**ASEN 5070**
**Exam No. 1**
**July 2, 2003**
**Open Book and Notes**

1. (35%) Given that the observations are related to the state by

$$y_i = (t_i - 1)x_1 + (t_i^2 + 1)x_2 + \varepsilon_i \qquad i = 1, 2$$

Observations $y_i$ are taken at $t_1=0$, $t_2=1$, and $y_1$ is as accurate as $y_2$.

The values of the observations are: $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. The state vector is $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Assume *a priori* information is given: $\bar{X} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\bar{W} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

a. Write the observation-state equation in the form Y=HX +ε.
b. Compute the least squares estimate of X including the *a priori* information.
c. Compute the best estimate of ε.

2. (35%) Given the system

$$\dot{x}_1 = \alpha x_1 + \beta x_2$$
$$\dot{x}_2 = \alpha x_2$$

a. Write the equations in state space form, $\dot{X} = AX$, where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

b. Determine the state transition matrix for this system.

c. Assuming $t_0 = 0$, write the expression for $X$ at $t = 1$ in terms of the initial conditions $x_1(t_0)$ and $x_2(t_0)$.

3. (30%) Answer the following questions true or false.

a. In problem 1 the observation state relationship is nonlinear_____

b. If the state vector is n x 1 and the observation vector in m x 1 the H matrix will be m x n _____

c. If there are fewer observations than unknowns but we are given apriori state information with a full rank weighting matrix it is possible to obtain a least squares estimate for the state _____

d. Range observations of a satellite from two different ground stations at the same instant in time generally will not be independent_____

e. The differential equation $\ddot{x} + 3\dot{x} + 2x^2 = 0$ is linear _____