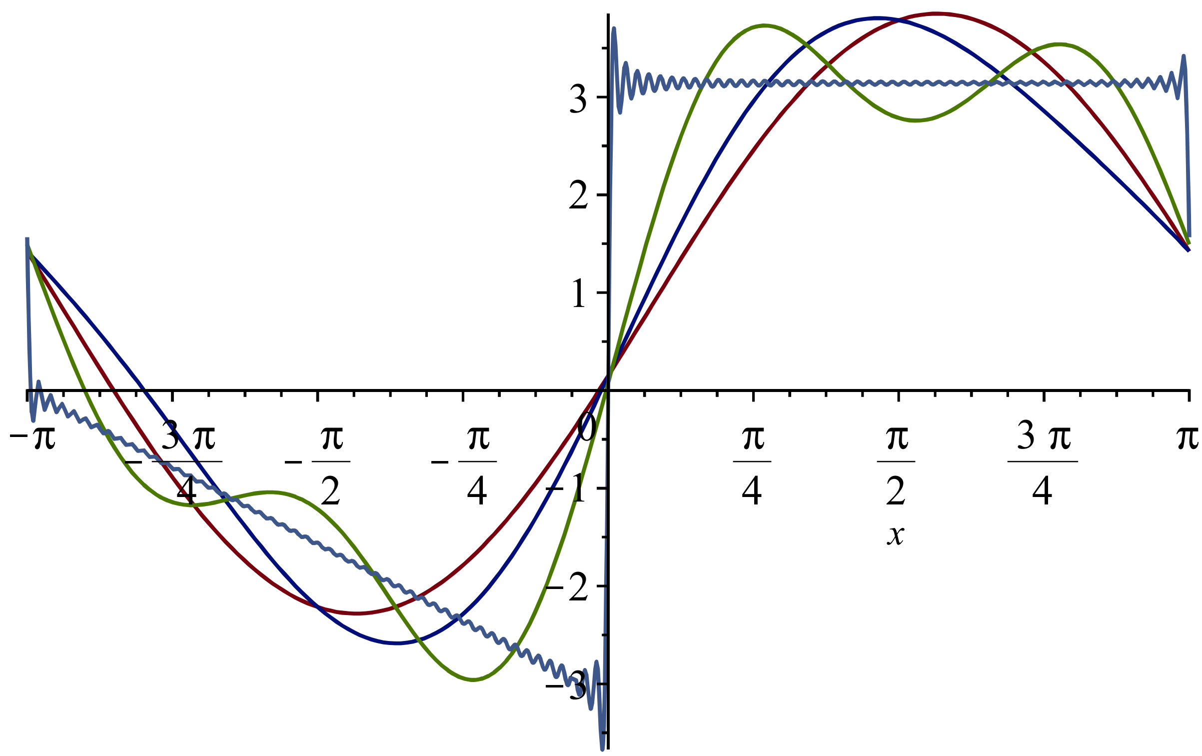
$p1 := plot([g(x, -1), g(x, 0), g(x, 1)], x = -3\,\mathrm{Pi}..3\,\mathrm{Pi}, discont = true, symbolsize = 1):$

$p2 := plots[pointplot]\left(\left[[-2\,\mathrm{Pi}, 0], [0, 0], [2\,\mathrm{Pi}, 0], \left[-3\,\mathrm{Pi}, \dfrac{\mathrm{Pi}}{2}\right], \left[-\mathrm{Pi}, \dfrac{\mathrm{Pi}}{2}\right], \left[\mathrm{Pi}, \dfrac{\mathrm{Pi}}{2}\right], \left[3\,\mathrm{Pi}, \dfrac{\mathrm{Pi}}{2}\right]\right], symbol = soliddiamond, symbolsize = 12\right):$

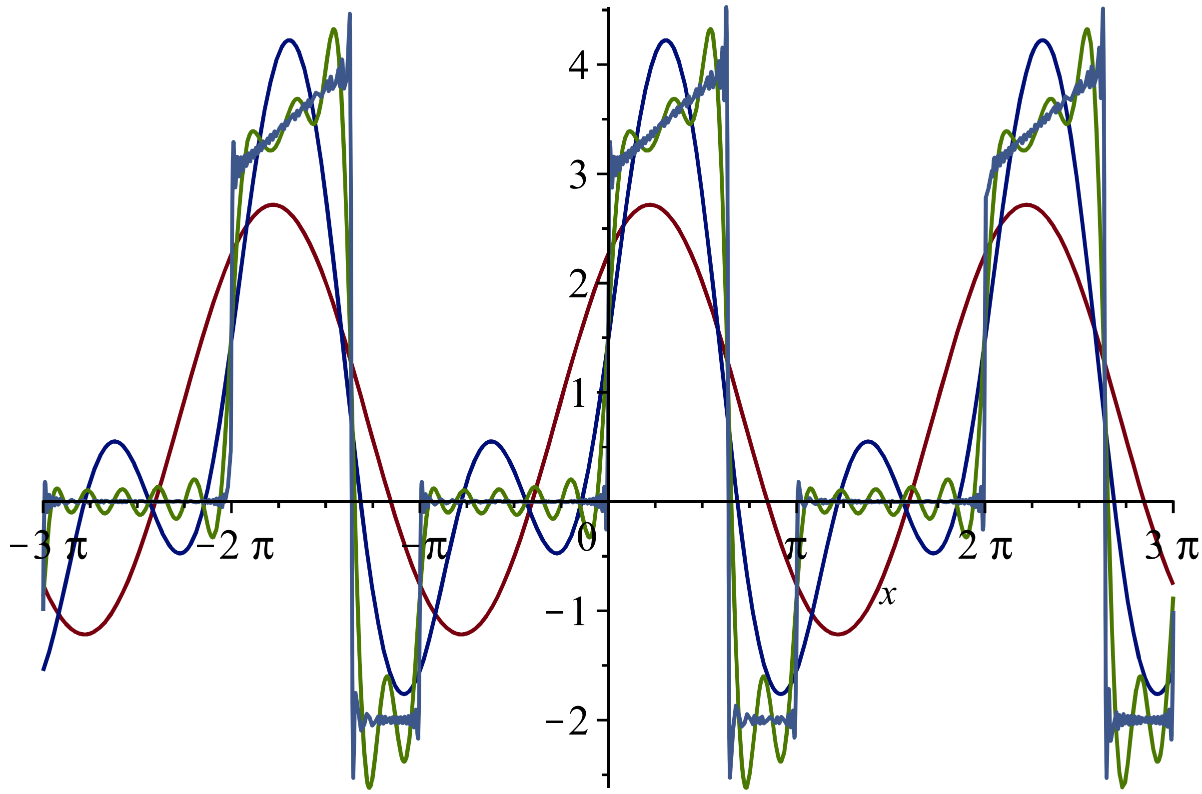
$plots[display](p1, p2);$



> *plot*([*FourierSeries*($f(x)$, 1, -Pi, Pi), *FourierSeries*($f(x)$, 2, -Pi, Pi), *FourierSeries*($f(x)$, 3, -Pi, Pi), *FourierSeries*($f(x)$, 100, -Pi, Pi)], x =-Pi ..Pi);



> *plot*([*FourierSeries*($f(x)$, 1, -Pi, Pi), *FourierSeries*($f(x)$, 2, -Pi, Pi), *FourierSeries*($f(x)$, 10, -Pi, Pi), *FourierSeries*($f(x)$, 100, -Pi, Pi)], x =-3 Pi ..3 Pi);



Задание 2

$$f := x \rightarrow piecewise\left(0 < x < 2, \frac{1}{2} \cdot x + 3, 2 \le x \le 6, -2\right):$$

$$a0 = \frac{1}{3} int(f(x), x = 0..6);$$

$$an = simplify\left(\frac{1}{3} int\left(f(x) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{3}\right), x = 0..6\right)\right) \text{ assuming } n :: posint;$$

$$bn = simplify\left(\frac{1}{3} int\left(f(x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{3}\right), x = 0..6\right)\right) \text{ assuming } n :: posint;$$

$$f = FourierSeries(f(x), \text{infinity}, 0, 6);$$

$$a0 = -\frac{1}{3}$$

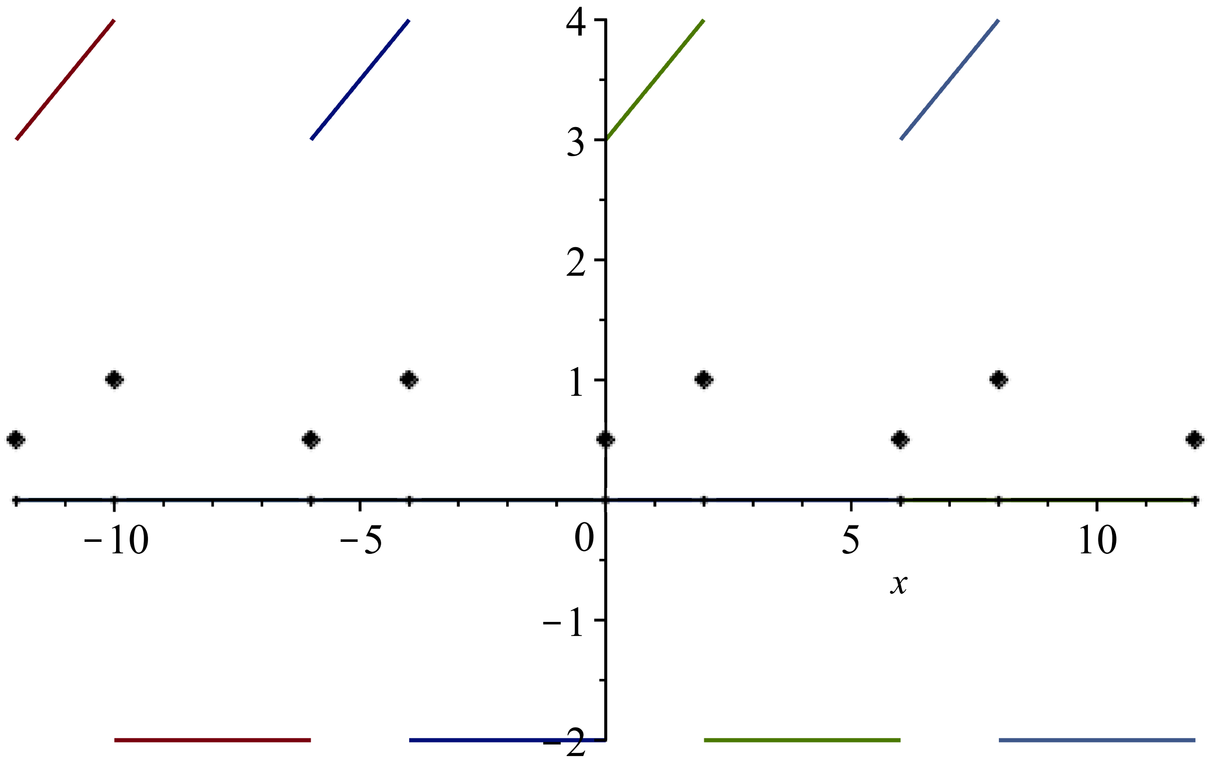
$$an = \frac{3\left(4\pi n \sin\left(\frac{2\pi n}{3}\right) + \cos\left(\frac{2\pi n}{3}\right) - 1\right)}{2 n^2 \pi^2}$$

$$bn = \frac{-12\pi n \cos\left(\frac{2\pi n}{3}\right) + 10\pi n + 3\sin\left(\frac{2\pi n}{3}\right)}{2 n^2 \pi^2}$$

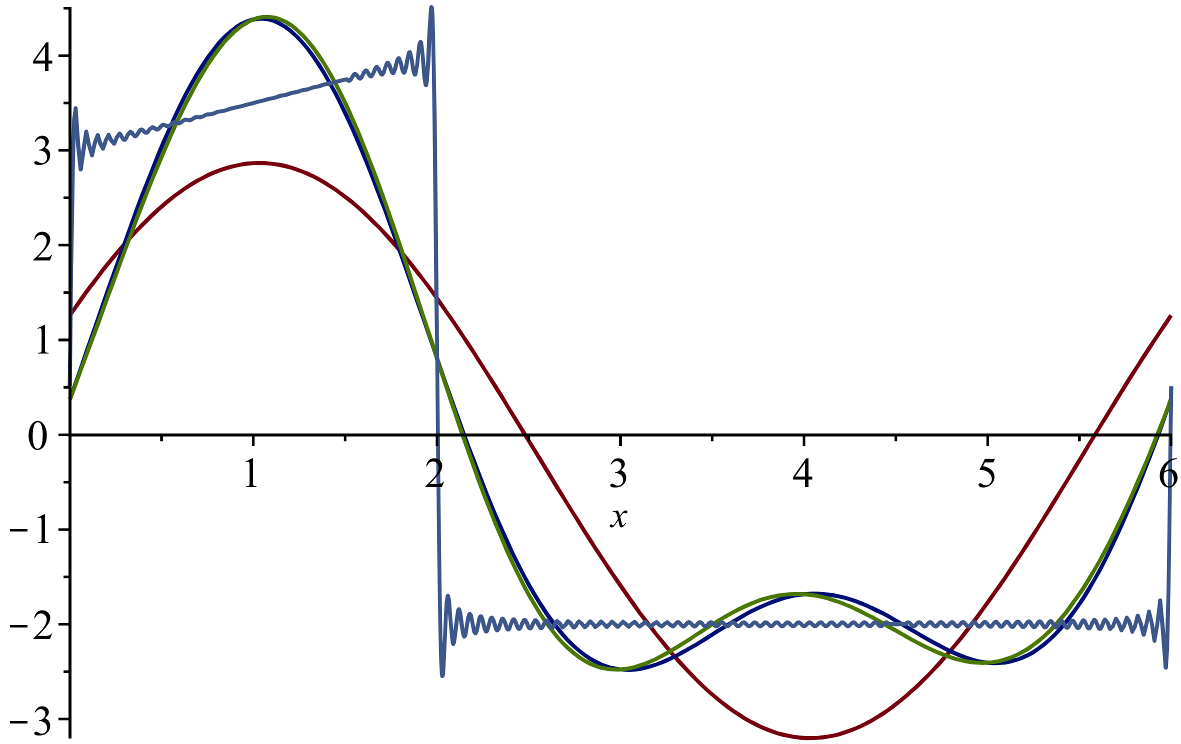
$$f = -\frac{1}{6} + \left(\sum_{k=1}^{\infty} \frac{1}{2 k^2 \pi^2}\left(\left(12 k \pi \sin\left(\frac{2 k \pi}{3}\right) + 3\cos\left(\frac{2 k \pi}{3}\right) - 3\right)\cos\left(\frac{k \pi x}{3}\right)\right.\right. \qquad \textbf{(2)}$$

$$-12\sin\left(\frac{k\pi x}{3}\right)\left(k\pi\cos\left(\frac{2k\pi}{3}\right)-\frac{5k\pi}{6}-\frac{\sin\left(\frac{2k\pi}{3}\right)}{4}\right)\Bigg)\Bigg)\Bigg)$$

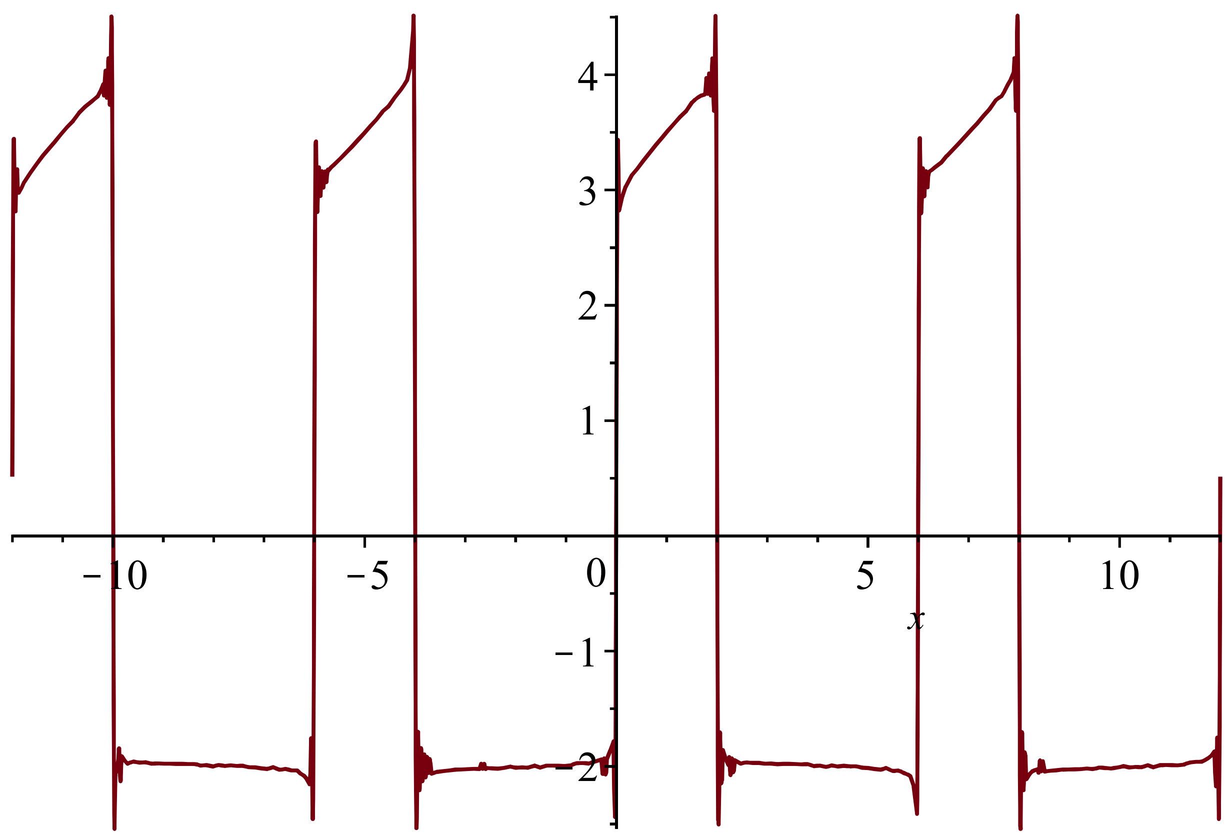
> $g := (x, n) \rightarrow piecewise\left(0 + 6n < x < 2 + 6n, \frac{1}{2}x - 3n + 3, 2 + 6n < x < 6 + 6n, -2\right):$
$p1 := plot([g(x,-2), g(x,-1), g(x,0), g(x,1)], x = -12..12, discont = true, symbolsize = 1):$
$p2 := plots[pointplot]\left(\left[\left[-12, \frac{1}{2}\right], [-10, 1], \left[-6, \frac{1}{2}\right], [-4, 1], \left[0, \frac{1}{2}\right], [2, 1], \left[6, \frac{1}{2}\right], [8, 1], \left[12, \frac{1}{2}\right]\right], symbol = soliddiamond, symbolsize = 12\right):$
$plots[display](p1, p2);$



> $plot([FourierSeries(f(x), 1, 0, 6), FourierSeries(f(x), 2, 0, 6), FourierSeries(f(x), 3, 0, 6), FourierSeries(f(x), 100, 0, 6)], x = 0..6);$



> plot(FourierSeries(f(x), 100, 0, 6), x=-12..12);



Задание 3

> $f := x \rightarrow piecewise\left(0 < x < 2, -\frac{3}{2}x^2 + 3x - \frac{3}{2}, 2 \le x \le 5, \frac{1}{2}x - \frac{5}{2}\right):$

$a0 = \frac{2}{5} int(f(x), x = 0 ..5);$
$an = simplify\left(\frac{2}{5} int\left(f(x) \cdot \cos\left(\frac{2 \cdot n \cdot \mathrm{Pi} \cdot x}{5}\right), x = 0 ..5\right)\right)$ assuming n :: *posint*;
$bn = simplify\left(\frac{2}{5} int\left(f(x) \cdot \sin\left(\frac{2 \cdot n \cdot \mathrm{Pi} \cdot x}{5}\right), x = 0 ..5\right)\right)$ assuming n :: *posint*;
$f = FourierSeries(f(x)$, infinity, 0, 5$)$;

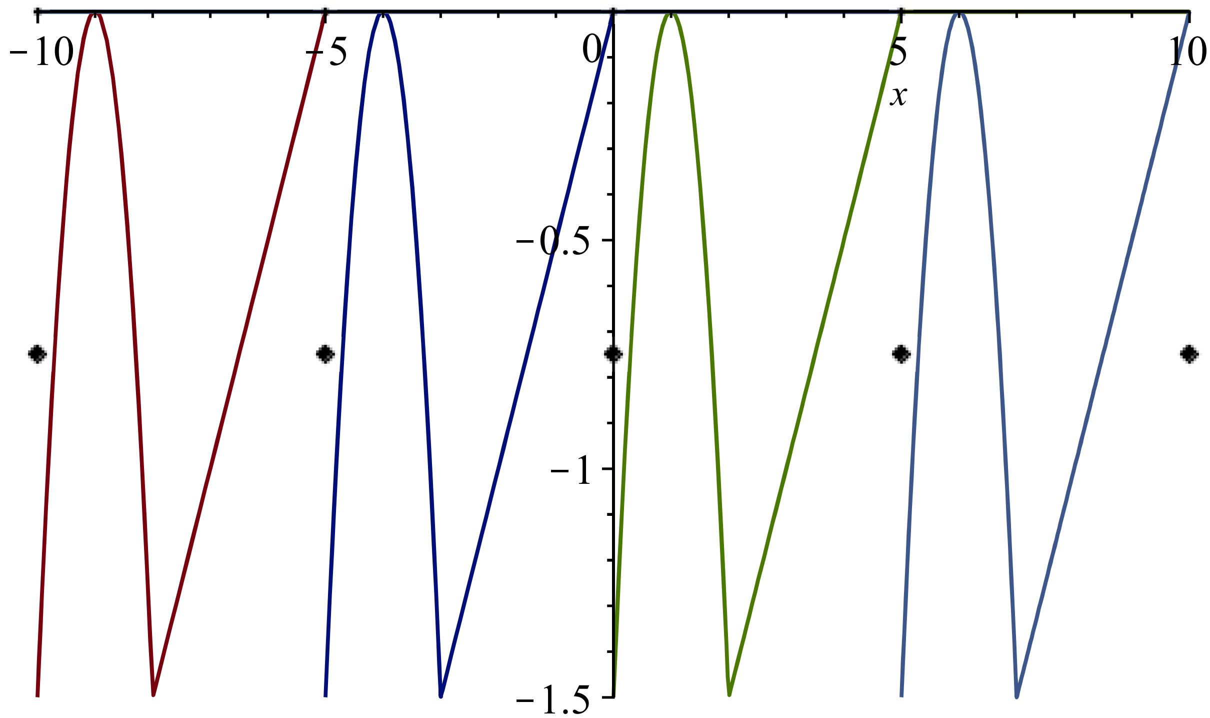
$$a0 = -\frac{13}{10}$$

$$an = -\frac{5\left(7 n \pi \cos\left(\frac{4 n \pi}{5}\right) + 5 n \pi - 15 \sin\left(\frac{4 n \pi}{5}\right)\right)}{4 n^3 \pi^3}$$

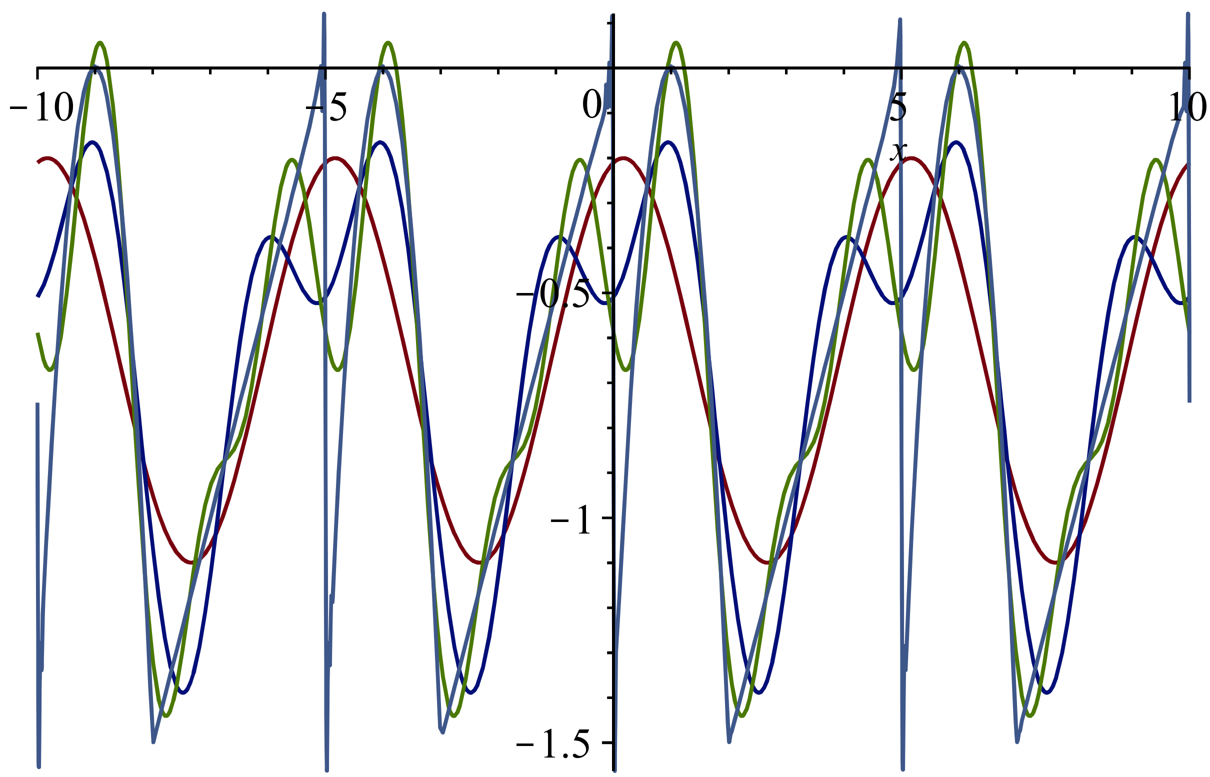
$$bn = \frac{-6 n^2 \pi^2 - 35 n \pi \sin\left(\frac{4 n \pi}{5}\right) - 75 \cos\left(\frac{4 n \pi}{5}\right) + 75}{4 n^3 \pi^3}$$

$$f = -\frac{13}{20} + \left(\sum_{k=1}^{\infty} \frac{1}{4 k^3 \pi^3} \left(\left(-6 k^2 \pi^2 - 35 k \pi \sin\left(\frac{4 k \pi}{5}\right) - 75 \cos\left(\frac{4 k \pi}{5}\right) + 75\right) \sin\left(\frac{2 k \pi x}{5}\right) - 35 \left(k \pi \cos\left(\frac{4 k \pi}{5}\right) + \frac{5 k \pi}{7} - \frac{15 \sin\left(\frac{4 k \pi}{5}\right)}{7}\right) \cos\left(\frac{2 k \pi x}{5}\right)\right)\right) \quad \textbf{(3)}$$

> $g := (x, n) \rightarrow piecewise\left(0 + 5 n < x < 2 + 5 n, -\frac{3}{2}(x - 5 n)^2 + 3(x - 5 n) - \frac{3}{2}, 2 + 5 n \leq x \leq 5 + 5 n, \frac{1}{2} \cdot (x - 5 n) - \frac{5}{2}\right):$
$p1 := plot([g(x, -2), g(x, -1), g(x, 0), g(x, 1)], x = -10 ..10, discont = true, symbolsize = 1):$
$p2 := plots[pointplot]\left(\left[\left[-10, -\frac{3}{4}\right], \left[-5, -\frac{3}{4}\right], \left[0, -\frac{3}{4}\right], \left[5, -\frac{3}{4}\right], \left[10, -\frac{3}{4}\right]\right], symbol = soliddiamond, symbolsize = 12\right):$
$plots[display](p1, p2);$



> $plot([FourierSeries(f(x), 1, 0, 5), FourierSeries(f(x), 2, 0, 5), FourierSeries(f(x), 3, 0, 5),$
$FourierSeries(f(x), 100, 0, 5)], x = -10 ..10);$



> $f := x \rightarrow piecewise\left(-5 \le x \le -2, -\frac{1}{2}x - \frac{5}{2}, -2 < x < 0, -\frac{3}{2}x^2 - 3x - \frac{3}{2}, 0 < x < 2, -\frac{3}{2}x^2 + 3x - \frac{3}{2}, 2 \le x \le 5, \frac{1}{2}x - \frac{5}{2}\right):$

$a0 = \frac{1}{5} int(f(x), x = -5 ..5);$

$an = simplify\left(\frac{1}{5} int\left(f(x) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{5}\right), x = -5 ..5\right)\right)$ assuming $n :: posint;$

$bn = simplify\left(\frac{1}{5} int\left(f(x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{5}\right), x = -5 ..5\right)\right)$ assuming $n :: posint;$

$f = FourierSeries(f(x), \text{infinity}, -5, 5);$

$$a0 = -\frac{13}{10}$$

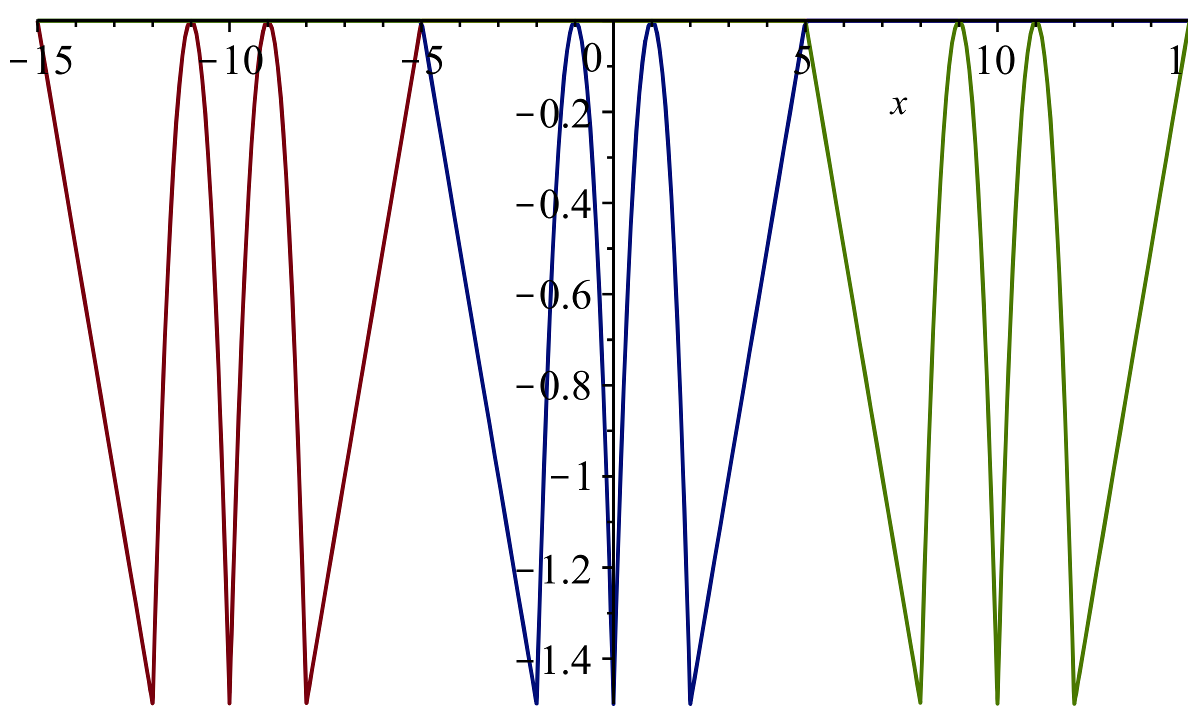
$$an = \frac{-35\, n\, \pi \cos\left(\frac{2\, n\, \pi}{5}\right) + 5\, \pi\, (-1)^n n - 30\, n\, \pi + 150 \sin\left(\frac{2\, n\, \pi}{5}\right)}{n^3\, \pi^3}$$

$$bn = 0$$

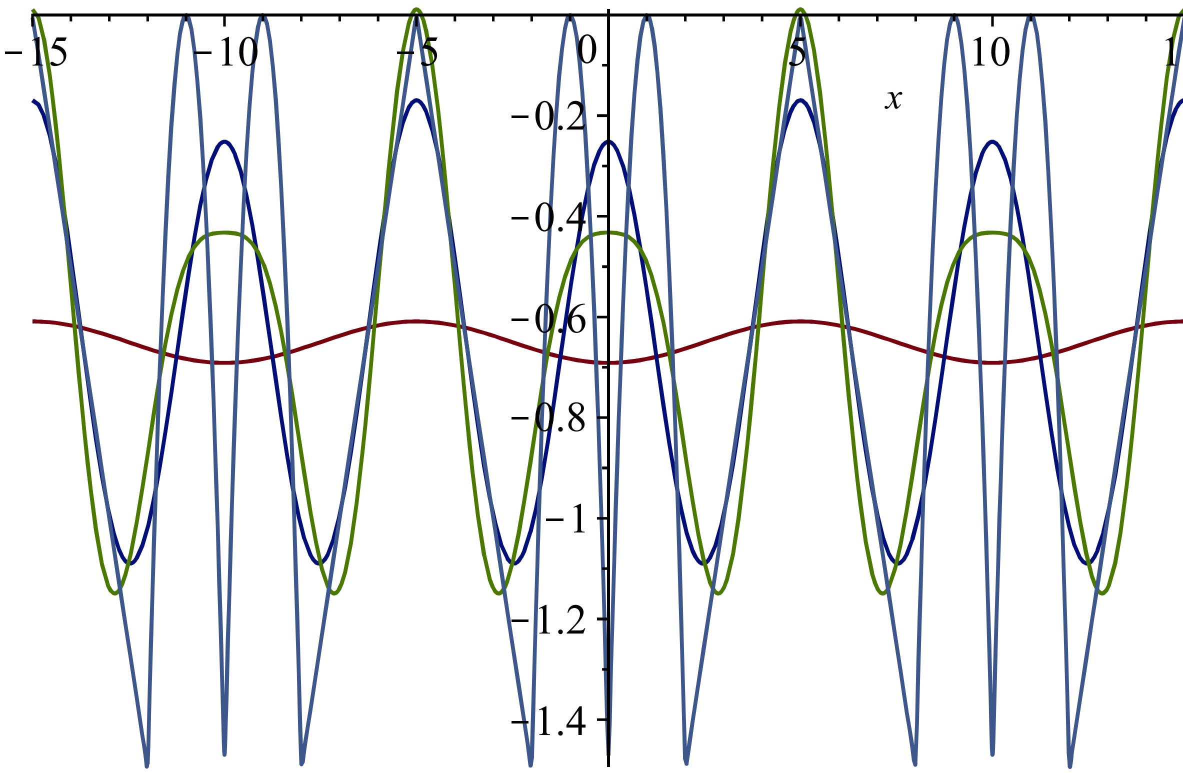
$$f = -\frac{13}{20} + \left(\sum_{k=1}^{\infty} \frac{5\left(\pi\,(-1)^k k - 7\, k\, \pi \cos\left(\frac{2\, k\, \pi}{5}\right) - 6\, k\, \pi + 30 \sin\left(\frac{2\, k\, \pi}{5}\right)\right) \cos\left(\frac{k\, \pi\, x}{5}\right)}{k^3\, \pi^3}\right) \quad (4)$$

> $g := (x, n) \rightarrow piecewise\left(-5 + 10\, n \le x \le -2 + 10\, n, -\frac{1}{2}(x - 10\, n) - \frac{5}{2}, -2 + 10\, n < x < 10\, n, -\frac{3}{2}(x - 10\, n)^2 - 3\,(x - 10\, n) - \frac{3}{2}, 10\, n < x < 2 + 10\, n, -\frac{3}{2}(x - 10\, n)^2 + 3(x - 10\, n) - \frac{3}{2}, 2 + 10\, n \le x \le 5 + 10\, n, \frac{1}{2}(x - 10\, n) - \frac{5}{2}\right):$

$plot([g(x, -1), g(x, 0), g(x, 1)], x = -15 ..15, discont = true, symbolsize = 1);$



> $plot([FourierSeries(f(x), 1, -5, 5), FourierSeries(f(x), 2, -5, 5), FourierSeries(f(x), 3, -5, 5), FourierSeries(f(x), 100, -5, 5)], x=-15..15);$



> $f := x \rightarrow piecewise\left(-5 \le x \le -2, \frac{1}{2}x + \frac{5}{2}, -2 < x < 0, \frac{3}{2}x^2 + 3x + \frac{3}{2}, 0 < x < 2, -\frac{3}{2}x^2 + 3x - \frac{3}{2}, 2 \le x \le 5, \frac{1}{2}x - \frac{5}{2}\right):$

$a0 = \frac{1}{5} int(f(x), x=-5..5);$

$an = simplify\left(\frac{1}{5} int\left(f(x) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{5}\right), x=-5..5\right)\right)$ assuming $n :: posint$;

$bn = simplify\left(\frac{1}{5} int\left(f(x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{5}\right), x=-5..5\right)\right)$ assuming $n :: posint$;

$f = FourierSeries(f(x), \text{infinity}, -5, 5);$

$$a0 = 0$$

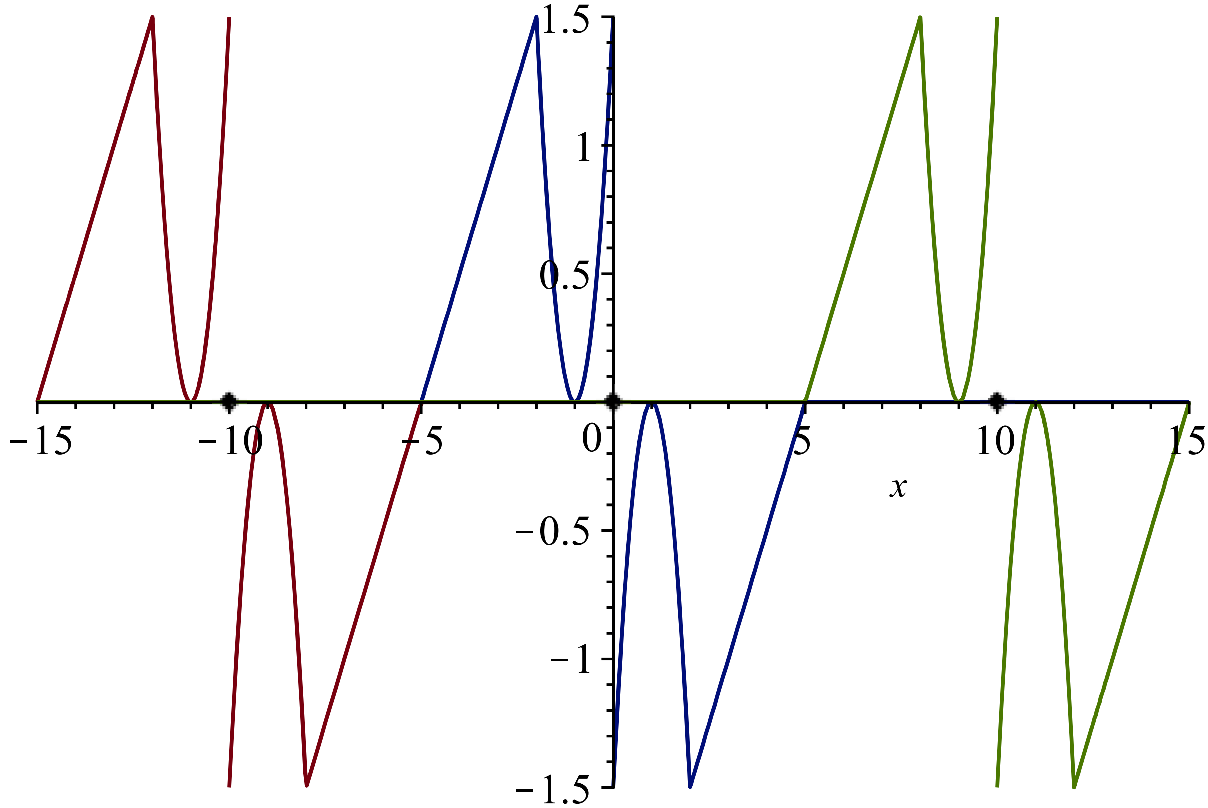
$$an = 0$$

$$bn = \frac{-3 n^2 \pi^2 - 35 n \pi \sin\left(\frac{2 n \pi}{5}\right) - 150 \cos\left(\frac{2 n \pi}{5}\right) + 150}{n^3 \pi^3}$$

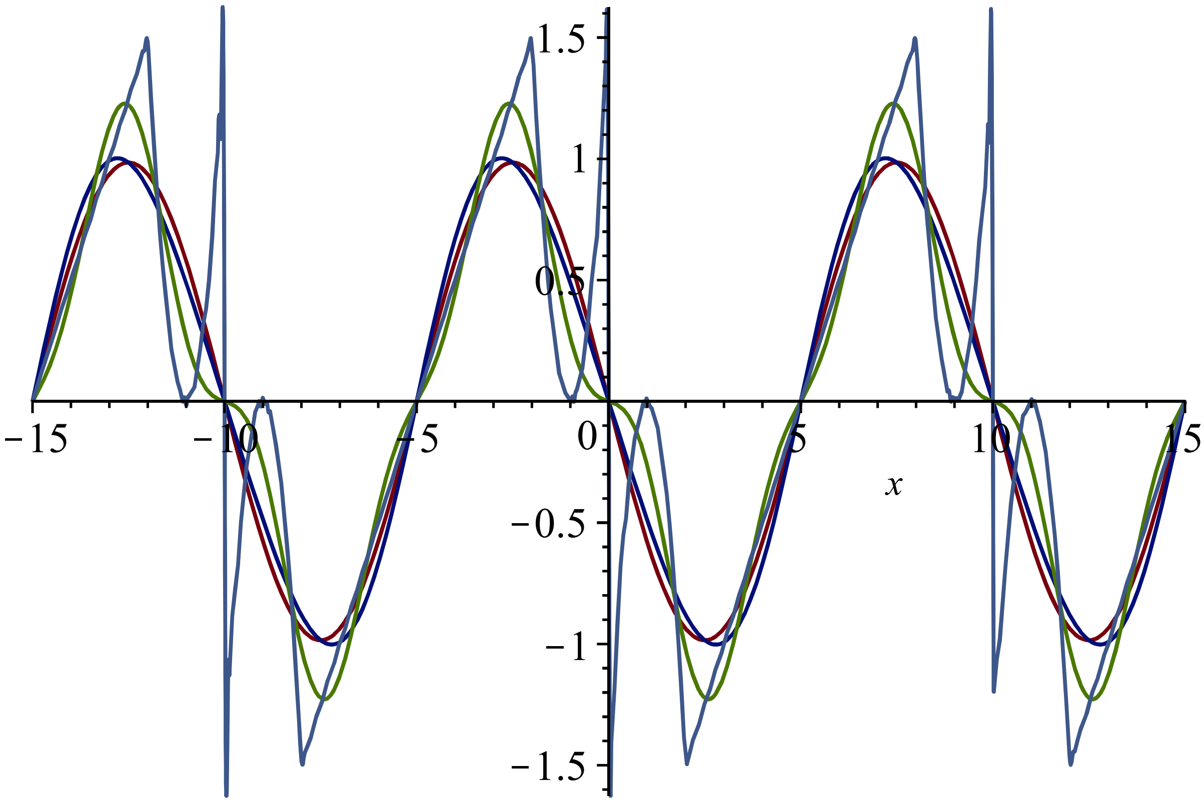
$$f = \sum_{k=1}^{\infty} -\frac{3\left(k^2 \pi^2 + \frac{35 k \pi \sin\left(\frac{2 k \pi}{5}\right)}{3} + 50 \cos\left(\frac{2 k \pi}{5}\right) - 50\right) \sin\left(\frac{k \pi x}{5}\right)}{k^3 \pi^3} \tag{5}$$

> $g := (x, n) \rightarrow piecewise\left(\left(-5 + 10\,n \le x \le -2 + 10\,n,\ \frac{1}{2}(x - 10\,n) + \frac{5}{2}, -2 + 10\,n < x < 10\,n,\ \frac{3}{2}(x - 10\,n)^2 + 3\,(x - 10\,n) + \frac{3}{2},\ 10\,n < x < 2 + 10\,n,\ -\frac{3}{2}(x - 10\,n)^2 + 3\,(x - 10\,n) - \frac{3}{2},\ 2 + 10\,n \le x \le 5 + 10\,n,\ \frac{1}{2}(x - 10\,n) - \frac{5}{2}\right)\right):$

$p1 := plot([g(x, -1), g(x, 0), g(x, 1)], x = -15..15, discont = true, symbolsize = 1):$
$p2 := plots[pointplot]([[0, 0], [-10, 0], [10, 0]],\ symbol = soliddiamond, symbolsize = 12):$
$plots[display](p1, p2);$



> $plot([FourierSeries(f(x), 1, -5, 5), FourierSeries(f(x), 2, -5, 5), FourierSeries(f(x), 3, -5, 5), FourierSeries(f(x), 100, -5, 5)], x = -15..15);$

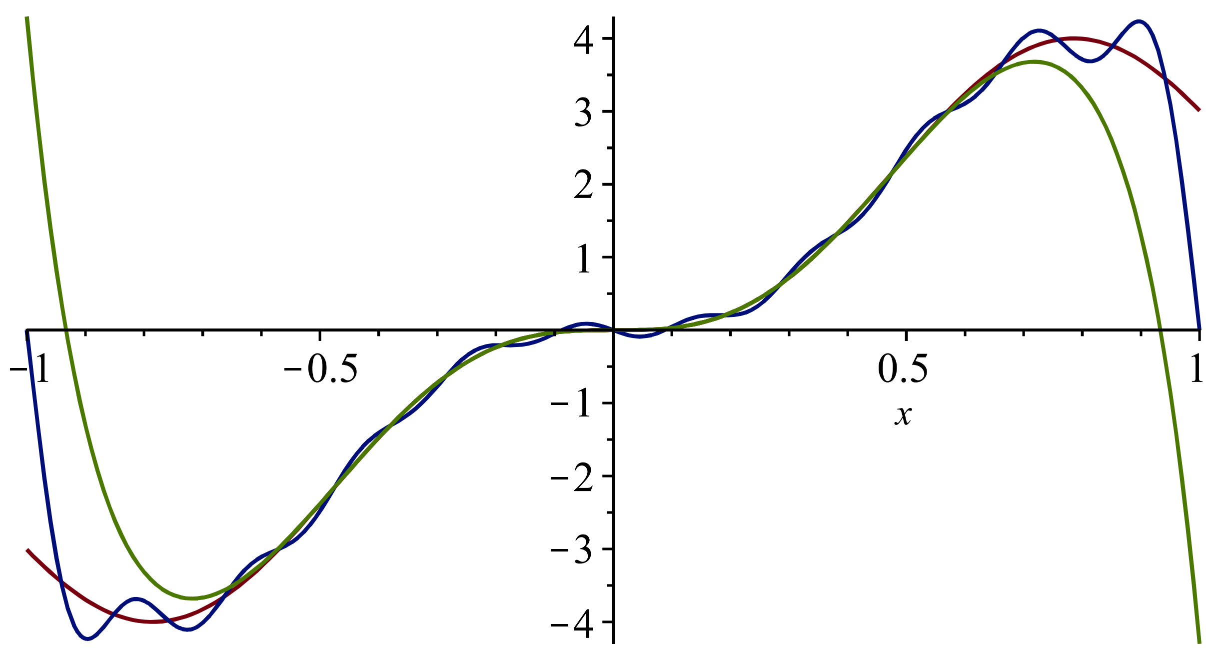


> $with(orthopoly):$
$FourierChebyshev := \mathbf{proc}(f, n)\ \mathbf{local}\ k;$
$$\mathbf{return}\ simplify\left(\frac{1}{\pi} int\left(\frac{f}{\sqrt{1-x^2}}, x=-1..1\right)\cdot T(0,x) + sum\left(\frac{2}{\pi}\ int\left(\frac{1}{\sqrt{1-x^2}}\cdot f\cdot T(k,x), x=-1..1\right)\cdot T(k,x), k=1..n\right)\right);$$
$\mathbf{end\ proc}:$

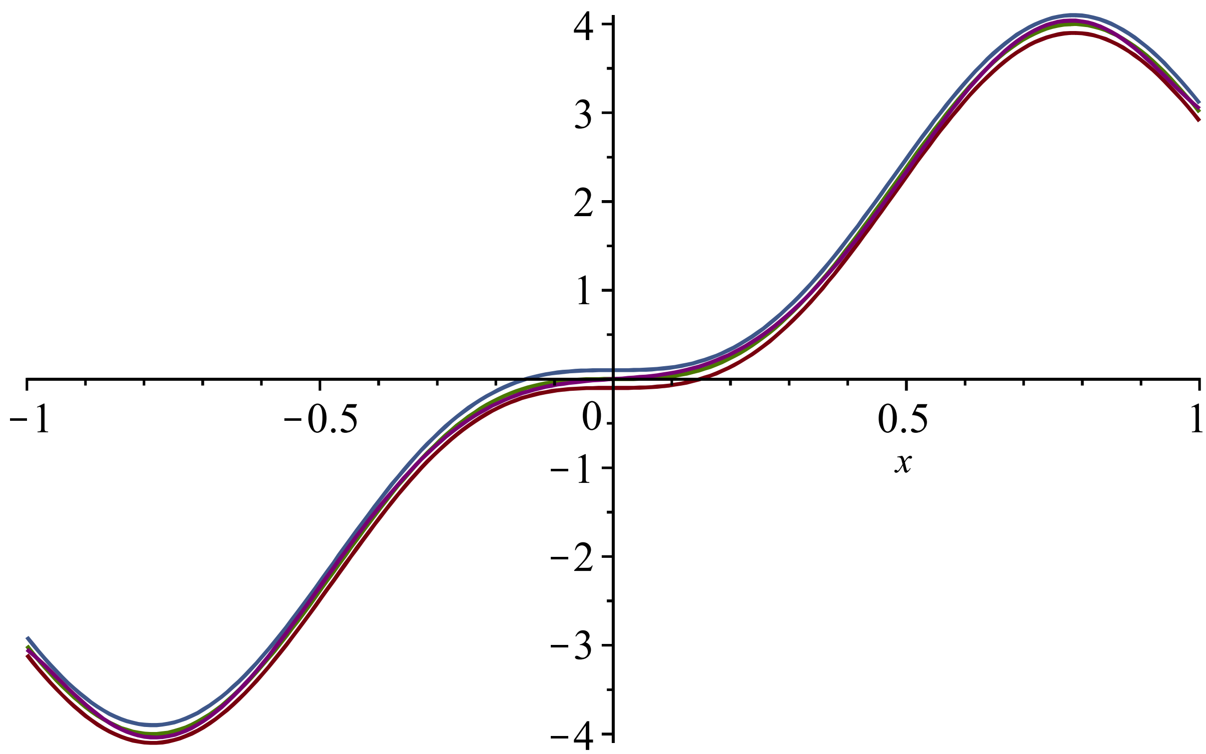
> $FourierLegendre := \mathbf{proc}(f, n)\ \mathbf{local}\ k;$
$$\mathbf{return}\ simplify\left(sum\left(\frac{2\,k+1}{2}\ int(f\cdot P(k,x), x=-1..1)\cdot P(k,x), k=0..n\right)\right);$$
$\mathbf{end\ proc}:$

> $FourierSeries := \mathbf{proc}(f, n, a, b)\ \mathbf{local}\ a0, ak, bk, l;$
$$l := \frac{b-a}{2};$$
$$a0 := \frac{1}{l} int(f, x=a..b);$$
$$ak := \frac{1}{l} int\left(f\cdot\cos\left(\frac{k\cdot \mathrm{Pi}\cdot x}{l}\right), x=a..b\right);$$
$$bk := \frac{1}{l} int\left(f\cdot\sin\left(\frac{k\cdot \mathrm{Pi}\cdot x}{l}\right), x=a..b\right);$$
$$\mathbf{return}\ \frac{a0}{2} + simplify\left(sum\left(\left(ak\cdot\cos\left(\frac{k\cdot \mathrm{Pi}\cdot x}{l}\right) + bk\cdot\sin\left(\frac{k\cdot \mathrm{Pi}\cdot x}{l}\right)\right), k=1..n\right)\right);$$
$\mathbf{end\ proc}:$

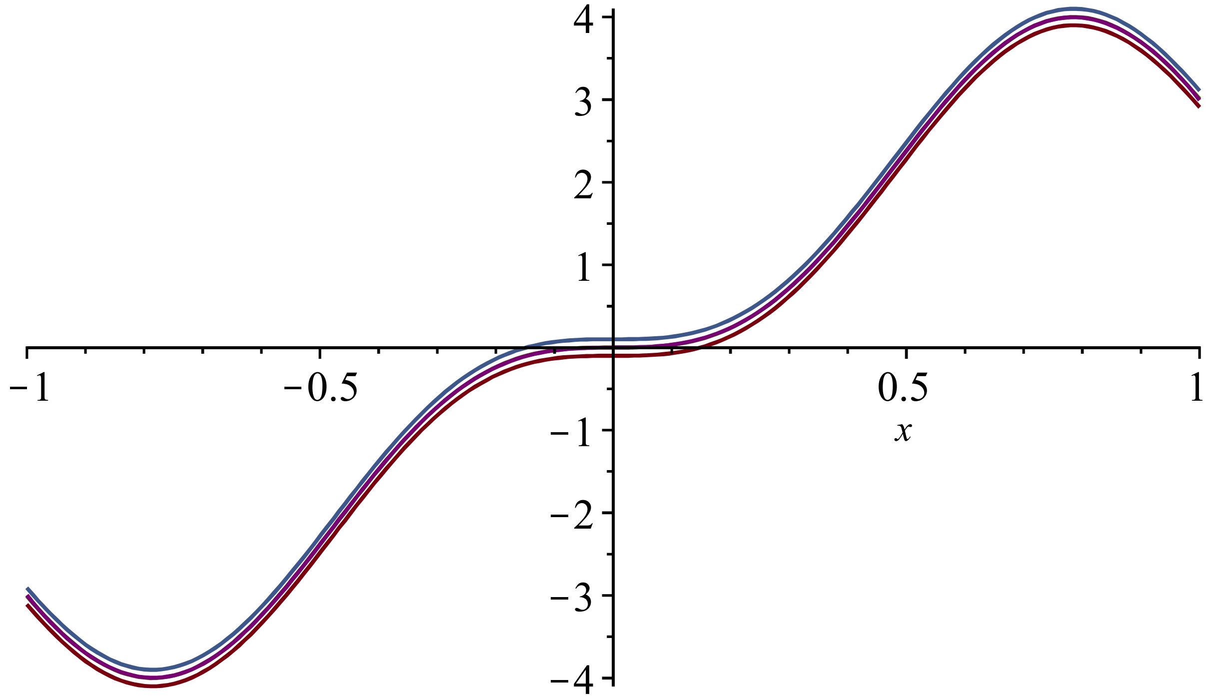
> $f := x \rightarrow 4\ (\sin(2\,x))^3 :$
$plot([f(x), FourierSeries(f(x), 10, -1, 1), taylor(f(x), x=0, 10)],\ x=-1..1);$



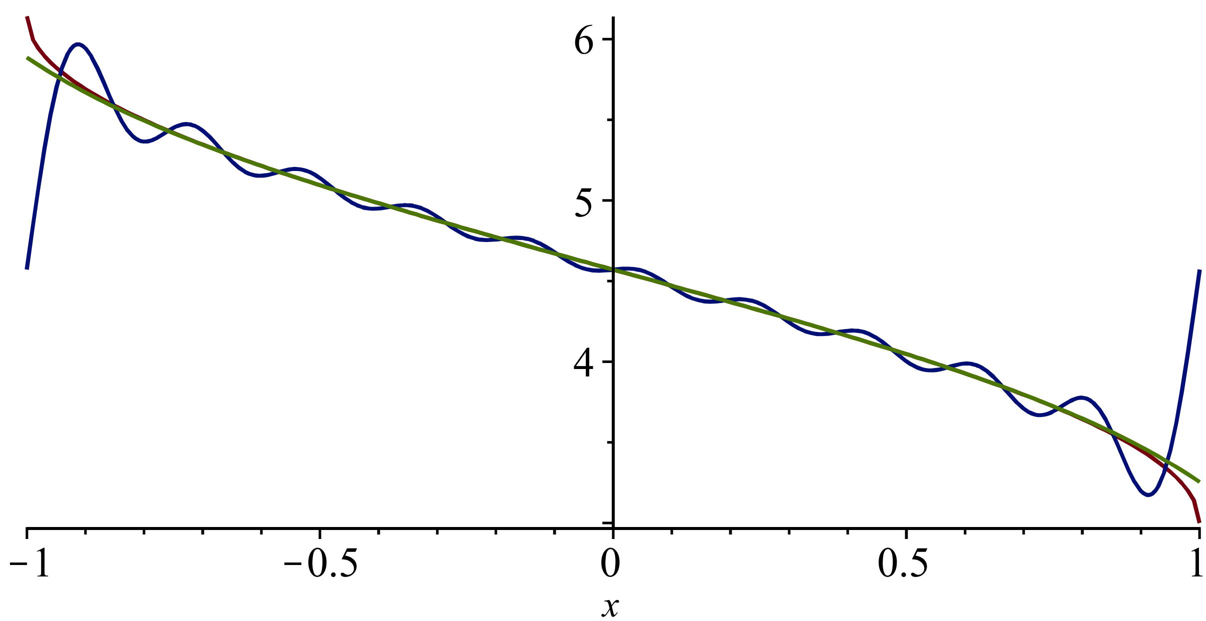
> $plot([f(x) - 0.1, f(x), f(x),\ f(x) + 0.1,\ FourierChebyshev(f(x), 7)], x = -1 ..1);$



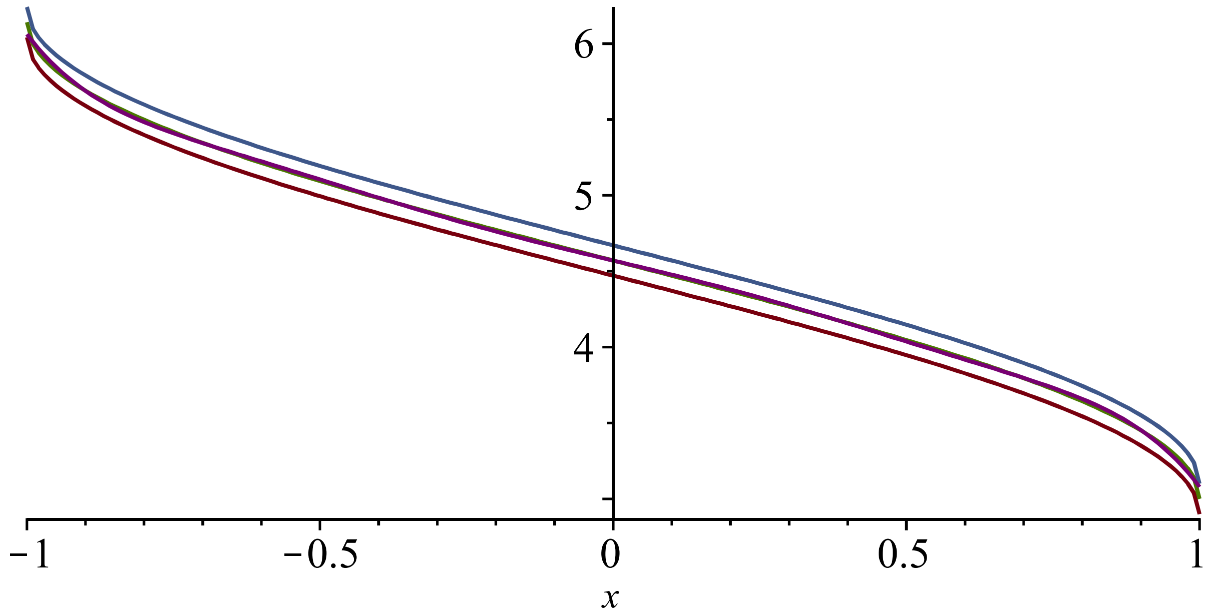
> $plot([f(x) - 0.1, f(x), f(x),\ f(x) + 0.1, FourierLegandre(f(x), 10)], x = -1 ..1);$



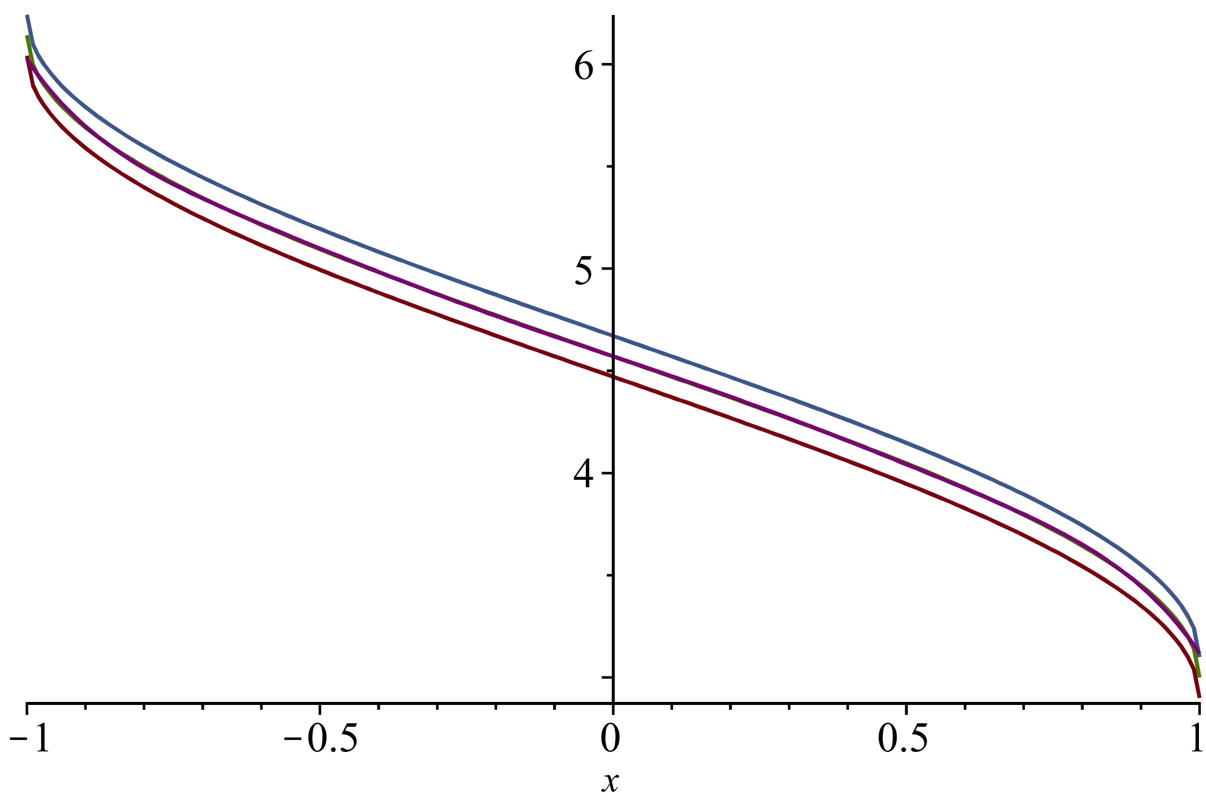
> $f := x \rightarrow \arccos(x) + 3$:
$plot([f(x), FourierSeries(f(x), 10, -1, 1), taylor(f(x), x=0, 10)], x=-1..1)$;



> $plot([f(x) - 0.1, f(x), f(x), f(x) + 0.1, FourierChebyshev(f(x), 8)], x=-1..1)$;



> $plot([f(x) - 0.1, f(x), f(x),\ f(x) + 0.1, FourierLegendre(f(x), 8)], x = -1..1);$



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