ASEN 5070:
Statistical Orbit Determination I

Homework Set #4

1. Two random variables have the joint density function given by:

$$f(x,y) = k(x^2 + y^2), \qquad 0 \le x \le 2, \quad 1 \le y \le 3$$
$$f(x,y) = 0, \qquad \text{elsewhere}$$

   a) Find k
   b) Find $p(1 < x \le 2,\ 2 < y \le 3)$
   c) Find $p(1 \le x \le 2)$
   d) Find $p(x + y \ge 4)$
   e) Find $p(x + y = 4)$
   f) Find $p(x \le 1 \ / \ y = 3)$
   g) Find σ_x
   h) Find $p(1 < x < 2 \ / \ 1 < y < 2)$

2. Show that the moment generating function for the univariate normal distribution

$$f(x) = \frac{1}{b\sqrt{2\pi}} e^{\left[-\frac{1}{2}\left(\frac{x-a}{b}\right)^2\right]} \qquad -\infty \le x \le \infty$$

is given by

$$M_x(\theta) = e^{\left[\frac{\theta^2 b^2}{2} + a\theta\right]}$$

3. If x and y are independent random variables, show that

$$\sigma^2(xy) = \sigma^2(x)\sigma^2(y) + \lambda^2(x)\sigma^2(y) + \lambda^2(y)\sigma^2(x)$$

using the notation of Appendix A:

$$\sigma^2(xy) = E\left[xy - E(xy)\right]^2,\ \sigma^2(x) = \mu_{20},\ \sigma^2(y) = \mu_{02},\ \lambda(x) = \lambda_{10},\ \lambda(y) = \lambda_{01}$$