ASEN 5005-Statistical Orbit Determination Homework 4

Zach Dischner

1 Problem 1

Given the joint density function:

$$f(x,y) = k * (x^2 + y^2) \quad 0 < x < 2, \quad 1 \le y \le 3$$

$$f(x,y) = 0 \qquad elsewhere$$

Several insights were to be found.

1.1 1a-Find k

To find k, I employed the rule that any joint density function must be equal to one when integrated across the number range.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k * (x^2 + y^2) dx dy = 1 \tag{1}$$

Since the validity of the function was limited to a specific number range for each variable, the joint density function becomes:

$$\int_{1}^{3} \int_{0}^{2} k * (x^{2} + y^{2}) dx dy = 1$$
 (2)

First, I integrated with respect to \mathbf{x}

$$k * \int_{1}^{3} \left(\frac{x^{3}}{3} + x * y^{2}\right) \Big|_{0}^{2} dy = 1$$
 (3)

Then after evaluating for the ${\bf x}$ range, I integrated with respect to ${\bf y}$

$$k * \left[y * \frac{8}{3} + 2 * \frac{y^3}{3} \right] \Big|_{1}^{3} dy = 1$$
 (4)

Which evaluates down to

$$k * \left[22.67\right] = 1 \tag{5}$$

Yielding a final value of

$$k = 0.0441$$
 (6)