ASEN 5070:
Statistical Orbit Determination I

Homework Set #5

Please turn in your code with your write-up. You are welcome to append it to your write-up, include it within the write-up, or submit it as a separate file to the D2L system (searchable PDF, txt, rtf, or doc file).

1. Assume an orbit plane coordinate system with a gravitational parameter of 1, i.e., $\mu = 1$. The equations of motion are:

$$\ddot{x} = -\frac{x}{r^3}$$
$$\ddot{y} = -\frac{y}{r^3}$$
$$r^2 = x^2 + y^2$$

(a) Generate a "true" solution by numerically integrating the equations of motion for the initial conditions:

$$\mathbf{X}(t_0) = \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix}_{t=t_0} = \begin{pmatrix} 1.0 \\ 0.0 \\ 0.0 \\ 1.0 \end{pmatrix}$$

Save the values of the state vector $\mathbf{X}(t_i)$ for $t_i = i * 10$ sec; $i = 0, \ldots, 10$.

Be sure to turn in your code. Apart from the code, please indicate which integrator you used, what the tolerance was set to, and any other details necessary. Note, if you use a fixed time-step integrator, set the time-step to be smaller than 10 seconds, but only save the data at 10 sec intervals.

(b) Perturb the previous set of initial conditions by an amount

$$\mathbf{X}^*(t_0) = \mathbf{X}(t_0) - \delta\mathbf{X}(t_0)$$

(notice that the perturbation is subtracted), where

$$\delta\mathbf{X}(t_0) = \begin{pmatrix} 1 \times 10^{-6} \\ -1 \times 10^{-6} \\ 1 \times 10^{-6} \\ 1 \times 10^{-6} \end{pmatrix}.$$

Numerically integrate this "nominal" trajectory along with the associated state transition matrix to find $\mathbf{X}^*(t_i)$ and $\Phi(t_i, t_0)$ at $t_i = i * 10$ sec; $i = 0, \dots, 10$.

Be sure to use the same integrator with the same tolerance as in part (a). Report what you get for $\mathbf{X}(t_{100})$, $\mathbf{X}^*(t_{100})$, and $\mathbf{X}(t_{100}) - \mathbf{X}^*(t_{100})$. Compare these values, along with the state transition matrix, to the solutions before you turn this in.

(c) For this problem, $\Phi(t_i, t_0)$ is symplectic. Demonstrate this for $\Phi(t_{100}, t_0)$ by multiplying it by $\Phi^{-1}(t_{100}, t_0)$, given by Equation 4.2.22 in the text. Show that the result is the identity matrix.

(d) Calculate the perturbation vector, $\delta\mathbf{X}(t_i)$, by the following two methods:

(1) $\delta\mathbf{X}(t_i) = \mathbf{X}(t_i) - \mathbf{X}^*(t_i)$
(2) $\delta\mathbf{X}(t_i) = \Phi(t_i, t_0)\delta\mathbf{X}(t_0)$

and compare the results of (1) and (2). How closely do they compare?

2. Given the observation state relation $y = Hx + \epsilon$, where x is a scalar and given that

$$y = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},$$

$$W = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and}$$

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

with *a priori* information, $\bar{x} = 2$ and $\bar{W} = 2$.

(a) Find $\hat{x}$ using the batch processing algorithm.

(b) What is the best estimate of the observation error, $\hat{\epsilon}$?

3. Problem 15 from Chapter 4. Please note the following!
    - **Please use the observation data file on the website**, so that you don't have to re-type each value from the book.
    - The problem states that you can build your own observation data set, but for the purpose of this homework use the file on the website.
    - Provide your answer after **FOUR** iterations rather than three.
    - Refer to section 4.8.2 for the algorithm.