ASEN 5070
Statistical Orbit Determination

Homework #5 Solutions

1.

```
i = 1 t = 0

x_true = 1.0000000000 0.0000000000 0.0000000000 1.0000000000

x_star = 0.9999990000 0.0000010000 -0.0000010000 0.9999990000

phi(t(0,0)
1.0000000000 0.0000000000 0.0000000000 0.0000000000
0.0000000000 1.0000000000 0.0000000000 0.0000000000
0.0000000000 0.0000000000 1.0000000000 0.0000000000
0.0000000000 0.0000000000 0.0000000000 1.0000000000

x_true_minus_x_star = 0.0000010000 -0.0000010000 0.0000010000 0.0000010000

phi_delx0 = 0.0000010000 -0.0000010000 0.0000010000 0.0000010000

difference = 0.0000000000 0.0000000000 0.0000000000 0.0000000000



i = 2 t = 10

x_true = -0.8390715275 -0.5440211133 0.5440211128 -0.8390715278

x_star = -0.8390310980 -0.5440714865 0.5440761204 -0.8390412442

phi(t(10,0)
-19.2963174688 -1.0005919538 -1.5446240957 -20.5922746748
 24.5395368966  2.5430400382  3.3820224397  24.9959638257
-26.6284485781 -1.2470410817 -2.0860289950 -27.5413748301
-15.0754226446 -1.4570972855 -2.0011442071 -14.6674122486

x_true_minus_x_star = -0.0000404295 0.0000503731 -0.0000550076 -0.0000302836

phi_delx0 = -0.0000404326 0.0000503745 -0.0000550088 -0.0000302869

difference = -0.0000000032 0.0000000013 -0.0000000012 -0.0000000033


i = 10 t = 100

x_true = 0.862318872290 -0.506365641106 0.506365641106 0.862318872290

x_star = 0.862623359658 -0.505843963213 0.505845689224 0.862623303043

phi(t(100,0)
-151.284032324145 -0.069643346052 -0.575183991319 -152.539455286171
-260.234514432991 0.881235606615 0.019132289455 -260.670088445794
259.154447540095 0.374643452777 1.236748437099 260.026380251548
-152.127910762943 0.366712857375 -0.138829570269 -151.639213161182

x_true_minus_x_star = -0.000304487368 -0.000521677893 0.000519951882 -0.000304430753

phi_delx0 = -0.000304329028 -0.000521766706 0.000520042933 -0.000304272666

difference = 0.000000158340 -0.000000088813 0.000000091051 0.000000158087
```

2.

$$\hat{x} = (H^T W H + \bar{W})^{-1}(H^T W y + \bar{W}\bar{x})$$

$$= \left[\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 2 \right]^{-1} \times \left[\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 4 \right] = \frac{1}{6}(9) = \frac{3}{2}$$

$$\hat{\epsilon} = y - H\hat{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \left(\frac{3}{2} \right) = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

3. Answers given in the book (after FOUR iterations, not three).