

Instructions:

1. Execute Cells 1 through 7 in that order (twice each if you want to get rid of annoying warning messages) to initialize all necessary modules.
Shortcut for Mathematica 4.0 or 5.0 users: select Kernel -> Evaluation -> Evaluate Initialization
To check individual cells, run test code (blue) for that cell only.
2. Cell 8 contains the driver program script for the bridge truss example.
Run by selecting the cell and executing.
3. Execute Cell 8A to generate deformed shape plot frames. Animate by double clicking one of the frames. Animation speed may be controlled by the playback button controls on the left side of the bottom window bar.
4. If the plots produced by running a program appear too small, click on the top one (only) with the mouse, grasp a corner "handle" and enlarge it. Then rerun the program.
5. To prepare a problem driver for the homework, open up a new cell, say 9, and write the script using Cell 8 as "template". Don't try to do the whole thing at once. Write a few statements and run the script up to that point (before running don't forget to have initialized Cells 1-7 as indicated above). Examine output and resolve error messages (if any) before proceeding.

Cell 0. Mathematica version-dependent settings (to get plots displayed correctly)

In[1]:=

```
DisplayChannel=$DisplayFunction;  
If [$VersionNumber>=6.0, DisplayChannel=Print];  
(* fix for Mathematica 6 & later *)  
  
Off[General::"spell1"];
```

Cell 1. Assembly module for space truss structure. The element stiffness module that supports the assembler has been described in Chapter 20.

In[6]:=

```

SpaceTrussMasterStiffness[nodxyz_,elenod_,
  elemat_,elefab_,prcopt_]:=Module[
  {numele=Length[elenod],numnod=Length[nodxyz],neldof,
  e,efstab,ni,nj,i,j,ii,jj,ncoor,Em,A,options,Ke,K},
  K=Table[0,{3*numnod},{3*numnod}];
  For [e=1, e<=numele, e++, {ni,nj}=elenod[[e]];
    efstab={3*ni-2,3*ni-1,3*ni,3*nj-2,3*nj-1,3*nj};
    ncoor={nodxyz[[ni]],nodxyz[[nj]]};
    Em=elemat[[e]]; A=elefab[[e]]; options=prcopt;
    Ke=SpaceBar2Stiffness[ncoor,Em,A,options];
    neldof=Length[Ke];
    For [i=1, i<=neldof, i++, ii=efstab[[i]];
      For [j=1, j<=neldof, j++, jj=efstab[[j]];
        K[[jj,ii]]=K[[ii,jj]]+Ke[[i,j]] ];
      ];
  ]; Return[K];
];
SpaceBar2Stiffness[ncoor_,Em_,A_,options_]:=Module[
  {x1,x2,y1,y2,z1,z2,x21,y21,z21,EA,numer,L,LL,LLL,Ke},
  {{x1,y1,z1},{x2,y2,z2}}=ncoor; {x21,y21,z21}={x2-x1,y2-y1,z2-z1};
  EA=Em*A; {numer,options}= {LL=x21^2+y21^2+z21^2; L=Sqrt[LL];
  IF [numer,{x21,y21,z21,EA,LL,L}=N[{x21,y21,z21,EA,LL,L}]];
  IF [!numer, L=PowerExpand[L]]; LLL=Simplify[LL*L];
  Ke=(Em*A/LLL)*
  {{ x21*x21, x21*y21, x21*z21, -x21*x21, -x21*y21, -x21*z21},
  { y21*x21, y21*y21, y21*z21, -y21*x21, -y21*y21, -y21*z21},
  { z21*x21, z21*y21, z21*z21, -z21*x21, -z21*y21, -z21*z21},
  {-x21*x21, -x21*y21, -x21*z21, x21*x21, x21*y21, x21*z21},
  {-y21*x21, -y21*y21, -y21*z21, y21*x21, y21*y21, y21*z21},
  {-z21*x21, -z21*y21, -z21*z21, z21*x21, z21*y21, z21*z21}};
  Return[Ke];

ClearAll[nodxyz,elemat,elefab,eleopt];
nodxyz={{0,0,0},{10,0,0},{10,10,0}};
elenod= {{1,2},{2,3},{1,3}};
elemat= Table[100,{3}]; elefab= {1,1/2,2*Sqrt[2]}; prcopt= {False};
K=SpaceTrussMasterStiffness[nodxyz,elenod,elemat,elefab,prcopt];
Print["Master Stiffness of Example Truss in 3D:\n",K//MatrixForm];
Print["eigs of K:",Chop[Eigenvalues[N[K]]]];

```

Master Stiffness of Example Truss in 3D:

$$\begin{pmatrix}
 20 & 10 & 0 & -10 & 0 & 0 & -10 & -10 & 0 \\
 10 & 10 & 0 & 0 & 0 & 0 & -10 & -10 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -10 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 5 & 0 & 0 & -5 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -10 & -10 & 0 & 0 & 0 & 0 & 10 & 10 & 0 \\
 -10 & -10 & 0 & 0 & -5 & 0 & 10 & 15 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{pmatrix}$$

eigs of K: {45.3577, 16.7403, 7.902, 0, 0, 0, 0, 0, 0}

Cell 2. These modules apply the displacement boundary conditions by modifying the force vector and the master stiffness matrix. This implementation of ModifyNodeForces handles nonzero prescribed displacements.

In[17]:=

```

ModifiedMasterStiffness[nodtag_,K_] := Module[
  {i,j,k,n=Length[K],pdof,np,Kmod=K},
  pdof=PrescDisplacementDOFTags[nodtag]; np=Length[pdof];
  For [k=1,k<=np,k++, i=pdof[[k]];
    For [j=1,j<=n,j++, Kmod[[i,j]]=Kmod[[j,i]]=0];
    Kmod[[i,i]]=1];
  Return[Kmod];

ModifiedNodeForces[nodtag_,nodval_,K_,f_] := Module[
  {i,j,k,n=Length[K],pdof,pval,np,d,c,fmod=f},
  pdof=PrescDisplacementDOFTags[nodtag]; np=Length[pdof];
  pval=PrescDisplacementDOFValues[nodtag,nodval]; c=Table[1,{n}];
  For [k=1,k<=np,k++, i=pdof[[k]]; c[[i]]=0];
  For [k=1,k<=np,k++, i=pdof[[k]]; d=pval[[k]];
    fmod[[i]]=d; If [d=0, Continue[]];
    For [j=1,j<=n,j++, fmod[[j]]-=K[[i,j]]*c[[j]]*d];
  ];
  Return[fmod];

PrescDisplacementDOFTags[nodtag_] := Module [
  {j,n,numnod=Length[nodtag],pdof={},k=0,m},
  For [n=1,n<=numnod,n++, m=Length[nodtag[[n]]];
    For [j=1,j<=m,j++, If [nodtag[[n,j]]>0,
      AppendTo[pdof,k+j]];
    ]; k+=m;
  ]; Return[pdof];

PrescDisplacementDOFValues[nodtag_,nodval_] := Module [
  {j,n,numnod=Length[nodtag],pval={},k=0,m},
  For [n=1,n<=numnod,n++, m=Length[nodtag[[n]]];
    For [j=1,j<=m,j++, If [nodtag[[n,j]]>0,
      AppendTo[pval,nodval[[n,j]]];
    ]; k+=m;
  ]; Return[pval];

ClearAll[K,f,v1,v2,v4]; Km=Array[K,{6,6}];
Print["Master Stiffness: ",Km/MatrixForm];
nodtag={{1,1},{0,1},{0,0}}; nodval={{v1,v2},{0,v4},{0,0}};
Kmod=ModifiedMasterStiffness[nodtag,Km];
Print["Modified Master Stiffness: ",Kmod/MatrixForm];
fm=Array[f,{6}]; Print["Master Force Vector: ",fm];
fmod=ModifiedNodeForces[nodtag,nodval,Km,fm];
Print["Modified Force Vector: ",fmod/MatrixForm];

(*nodtag= {{0,1,0},{0,0,0},{1,0,1},{0,2,1},{0,0,1}};
nodval={{1,2,3},{4,5,6},{7,8,9},{10,11,12},{13,14,15}};
Print[PrescDisplacementDOFTags[nodtag]];
Print[PrescDisplacementDOFValues[nodtag,nodval]];*)

```

$$\text{Master Stiffness: } \begin{pmatrix} K[1,1] & K[1,2] & K[1,3] & K[1,4] & K[1,5] & K[1,6] \\ K[2,1] & K[2,2] & K[2,3] & K[2,4] & K[2,5] & K[2,6] \\ K[3,1] & K[3,2] & K[3,3] & K[3,4] & K[3,5] & K[3,6] \\ K[4,1] & K[4,2] & K[4,3] & K[4,4] & K[4,5] & K[4,6] \\ K[5,1] & K[5,2] & K[5,3] & K[5,4] & K[5,5] & K[5,6] \\ K[6,1] & K[6,2] & K[6,3] & K[6,4] & K[6,5] & K[6,6] \end{pmatrix}$$

$$\text{Modified Master Stiffness: } \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K[3,3] & 0 & K[3,5] & K[3,6] \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & K[5,3] & 0 & K[5,5] & K[5,6] \\ 0 & 0 & K[6,3] & 0 & K[6,5] & K[6,6] \end{pmatrix}$$

$$\text{Master Force Vector: } \{f[1], f[2], f[3], f[4], f[5], f[6]\}$$

$$\text{Modified Force Vector: } \begin{pmatrix} v1 \\ v2 \\ f[3] - v1 K[1,3] - v2 K[2,3] - v4 K[4,3] \\ v4 \\ f[5] - v1 K[1,5] - v2 K[2,5] - v4 K[4,5] \\ f[6] - v1 K[1,6] - v2 K[2,6] - v4 K[4,6] \end{pmatrix}$$

Cell 3. Computation of internal forces (axial forces) for all elements of a space truss, from the computed nodal displacements.

In[35]:=

```

SpaceTrussIntForces[nodxyz_,elenod_,elemat_,elefab_,
  noddis_,prcopt_]:= Module[{ numnod=Length[nodxyz],
  numele=Length[elenod],e,ni,nj,ncoor,Em,A,options,ue,p},
  p=Table[0,{numele}];
  For [e=1, e<=numele, e++, {ni,nj}=elenod[[e]];
    ncoor={nodxyz[[ni]],nodxyz[[nj]]};
    ue=Flatten[{ noddis[[ni]],noddis[[nj]] }];
    Em=elemat[[e]]; A=elefab[[e]]; options=prcopt;
    p[[e]]=SpaceBar2IntForce[ncoor,Em,A,ue,options]
  ];
  Return[p];

SpaceBar2IntForce[ncoor_,Em_,A_,ue_,options_]:= Module[
  {x1,x2,y1,y2,z1,z2,x21,y21,z21,EA,numer,LL,pe},
  {{x1,y1,z1},{x2,y2,z2}}=ncoor; {x21,y21,z21}={x2-x1,y2-y1,z2-z1};
  EA=Em*A; {numer}=options; LL=x21^2+y21^2+z21^2;
  If [numer,{x21,y21,z21,EA,LL}=N[{x21,y21,z21,EA,LL}]];
  pe=(EA/LL)*(x21*(ue[[4]]-ue[[1]])+y21*(ue[[5]]-ue[[2]])+
    +z21*(ue[[6]]-ue[[3]]));
  Return[pe];

SpaceTrussStresses[elefab_,elefor_,prcopt_]:= Module[
  {numele=Length[elefab],e,esig}, elesig=Table[0,{numele}];
  For [e=1, e<=numele, e++, elesig[[e]]=elefor[[e]]/elefab[[e]] ];
  Return[elesig];

ClearAll[nodxyz,elenod,elemat,elefab,noddis];
nodxyz={{0,0,0},{10,0,0},{10,10,0}}; elenod={{1,2},{2,3},{1,3}};
elemat=Table[100,{3}]; elefab={1,1/2,2*Sqrt[2]};
noddis={{0,0,0},{0,0,0},{4/10,-2/10,0}}; prcopt={False};
elefor=SpaceTrussIntForces[nodxyz,elenod,elemat,elefab,noddis,prcopt];
Print["Int Forces of Example Truss:",elefor];
Print["Stresses:",SpaceTrussStresses[elefab,elefor,prcopt]];

```

Int Forces of Example Truss: $\{0, -1, 2\sqrt{2}\}$

Stresses: $\{0, -2, 1\}$

Cell 4 has groups plot modules that support preprocessing, that is, draw pictures of the FEM model.

Modules to plot loads and BCs removed to keep it short. PlotSpaceTrussElements draws only the element (bars) without any labels. PlotSpaceTrussElementsAndNodes draws elements and nodes, and optionally labels them.

In[48]:=

```

WorldToScreenMatrix[view_]:=Module[{VW={{0,1,0},{0,0,1}},
  Vx,Vy,Vz,Wx,Wy,Wz,xden,yden,Q},
  If [Length[view]==2, VW=view];
  If [Length[view]==3, VW[[2]]=view];
  {{Vx,Vy,Vz},{Wx,Wy,Wz}}=VW;
  xden=Sqrt[Wz^2*(Vx^2+Vy^2)-2*Wx*Wz*Vx*Vz-
    2*Wy*Vy*(Wx*Vx+Wz*Vz)+Wy^2*(Vx^2+Vz^2)+Wx^2*(Vy^2+Vz^2)];
  yden=Sqrt[(Wx^2+Wy^2+Wz^2)*(Wz^2*(Vx^2+Vy^2)-2*Wx*Wz*Vx*Vz-
    2*Wy*Vy*(Wx*Vx+Wz*Vz)+Wy^2*(Vx^2+Vz^2)+Wx^2*(Vy^2+Vz^2))];
  Q={{(Wz*Vy-Wy*Vz,-Wz*Vx+Wx*Vz,Wy*Vx-Wx*Vy)/xden,
    {Wy^2*Vx-Wx*Wy*Vy+Wz*(Wz*Vx-Wx*Vz),-Wx*Wy*Vx+Wx^2*Vy+
    Wz*(Wz*Vy-Wy*Vz),-Wz*(Wx*Vx+Wy*Vy)+(Wx^2+Wy^2)*Vz}/yden};
  Return[Q];

PlotSpaceTrussElements[nodxyz_,elenod_,title_,{view_,aspect_,labels_}]:=
Module[{numnod=Length[nodxyz],numele=Length[elenod],
  e,n,ni,nj,Q,x,y,xmin,xmax,ymin,ymax,dx,dy,xyz,ar,pbars={}},
  x=y=Table[0,{numnod}]; Q=WorldToScreenMatrix[view];
  For [n=1,n<=numnod,n++, {x[[n]],y[[n]]}=Q.nodxyz[[n]] ];
  {xmin,xmax,ymin,ymax}=N[{Min[x],Max[x],Min[y],Max[y]}];
  {dx,dy}={xmax-xmin,ymax-ymin};
  For [e=1,e<=numele,e++, {ni,nj}=elenod[[e]];
    xyz={{x[[ni]],y[[ni]]},{x[[nj]],y[[nj]]}};
    AppendTo[pbars,Graphics[Line[xyz]]]
  ];
  arat=AspectRatio->Automatic; If [aspect>0, arat=AspectRatio->aspect];
  If [aspect==0&&dx>0, arat=AspectRatio->dy/dx];
  Show[Graphics[AbsoluteThickness[2]],
    Graphics[RGBColor[0,0,0]],
    pbars, arat, PlotLabel->title,
    DisplayFunction->DisplayChannel ];
  ClearAll[pbars];
];

PlotSpaceTrussElementsAndNodes[nodxyz_,elenod_,title_,
  {view_,aspect_,labels_}]:= Module[{numnod=Length[nodxyz],
  numele=Length[elenod],k=Length[labels],e,n,ni,nj,Q,x,y,xyz,
  xy0,xyn,xmin,xmax,ymin,ymax,elabels,nlabels,fntinfo,
  labnod,frn,fex,fey,labels,fre,fntnam,fntsiz,fntwgt,fntslt,
  black,red,green,blue,yellow,fill,gray,nofill,arat,
  style,dx,dy,dmin,rn,re,ex,ey,elab,nlab,pbars={},
  pecirc={},pelab={},pedisk={},pncirc={},pndisk={},pnlab={} },
  x=y=Table[0,{numnod}]; Q=WorldToScreenMatrix[view];
  {labnod,frn,fex,fey,labels,fre,fntnam,fntsiz,fntwgt,fntslt}=
  {True,0.03,-1.5,0.8,True,0.03,"Times",12,"Plain","Plain"};

```

```

If [k>=1, nlabels=labels[[1]] ];
If [k>=2, elabels=labels[[2]] ];
If [k>=3, fntinfo=labels[[3]] ];
If [Length[nlabels]>=1, labnod=nlabels[[1]] ];
If [Length[nlabels]>=2, frn= nlabels[[2]] ];
If [Length[nlabels]>=3, fex= nlabels[[3]] ];
If [Length[nlabels]>=4, fey= nlabels[[4]] ];
If [Length[elabels]>=1, labels=elabels[[1]] ];
If [Length[elabels]>=2, fre= elabels[[2]] ];
If [Length[fntinfo]>=1, fntnam=fntinfo[[1]] ];
If [Length[fntinfo]>=2, fntsiz=fntinfo[[2]] ];
If [Length[fntinfo]>=3, fntwtg=fntinfo[[3]] ];
If [Length[fntinfo]>=4, fntslt=fntinfo[[4]] ];
For [n=1,n<=numnod,n++, {x[[n]],y[[n]]}=Q.nodxyz[[n]] ];
{xmin,xmax,ymin,ymax}=N[{Min[x],Max[x],Min[y],Max[y]}];
{dx,dy}={xmax-xmin,ymax-ymin}; dmin=Min[dx,dy];
rn=frn*dmin; re=fre*dmin;
style=TextStyle->{FontFamily->fntnam,FontSize-> fntsiz,
FontWeight->fntwtg,FontSlant->fntslt};
For [e=1,e<=numele,e++, {ni,nj}=elenod[[e]];
xyc={x[[ni]],y[[ni]],{x[[nj]],y[[nj]]}};
xy0=(xyc[[1]]+xyc[[2]])/2;
AppendTo[pbars,Graphics[Line[xyc]]];
If [labele, elab=ToString[e];
AppendTo[pdisk, Graphics[Disk[xy0,re]]];
AppendTo[pecirc, Graphics[Circle[xy0,re]]];
AppendTo[peleb, Graphics[Text[elab,xy0-{0,0.2*re},style]]];
];
style=TextStyle->{FontFamily->fntnam,FontSize-> fntsiz,
FontWeight->"Bold",FontSlant->"Plain"};
For [n=1,n<=numnod,n++, xyn={x[[n]],y[[n]]};
AppendTo[pndisk, Graphics[Disk[xyn,rn]]];
AppendTo[pncirc, Graphics[Circle[xyn,rn]]];
If [labnod, nlab=ToString[n]; xy0=xyn+{fex,fey}*rn;
AppendTo[pnlab, Graphics[Text[nlab,xy0,style]]];
];
{black,red,green,blue,yellow}={RGBColor[0,0,0],RGBColor[1,0,0],
RGBColor[0,1,0],RGBColor[0,0,1],RGBColor[1,1,0]};
{nofill,gray,fill}={GrayLevel[0],GrayLevel[.8],GrayLevel[1]};
arat=AspectRatio->Automatic;
If [aspect>0, arat=AspectRatio->aspect];
If [aspect==0&&dx>0, arat=AspectRatio->dy/dx];
Show[Graphics[nofill],Graphics[AbsoluteThickness[3]],
Graphics[red],pbars, Graphics[AbsoluteThickness[1]],
Graphics[fill],pndisk, Graphics[nofill],pncirc,
Graphics[yellow], pedisk, Graphics[nofill],
Graphics[black], pecirc, pelab, pnlab,
arat, PlotLabel->title,
DisplayFunction->DisplayChannel ];
ClearAll[pbars,pndisk,pncirc,pedisk,pecirc,pelab];
];

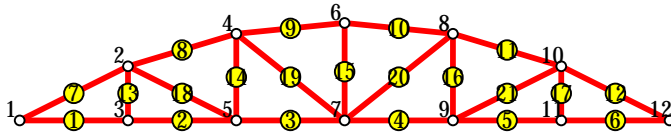
nodxyzs={{0,0,0},{10,5,0},{10,0,0},{20,8,0},{20,0,0},{30,9,0},
{30,0,0},{40,8,0},{40,0,0},{50,5,0},{50,0,0},{60,0,0}};
nodxyzs=N[nodxyzs];
elenods={{1,3},{3,5},{5,7},{7,9},{9,11},{11,12},
{1,2},{2,4},{4,6},{6,8},{8,10},{10,12},
{2,3},{4,5},{6,7},{8,9},{10,11},
{2,5},{4,7},{6,9},{8,11}};
view={{0,1,0},{0,0,1}};
PlotSpaceTrussElements[nodxyzs,elenods,"plain mesh",{view,0,{}]];
labels={{True,0.05,-1.6,1.8},{True,0.10},{Times,12,"Plain","Italic"}};
PlotSpaceTrussElementsAndNodes[nodxyzs,elenods,"mesh with labels",{view,-1,labels}];

```

plain mesh



mesh with labels



Cell 5. Plot modules that support postprocessing. These are subdivided into Cells 5A and 5C to keep them short.

Cell 5A. PlotSpaceTrussDeformedShape does exactly what it says. Note: argument amplif can be a scalar amplification, or a list of amplifications. If the latter, a sequence of colors is used.

In[61]:=

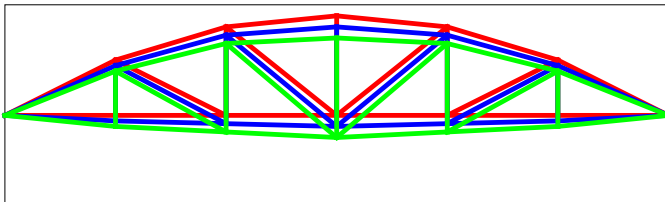
```

PlotSpaceTrussDeformedShape[nodxyz_,elenod_,noddiss_,
  amplif_,box_,title_,{view_,aspect_,colors_}]:=
Module[{numnod=Length[nodxyz],numele=Length[elenod],a={},c={},e,k,m,n,
  ni,nj,nf,Q,xbox,ybox,x,y,x0,y0,xmin,xmax,ymin,ymax,dx,dy,xyz,arat,
  black,red,green,blue,yellow,white,fill,gray,grayfill,f,pbars={},p={}},
  Q=WorldToScreenMatrix[view]; x=y=Table[0,{numnod}]; nf=Length[box];
  If [nf>0, xbox=ybox=Table[0,{nf}];
    For [n=1,n<=nf,n++, {xbox[n],ybox[n]}=Q.box[n] ];
    {xmin,xmax,ymin,ymax}=N[{Min[xbox],Max[xbox],Min[ybox],Max[ybox]}];
  If [nf==0,
    For [n=1,n<=numnod,n++, {x[n],y[n]}=Q.nodxyz[n] ];
    {xmin,xmax,ymin,ymax}=N[{Min[x],Max[x],Min[y],Max[y]}];
  {dx,dy}={xmax-xmin,ymax-ymin};
  {black,red,green,blue,yellow,white}={RGBColor[0,0,0],RGBColor[1,0,0],
    RGBColor[0,1,0],RGBColor[0,0,1],RGBColor[1,1,0],RGBColor[1,1,1]};
  {nofill,gray,fill}={GrayLevel[0],GrayLevel[.8],GrayLevel[1]};
  arat=AspectRatio->Automatic;
  If [aspect>0, arat=AspectRatio->aspect];
  If [aspect==0&&dx>0, arat=AspectRatio->dy/dx];
  AppendTo[p,Graphics[AbsoluteThickness[.5]]]; AppendTo[p,Graphics[nofill]];
  enclosingbox={ {xmin,ymin}, {xmax,ymax}, {xmin,ymax}, {xmin,ymin} };
  AppendTo[p,Graphics[Line[enclosingbox]]];
  AppendTo[p,Graphics[AbsoluteThickness[2.5]]];
  If [Length[amplif]==0,a={amplif},a=amplif];
  If [Length[colors]==0,c={colors},c=colors]; k=Length[c];
  AppendTo[p,Graphics[black]];
  For [m=1,m<=Length[a],m++, f=a[m]; color=c[[Min[m,k]]];
    If [color=="black", AppendTo[p,Graphics[black]]];
    If [color=="red", AppendTo[p,Graphics[red]]];
    If [color=="blue", AppendTo[p,Graphics[blue]]];
    If [color=="green", AppendTo[p,Graphics[green]]];
    If [color=="yellow", AppendTo[p,Graphics[yellow]]];
    If [color=="white", AppendTo[p,Graphics[white]]];
    For [n=1,n<=numnod,n++,
      {x[n],y[n]}=Q.(nodxyz[n]+f*noddiss[n]);
    For [e=1,e<=numele,e++, {ni,nj}=elenod[e];
      xyz={ {x[ni],y[ni]}, {x[nj],y[nj]} };
      AppendTo[p,Graphics[Line[xyz] ]]];
  ];
  Show[p, arat, PlotLabel->title, DisplayFunction->DisplayChannel ];
  ClearAll[x,y,xbox,ybox,p];
  ];

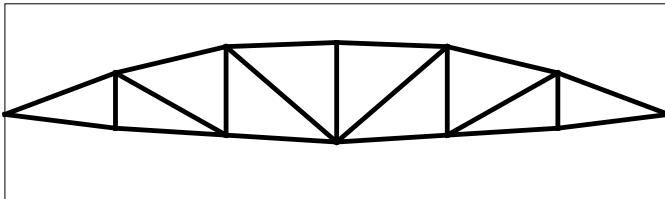
nodxyz={{0,0,0},{10,5,0},{10,0,0},{20,8,0},{20,0,0},{30,9,0},
  {30,0,0},{40,8,0},{40,0,0},{50,5,0},{50,0,0},{60,0,0}};
noddiss={{0,0,0},{0,-10,0},{0,-10,0},{0,-15,0},{0,-15,0},{0,-20,0},
  {0,-20,0},{0,-15,0},{0,-15,0},{0,-10,0},{0,-10,0},{0,0,0}}/20;
nodxyz=N[nodxyz];
elenod={{1,3},{3,5},{5,7},{7,9},{9,11},{11,12},
  {1,2},{2,4},{4,6},{6,8},{8,10},{10,12},
  {2,3},{4,5},{6,7},{8,9},{10,11},
  {2,5},{4,7},{7,8},{9,10}};
view={{0,1,0},{0,0,1}}; box={{0,-8,0},{60,-8,0},{60,10,0},{0,10,0}};
PlotSpaceTrussDeformedShape[nodxyz,elenod,noddiss,{0,1,2},
  box,"orig & 2 deformed shapes",{view,-1,{"red","blue","green"}}];
PlotSpaceTrussDeformedShape[nodxyz,elenod,noddiss,2.5,box,
  "deformed shape @ mag 2.5",{view,-1,"black"}];
PlotSpaceTrussDeformedShape[nodxyz,elenod,noddiss,4.0,box,
  "deformed shape @ mag 4",{view,-1,"black"}];

```

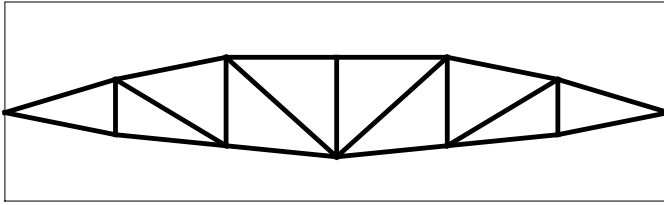
orig & 2 deformed shapes



deformed shape @ mag 2.5



deformed shape @ mag 4



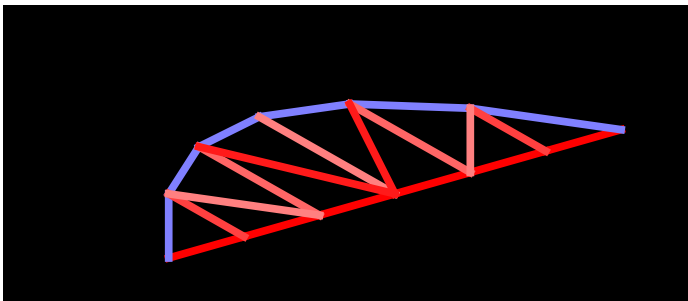
Cell 5B. Plot Axial Stress level displays stress level and sign using color: red for tension, blue for compression, white for zero stress. A black background is used.

In[72]:=

```
PlotSpaceTrussStresses[nodxyz_,elenod_,elesig_,sigfac_,box_,
  title_,{view_,aspect_,labels_}]:= Module[
{numele=Length[elenod],numnod=Length[nodxyz],
e,n,ni,nj,nc,nf,xyz,x,y,xbox,ybox,xmin,xmax,ymin,ymax,dx,dy,
smin,smax,fmax,sval,c1,c2,c3,arat,pbox={},pbars={}},
x=y=Table[0,{numnod}]; Q=WorldToScreenMatrix[view]; nf=Length[box];
For [n=1,n<=numnod,n++, {x[[n]],y[[n]]}=Q.nodxyz[[n]] ];
If [nf>0, xbox=ybox=Table[0,{nf}];
For [n=1,n<=nf,n++, {xbox[[n]],ybox[[n]]}=Q.box[[n]] ];
{xmin,xmax,ymin,ymax}=N[{Min[xbox],Max[xbox],Min[ybox],Max[ybox]}] ];
If [nf==0,
{xmin,xmax,ymin,ymax}=N[{Min[x],Max[x],Min[y],Max[y]}] ];
{dx,dy}={xmax-xmin,ymax-ymin};
AppendTo[pbox,Graphics[AbsoluteThickness[.5]]];
AppendTo[pbox,Graphics[GrayLevel[0]]];
enclosingbox={xmin,ymin},{xmax,ymin},{xmax,ymax},{xmin,ymax},{xmin,ymin}};
AppendTo[pbox,Graphics[Line[enclosingbox]]];
AppendTo[pbox,Graphics[AbsoluteThickness[4]]];
smin=Min[elesig]; smax=Max[elesig]; fmax=Max[Abs[smax],Abs[smin]];
For [e=1,e<=numele,e++,
sval=N[elesig[[e]]*sigfac]; {c1,c2,c3}=LineColor[sval,fmax];
{ni,nj}=elenod[[e]]; xyz={{x[[ni]],y[[ni]]},{x[[nj]],y[[nj]]}};
AppendTo[pbars,Graphics[RGBColor[c1,c2,c3]]];
AppendTo[pbars,Graphics[Line[xyz]]];
];
arat=AspectRatio->Automatic;
If [aspect>0, arat=AspectRatio->aspect];
If [aspect==0&&dx>0, arat=AspectRatio->dy/dx];
Show[pbox,Graphics[RGBColor[0,0,0],pbars,
Background->GrayLevel[0], arat, PlotLabel->title,
DisplayFunction->DisplayChannel ];
ClearAll[pbars,x,y];
];

LineColor[f_,fmax_]:= Module[{r,RGBmax={1,0,0},
RGBmin={0,0,1}, RGBzero={1,1,1}, RGBout={0,0,0}},
If [f==0 || fmax==0,
Return[RGBzero]]; (* White if f=0 *)
If [f>fmax || f<-fmax,
Return[RGBout]]; (* Black if outside range *)
If [f>0, r= N[f/fmax];
Return[r*RGBmax+(1-r)*RGBzero]]; (* positive *)
If [f<0, r=-N[f/fmax];
Return[r*RGBmin+(1-r)*RGBzero]]; (* negative *)
];

nodxyz={{0,0,0},{10,5,0},{10,0,0},{20,8,0},{20,0,0},{30,9,0},
{30,0,0},{40,8,0},{40,0,0},{50,5,0},{50,0,0},{60,0,0}};
elenod= {{1,3},{3,5},{5,7},{7,9},{9,11},{11,12},
{1,2},{2,4},{4,6},{6,8},{8,10},{10,12},
{2,3},{4,5},{6,7},{8,9},{10,11},
{2,5},{4,7},{7,8},{9,10}};
elesig={20,20,20,20,20,20,-10,-10,-10,-10,-10,-10,
15,12,10,12,15,10,18,18,10};
view={{2,1,0},{-3,-1,1}}; box={{0,-4,0},{60,-4,0},{60,10,0},{0,10,0}};
PlotSpaceTrussStresses[nodxyz,elenod,elesig,1,box,
"axial stress in truss members",{view,0,labels}];
```



Cell 6. Miscellaneous utilities supporting print output and some array transformations

In[81]:=

```
PrintSpaceTrussNodeCoordinates[nodxyz_,title_,digits_]:= Module[
{numnod=Length[nodxyz],n,x,y,z,d=6,f=6,tab},
tab=Table[0,{numnod}]; If [Length[digits]==2,{d,f}=digits];
For [n=1,n<=numnod,n++,{x,y,z}=nodxyz[[n]];
tab[[n]]={ToString[n],PaddedForm[x,{d,f}],PaddedForm[y,{d,f}],
PaddedForm[z,{d,f}]}];
If [StringLength[title]>0, Print[title]];
Print[TableForm[tab, TableAlignments->{Right},
TableDirections->{Column,Row},TableSpacing->{0,2},
TableHeadings ->{None,{"node", "x-coor", "y-coor", "z-coor"}}]];
];

PrintSpaceTrussElementData[elenod_,elemat_,elefab_,title_,digits_]:= Module[
{e,numele=Length[elenod],Em,A,d=6,f=3,tab},
tab=Table[0,{numele}]; If [Length[digits]==2,{d,f}=digits];
For [e=1,e<=numele,e++, Em=elemat[[e]]; A=elefab[[e]];
tab[[e]]={ToString[e],ToString[elenod[[e]]],
PaddedForm[Em,{d,f}],PaddedForm[A,{d,f}]}];
If [StringLength[title]>0, Print[title]];
Print[TableForm[tab, TableAlignments->{Right},
TableDirections->{Column,Row},TableSpacing->{0,2},
TableHeadings ->{None,{"elem", "nodes", "modulus", "area"}}]];
];

PrintSpaceTrussFreedomActivity[nodtag_,nodval_,title_,digits_]:= Module[
{numnod=Length[nodtag],n,tag,val,tab},
tab=Table[0,{numnod}]; If [Length[digits]==2,{d,f}=digits];
For [n=1,n<=numnod,n++, t=nodtag[[n]]; v=nodval[[n]];
tab[[n]]={ToString[n],
PaddedForm[t[[1]]],PaddedForm[t[[2]]],PaddedForm[t[[3]]],
PaddedForm[v[[1]]],PaddedForm[v[[2]]],PaddedForm[v[[3]]]}];
If [StringLength[title]>0, Print[title]];
Print[TableForm[tab,TableAlignments->{Right},
TableDirections->{Column,Row},TableSpacing->{0,1},
TableHeadings ->{None,{"node", "x-tag", "y-tag", "z-tag",
"x-value", "y-value", "z-value"}}]];
];

PrintSpaceTrussNodeDisplacements[noddis_,title_,digits_]:= Module[
{numnod=Length[noddis],n,ux,uy,uz,tab,d=6,f=6},
tab=Table[0,{numnod}]; If [Length[digits]==2,{d,f}=digits];
For [n=1,n<=numnod,n++, {ux,uy,uz}=noddis[[n]];
tab[[n]]={ToString[n],PaddedForm[ux,{d,f}],
PaddedForm[uy,{d,f}],PaddedForm[uz,{d,f}]}];
If [StringLength[title]>0, Print[title]];
Print[TableForm[tab,TableAlignments->{Right},
TableDirections->{Column,Row},TableSpacing->{0,1},
TableHeadings ->{None,{"node", "x-displ", "y-displ", "z-displ"}}]];
];

PrintSpaceTrussNodeForces[nodfor_,title_,digits_]:= Module[
{numnod=Length[nodfor],n,fx,fy,fz,tab,d=4,f=4},
tab=Table[0,{numnod}]; If [Length[digits]==2,{d,f}=digits];
For [n=1,n<=numnod,n++, {fx,fy,fz}=nodfor[[n]];
tab[[n]]={ToString[n],PaddedForm[fx,{d,f}],
PaddedForm[fy,{d,f}],PaddedForm[fz,{d,f}]}];
If [StringLength[title]>0, Print[title]];
Print[TableForm[tab,TableAlignments->{Right},
TableDirections->{Column,Row},TableSpacing->{0,1},
TableHeadings ->{None,{"node", "x-force", "y-force", "z-force"}}]];
];

PrintSpaceTrussElemForcesAndStresses[elefor_,elesig_,title_,digits_]:=
Module[{numele=Length[elefor],e,p,s,d=4,f=4,tab},
tab=Table[0,{numele}]; If [Length[digits]==2,{d,f}=digits];
For [e=1,e<=numele,e++, p=elefor[[e]]; s=elesig[[e]];
tab[[e]]={ToString[e],PaddedForm[p,{d,f}],PaddedForm[s,{d,f}]}];
If [StringLength[title]>0, Print[title]];
Print[TableForm[tab,TableAlignments->{Right},
TableDirections->{Column,Row},TableSpacing->{0,1},
TableHeadings ->{None,{"elem", "axial force", "axial stress"}}]];
];

FlatNodePartVector[nv_]:=Flatten[nv];

NodePartFlatVector[nfc_,v_]:= Module [
{i,k,m,n,nv={},numnod},
If [Length[nfc]==0, nv=Partition[v,nfc]];
If [Length[nfc]>0, numnod=Length[nfc]; m=0;
nv=Table[0,{numnod}];
For [n=1,n<=numnod,n++, k=nfc[[n]];
nv[[n]]=Table[v[[m+1]],{i,1,k}];
m+=k];
Return[nv]];
];
```

Cell 7. Solution driver module

In[96]:=

```

SpaceTrussSolution[nodxyz_,elenod_,elemat_,elefab_,nodtag_,nodval_,
prcopt_]:= Module[{K,Kmod,f,fmod,u,noddis,nodfor,elefor,esig},
K=SpaceTrussMasterStiffness[nodxyz,elenod,elemat,elefab,prcopt];
(* Print["size of K=",Chop[Eigenvalues[N[K]]]]; *)
Kmod=ModifiedMasterStiffness[nodtag,K];
f=FlatNodePartVector[nodval];
fmod=ModifiedNodeForces[nodtag,nodval,K,f];
(* Print["size of Kmod=",Chop[Eigenvalues[N[Kmod]]]]; *)
u=LinearSolve[Kmod,fmod]; u=Chop[u]; f=Chop[K.u, 10.0^(-8)];
nodfor=NodePartFlatVector[3,f]; noddis=NodePartFlatVector[3,u];
elefor=Chop[SpaceTrussIntForces[nodxyz,elenod,elemat,elefab,
noddis,prcopt]];
esig=SpaceTrussStresses[elefab,elefor,prcopt];
Return[{noddis,nodfor,elefor,esig}];
];

```

Cell 8. Driver program for bridge truss model

Cell 8A. Animation of deflections. Uses the displacements u computed by driver program to plot a sequence of deformed shapes with amplitude varying as sine of pseudotime t.

```

view={{0,1,0},{0,0,1}}; box={{0,-4,0},{65,-4,0},{65,10,0},{0,10,0}};
For [t=0.,t<=N[Pi],t=t+N[Pi/6], amp=Sin[t];
PlotSpaceTrussDeformedShape[NodeCoordinates,ElemNodes,
NodeDisplacements,amp,box,"deformed bridge animation",
{view,-1,"red"}]];

```

Cell 9. 25-member transmission tower driver script

Cell 10. Module to compute space truss weight as model check. Run only after model is defined in Cell 10

```

SpaceTrussWeight[nodxyz_,elenod_,elefab_,specw_]:=Module[
{e,ni,nj,w=0,x1,x2,y1,y2,z1,z2,x21,y21,z21,
numele=Length[elenod],LL},
For [e=1,e<=numele,e++, {ni,nj}=elenod[[e]];
ncoor={nodxyz[[ni]],nodxyz[[nj]]};
{{x1,y1,z1},{x2,y2,z2}}=ncoor;
{x21,y21,z21}={x2-x1,y2-y1,z2-z1};LL=x21^2+y21^2+z21^2;
w=w+Sqrt[LL]*elefab[[e]]*specw];
Return[w];

w=SpaceTrussWeight[NodeCoordinates,ElemNodes,ElemFabrications,0.1];
Print["weight=",w];

```

Homework Problem 1: Book 20.6

See written attachment

Homework Problem 2: Book 21.2

This logic of ModifiedMasterStiffness.nb and ModifiedNodeForces.nb is no restricted to just space trusses because the two modules only requires a DOF state vector and a forcing terms vector. The forcing terms are the conjugate term representing a generalizing force. An example of this is Heat conduction where the DOF state vector is the temperature at each node and the conjugate forcing term vector is the heat flux. You could send in a K matrix along with temperature and flux values and get out a FEM result for heat conduction.

Homework Problem 3: Book 21.5 - Super Tower

In[412]:=

```
NodeCoordinates={{-37.5,0,200}, (*1*)
                 {37.5,0,200}, (*2*)
                 {-37.5,37.5,100}, (*3*)
                 {37.5,37.5,100}, (*4*)
                 {37.5,-37.5,100}, (*5*)
                 {-37.5,-37.5,100}, (*6*)
                 {-100,100,0}, (*7*)
                 {100,100,0}, (*8*)
                 {100,-100,0}, (*9*)
                 {-100,-100,0} (*10*)
};
ElemNodes={ {1,2}, (*1*)
            {1,4}, (*2*)
            {2,3}, (*3*)
            {1,5}, (*4*)
            {2,6}, (*5*)
            {2,4}, (*6*)
            {2,5}, (*7*)
            {1,3}, (*8*)
            {1,6}, (*9*)
            {3,6}, (*10*)
            {4,5}, (*11*)
            {3,4}, (*12*)
            {5,6}, (*13*)
            {3,10}, (*14*)
            {6,7}, (*15*)
            {4,9}, (*16*)
            {5,8}, (*17*)
            {4,7}, (*18*)
            {3,8}, (*19*)
            {5,10}, (*20*)
            {6,9}, (*21*)
            {6,10}, (*22*)
            {3,7}, (*23*)
            {5,9}, (*24*)
            {4,8} (*25*)
};
PrintSpaceTrussNodeCoordinates[NodeCoordinates,"Node coordinates:",{}];
numnod=Length[NodeCoordinates]; numele=Length[ElemNodes];
Em=10^7;
ElemMaterials= Table[Em,{numele}];
ElemFabrications={ 0.033,
                  2.015,
                  2.015,
                  2.015,
                  2.015,
                  2.823,
                  2.823,
                  2.823,
                  2.823,
                  2.823,
```

```

0.010,
0.010,
0.014,
0.014,
0.980,
0.980,
0.980,
0.980,
1.760,
1.760,
1.760,
1.760,
2.440,
2.440,
2.440,
2.440];
PrintSpaceTrussElementData[ElemNodes,ElemMaterials,ElemFabrications,
  "Element data:",{}];
ProcessOptions= {True};

view={{0,0,1},{1,-3,1}};
labels={{True,0.014,-1.5,1.5},{True,0.020},{ "Times",12,"Roman"}};
PlotSpaceTrussElementsAndNodes[NodeCoordinates,ElemNodes,
  "tower mesh in perspective",{view,-1,labels}];

NodeDOFTags= Table[{0,0,0},{numnod}];
NodeDOFValues=Table[{0,0,0},{numnod}];
NodeDOFValues[[1]]={1000,10000,-5000};
NodeDOFValues[[2]]={0,10000,-5000};
NodeDOFValues[[3]]={500,0,0};
NodeDOFValues[[6]]={500,0,0};
NodeDOFTags[[7]]={1,1,1}; (* fixed node 7,8,9,10 *)
NodeDOFTags[[8]]={1,1,1};
NodeDOFTags[[9]]={1,1,1};
NodeDOFTags[[10]]={1,1,1};

PrintSpaceTrussFreedomActivity[NodeDOFTags,NodeDOFValues,
  "DOF Activity:",{}];

{NodeDisplacements,NodeForces,ElemForces,ElemStresses}=
  SpaceTrussSolution[ NodeCoordinates,ElemNodes,ElemMaterials,
    ElemFabrications,NodeDOFTags,NodeDOFValues,ProcessOptions ];

PrintSpaceTrussNodeDisplacements[NodeDisplacements,
  "Computed node displacements:",{}];
PrintSpaceTrussNodeForces[NodeForces,
  "Node forces including reactions:",{}];
PrintSpaceTrussElemForcesAndStresses[ElemForces,ElemStresses,
  "Int Forces and Stresses:",{}];

view={{0,0,1},{2,-3,1}};
box={{-100,100,0},{100,100,0},{100,-100,0},{-100,-100,0},
  {-100,100,200},{100,100,200},{100,-100,200},{-100,-100,200}};
PlotSpaceTrussDeformedShape[NodeCoordinates,ElemNodes,NodeDisplacements,
  {0,50},box,"deformed shape (mag x250",{view,-1,{"black","red"}}];
PlotSpaceTrussStresses[NodeCoordinates,ElemNodes,ElemStresses,1,box,
  "axial stresses in truss members",{view,0,labels}];

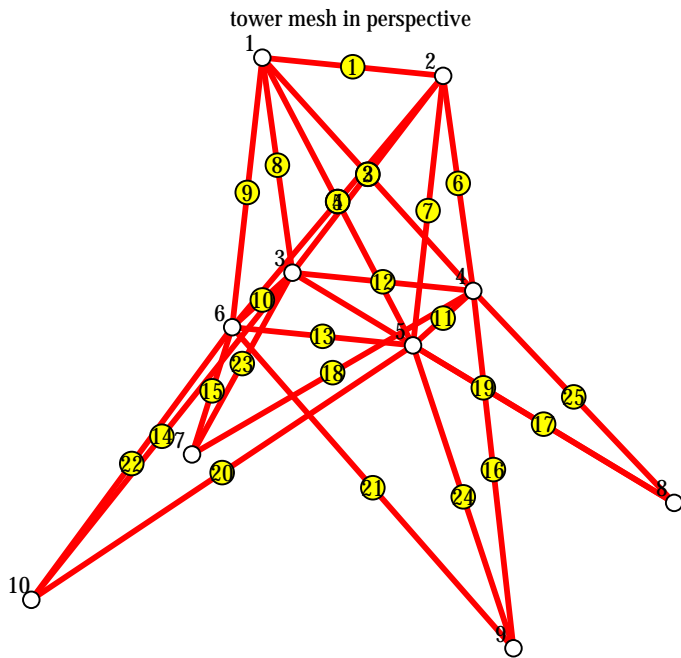
```

Node coordinates:

node	x-coor	y-coor	z-coor
1	-37.500000	0.000000	200.000000
2	37.500000	0.000000	200.000000
3	-37.500000	37.500000	100.000000
4	37.500000	37.500000	100.000000
5	37.500000	-37.500000	100.000000
6	-37.500000	-37.500000	100.000000
7	-100.000000	100.000000	0.000000
8	100.000000	100.000000	0.000000
9	100.000000	-100.000000	0.000000
10	-100.000000	-100.000000	0.000000

Element data:

elem	nodes	modulus	area
1	{1, 2}	10000000.000	0.033
2	{1, 4}	10000000.000	2.015
3	{2, 3}	10000000.000	2.015
4	{1, 5}	10000000.000	2.015
5	{2, 6}	10000000.000	2.015
6	{2, 4}	10000000.000	2.823
7	{2, 5}	10000000.000	2.823
8	{1, 3}	10000000.000	2.823
9	{1, 6}	10000000.000	2.823
10	{3, 6}	10000000.000	0.010
11	{4, 5}	10000000.000	0.010
12	{3, 4}	10000000.000	0.014
13	{5, 6}	10000000.000	0.014
14	{3, 10}	10000000.000	0.980
15	{6, 7}	10000000.000	0.980
16	{4, 9}	10000000.000	0.980
17	{5, 8}	10000000.000	0.980
18	{4, 7}	10000000.000	1.760
19	{3, 8}	10000000.000	1.760
20	{5, 10}	10000000.000	1.760
21	{6, 9}	10000000.000	1.760
22	{6, 10}	10000000.000	2.440
23	{3, 7}	10000000.000	2.440
24	{5, 9}	10000000.000	2.440
25	{4, 8}	10000000.000	2.440



DOF Activity:

node	x-tag	y-tag	z-tag	x-value	y-value	z-value
1	0	0	0	1000	10000	-5000
2	0	0	0	0	10000	-5000
3	0	0	0	500	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	500	0	0
7	1	1	1	0	0	0
8	1	1	1	0	0	0
9	1	1	1	0	0	0
10	1	1	1	0	0	0

Computed node displacements:

node	x-displ	y-displ	z-displ
1	0.008515	0.349956	-0.022128
2	0.031916	0.349956	-0.032242
3	0.011530	-0.009770	-0.108526
4	-0.004039	-0.008781	-0.115393
5	0.000448	-0.005089	0.070508
6	0.007042	-0.004100	0.077375
7	0.000000	0.000000	0.000000
8	0.000000	0.000000	0.000000
9	0.000000	0.000000	0.000000
10	0.000000	0.000000	0.000000

Node forces including reactions:

PaddedForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. >>

PaddedForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. >>

PaddedForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. >>

General::stop : Further output of PaddedForm::sigz will be suppressed during this calculation. >>

node	x-force	y-force	z-force
1	1000.0000	10000.0000	-5000.0000
2	0.0000	10000.0000	-5000.0000
3	500.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000
6	500.0000	0.0000	0.0000
7	9908.0000	-6263.0000	11750.0000
8	-10910.0000	-7326.0000	13250.0000
9	5665.0000	-2674.0000	-6750.0000
10	-6665.0000	-3737.0000	-8250.0000

Int Forces and Stresses:

PaddedForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. >>

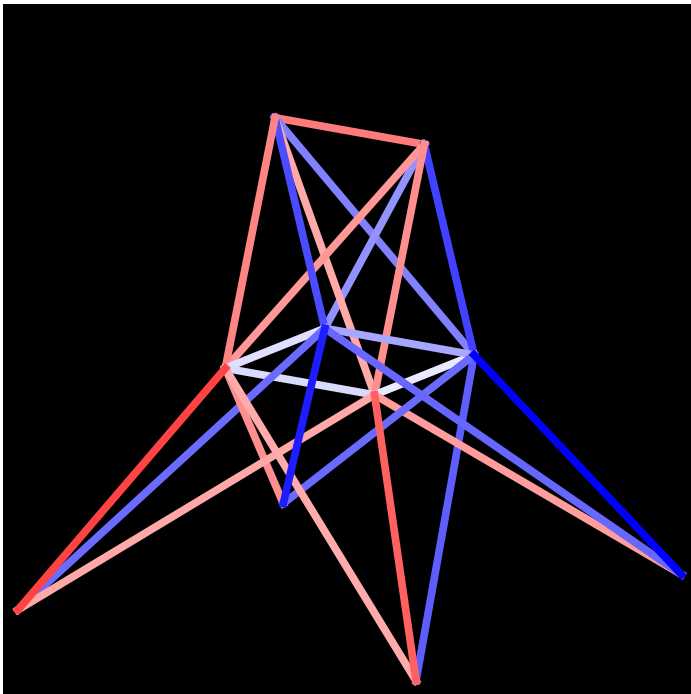
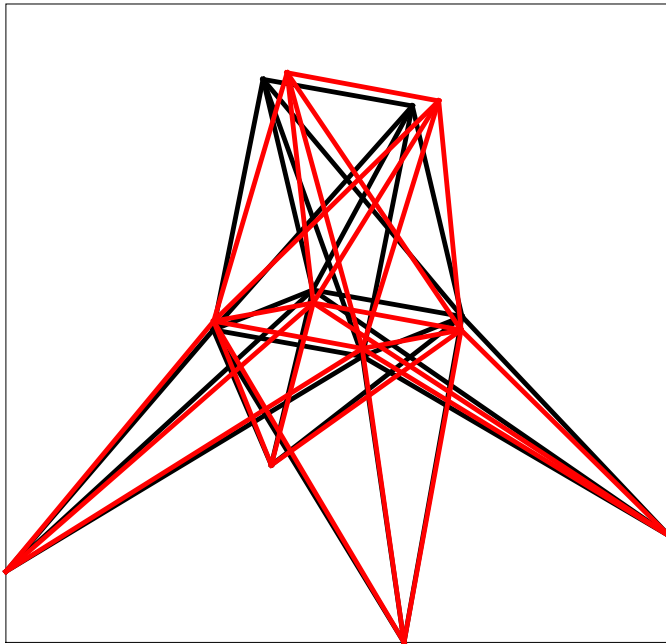
PaddedForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. >>

PaddedForm::sigz : In addition to the number of digits requested, one or more zeros will appear as placeholders. >>

General::stop : Further output of PaddedForm::sigz will be suppressed during this calculation. >>

elem	axial force	axial stress
1	103.0000	3120.0000
2	-5996.0000	-2976.0000
3	-5126.0000	-2544.0000
4	4077.0000	2023.0000
5	4947.0000	2455.0000
6	-12720.0000	-4504.0000
7	7522.0000	2665.0000
8	-12000.0000	-4252.0000
9	8234.0000	2917.0000
10	-7.5590	-755.9000
11	-4.9230	-492.3000
12	-29.0600	-2076.0000
13	-12.3100	-879.3000
14	-3427.0000	-3497.0000
15	2611.0000	2664.0000
16	-3732.0000	-3808.0000
17	2306.0000	2354.0000
18	-6193.0000	-3519.0000
19	-6344.0000	-3605.0000
20	3644.0000	2071.0000
21	3493.0000	1985.0000
22	10850.0000	4447.0000
23	-13040.0000	-5345.0000
24	9184.0000	3764.0000
25	-14710.0000	-6028.0000

deformed shape (mag x250)



In[449]:=

```

SpaceTrussWeight[nodxyz_,elenod_,elefab_,specw_]:=Module[
{e,ni,nj,w=0,x1,x2,y1,y2,z1,z2,x21,y21,z21,
 numele=Length[elenod],LL},
  For [e=1,e<=numele,e++, {ni,nj}=elenod[[e]],
    ncoor={nodxyz[[ni]],nodxyz[[nj]]};
    {{x1,y1,z1},{x2,y2,z2}}=ncoor;
    {x21,y21,z21}={x2-x1,y2-y1,z2-z1};LL=x21^2+y21^2+z21^2;
    w=w+Sqrt[LL]*elefab[[e]]*specw];
  Return[w];

w=SpaceTrussWeight[NodeCoordinates,ElemNodes,ElemFabrications,0.1];
Print["weight=",w];

```

weight=555.184