ASEN 5007-Homework 2

Zach Dischner && Andrew Haynes

Problem 1

Initializer Modules

Cell 1: module to form element stiffness of two-dimensional two-node bar element in global coordinates.

In[29]:=
```
ElemStiff2DTwoNodeBar[{{x1_, y1_}, {x2_, y2_}}, {Em_, A_}] := Module[
 {c, s, dx = x2 - x1, dy = y2 - y1, L, Ke},
  L = Sqrt[dx^2 + dy^2]; c = dx / L; s = dy / L;
  Ke = (Em * A / L) * {{ c^2, c * s, -c^2, -c * s},
        { c * s, s^2, -s * c, -s^2},
        {-c^2, -s * c, c^2, s * c},
        {-s * c, -s^2, s * c, s^2}};
  Return[Ke]
 ];
```

Cell 2 : module to merge two - node bar element stiffness into the master stiffness matrix.

In[30]:=
```
MergeElemIntoMasterStiff[Ke_, eftab_, Kin_] := Module[ {i, j, ii, jj, K = Kin},
  For [i = 1, i <= 4, i++, ii = eftab[[i]];
   For [j = i, j <= 4, j++, jj = eftab[[j]];
     K[[jj, ii]] = K[[ii, jj]] += Ke[[i, j]]
   ]
  ]; Return[K]
];
```

Cell 3 : module to assemble master stiffness matrix of example truss.

In[31]:=
```
AssembleMasterStiffOfExampleTruss[] :=
   Module[{Ke, K = Table[0, {6}, {6}], sl = 8, h = 3, Em = 1000, A1 = 2, A2 = 4},
  Ke = ElemStiff2DTwoNodeBar[{{0, 0}, {sl / 2, h}}, {Em, A1}];
  K = MergeElemIntoMasterStiff[Ke, {1, 2, 3, 4}, K];
  Ke = ElemStiff2DTwoNodeBar[{{sl / 2, h}, {sl, 0}}, {Em, A2}];
  K = MergeElemIntoMasterStiff[Ke, {3, 4, 5, 6}, K];
  (*Ke=ElemStiff2DTwoNodeBar[{{0,0},{10,10}},{100,2*Sqrt[2]}];
    K= MergeElemIntoMasterStiff[Ke,{1,2,5,6},K]; *)
  Return[K]
 ];
```

Cell 4 : modules to apply homogeneous, single - freedom displacement BCs on master stiffness and

forces.

In[32]:=
```
ModifiedMasterStiffForDBC[pdof_, K_] := Module[
 {i, j, k, nk = Length[K], np = Length[pdof], Kmod = K},
   For [k = 1, k <= np, k++, i = pdof[[k]];
    For [j = 1, j <= nk, j++, Kmod[[i, j]] = Kmod[[j, i]] = 0];
     Kmod[[i, i]] = 1];
 Return[Kmod]
];
ModifiedMasterForcesForDBC[pdof_, f_] := Module[
 {i, k, np = Length[pdof], fmod = f},
   For [k = 1, k <= np, k++, i = pdof[[k]]; fmod[[i]] = 0];
 Return[fmod]
];
```

Cell 5 : module to compute the internal force in a two - dimensional two - node bar element.

In[34]:=
```
IntForce2DTwoNodeBar[{{x1_, y1_}, {x2_, y2_}}, {Em_, A_}, eftab_, u_] :=
 Module[ {c, s, dx = x2 - x1, dy = y2 - y1, L, ix, iy, jx, jy, ubar, e},
  L = Sqrt[dx^2 + dy^2]; c = dx / L; s = dy / L; {ix, iy, jx, jy} = eftab;
  ubar = {c * u[[ix]] + s * u[[iy]], -s * u[[ix]] + c * u[[iy]],
     c * u[[jx]] + s * u[[jy]], -s * u[[jx]] + c * u[[jy]]};
  e = (ubar[[3]] - ubar[[1]]) / L; Return[Em * A * e]
];
```

Cell 6 : module to get internal forces in members of example truss.

In[109]:=
```
IntForcesOfExampleTruss[u_] :=
   Module[{f = Table[0, {2}], sl = 8, h = 3, Em = 1000, A1 = 2, A2 = 4},
 f[[1]] = IntForce2DTwoNodeBar[{{0, 0}, {sl / 2, h}}, {Em, A1}, {1, 2, 3, 4}, u];
 f[[2]] = IntForce2DTwoNodeBar[{{sl / 2, h}, {sl, 0}}, {Em, A2}, {3, 4, 5, 6}, u];
(*  f[[3]]=IntForce2DTwoNodeBar[{{0,0},{10,10}},{100,2*Sqrt[2]},
         {1,2,5,6},u]; *)
 Return[f]
];
```

Cell 7: Simple function to print output for solutions in a stylazed way

In[36]:=
```
PrintWithStyle[x_] :=
 Module[{color = LightGreen}, Framed[Style[x, 18, Bold, Background → color],
   Background → color]
 ]
```

Problem 1 Driver Script

In[110]:=

```
f = {0, 0, 12, 0, 0, 0};
K = AssembleMasterStiffOfExampleTruss[];
Print["Master Stiffness Matrix: "]; Print[K // MatrixForm];

(*Nodes 1,2,5, and 6 are fixed in position. No movement can occur*)
Kmod = ModifiedMasterStiffForDBC[{1, 2, 5, 6}, K];
fmod = ModifiedMasterForcesForDBC[{1, 2, 5, 6}, f];
u = Simplify[Inverse[Kmod].fmod];
Print["Computed nodal displacements:"];
PrintWithStyle[u]
f = Simplify[K.u];
Print["External node forces including reactions:"];
PrintWithStyle[f]
 p = Simplify[IntForcesOfExampleTruss[u]];
Print["Internal member forces:"]
PrintWithStyle[p]
```

Master Stiffness Matrix:

$$\begin{pmatrix} 256 & 192 & -256 & -192 & 0 & 0 \\ 192 & 144 & -192 & -144 & 0 & 0 \\ -256 & -192 & 768 & -192 & -512 & 384 \\ -192 & -144 & -192 & 432 & 384 & -288 \\ 0 & 0 & -512 & 384 & 512 & -384 \\ 0 & 0 & 384 & -288 & -384 & 288 \end{pmatrix}$$

Computed nodal displacements:

Out[117]=

$$\left\{0, 0, \frac{9}{512}, \frac{1}{128}, 0, 0\right\}$$

External node forces including reactions:

Out[120]=

$$\left\{-6, -\frac{9}{2}, 12, 0, -6, \frac{9}{2}\right\}$$

Internal member forces:

Out[123]=

$$\left\{\frac{15}{2}, -\frac{15}{2}\right\}$$