ECI CARTESIAN COORDINATES TO KEPLER ORBIT ELEMENTS CONVERSION
Elliptical Case (02/02/02)

Compute orbital elements given ECI position and velocity at time t for elliptical motion

$$X, Y, Z, \dot{X}, \dot{Y}, \dot{Z} \Rightarrow a, e, i, \omega, \Omega, T$$

1. Compute the specific angular momentum and check for a degenerate orbit,

$$\vec{r} \times \vec{v} = \vec{h} = h_X \vec{u}_X + h_Y \vec{u}_Y + h_Z \vec{u}_Z$$
$$h = (h_X^2 + h_Y^2 + h_Z^2)^{1/2}$$

2. Compute the radius, *r*, and velocity, v,

$$r = (X^2 + Y^2 + Z^2)^{1/2}$$
$$\mathrm{v} = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}$$

3. Compute the specific energy, *E*, and verify elliptical motion,

$$E = \frac{\mathrm{v}^2}{2} - \frac{\mu}{r}$$

4. Compute semi-major axis, *a*,

$$a = -\frac{\mu}{2E}$$

5. Compute eccentricity, *e*,

$$\mathrm{e} = \left(1 - \frac{\mathrm{h}^2}{\mathrm{a}\mu}\right)^{1/2}$$

6. Compute inclination, *i*, (0° → 180°),

$$\cos \mathrm{i} = \frac{\mathrm{h}_Z}{\mathrm{h}}$$

7. Compute right ascension of the ascending node, Ω, (0° → 360°),

$$\Omega = \mathrm{atan2}(\mathrm{h}_X, -\mathrm{h}_Y)$$

8. Compute argument of latitude, $\omega+\nu$, (0° → 360°),

$$\omega+\nu = \operatorname{atan2}\left(\frac{Z}{\sin i}, (X\cos\Omega + Y\sin\Omega)\right)$$

9. Compute true anomaly, ν, (0° → 360°),

$$\cos\nu = \frac{a(1-e^2)-r}{er}$$

If $\vec{r}\cdot\vec{v} > 0$ then $\nu < 180^\circ$

Or use

$$\nu = \operatorname{atan2}\left(\sqrt{\frac{p}{\mu}}(\dot{\vec{r}}\bullet\vec{r}), p-r\right), \quad \text{where } p = a(1-e^2)$$

10. Compute argument of periapse, ω, (0° → 360°),

$$\omega = (\omega+\nu)-\nu$$

11. Compute eccentric anomaly, *EA*, (0° → 360°),

$$\tan\frac{EA}{2} = \left[\frac{(1-e)}{(1+e)}\right]^{1/2}\tan\frac{\nu}{2}$$

EA is in the same half plane as ν. This equation will yield the correct quadrant for EA.

12. Compute the time of periapse passage, *T* (note that EA must be in radians),

$$T = t - \frac{1}{n}(EA - e\sin EA), \quad n = \sqrt{\frac{\mu}{a^3}}$$