ASEN 5070

Exam 3

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1. See following scans
2. See following scans
3. See following scans

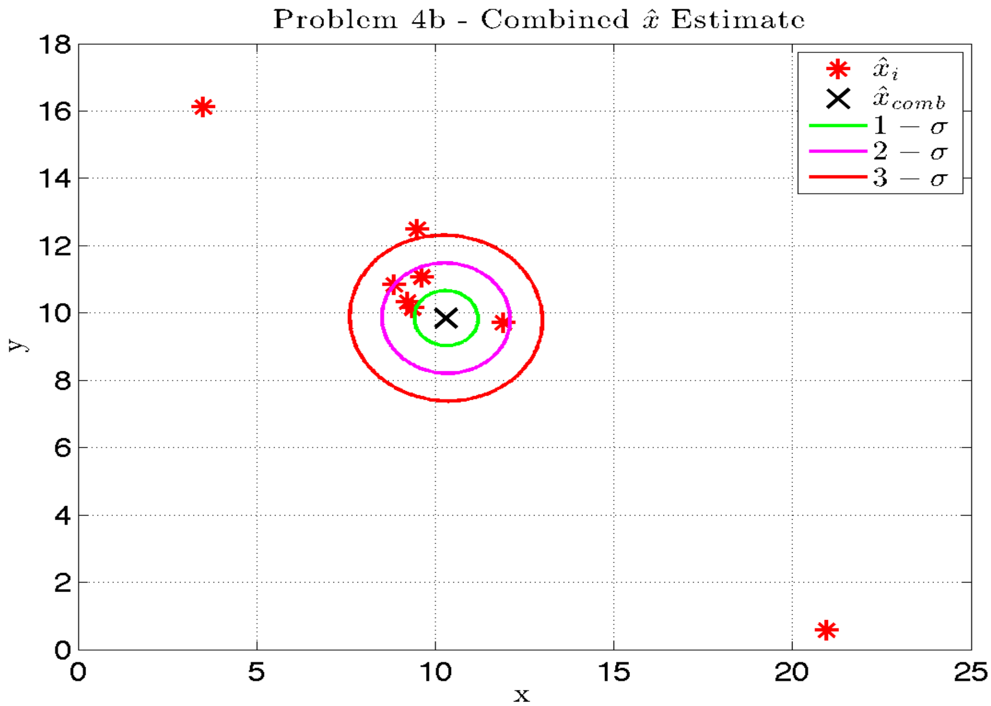
4. Multiple Estimates (See Attached Matlab Publishing Sheet for Code and Direct Output). For plotting of covariance ellipses, I used a function I had already obtained from Mathworks called *ellipse.m***.** It is available from: http://www.mathworks.com/matlabcentral/fileexchange/289-ellipse-m
It is covered by the standard BSD license, which allows use and modification.

I also tried another function, covered by a similar BSD license.
http://www.mathworks.com/matlabcentral/fileexchange/4705
However, when I used it, ellipses ended up being scaled by an unknown factor without my asking. I can't speak to why, but it is notable in case the code is used elsewhere by other people.

a. The Best Estimate for the state deviation vector and Covariance Matrix is:

$\hat{\boldsymbol{x}} = [\mathbf{10.3\ 9.848}]$
$\mathbf{P} = \begin{bmatrix} \mathbf{0.813} & \mathbf{-0.083} \\ \mathbf{-0.083} & \mathbf{0.675} \end{bmatrix}$

b. Plot of Overlapping Estimates:



Just using this plot, it appears that there are three questionable estimates, with two estimates which fall far outside of the 3-σ ellipse. Without knowing their covariances, the two far-outliers seem to be a definite result of faulty model/assumptions.

c. Next I found the distance of $\hat{x}_6$ from $\hat{x}$ found in (a). This was done in terms of covariances belonging to the combined estimate from (a). The distance in standard deviation of the combined covariance matrix estimate is:

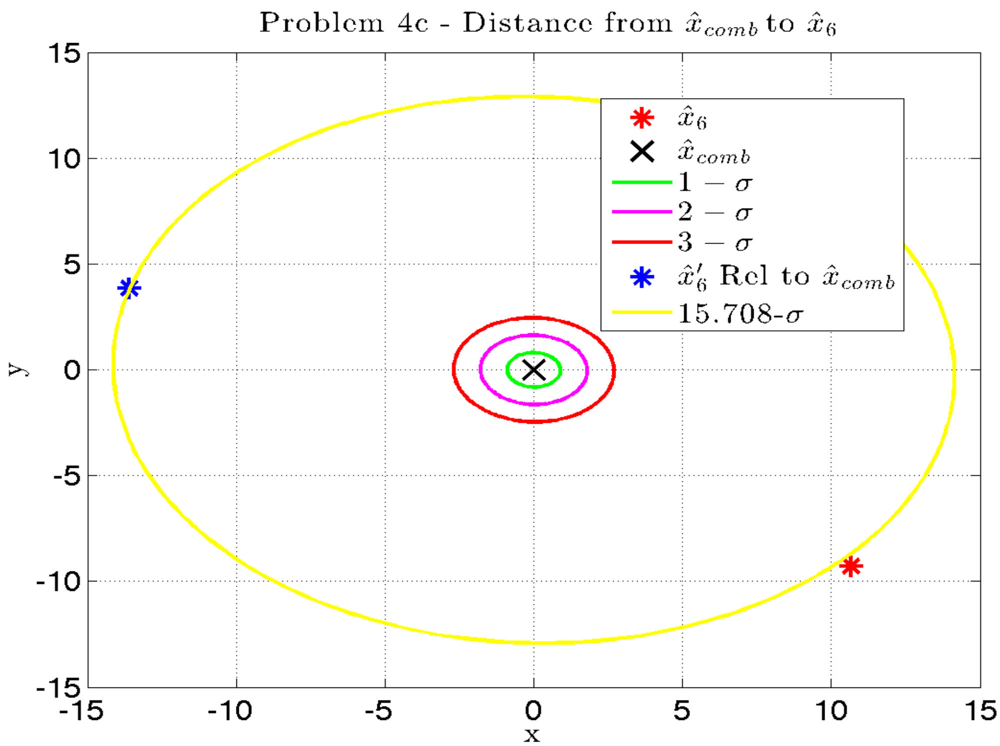
$$\boxed{\boldsymbol{dist = 15.708\ \sigma_{comb}}}$$

To do this, I just performed simple geometric analysis. As a general outline:

- Move point of interest to origin
- Rotate $\hat{x}_6$ about $\hat{x}$ according to eigenvectors
- Find angle of the points relative to principal axes
- Find distance between points in terms of X and Y σ_{comb}, along principal axis directions.

- Take norm.

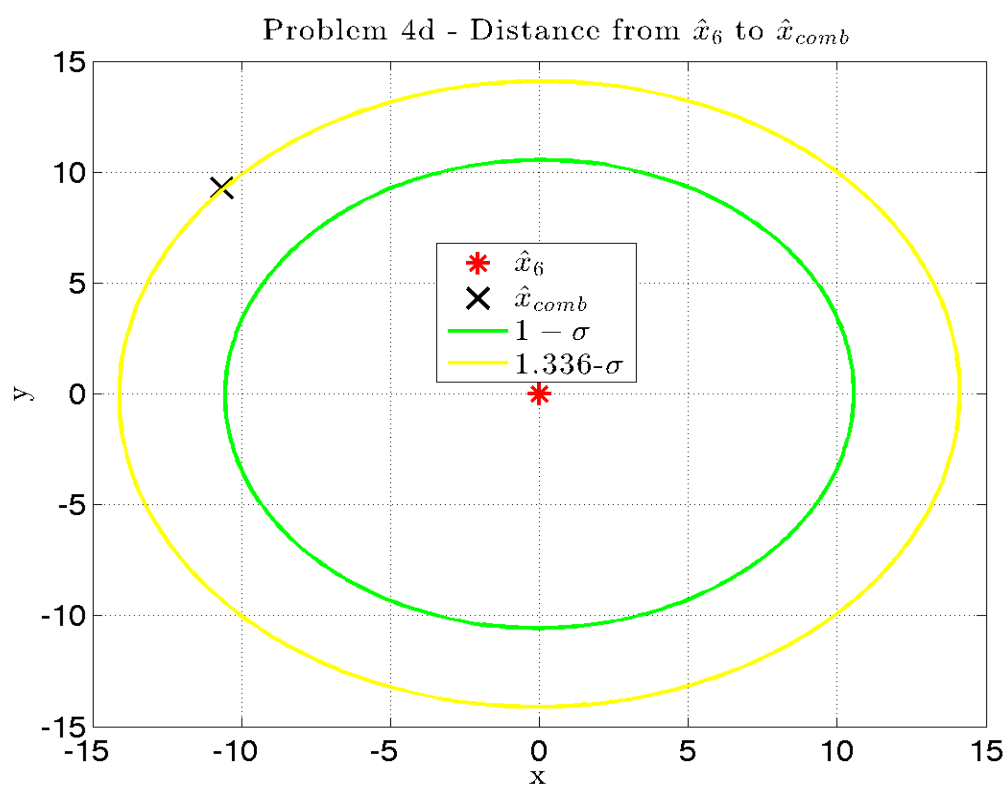
The code for this is provided as a Matlab published section following this problem answer sheet. As a sanity check, I plotted the points moved to the origin, with the computed σ distance overlaid. Indeed, the new covariance ellipse bisects the rotated point for $\hat{x}_6$.



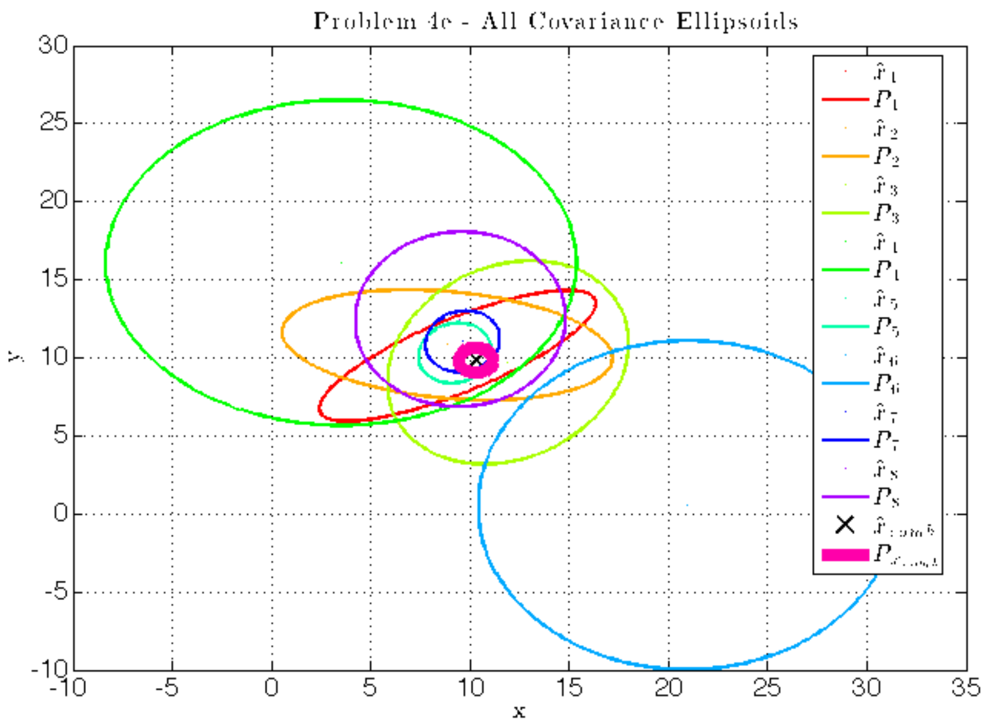
d. Finding the distance in σ_6 from $\hat{x}_6$ to $\hat{x}_{comb}$ was performed in the same manner as above, in problem 4c. The resulting distance between the two points is:

$$\boxed{\boldsymbol{dist = 1.336\,\sigma_6}}$$

This result makes sense, when looking at the size of each covariance matrix. The combined estimate had a much tighter covariance matrix associated with it than $\hat{x}_6$ did. So even though the Cartesian distances are the same, the distance in terms of standard deviations is different. Again, a plot with the overlaid covariance matrix on top of the rotated $\hat{x}_{comb}$ is provided below. The intersection is clear, as is the fact that the original 1-σ covariance ellipse is much larger than the one associated with $\hat{x}_{comb}$. Also, note that the ellipse appears to be horizontal. It is only that way in appearance, where it really does have some angle associated with the correlation coefficient.

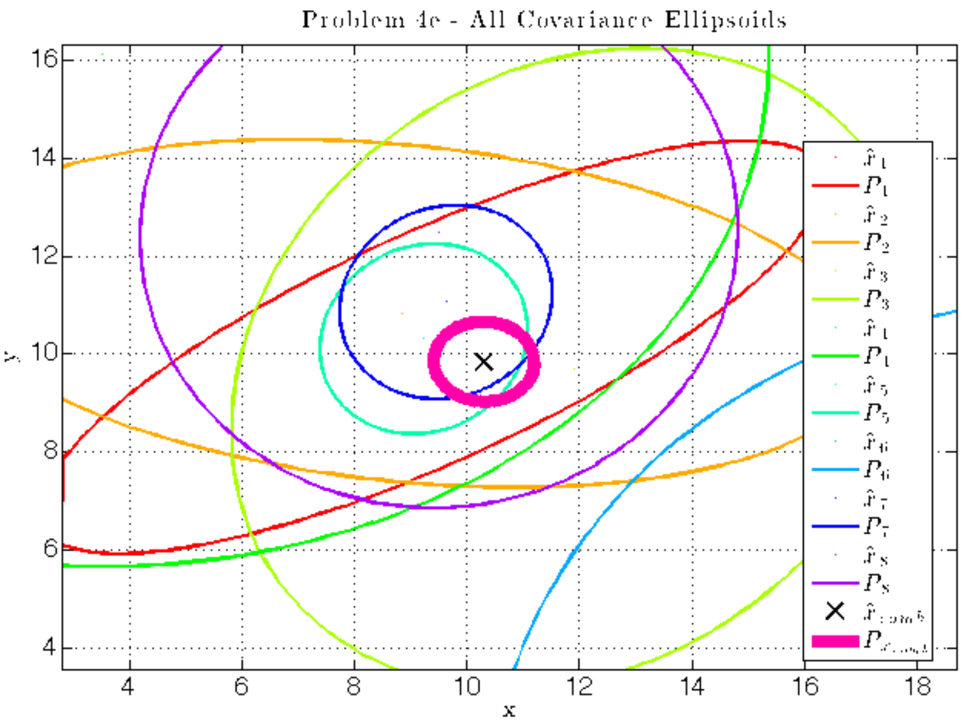


e. All points and covariances are plotted below.



Similar to the plot in 4a, it is clear that while most estimates are near, and have covariances that encircle the combined estimate, there are two points which are still far enough away to raise concern. $\hat{x}_4$ (green) was originally thought to be an outlier, but its large covariance ellipse still encompasses the combined estimate. However, $\hat{x}_6$ (blue) still seems to be a result of a bad model or poor assumptions. Not only is it relatively far from the combined estimate, but its 1-σ error ellipse does not even overlap with the combined one. I would assume this estimate is a poor one, and its methodology would need to be re-examined.

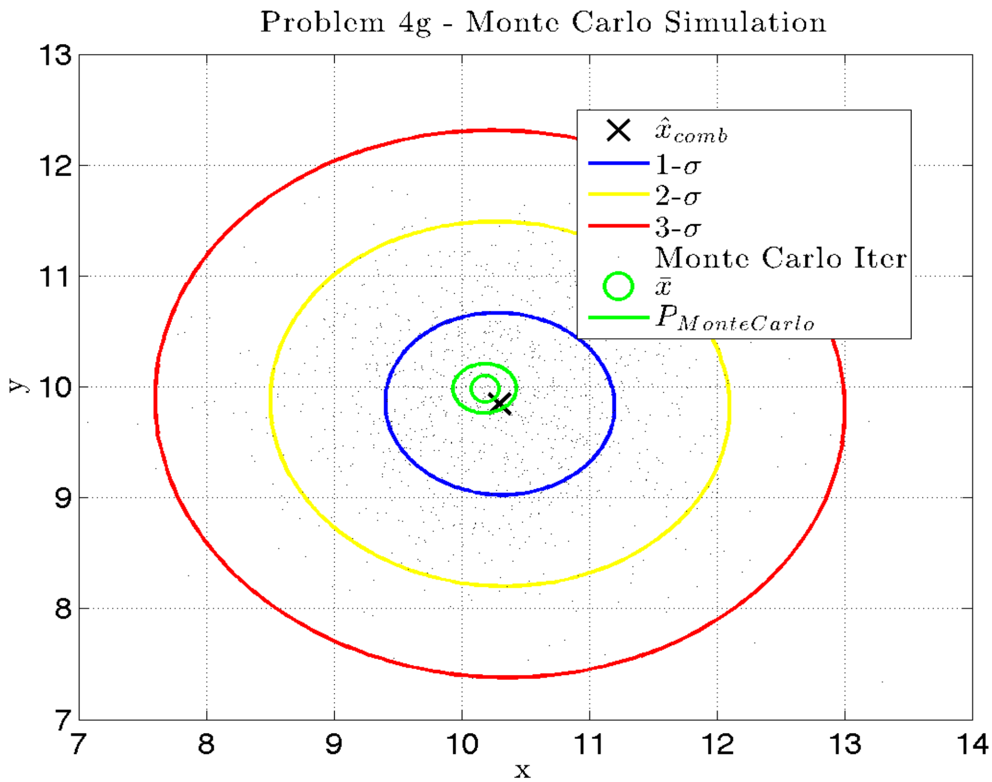
Here is a close up of the critical area:



The combined estimate is shown as a thicker pink ellipse, with an X for its point estimate. A good reinforcement of our estimate is the fact that the point (X) lies inside the two tightest covariances (dark blue and turquois). This is good, since we want to weight our combined estimate with the more confident covariances. In fact, the combined estimate lies within the 1-σ estimates for all points, with the exception of $\hat{x}_6$ as previously discussed.

f. As a navigator, I would be mostly satisfied with the solutions given, with the exception of solution number 6. As discussed above, the combined solution lies within the 1-σ error ellipses (confidence region), of each individual solution except $\hat{x}_6$. Solution 4 also has a very large covariance (relatively), and its point estimate is far from the rest of the estimates. I would be satisfied to fly the *best* (combined) estimate if I were asked to, since it agrees with most solutions, and especially agrees with the 'most confident' ones. But I would probably like to disregard solution 4 and 6, and ask those teams to re-evaluate their filter methods and dynamics.

g. Next, I performed a Monte Carlo analysis by sampling the combined P matrix 1000 times with guassian noise added in. This procedure is outlined in Lecture slides for number 27.



Black dots represent the point estimates for each sample, and green parts represent the computed numerical covariance and average position reading $\bar{x}$. You can see how most (about %66) of the estimates fall within the 1-σ curve, which is to be expected.

h. The estimates for the Monte Carlo $\bar{x}$ and P values were calculated using the given formula, and is below.

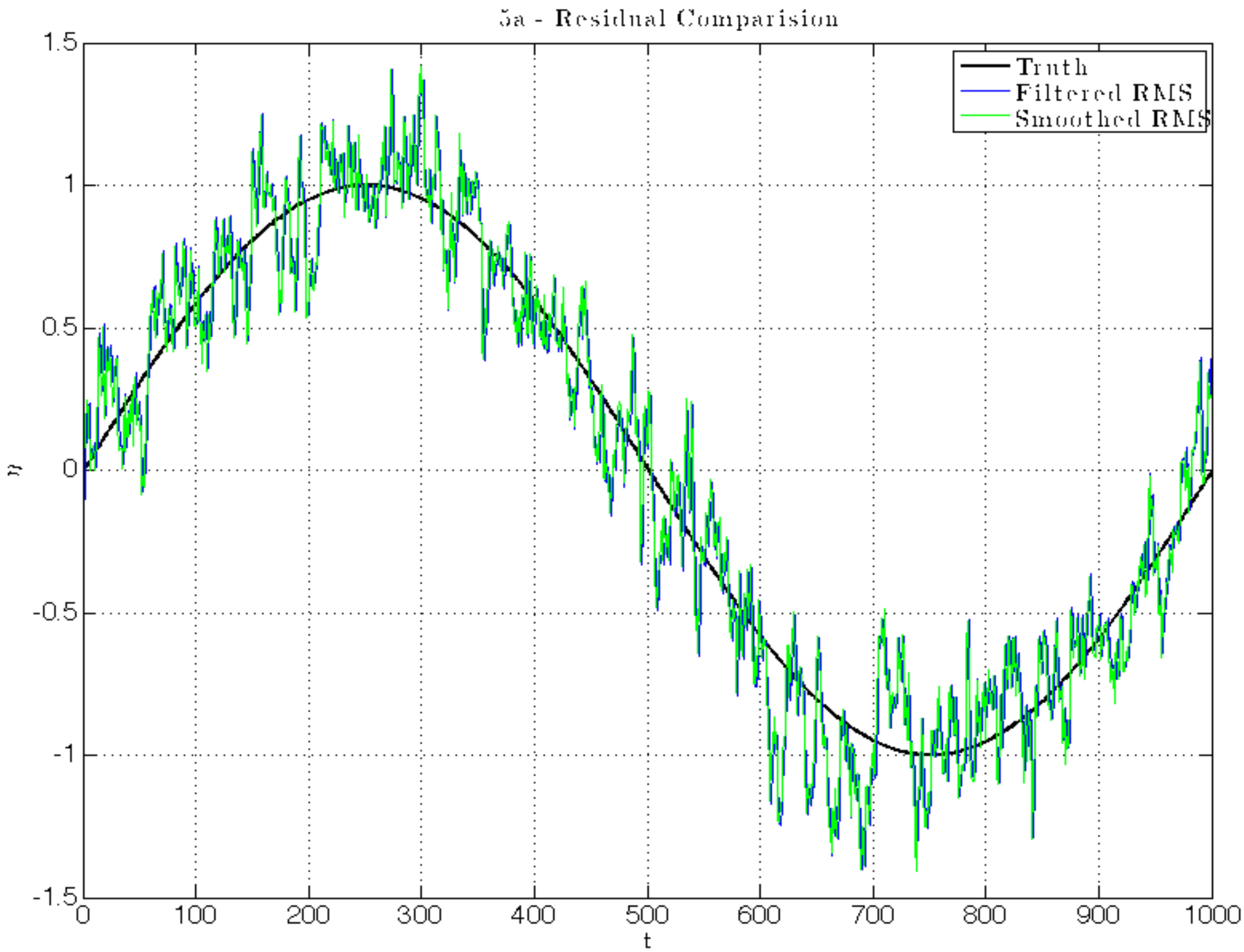
$\overline{\boldsymbol{x}}_{\boldsymbol{monte}} = [\mathbf{10.343\ 9.8894}]$
$\boldsymbol{P}_{\boldsymbol{monte}} = \begin{bmatrix} \mathbf{0.17011} & \mathbf{-0.084679} \\ \mathbf{-0.084679} & \mathbf{0.077051} \end{bmatrix}$

The average position is very near the expected $\hat{x}$, and the covariance is much tighter due to the fact that we now have many samples going into covariance calculations. Note that since this is a random process, the exact values of the position and covariance change slightly with each trial.

Output for all Problem 4 Code is in the following pages:

5. Problem 4-42 from the book. Computation is performed in Matlab, and code is included after this short write up.

a. Below is a plot resembling that from the book, Figure 4.19.2.



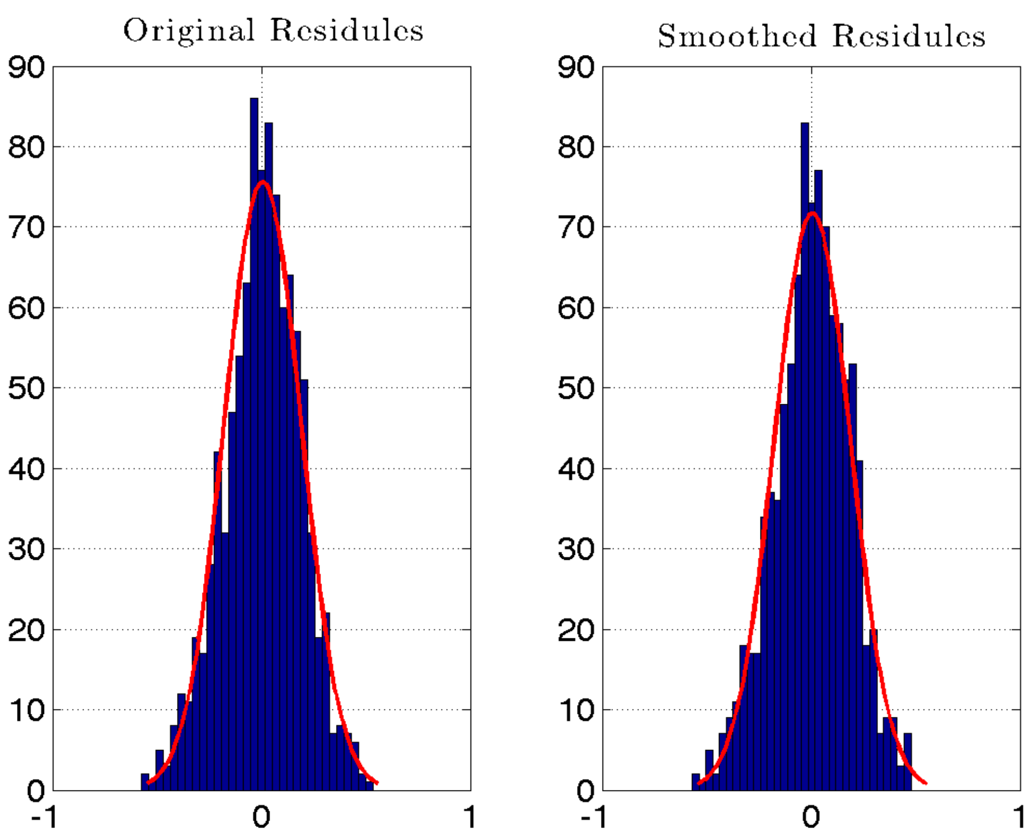
The smoothing algorithm ran successfully, but the smoothed force estimate does not depart from the original estimation at any meaningful level. This is not to say that the smoothing algorithm does not work, only that it is not especially well suited for this dataset. In fact, it looks very similar to that seen in the lecture notes, so evidently smoothing is not always necessary.

b. Next, I computed the rms of each residual calculation. The results are below.

$\boldsymbol{rms_{filt} = 0.18317}$
$\boldsymbol{rms_{smooth} = 0.18195}$
$\boldsymbol{difference = \%0.1272}$

While there is a drop in the rms, it is almost negligible. I tried the smoothing algorithm on multiple datasets, formed by using different β and σ values in the original filter. All explorations yielded similar results, where the smoothing only helps drop the rms by a small amount. Each time, the smoothed rms is slightly smaller, but only slightly.

c. Histograms comparing the two residual calculations are provided below.



If pressed, I really couldn't say that the smoothed residuals look any more normal. Both histograms have a definite normal shape, but neither stands out as being more normal. Again, this could be because the smoothing algorithm is not well formulated for this type of problem, but that is just speculation. Either way, there is no immediately apparent difference in normality for me to say that smoothing has any effect therein.

Matlab publishing direct output is provided below, so the code and results are visible.