ASEN 5007-Homework 8

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Helpful Modules

In[1]:=
```
PrintWithStyle[x_] :=
 Module[{color = LightGreen}, Framed[Style[x, 18, Bold, Background → color],
   Background → color]
 ]
```

Cell 7: Simple function to print output for solutions in a stylazed way

In[2]:=
```
PrintWithStyleMat[x_] := Module[{color = LightGreen}, Style[x,
   Background → color]
 ]
```

The following modules compute the stiffness matrix, consistent node body forces,
and corner stresses of the 4-node bilinear iso-P quad in plane stress. For Exercises
in Chapter 17 only the stiffness module is necessary.

Compute element stiffness matrix of 4-node bilinear quadrilateral

Problem 1-Book 19.2

Show that the minimum α's (minimal in absolute value sense) for which J = d$\bar{x}$/dξ vanishes at a point in the element are ±1/4 (the quarter-points)

In[716]:=
```
ClearAll[ξ, N1, N2, N3, x1, x2, x3, xx, NN, xbar, J]
N1[ξ_] := -1 / 2 * ξ * (1 - ξ);   N2[ξ_] := 1 / 2 * ξ * (1 + ξ);   N3[ξ_] := (1 - ξ^2);
x1 = 0;   x2 = L;   x3 = 1 / 2 * L + α * L;
NN[ξ_] := {N1[ξ], N2[ξ], N3[ξ]};
xx = {x1, x2, x3};
xbar[ξ_] := xx.NN[ξ]
J[ξ_] = D[xbar[ξ], ξ];
Print["J = ", J[ξ]]
soln1 = Solve[J[-1] == 0, α];
soln2 = Solve[J[1] == 0, α];
PrintWithStyle[
  "The points for which J vanishes within the isoparametric straight 3-node bar
    element can be found at: "] PrintWithStyle[soln1[[1, 1]]]
 PrintWithStyle[soln2[[1, 1]]]
PrintWithStyle["This is the quarter point for the 3 node bar element"]
```

$$J = \frac{L\,\xi}{2} - 2\left(\frac{L}{2} + L\,\alpha\right)\xi + \frac{1}{2}L\,(1+\xi)$$

Out[726]= **The points for which J vanishes within the isoparametric straight 3-node bar element can be found at:**

$\alpha \to -\frac{1}{4}$ $\alpha \to \frac{1}{4}$

Out[727]= **This is the quarter point for the 3 node bar element**

In[2695]:=

Problem 2-Book 19.3

Using 19.7, find the minimal rank-preserving Gauss integration rules with *p* points in the Longitudnal direction if the number of node points is n=2,3, or 4 for a 1 dimensional bar-like element

In[1363]:=
```
ClearAll[n, nE, nF, nG, nR, gmin]
(*Number of Nodes to consider for the bar like element*)
n = {2, 3, 4};

(*nR: Number of independant rigid body nodes*)
nR = 1;

(*For bar-like element, [E] matrix is 1x1. So nE=1 (rank of E) *) (*???*)
nE = 1;
gmin = {};
For[ii = 1, ii <= Length[n], ii++,
 (*nF: Element degrees of freedom*)
 nF = n[[ii]];
 tmp = {(nF - nR) / nE};
 (*gmin[[ii]]=(nF-nR)/nE;*)
 (*gmin=Join[gmin,tmp];*)
 gmin = Join[gmin, {Reduce[nE * nG >= (nF - nR), nG]}];
]
PrintWithStyle[
 "The minimum number of Gauss integration points for a bar element with nodes: "]
PrintWithStyle["n="] PrintWithStyle[n // MatrixForm]
 PrintWithStyle[gmin // MatrixForm]
```

Out[1369]= **The minimum number of Gauss integration points for a bar element with nodes:**

Out[1370]= **n=** $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $\begin{pmatrix} nG \geq 1 \\ nG \geq 2 \\ nG \geq 3 \end{pmatrix}$

Problem 3-Book 19.4

Perform the same analysis as above, but now considering a 3 dimensional brick element with n nodes and 3 degrees of freedom. Now gaussian points are uniform in 3 dimensions, so [nG x nG x nG]. Peforrm for 4 nodes listed below.

In[2682]:=
```
ClearAll[n, nE, nF, nG, nR, gmin]
(*Number of Nodes to consider for the bar like element*)
n = {8, 20, 27, 64};

dof = 3;

(*nR: Number of independant rigid body nodes*)
nR = 6;

(*For bar-like element, [E] matrix is 1x1. So nE=1 (rank of E) *) (*???*)
nE = 6;
gmin = {};
For[ii = 1, ii <= Length[n], ii++,
  (*nF: Element degrees of freedom*)
  nF = n[[ii]] * dof;
  tmp = {(nF - nR) / nE};
  (*gmin[[ii]]=(nF-nR)/nE;*)
  (*gmin=Join[gmin,tmp];*)
  gmin = Join[gmin, {Reduce[nE * nG >= (nF - nR), nG]}]];

(*Adjust for cubic gauss point schemas*)
nposs = {1, 2, 3, 4, 5, 6, 7, 8, 9}^3;
gvals = gmin[[All, 2]]; (*Extract values from inequalities*)
(*gvals[[2]]*)
newGmin = {};
For [ii = 1, ii <= Length[gvals], ii++,
 diff = gvals[[ii]] - nposs;
 tmp = If[Count[diff, 0] == 1, gvals[[ii]], Count[diff, _?Positive] + 1];
 newGmin = Join[newGmin, {tmp }];
]

PrintWithStyle["For the 3d brick with n nodes and 3 dof,
   the gaussian rule minimum points are chosen as shown below:"]
PrintWithStyle["n="] PrintWithStyle[n // MatrixForm]
Print["Gauss points =", newGmin // MatrixForm,
 "x", newGmin // MatrixForm, "x", newGmin // MatrixForm]
```

Out[2692]= **For the 3d brick with n nodes and 3 dof, the gaussian rule minimum points are chosen as shown below:**

Out[2693]= **n=** $\begin{pmatrix} 8 \\ 20 \\ 27 \\ 64 \end{pmatrix}$

Gauss points = $\begin{pmatrix} 2 \\ 3 \\ 3 \\ 4 \end{pmatrix}$ x $\begin{pmatrix} 2 \\ 3 \\ 3 \\ 4 \end{pmatrix}$ x $\begin{pmatrix} 2 \\ 3 \\ 3 \\ 4 \end{pmatrix}$