ASEN 5070:
Statistical Orbit Determination I

Homework Set #2

1. Integrate the equations of motion for one day using the same initial conditions as in Homework Set #1, however, now include the Earth's oblateness, i.e., the J_2 term. The equations of motion are still $\ddot{\vec{r}} = \nabla U$, but now U includes J_2:

$$U = U_{\text{point mass}} + U_{J_2} = \frac{\mu}{r}\left[1 - J_2\left(\frac{R_{\text{Earth}}}{r}\right)^2\left(\frac{3}{2}\sin^2\varphi - \frac{1}{2}\right)\right].$$

Use the following (note, some of these are different than in Homework Set #1):

$J_2 = 0.00108248$
$\mu = 398{,}600.4 \text{ km}^3/\text{s}^2$
$R_{\text{Earth}} = 6378.145$ km
φ = Latitude → $\sin\varphi = z/r$
$r = \sqrt{x^2 + y^2 + z^2}$

The initial conditions from HW1 are:

$$\vec{R} = -2436.45\hat{\imath} - 2436.45\hat{\jmath} + 6891.037\hat{k} \text{ km}$$
$$\vec{V} = \dot{\vec{R}} = 5.088611\hat{\imath} - 5.088611\hat{\jmath} + 0.0\hat{k} \text{ km/s}$$

1a. Use the Matlab symbolic toolbox to compute the Cartesian partial derivatives of U. Compute $\partial U / \partial x$ by hand and compare your results with Matlab.

1b. Plot the orbital elements a, e, i, Ω, ω, and T_p for one day at 20 second intervals, where T_p is the time of perigee passage. Be sure to label your axes and ensure that everything in each figure is easily readable. Using your insight from the two-body model, what conclusions can you draw about the J_2 effect on Keplerian orbital elements?

1c. Compute the specific energy (energy/mass) and show that it is conserved around the orbit by plotting $dE = E(t) - E(t_0)$. Use the following equation using the U from above:

$$E = \frac{v^2}{2} - U$$

1d. Compute h_k, the k-component of the angular momentum vector ($\vec{h} = \vec{r} \times \vec{v}$) and plot $dh_k = h_k(t) - h_k(t_0)$ to show that it remains constant.

2. Integrate the equations of motion with the conditions given in Problem 1 that include the Earth point-mass, J_2, and now also drag. Use the following relationship for the acceleration due to drag:

$$\ddot{\vec{r}}_{\text{drag}} = -\frac{1}{2} C_D \left(\frac{A}{m}\right) \rho_A V_A \vec{V}_A$$

where:

$C_D = 2.0$
$A = 3.6 \text{ m}^2$
$m = 1350 \text{ kg}$
$\rho_0 = 4.0 \times 10^{-13} \text{ kg/m}^3$
$r_0 = 7298.145 \text{ km}$
$H = 200.0 \text{ km}$
$\dot{\theta} = 7.29211\ 58553\ 0066 \times 10^{-5} \text{ rad/s}$

$$\rho_A = \rho_0 e^{\frac{-(r-r_0)}{H}}$$

$$\vec{V}_A = \begin{bmatrix} \dot{x} + \dot{\theta} y \\ \dot{y} - \dot{\theta} x \\ \dot{z} \end{bmatrix}$$

$$V_A = \sqrt{\left(\dot{x} + \dot{\theta} y\right)^2 + \left(\dot{y} - \dot{\theta} x\right)^2 + \dot{z}^2}$$

2a. Compute the specific energy at 20 second intervals and plot $dE = E(t) - E(t_0)$. What can you infer from the plot? Is the total energy conserved? Why or why not?

2b. Compute the same Keplerian orbital elements generated in Problem 1b at 20 second intervals (a, e, i, Ω, ω, and T_p) and plot the differences in these elements from those computed in Problem 1. That is, generate time histories of each orbital element with and without drag and plot the differences, e.g., $a_{(2B+J2+Drag)} - a_{(2B+J2)}$. What can you observe in these plots? Which orbital elements are impacted by drag and how are they affected?