Spacecraft Dynamics and Control – ASEN 5010

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University of Colorado
Boulder

ASEN 5010
Spacecraft Dynamics and Control
Spring 2013

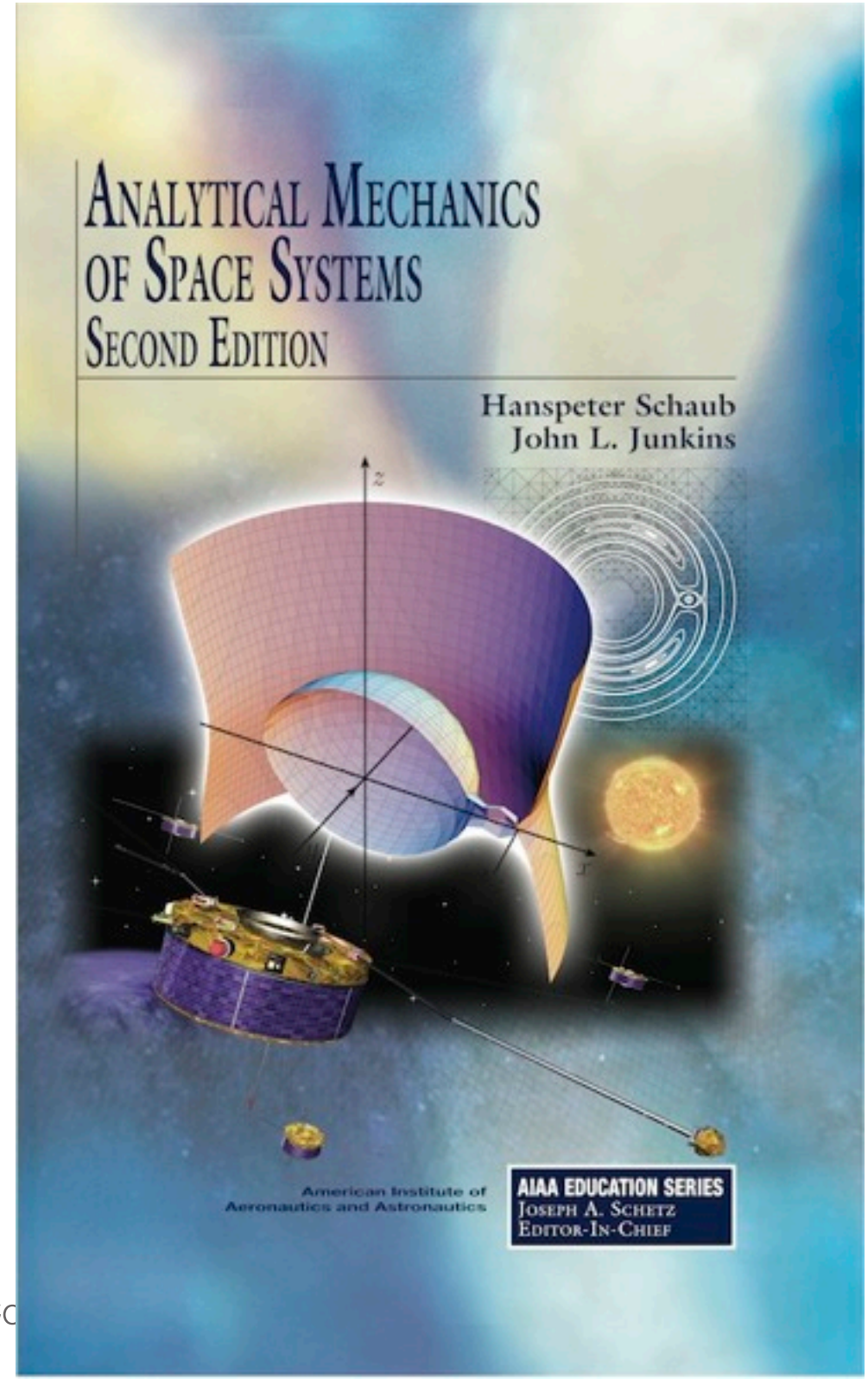
Instructor: Dr. Hanspeter Schaub, Office: ECNT 321, Phone: (303) 492-2767, Email: hanspeter.schaub@colorado.edu

Lectures: TR 8-9:15am, ECCS 1B28

Office Hours: M 1:30-3:00pm, W 9:00-10:00am (or by appointment)

Text: H. Schaub and J. L. Junkins, *Analytical Mechanics of Space Systems*, AIAA Education Series, 2nd Edition, 2009. (please download the errata sheet from the web page `http://hanspeterschaub.info/books.html`
Course notes supplied on the class blackboard web site.

Course Web Page: `http://learn.colorado.edu`



Discounts available from the AIAA online store for AIAA members, even student member!

Overview: Studies the rotational motion of spacecrafts, including attitude parameters and spacecraft torques. Applies Euler equations to the attitude motions of simple spacecrafts and their stability. Pre: ASEN 3200 or equivalent, or permission of instructor. (3H, 3C)

Goal: To introduce students to the spacecraft attitude dynamics, kinematics, as well as control.

Homework Policy: Each homework assignment is due on the specified due date and must be turned in at the beginning of the lecture. Normally, late homework will not be accepted. Some homework will require simple programs to be created. These can be done in Matlab, Maple, Mathematica, C, or Fortran. See instructor if not sure about the software package being used. If a homework has been graded incorrectly, you need to see me within 2 weeks of having the homework returned to you.

Exams: There will be a mid-term exam and one comprehensive final exam. If you have exam grading issues, you must see me within 2 weeks of having the exam returned to you. There will also be one course project which will require you to write a technical report. These reports must be type written and composed as a professional technical report.

Class Attendance: You are expected to attend class. If you need to miss a lecture, it is your responsibility to catch up on the material. Don't go to the instructor to catch up on missed material, speak with class mates and get the notes from them. Campus policy regarding religious observances requires that faculty make every effort to reasonably and fairly deal with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. If you cannot attend a regularly scheduled class, it is up to the student to catch up on the missed material. If you cannot take an exam on a particular day, please let the instructor know at the time the exam is being scheduled.

Make-Up Policy: There are no make-up homework assignments. If you miss the assignment, you get a zero for it. If you can't make an exam or a pressing reason, you need to contact the instructor *one week prior* to the exam date. If you can't take the exam for some emergency reason, you still need to notify the instructor prior to the exam. Without prior consent, there will be no make-up exams.

Grading Policy: A conventional ten-point system will be used for grading. If I feel it necessary, I will curve the exam scores to reflect the difficulty level of the problems assigned. Thus, your final assigned scores on each set of papers is your true grade and should be interpreted on a 100 point scale (i.e. A(90-100), B(80-89), C(70-79), D(60-69), F(below 60)). I will assign "+" and "-" grades at my discretion The exam with your *highest* score will be weighted with an additional 5%. The percent worth of exams and class assignments are:

Homework/Quizzes – 20%
Project – 25%
Mid-Term – 25%
Final Exam – 25%
Mystery Points – 5%

Honor Code: All students of the University of Colorado at Boulder are responsible for knowing and adhering to the academic integrity policy of this institution. Violations of this policy may include: cheating, plagiarism, aid of academic dishonesty, fabrication, lying, bribery, and threatening behavior. All incidents of academic misconduct shall be reported to the Honor Code Council (honor@colorado.edu; 303-725-2273). Students who are found to be in violation of the academic integrity policy will be subject to both academic sanctions from the faculty member and non-academic sanctions (including but not limited to university probation, suspension, or expulsion). Other information on the Honor Code can be found at
http://www.colorado.edu/policies/honor.html
and at
http://www.colorado.edu/academics/honorcode/

Students with Disabilities If you qualify for accommodations because of a disability, please submit to me a letter from Disability Services in a timely manner so that your needs may be addressed. Disability Services determines accommodations based on documented disabilities. Contact: 303-492-8671, Willard 322, and
http://www.Colorado.EDU/disabilityservices

Disability Services' letters for students with disabilities indicate legally mandated reasonable accommodations. The syllabus statements and answers to Frequently Asked Questions can be found at
http://www.colorado.edu/disabilityservices

Class Room Behavior Students and faculty each have responsibility for maintaining an appropriate learning environment. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with differences of race, culture, religion, politics, sexual orientation, gender, gender variance, and nationalities. Class rosters are provided to the instructor with the student's legal name. I will gladly honor your request to address you by an alternate name or gender pronoun. Please advise me of this preference early in the semester so that I may make appropriate changes to my records. See polices at
http://www.colorado.edu/policies/classbehavior.html
and at
http://www.colorado.edu/studentaffairs/judicialaffairs/code.html#student_code

Religious Observances Campus policy regarding religious observances requires that faculty make every effort to reasonably and fairly deal with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. In this class I will attempt to accommodate any conflicts with exam times if you let me know at least 2 weeks in advance See full details at
http://www.colorado.edu/policies/fac_relig.html

Discrimination and Harassment The University of Colorado at Boulder policy on Discrimination and Harassment, the University of Colorado policy on Sexual Harassment and the University of Colorado policy on Amorous Relationships apply to all students, staff and faculty. Any student, staff or faculty member who believes s/he has been the subject of discrimination or harassment based upon race, color, national origin, sex, age, disability, religion, sexual orientation, or veteran status should contact the Office of Discrimination and Harassment (ODH) at 303-492-2127 or the Office of Judicial Affairs at 303-492-5550. Information about the ODH, the above referenced policies and the campus resources available to assist individuals regarding discrimination or harassment can be obtained at
http://www.colorado.edu/odh

Estimate of Topics Covered

Introduction Review of vector notation, Vector Differentiation, Euler angles

Spacecraft attitude coordinate choices direction cosine matrix, Euler parameters, modified and classical Rodrigues parameters

Spacecraft equations of motion Use momentum and energy equations for rigid bodies

Linear and nonlinear attitude control of rigid bodies Learn how to exploit attitude coordinate descriptions to create regulator and tracking feedback control laws.

Momentum exchange devices Develop equations of motion of satellite with multiple VSCMGs.

Particle Kinematics

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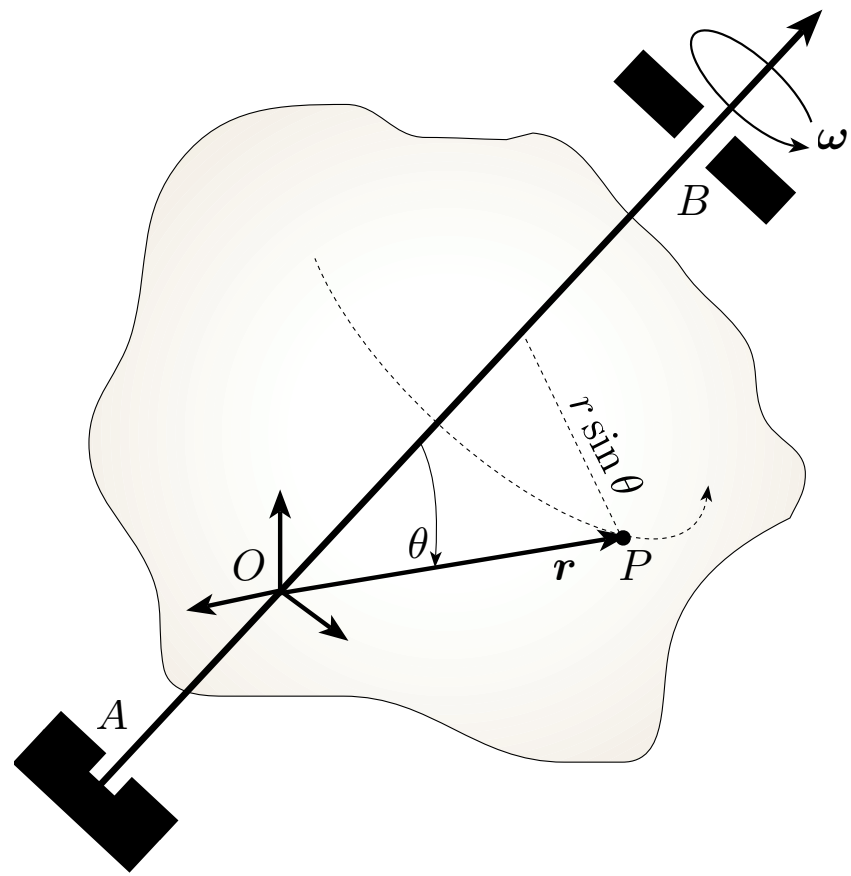
Outline

- Vector Notation
- Vector Differentiation
- Lots of brushing up on this material on your own!



Vector Notation

Hopefully a boring topic for you by now...

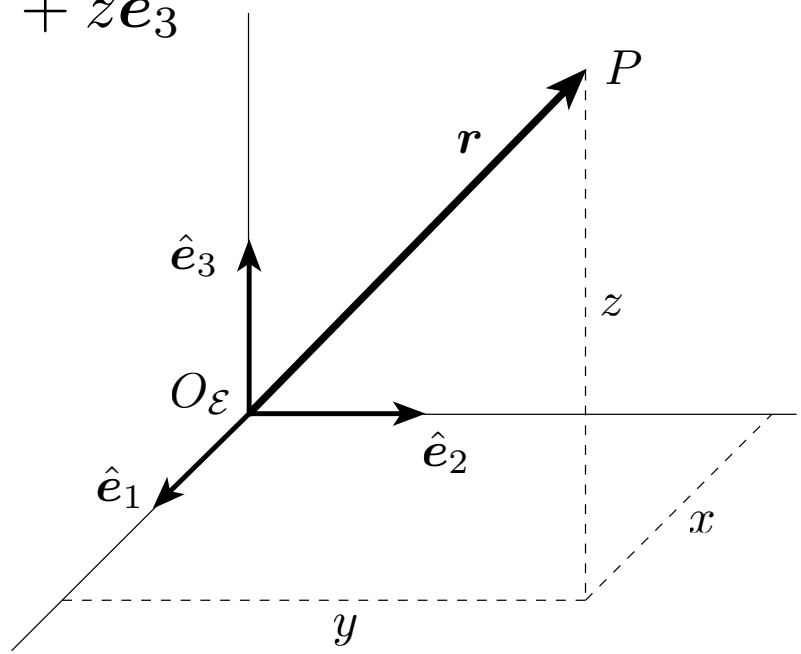
What is a vector?

- Something with a direction and magnitude.
- A vector can be written as

$$\boldsymbol{r} = x\hat{\boldsymbol{e}}_1 + y\hat{\boldsymbol{e}}_2 + z\hat{\boldsymbol{e}}_3$$

$$= r\hat{\boldsymbol{e}}_r$$

$$= \mathcal{E}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



Vector Addition

$$\boldsymbol{q} = \boldsymbol{r} + \boldsymbol{p} \quad \Rightarrow \quad \text{True}$$

$$\mathcal{E}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \mathcal{E}\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} + \mathcal{B}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \Rightarrow \begin{aligned} q_1 &= r_1 + p_1 \\ q_2 &= r_2 + p_2 \\ q_3 &= r_3 + p_3 \end{aligned} \quad \Rightarrow \quad \text{False}$$

$$^{\mathcal{E}}\boldsymbol{q} = {}^{\mathcal{E}}\boldsymbol{r} + {}^{\mathcal{B}}\boldsymbol{p} \quad \Rightarrow \quad \text{False}$$

Coordinate frame

- Let a coordinate frame *B* be defined through the three unit orthogonal vectors:

$$\hat{\boldsymbol{b}}_1 \qquad \hat{\boldsymbol{b}}_2 \qquad \hat{\boldsymbol{b}}_3$$

- Let the origin of this frame be given by $\mathcal{O}_{\mathcal{B}}$

- The frame is then defined through $\mathcal{B} : \{\mathcal{O}_{\mathcal{B}}, \hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$

- If we can ignore the frame origin, then we often use the shorthand notation

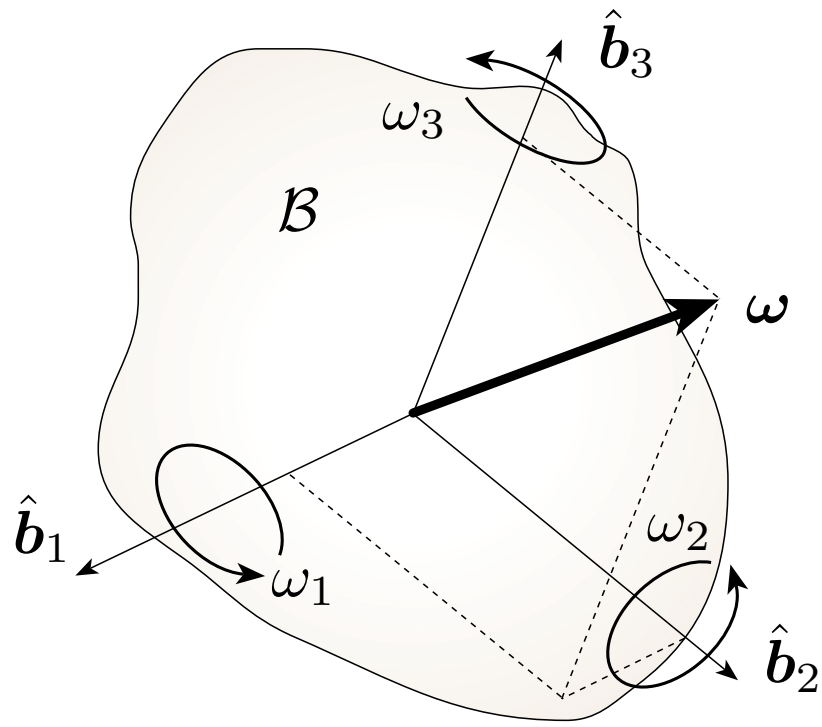
$$\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$$

Angular Velocity Vector

- Angular velocity vector can be expressed as

$$\boldsymbol{\omega} = \omega_1 \hat{\boldsymbol{b}}_1 + \omega_2 \hat{\boldsymbol{b}}_2 + \omega_3 \hat{\boldsymbol{b}}_3$$

$$^{\mathcal{B}}\boldsymbol{\omega} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}^{\mathcal{B}}$$



- ω_i are instantaneous body rates about the orthogonal $\hat{\boldsymbol{b}}_i$ axes.

Vector Differentiation

A crucial ability for attitude dynamics research...

Fixed Axis Rotation

- The rigid body is rotating about a fixed axis.
- The speed of P is given by

$$|\dot{\boldsymbol{r}}| = (r \sin\theta)\,\omega$$

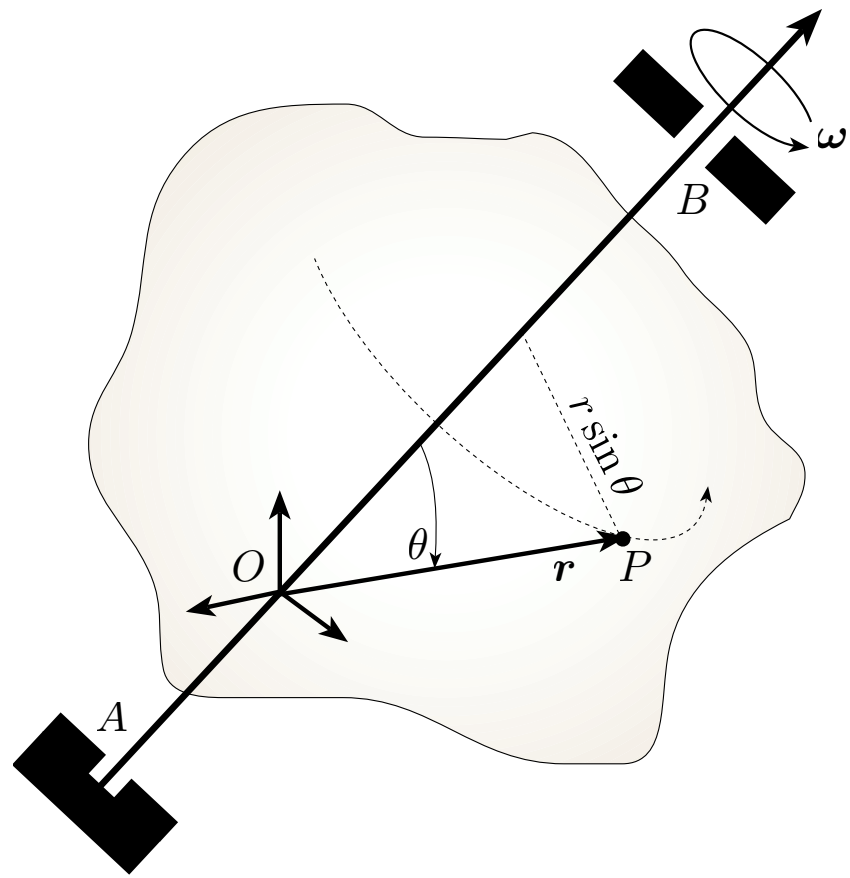
$$\dot{\boldsymbol{r}} = (r \sin\theta)\,\omega \left(\frac{\boldsymbol{\omega} \times \boldsymbol{r}}{|\boldsymbol{\omega} \times \boldsymbol{r}|} \right)$$

- note that

$$|\boldsymbol{\omega} \times \boldsymbol{r}| = \omega r \sin\theta$$

- thus the transport velocity is

$$\boxed{\dot{\boldsymbol{r}} = \boldsymbol{\omega} \times \boldsymbol{r}}$$



Transport Theorem

- Let a position vector be written as

$$\boldsymbol{r} = r_1\hat{\boldsymbol{b}}_1 + r_2\hat{\boldsymbol{b}}_2 + r_3\hat{\boldsymbol{b}}_3$$

  while the angular velocity vector is written as

$$\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} = \omega_1\hat{\boldsymbol{b}}_1 + \omega_2\hat{\boldsymbol{b}}_2 + \omega_3\hat{\boldsymbol{b}}_3$$

- The derivative of a vector with respect to the $\mathcal{B}$ frame is written as

$$\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) = \dot{r}_1\hat{\boldsymbol{b}}_1 + \dot{r}_2\hat{\boldsymbol{b}}_2 + \dot{r}_3\hat{\boldsymbol{b}}_3$$

  since

$$\frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}\left(\hat{\boldsymbol{b}}_i\right) = 0$$

Transport Theorem

- The inertial derivative of the position vector is

$$\frac{\mathcal{N}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) = \dot{r}_1\hat{\boldsymbol{b}}_1 + \dot{r}_2\hat{\boldsymbol{b}}_2 + \dot{r}_3\hat{\boldsymbol{b}}_3 + r_1\frac{\mathcal{N}\mathrm{d}}{\mathrm{d}t}\left(\hat{\boldsymbol{b}}_1\right) + r_2\frac{\mathcal{N}\mathrm{d}}{\mathrm{d}t}\left(\hat{\boldsymbol{b}}_2\right) + r_3\frac{\mathcal{N}\mathrm{d}}{\mathrm{d}t}\left(\hat{\boldsymbol{b}}_3\right)$$

- Note that $\hat{\boldsymbol{b}}_i$ are body fixed vectors, thus we find

$$\frac{\mathcal{N}\mathrm{d}}{\mathrm{d}t}\left(\hat{\boldsymbol{b}}_i\right) = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \hat{\boldsymbol{b}}_i$$

- This allows us to write the inertial derivative of the position vector as

$$\frac{\mathcal{N}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) = \frac{\mathcal{B}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}$$

Transport Theorem

$$\frac{{}^{\mathcal{N}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) = \frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{r}) + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}$$

Learn to be one with this equation, and three-dimensional rotations will never haunt you again!

Comments

- Another noted otherwise, the following short-hand notation is used to denote inertial vector derivatives:

$$\frac{{}^{\mathcal{N}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{x}) \equiv \dot{\boldsymbol{x}}$$

- Note that we can analytically differentiate vectors, without first assigning specific coordinate frame. In fact, it is typically easier to wait until the very last steps before specifying a vectors through the vector components.