ASEN 5070
Exam #2
November 12,1999
Open book and notes

I. (50%) Given the following function:

$$f(x, y) = c(x^2 + y^2 - 2xy), \quad 0 < x < 1$$
$$0 < y < 1$$
$$0 \text{ elsewhere}$$

1. What is the value of c in order for f(x,y) to be a probability density function?
2. What is h(y), the marginal probability density function of y?
3. What is the conditional density function, f(y/x), of y?
4. Determine p(0<y<1/2 / x=0)
5. Determine p(x+y<1)
6. Are x and y independent? Why or why not?

II. (50%) Given the following system. Assume we wish to estimate X(t) from observations of the position, x(t), using a Kalman (sequential) filter

$$x(t) = x_0 + \dot{x}_0(t - t_0) + 1/2 a(t - t_0)^2$$
$$\dot{x}(t) = \dot{x}_0 + a(t - t_0)$$

$$X(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ a \end{bmatrix}, \quad \overline{X}_0 = \begin{bmatrix} \overline{x}_0 \\ \overline{\dot{x}}_0 \\ \overline{a} \end{bmatrix}, \quad \overline{P}_0 = I$$

1. Write the expression for the state transition matrix $\Phi(t, t_0)$, and the mapping matrix $\tilde{H}$.

Assume there is an observation, x_1, of x at t_1. The variance of the measurement is, $\sigma^2 = 1$. Assume $t_0 = 0$.

2. Write the time update equations at t_1.
3. Write the measurement update equations.

For questions 2 and 3 do evaluate the matrices in detail but show the correct equations and define each term in the equation.