ASEN 5070:
Statistical Orbit Determination I

Homework Set #8

1. Beginning with Equation 4.7.20 from the text and the *a priori* information given in the text, write a program (Matlab, Python, etc) to compute the following:

   1-a) The exact value of P2 using Equation 4.7.24.

   1-b) P2 using a conventional Kalman algorithm.

   1-c) P2 using the Joseph formulation.

   1-d) P2 using the Potter algorithm (Equation 5.7.17).

   1-e) P2 using the Batch processor.

Problem 1 does not require any written response, but be sure to include your code in your submission (D2L accepts .txt, .rtf, .doc, and .pdf formats, but not .m, .py, .c, or any other useful formats). You may copy and paste the code into the same file or upload many files. We expect your code to have at least a minimal header stating that you wrote it, etc., as well as a few comments throughout making it clear the purpose for each part of the code. (points will be awarded accordingly).

2. Plot the trace of the exact value of P2 minus the trace of P2 vs ε for each of the following:

   2-a) Conventional Kalman

   2-b) Joseph formulation

   2-c) Potter algorithm

   2-d) Batch processor

Generate the plot for ε values ranging from $1\text{x}10^{-6}$ to $1\text{x}10^{-15}$ in logarithmic steps ($1\text{x}10^{-6}$, $1\text{x}10^{-7}$,…,$1\text{x}10^{-15}$ – include smaller steps if you have interest). The plots will most likely be more informative if you plot the logarithm of the absolute value of the difference. Explore Matlab's "semilogx" and "loglog" plotting functions. However you plot them, make sure you make it clear what is being plotted! Full credit will be awarded to submissions that include at least a brief description as well as clear plots that are well labeled. It's not necessary to include the code for the plots.

3. Compare and contrast the behavior and stability of the various filters. Which was the most accurate for the smallest ε values?

4. Assume that two observations, Y_1 and Y_2, are made of two constants X_1 and X_2. Observations are of the form

$$Y = \begin{bmatrix} X_1 + 2\epsilon X_2 \\ X_1 + 3\epsilon X_2 \end{bmatrix} \tag{1}$$

where ϵ is a constant such that $0 < \epsilon \ll 1$ and

$$1 + \epsilon \neq 1$$
$$1 + \epsilon^2 = 1$$

when limited by the finite precision of a computer. Observations have zero mean and unit variance. The constants do not change with time, so $\Phi(t, t_0) = I$. The *a priori* estimated state vector is

$$\bar{X} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

with covariance $\bar{P}_0 = \sigma^2 I$ where $\sigma^2 = 1/\epsilon^2$ and I is the 2x2 identity matrix. The true state is

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}. \tag{2}$$

4-a) Derive the true covariance (P) in terms of ϵ using the batch equation (inverse of Equation 4.6.7).

4-b) Estimate the state using a variety of algorithms as described in Problems 1-3 above for many values of ϵ, ranging from 10^{-6} to 10^{-15}. You may set $\epsilon = 10^{-6}, 10^{-7}, \ldots, 10^{-15}$ or use more values to your heart's desire. Use two perfect observations computed from Eq. 1 using the true state given in Eq. 2. Turn in the code that performs these estimates:

- Estimate the state using the batch processor for all values of ϵ.
- Estimate the state using the conventional Kalman filter for all values of ϵ. Process the observations one at a time.
- Estimate the state using the conventional Kalman filter with the Joseph formulation for all values of ϵ. Process the observations one at a time.
- Estimate the state using the Potter algorithm for all values of ϵ. Process the observations one at a time.

4-c) Plot the estimated state errors for all filters in 4-b as functions of ϵ.

4-d) Compare and contrast the performance of all filters, especially for small ϵ.