1. The A matrix can be determined by diffentiating Φ and using $\dot{\Phi} = A\Phi$

$$\dot{\Phi} = \begin{bmatrix} -2ae^{-2at} & 0 \\ 0 & be^{bt} \end{bmatrix} = \begin{bmatrix} -2a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} e^{-2at} & 0 \\ 0 & e^{bt} \end{bmatrix}.$$

Hence,

$$A = \begin{bmatrix} -2a & 0 \\ 0 & b \end{bmatrix}.$$

2. Show that

   a. $\dot{\Phi}(t_i, t_k) = A(t_i)\Phi(t_i, t_k)$
   From Eq. (4.2.7)

$$x(t) = \Phi(t, t_k)x(t_k).$$

   Differentiating Eq. (4.2.7) yields

$$\dot{x}(t) = \dot{\Phi}(t, t_k)x(t_k)$$

   ($x(t_k)$ is a constant). From Eq. (4.2.6)

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) \\ &= \dot{\Phi}(t, t_k)x(t_k). \end{aligned}$$

   Substituting for $x(t)$ from Eq. (4.2.7)

$$\dot{x}(t) = A(t)\Phi(t, t_k)x(t_k) = \dot{\Phi}(t, t_k)x(t_k)$$

   hence,

$$\dot{\Phi}(t, t_k) = A(t)\Phi(t, t_k).$$

   b. $\Phi(t_i, t_j) = \Phi(t_i, t_k)\Phi(t_k, t_j)$
   Use Eq. (4.2.7)

$$\begin{aligned} x(t_i) &= \Phi(t_i, t_k)x(t_k) \\ &= \Phi(t_i, t_k)\Phi(t_k, t_j)x(t_j) \\ &= \Phi(t_i, t_j)x(t_j). \end{aligned}$$