**Transport Theorem:**

$$\frac{{}^{\mathcal{N}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) = \frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}$$

**Rotation Matrix:**

$$\{\hat{\boldsymbol{b}}\} = [C]\{\hat{\boldsymbol{n}}\} = [BN]\{\hat{\boldsymbol{n}}\}$$

$$C_{ij} = \hat{\boldsymbol{b}}_i \cdot \hat{\boldsymbol{n}}_j$$

$$[\dot{C}] = -[\tilde{\boldsymbol{\omega}}][C]$$

$$[\tilde{\boldsymbol{x}}] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

$$[M_1(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$[M_2(\theta)] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$[M_3(\theta)] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**3-2-1 Euler Angles:**

$$[C] = \begin{bmatrix} c\theta_2 c\theta_1 & c\theta_2 s\theta_1 & -s\theta_2 \\ s\theta_3 s\theta_2 c\theta_1 - c\theta_3 s\theta_1 & s\theta_3 s\theta_2 s\theta_1 + c\theta_3 c\theta_1 & s\theta_3 c\theta_2 \\ c\theta_3 s\theta_2 c\theta_1 + s\theta_3 s\theta_1 & c\theta_3 s\theta_2 s\theta_1 - s\theta_3 c\theta_1 & c\theta_3 c\theta_2 \end{bmatrix}$$

$$\psi = \theta_1 = \tan^{-1}\left(\frac{C_{12}}{C_{11}}\right)$$

$$\theta = \theta_2 = -\sin^{-1}(C_{13})$$

$$\phi = \theta_3 = \tan^{-1}\left(\frac{C_{23}}{C_{33}}\right)$$

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{bmatrix} -\sin\theta & 0 & 1 \\ \sin\phi\cos\theta & \cos\phi & 0 \\ \cos\phi\cos\theta & -\sin\phi & 0 \end{bmatrix} \begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix}$$

$$\begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \frac{1}{\cos\theta} \begin{bmatrix} 0 & \sin\phi & \cos\phi \\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta \\ \cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta \end{bmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

**3-1-3 Euler Angles:**

$$[C] = \begin{bmatrix} c\theta_3 c\theta_1 - s\theta_3 c\theta_2 s\theta_1 & c\theta_3 s\theta_1 + s\theta_3 c\theta_2 c\theta_1 & s\theta_3 s\theta_2 \\ -s\theta_3 c\theta_1 - c\theta_3 c\theta_2 s\theta_1 & -s\theta_3 s\theta_1 + c\theta_3 c\theta_2 c\theta_1 & c\theta_3 s\theta_2 \\ s\theta_2 s\theta_1 & -s\theta_2 c\theta_1 & c\theta_2 \end{bmatrix}$$

$$\Omega = \theta_1 = \tan^{-1}\left(\frac{C_{31}}{-C_{32}}\right)$$

$$i = \theta_2 = \cos^{-1}(C_{33})$$

$$\omega = \theta_3 = \tan^{-1}\left(\frac{C_{13}}{C_{23}}\right)$$

$$\boldsymbol{\omega} = \begin{bmatrix} \sin\theta_3\sin\theta_2 & \cos\theta_3 & 0 \\ \cos\theta_3\sin\theta_2 & -\sin\theta_3 & 0 \\ \cos\theta_2 & 0 & 1 \end{bmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix}$$

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = \frac{1}{\sin\theta_2} \begin{bmatrix} \sin\theta_3 & \cos\theta_3 & 0 \\ \cos\theta_3\sin\theta_2 & -\sin\theta_3\sin\theta_2 & 0 \\ -\sin\theta_3\cos\theta_2 & -\cos\theta_3\cos\theta_2 & \sin\theta_2 \end{bmatrix} \boldsymbol{\omega}$$

**Principal Rotation Parameters:**

$$[C] = \begin{bmatrix} e_1^2\Sigma + c\Phi & e_1 e_2\Sigma + e_3 s\Phi & e_1 e_3\Sigma - e_2 s\Phi \\ e_2 e_1\Sigma - e_3 s\Phi & e_2^2\Sigma + c\Phi & e_2 e_3\Sigma + e_1 s\Phi \\ e_3 e_1\Sigma + e_2 s\Phi & e_3 e_2\Sigma - e_1 s\Phi & e_3^2\Sigma + c\Phi \end{bmatrix}$$

$$\cos\Phi = \frac{1}{2}(C_{11} + C_{22} + C_{33} - 1) \qquad \Sigma = 1 - \cos\Phi$$

$$[C] = e^{-\Phi[\tilde{\boldsymbol{e}}]} = [I_{3\times3}]\cos\Phi - \sin\Phi[\tilde{\boldsymbol{e}}] + (1-\cos\Phi)\hat{\boldsymbol{e}}\hat{\boldsymbol{e}}^T$$

$$\hat{\boldsymbol{e}} = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \frac{1}{2\sin\Phi} \begin{pmatrix} C_{23} - C_{32} \\ C_{31} - C_{13} \\ C_{12} - C_{21} \end{pmatrix}$$

**Euler Parameters:**

$$[C] = \begin{bmatrix} \beta_0^2+\beta_1^2-\beta_2^2-\beta_3^2 & 2(\beta_1\beta_2 + \beta_0\beta_3) & 2(\beta_1\beta_3 - \beta_0\beta_2) \\ 2(\beta_1\beta_2 - \beta_0\beta_3) & \beta_0^2-\beta_1^2+\beta_2^2-\beta_3^2 & 2(\beta_2\beta_3 + \beta_0\beta_1) \\ 2(\beta_1\beta_3 + \beta_0\beta_2) & 2(\beta_2\beta_3 - \beta_0\beta_1) & \beta_0^2-\beta_1^2-\beta_2^2+\beta_3^2 \end{bmatrix}$$

$$[C] = (\beta_0^2 - \boldsymbol{\epsilon}^T\boldsymbol{\epsilon})[I_{3\times3}] + 2\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T - 2\beta_0[\tilde{\boldsymbol{\epsilon}}]$$

$$\boldsymbol{\epsilon} = (\beta_1, \beta_2, \beta_3)$$

$$\beta_0 = \pm\frac{1}{2}\sqrt{C_{11} + C_{22} + C_{33} + 1}$$

$$\beta_1 = \frac{C_{23} - C_{32}}{4\beta_0}$$

$$\beta_2 = \frac{C_{31} - C_{13}}{4\beta_0}$$

$$\beta_3 = \frac{C_{12} - C_{21}}{4\beta_0}$$

**Classical Rodrigues Parameters:**

$$q_i = \frac{\beta_i}{\beta_0} \qquad \beta_0 = \frac{1}{\sqrt{1 + \boldsymbol{q}^T\boldsymbol{q}}} \qquad \beta_i = \frac{q_i}{\sqrt{1 + \boldsymbol{q}^T\boldsymbol{q}}}$$

$$[C] = \frac{1}{1 + \boldsymbol{q}^T\boldsymbol{q}} \begin{bmatrix} 1+q_1^2-q_2^2-q_3^2 & 2(q_1q_2 + q_3) & 2(q_1q_3 - q_2) \\ 2(q_2q_1 - q_3) & 1-q_1^2+q_2^2-q_3^2 & 2(q_2q_3 + q_1) \\ 2(q_3q_1 + q_2) & 2(q_3q_2 - q_1) & 1-q_1^2-q_2^2+q_3^2 \end{bmatrix}$$

$$[C] = \frac{1}{1 + \boldsymbol{q}^T\boldsymbol{q}}\left(\left(1 - \boldsymbol{q}^T\boldsymbol{q}\right)[I_{3\times3}] + 2\boldsymbol{q}\boldsymbol{q}^T - 2[\tilde{\boldsymbol{q}}]\right)$$

$$\boldsymbol{q} = \frac{\boldsymbol{q}'' + \boldsymbol{q}' - \boldsymbol{q}'' \times \boldsymbol{q}'}{1 - \boldsymbol{q}'' \cdot \boldsymbol{q}'}$$

$$\dot{\boldsymbol{q}} = \frac{1}{2}\left[[I_{3\times3}] + [\tilde{\boldsymbol{q}}] + \boldsymbol{q}\boldsymbol{q}^T\right]\boldsymbol{\omega}$$

**Modified Rodrigues Parameters:**

$$\sigma_i = \frac{\beta_i}{1+\beta_0} \qquad \beta_0 = \frac{1-\sigma^2}{1+\sigma^2} \qquad \beta_i = \frac{2\sigma_i}{1+\sigma^2}$$

$$[C] = [I_{3\times3}] + \frac{8[\tilde{\boldsymbol{\sigma}}]^2 - 4\left(1-\sigma^2\right)[\tilde{\boldsymbol{\sigma}}]}{\left(1+\sigma^2\right)^2}$$

$$\boldsymbol{\sigma} = \frac{(1-|\boldsymbol{\sigma}'|^2)\boldsymbol{\sigma}'' + (1-|\boldsymbol{\sigma}''|^2)\boldsymbol{\sigma}' - 2\boldsymbol{\sigma}''\times\boldsymbol{\sigma}'}{1+|\boldsymbol{\sigma}'|^2|\boldsymbol{\sigma}''|^2 - 2\boldsymbol{\sigma}'\cdot\boldsymbol{\sigma}''}$$

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4}\left[\left(1-\sigma^2\right)[I_{3\times3}] + 2[\tilde{\boldsymbol{\sigma}}] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T\right]\boldsymbol{\omega} = \frac{1}{4}[B(\boldsymbol{\sigma})]\boldsymbol{\omega}$$

**Chapter 4:**

$$M\boldsymbol{R}_c = \int_B \boldsymbol{R}\mathrm{d}m$$

$$\int_B \boldsymbol{r}\mathrm{d}m = 0$$

$$\dot{\boldsymbol{H}} = \boldsymbol{L}$$

$$\boldsymbol{H} = [I]\boldsymbol{\omega} \qquad T = \frac{1}{2}\boldsymbol{\omega}^T[I]\boldsymbol{\omega}$$

$$I_1\dot{\omega}_1 = -(I_3-I_2)\omega_2\omega_3 + L_1$$

$$I_2\dot{\omega}_2 = -(I_1-I_3)\omega_1\omega_3 + L_2$$

$$I_3\dot{\omega}_3 = -(I_2-I_1)\omega_1\omega_2 + L_3$$

$$H^2 = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$$

$$2T = I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$$