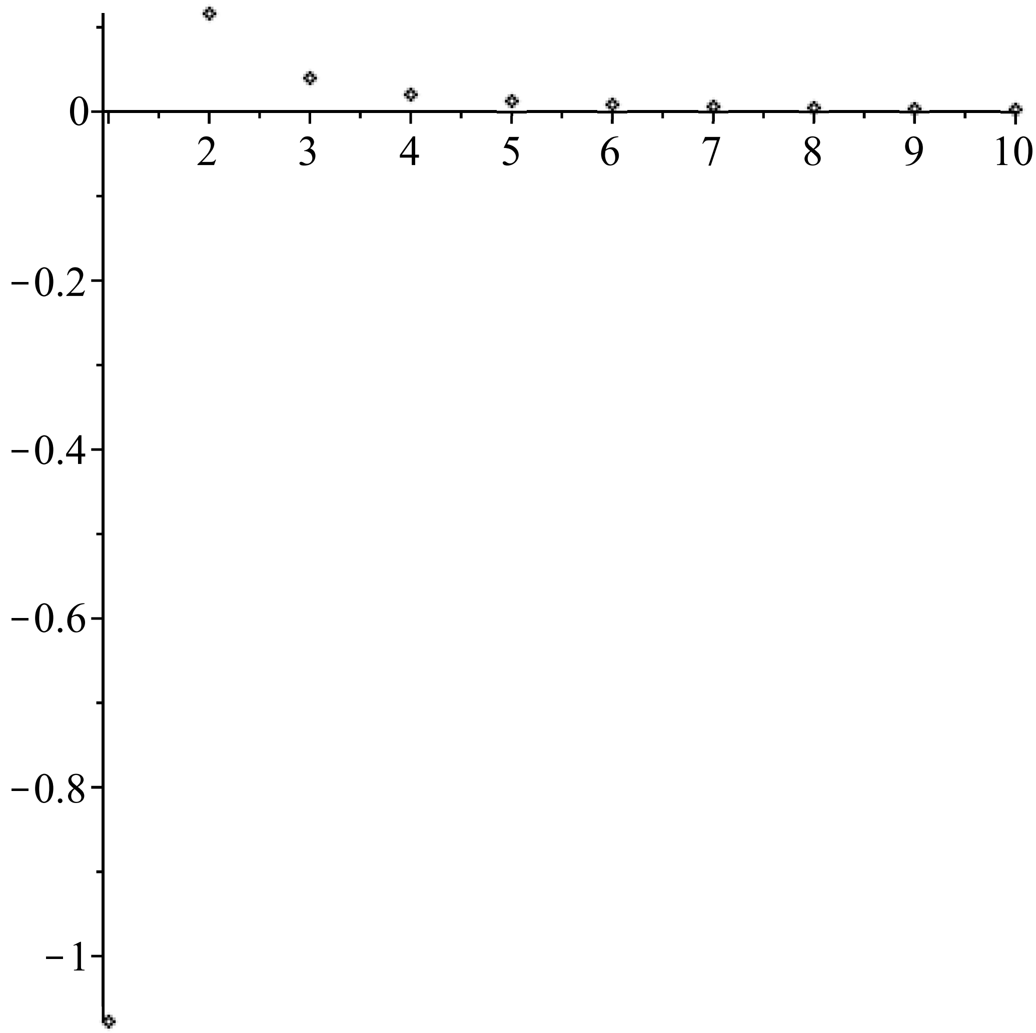
> *Задание* 1 :

> $f := n \rightarrow \dfrac{14}{49\,n^2 - 14\,n - 48}$:

$Limit(f(n), n = \text{infinity}) = limit(f(n), n = \text{infinity});$

$$\lim_{n \to \infty} \frac{14}{49\,n^2 - 14\,n - 48} = 0 \qquad (1)$$

> $plots[pointplot](\{seq([n, f(n)], n = 1..10)\});$



> $Sum(f(n), n = 1..\text{infinity}) = sum(f(n), n = 1..\text{infinity});$

$$\sum_{n=1}^{\infty} \frac{14}{49\,n^2 - 14\,n - 48} = -\frac{5}{6} \qquad (2)$$

> $Int(f(x), x = n + 1..\text{infinity}) = int(f(x), x = n + 1..\text{infinity})$ assuming $n \geq 1$;

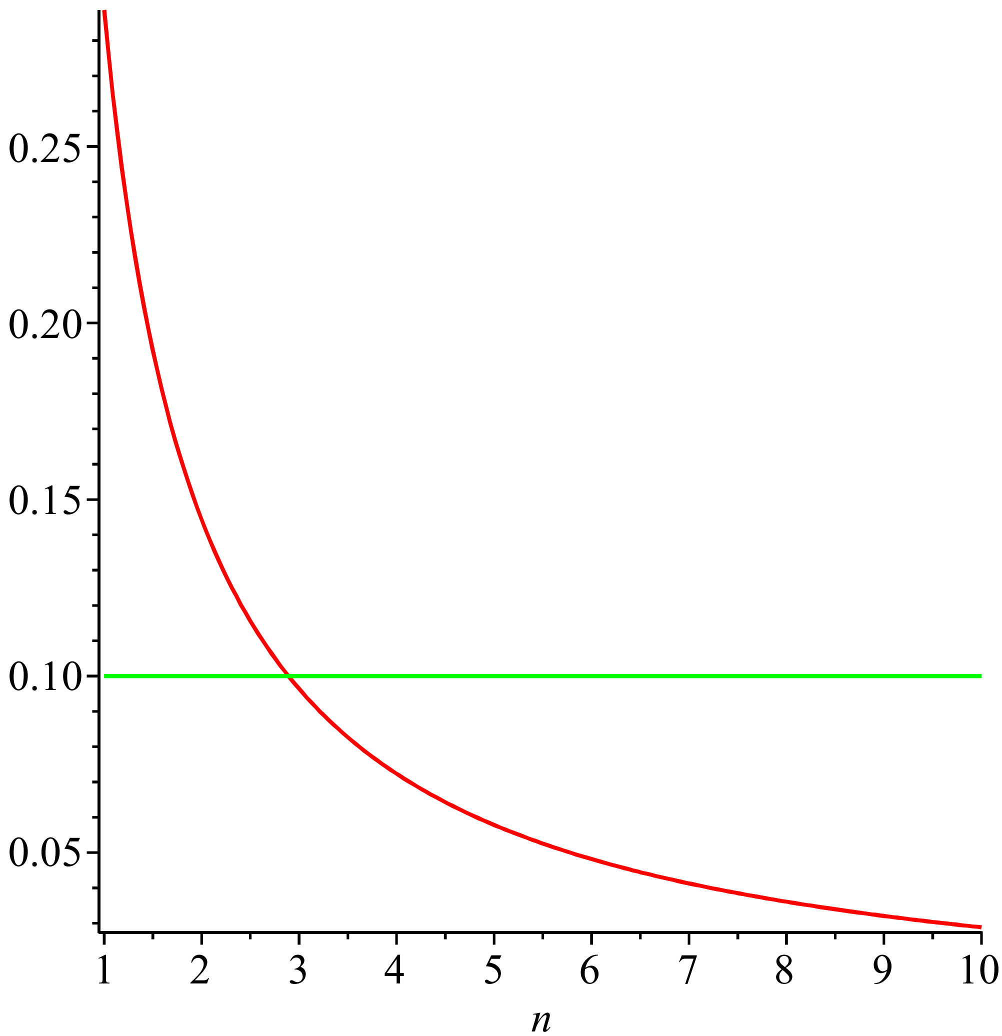
$m1 := plot\left(-\dfrac{\ln(-1 + 7\,n)}{7} + \dfrac{\ln(13 + 7\,n)}{7} + f(n + 1), n = 1..10, color = red\right)$:

$m2 := plot(0.1, n = 1..10, color = green)$:

$plots[display](m1, m2);$

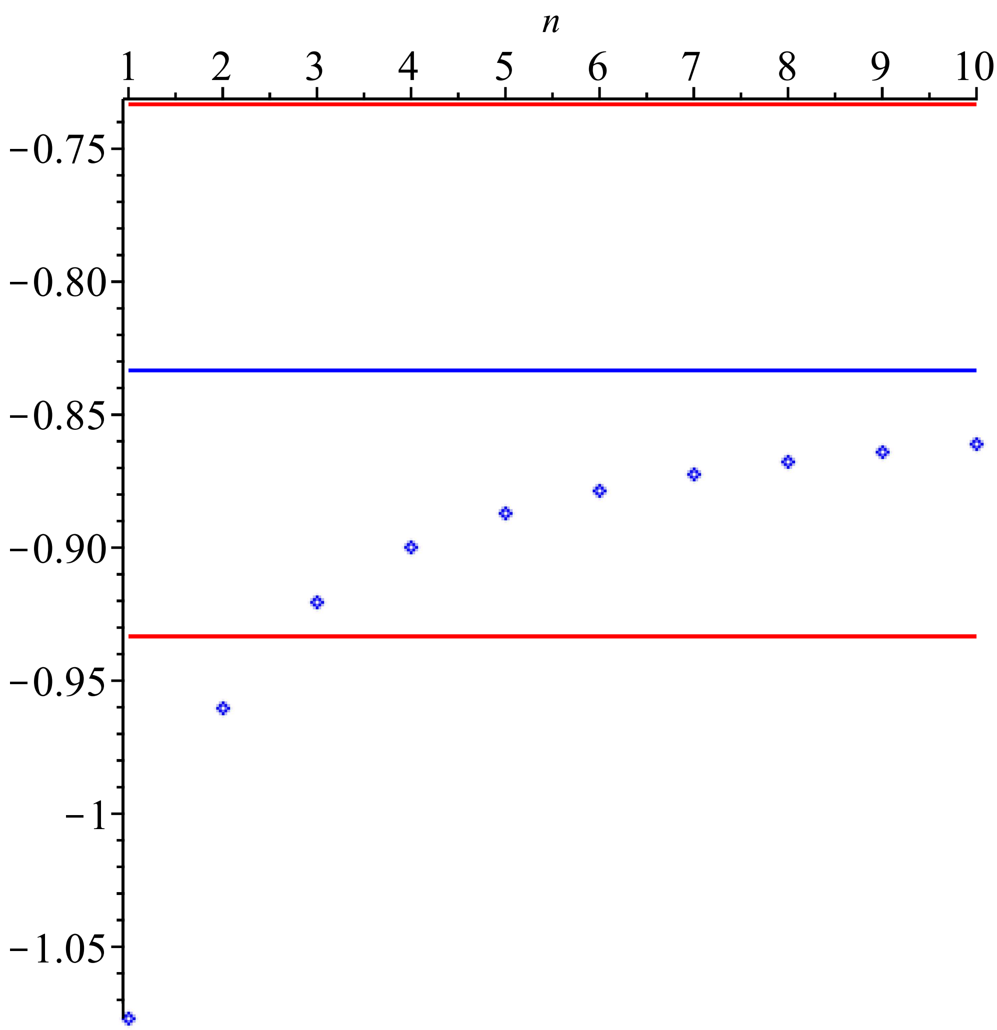
$n \geq fsolve\left(-\dfrac{\ln(-1 + 7\,n)}{7} + \dfrac{\ln(13 + 7\,n)}{7} + f(n + 1) = 0.1, n\right);$

$$\int_{n+1}^{\infty} \frac{14}{49 x^2 - 14 x - 48}\, dx = -\frac{\ln(-1+7n)}{7} + \frac{\ln(13+7n)}{7}$$



$$2.891421266 \le n \qquad \textbf{(3)}$$

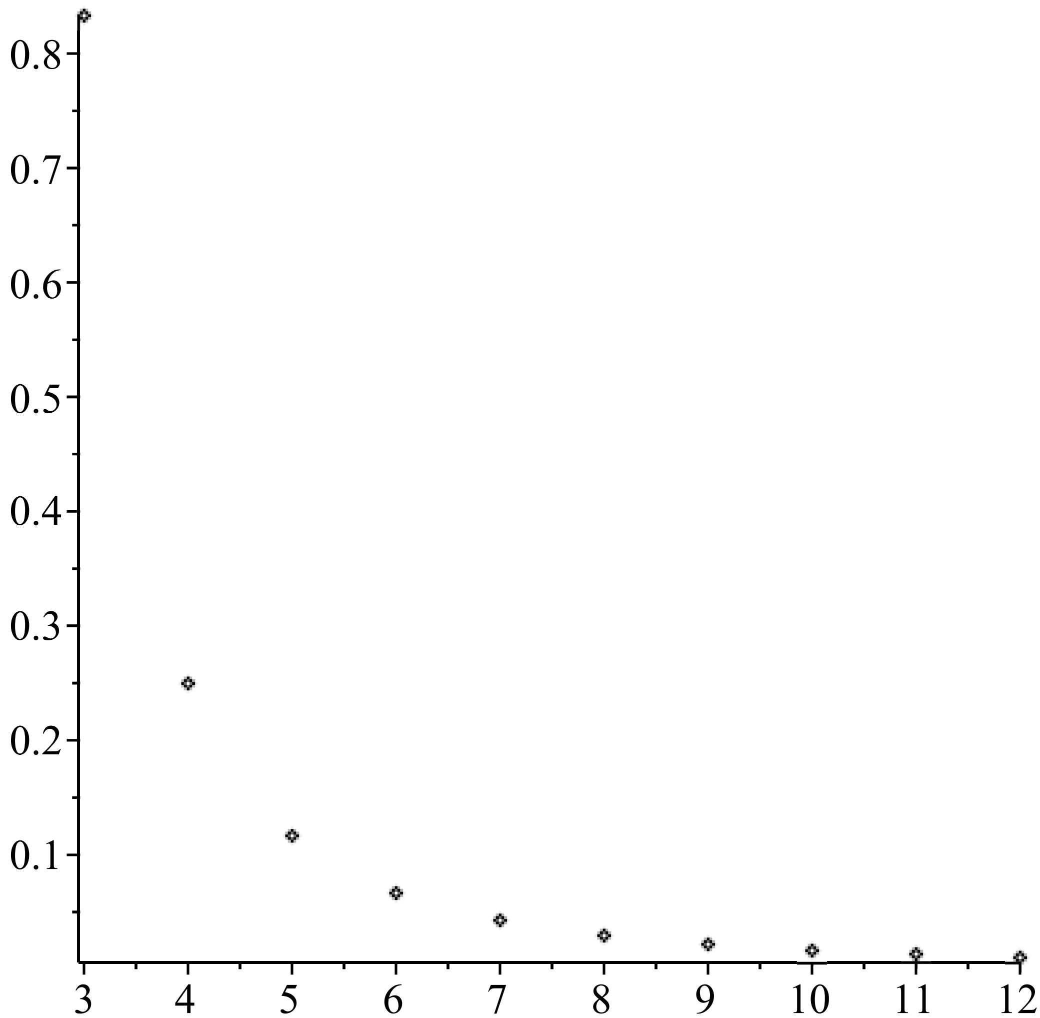
```
> S := n→sum(f(x), x = 1 ..n) :
  m1 := plot([-5/6 + 0.1, -5/6, -5/6 - 0.1], n = 1 ..10, color = [red, blue, red]) :
  m2 := plots[pointplot]({seq([n, S(n)], n = 1 ..10)}, color = blue) :
  plots[display](m1, m2);
```



> $f := n \to \dfrac{n+2}{n \cdot (n-1) \cdot (n-2)}$:
$Limit(f(n), n = \text{infinity}) = limit(f(n), n = \text{infinity})$;

$$\lim_{n \to \infty} \frac{n+2}{n\,(n-1)\,(n-2)} = 0 \tag{4}$$

> $plots[pointplot](\{seq([n, f(n)], n = 3..12)\})$;

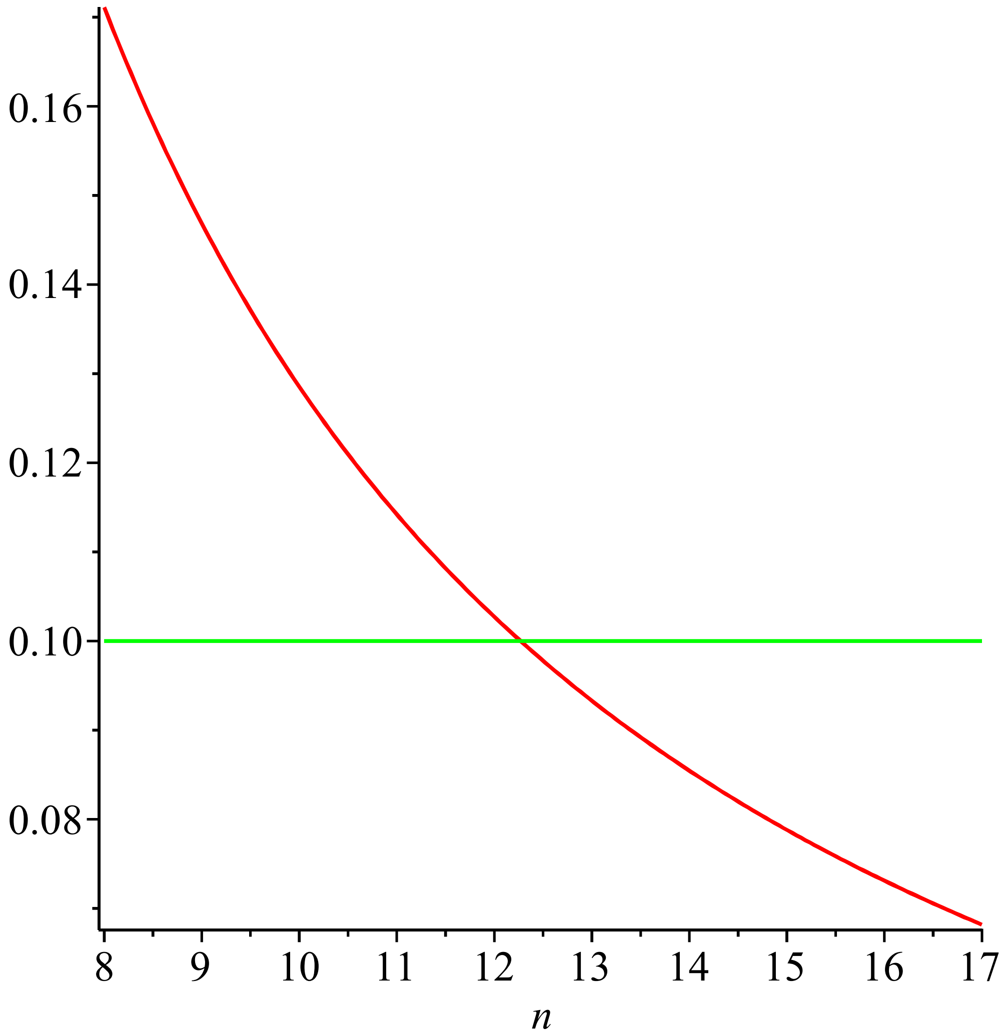


> Sum(f(n), n = 3 ..infinity) = sum(f(n), n = 3 ..infinity);

$$\sum_{n=3}^{\infty} \frac{n+2}{n\,(n-1)\,(n-2)} = \frac{3}{2} \tag{5}$$

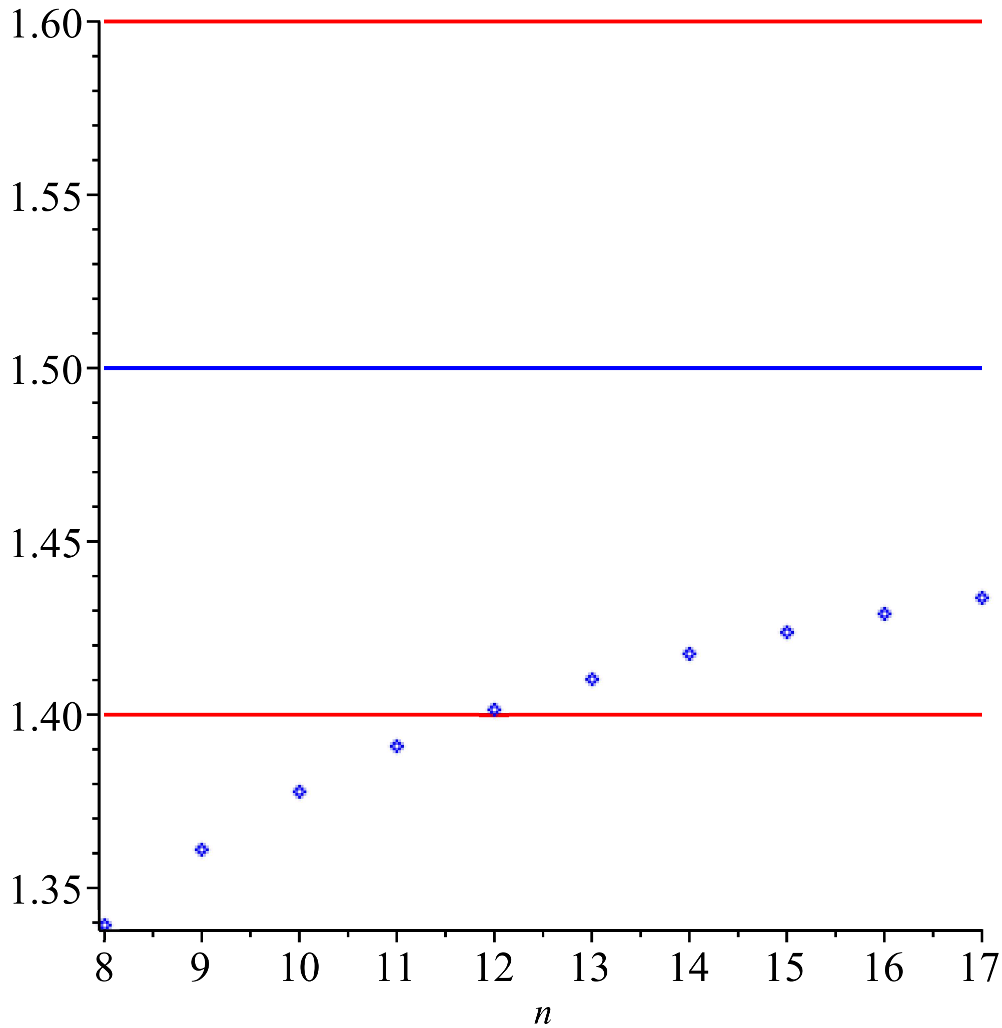
> Int(f(x), x = n + 1 ..infinity) = int(f(x), x = n + 1 ..infinity) assuming n ≥ 3;
m1 := plot(-ln(n + 1) − 2 ln(n − 1) + 3 ln(n) + f(n + 1), n = 8 ..17, color = red) :
m2 := plot(0.1, n = 8 ..17, color = green) :
plots[display](m1, m2);
n ≥ fsolve(-ln(n + 1) − 2 ln(n − 1) + 3 ln(n) + f(n + 1) = 0.1, n);

$$\int_{n+1}^{\infty} \frac{x+2}{x\,(x-1)\,(x-2)}\,\mathrm{d}x = 3\ln(n) - \ln(n+1) - 2\ln(n-1)$$



$$12.26987106 \leq n \tag{6}$$

```
> S := n→sum(f(x), x=3..n) :
  m1 := plot([3/2 + 0.1, 3/2, 3/2 - 0.1], n=8..17, color=[red, blue, red]) :
  m2 := plots[pointplot]({seq([n, S(n)], n=8..17)}, color=blue) :
  plots[display](m1, m2);
```



> *Задание* 2 :

> $f := n \to \dfrac{(-1)^n}{\text{doublefactorial}(2\,n+1)}$:

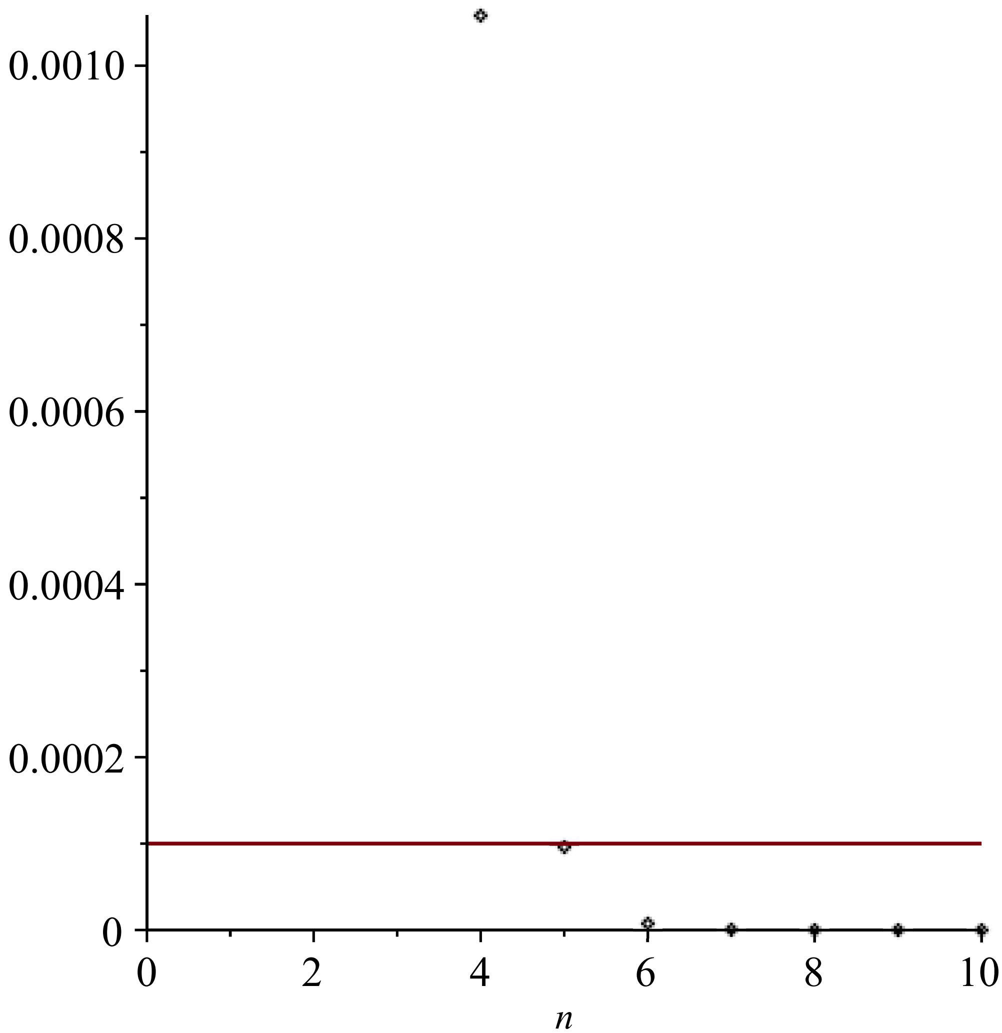
$Limit(\text{abs}(f(n)), n = \text{infinity}) = limit(\text{abs}(f(n)), n = \text{infinity});$

$Limit\left(\dfrac{\text{abs}(f(n+1))}{\text{abs}(f(n))}, n = \text{infinity}\right) = limit\left(\dfrac{\text{abs}(f(n+1))}{\text{abs}(f(n))}, n = \text{infinity}\right);$

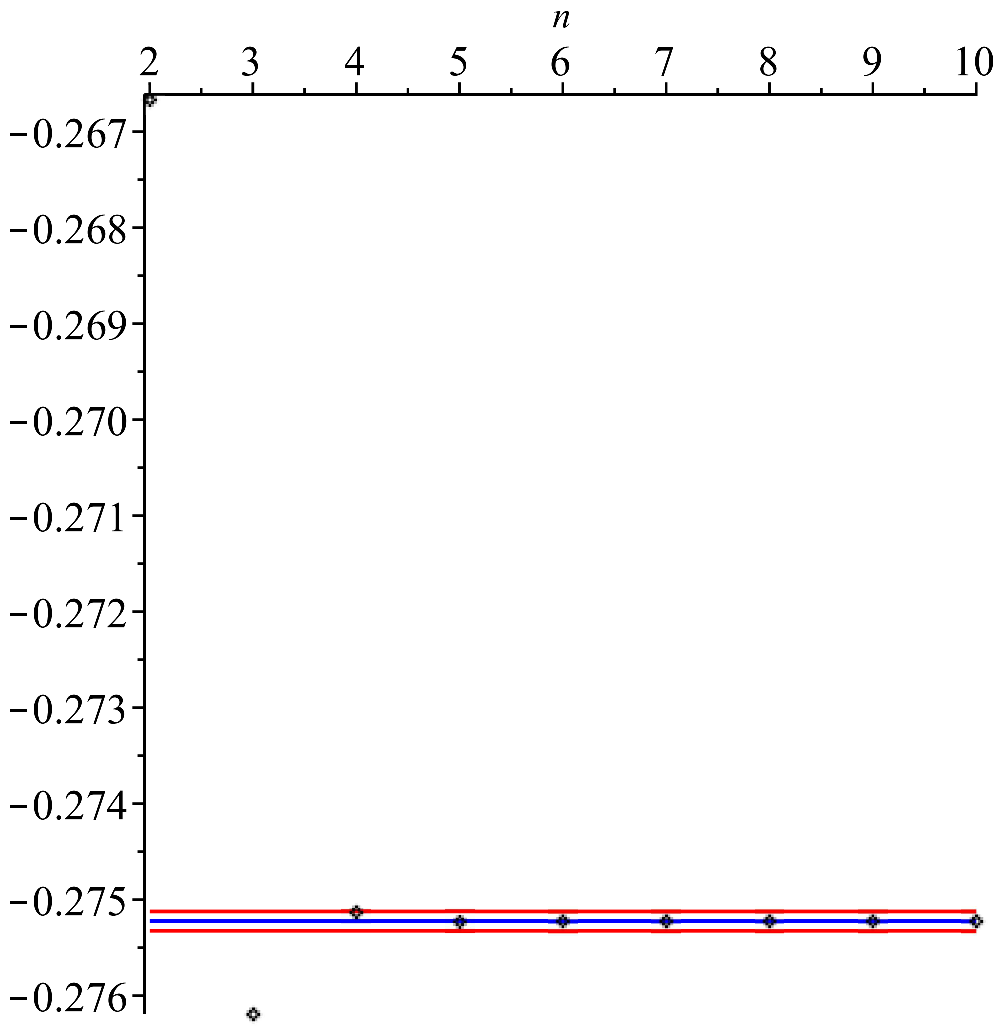
$$\lim_{n \to \infty} \left| \frac{(-1)^n}{\text{doublefactorial}(2\,n+1)} \right| = 0$$

$$\lim_{n \to \infty} \frac{\left| \dfrac{(-1)^{n+1}}{\text{doublefactorial}(2\,n+3)} \right|}{\left| \dfrac{(-1)^n}{\text{doublefactorial}(2\,n+1)} \right|} = 0 \qquad \textbf{(7)}$$

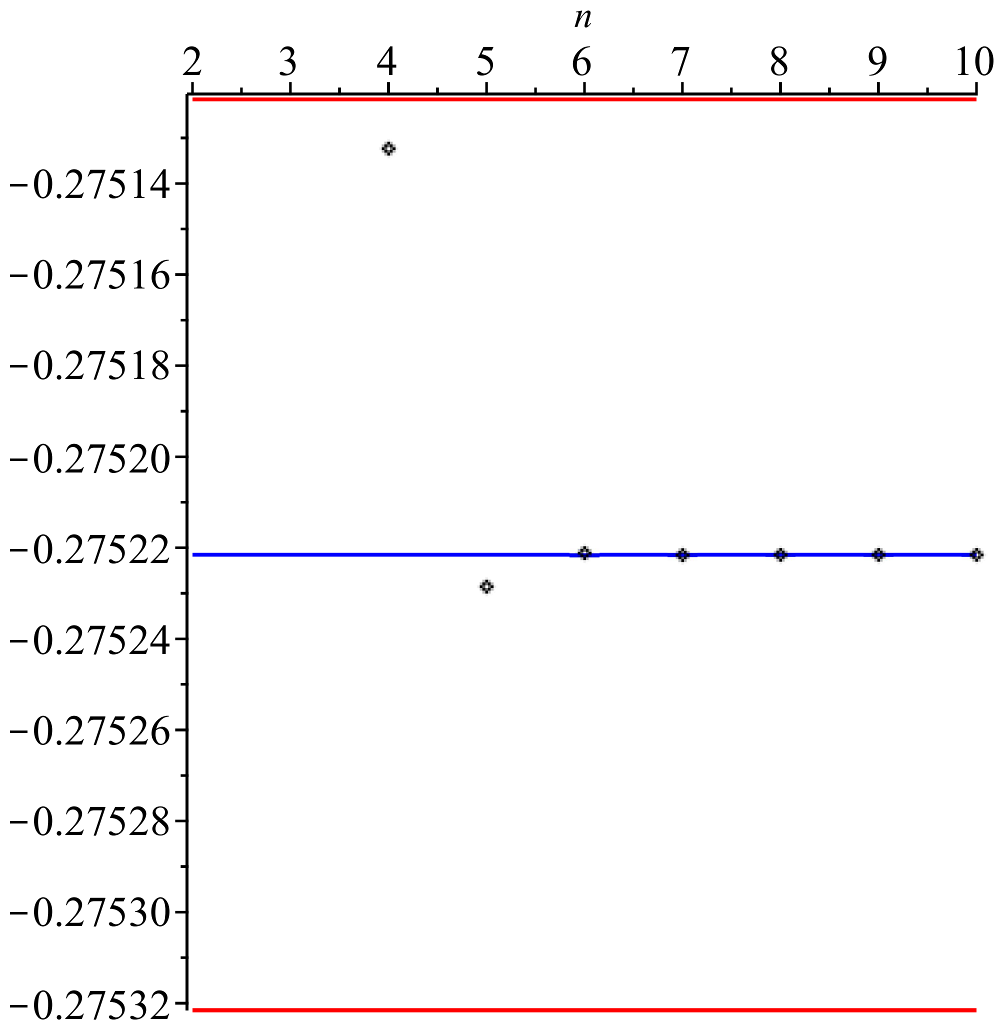
> $m1 := plots[pointplot](\{seq([n, |f(n)|], n = 4..10)\})$:
$m2 := plot(0.0001, n = 0..10)$:
$plots[display](m1, m2);$




```
> S := n→sum(f(x), x = 1 ..n) :
  m1 := plot([S(infinity) + 0.0001, S(infinity), S(infinity) − 0.0001], n = 2 ..10, color = [red,
      blue, red]) :
  m2 := plots[pointplot]({seq([n, S(n)], n = 2 ..10)}) :
  m3 := plots[pointplot]({seq([n, S(n)], n = 4 ..10)}) :
  plots[display](m1, m2);
  plots[display](m1, m3);
  n ≥ fsolve(abs(f(n + 1)) = 0.0001, n);
```



n
2
3
4
5
6
7
8
9
10
−0.267
−0.268
−0.269
−0.270
−0.271
−0.272
−0.273
−0.274
−0.275
−0.276



$$3.984184776 \le n \quad \textbf{(8)}$$

> *Задание* 3 :

> $f := n \rightarrow \dfrac{n^n}{(2n+1)!}$:

$Limit\left(\dfrac{f(n+1)}{f(n)}, n = \text{infinity}\right) = limit\left(\dfrac{f(n+1)}{f(n)}, n = \text{infinity}\right);$

$$\lim_{n \to \infty} \frac{(n+1)^{n+1}\,(2n+1)!}{(2n+3)!\,n^n} = 0 \quad \textbf{(9)}$$

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