ASEN 5070
Exam No. 1
Oct. 3, 2001
Open Book and Notes

1. (35%) Given that the observations are related to the state by

$$y_i = (t_i + 1)x_1 + t_i^{\,2} x_2 + \varepsilon_i \qquad \text{i = 1, 2, 3}$$

Observations y_i are taken at t_1=0, t_2=1, t_3=2, and

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}. \text{ Let the state vector be } X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   a. Write the observation-state equation in the form Y=HX +ε.
   b. Compute the best (least squares) estimate of X
   c. Compute the best estimate of ε.

2. (35%) The differential equations for a linear feedback control system are

$$\dot{u} = -a(u - v) \qquad t_0 = 0,\ u(t_0) = u_0,\ v(t_0) = v_0$$
$$\dot{v} = 0$$

   a. Write the equations in state space form, $\dot{X} = AX$, where $X = \begin{bmatrix} u \\ v \end{bmatrix}$.
   b. Determine the state transition matrix for this system.

3. (30%) Answer the following questions true or false.

   a. In problem 1 the observation state relationship is nonlinear______

   b. If the state vector is n x 1 and the observation vector in m x 1 the H matrix will be n x m _______

   c. If there are fewer observations than unknowns but we are given apriori state information with a full rank weighting matrix it is possible to obtain a least squares estimate for the state ______

   d. The state transition matrix is always square______

   e. If the differential equations for the state and the equations relating the observations and the state are linear there is no need to use a state or observation deviation vector_____

   f. Range observations of a satellite from two different ground stations at the same instant in time generally will not be independent_____