$$= \mathcal{L}^{-1}\begin{bmatrix} \dfrac{1}{s-a} & 0 \\ \dfrac{b}{(s-a)(s-g)} & \dfrac{1}{s-g} \end{bmatrix}$$

$$= \begin{bmatrix} e^{at} & 0 \\ \dfrac{b}{a-g}(e^{at} - e^{gt}) & e^{gt} \end{bmatrix}$$

We have assumed that $t_0 = 0$. If not, replace t by $t - t_0$.

5.

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -ab & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \\ \dot{\Phi} &= \begin{bmatrix} 0 & 1 \\ -ab & 0 \end{bmatrix} \Phi \end{aligned}$$

Using Laplace Transforms

$$\begin{aligned} \Phi &= \mathcal{L}^{-1}[SI - A] = \mathcal{L}^{-1}\begin{bmatrix} s & -1 \\ ab & s \end{bmatrix}^{-1} \\ &= \mathcal{L}^{-1}\left\{ \frac{1}{s^2+ab} \begin{bmatrix} s & 1 \\ -ab & s \end{bmatrix} \right\} \\ &= \begin{bmatrix} \cos\sqrt{ab}\ t & \frac{1}{\sqrt{ab}} \sin\sqrt{ab}\ t \\ -\sqrt{ab}\ \sin\sqrt{ab}\ t & \cos\sqrt{ab}\ t \end{bmatrix} \end{aligned}$$

6. Show that $\dot{\Phi} = A\Phi$, i.e.

$$\dot{\Phi} = \begin{bmatrix} 3ae^{at} & 0 \\ 0 & -2be^{-bt} \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & -b \end{bmatrix} \begin{bmatrix} 3e^{at} & 0 \\ 0 & 2e^{-bt} \end{bmatrix}$$

Hence $\dot{\Phi} = A\Phi$; however, Φ is not a state transition matrix because $\Phi(t_0, t_0) \neq I (t_0 = 0)$.