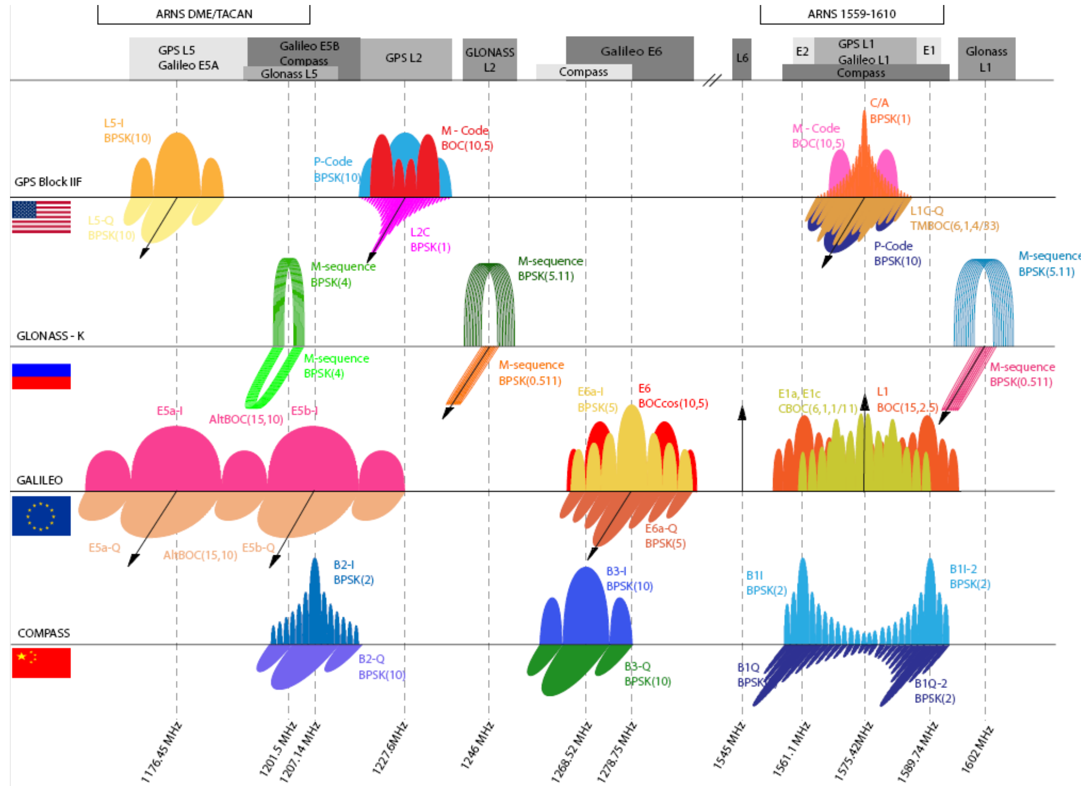


Courtesy He Lin, University of Colorado

ASEN 5090

Signal Structure

Outline

- L1 and L2 carriers
- PRN Codes, CDMA, BPSK
- Navigation Message
- Correlation properties
- Spectrum
- Signal power

- **Reading Assignment**
  – Ch 2, problems 2-1 and 2-2 (See assignment page)
- **Questions to think about**
  – What features of GPS are enabled by the codes?
  – Why 2 frequencies?
  – Would there be an advantage to substantially increasing the GPS signal power? Any downsides?

SIGNAL COMPONENTS

- Base frequency is f_0 = 10.23 MHz = 10,230,000 Hz (cycles/sec)
- L1 = 154 f_0 = 1575.42 MHz
- L2 = 120 f_0 = 1227.6 MHz

- C/A code chip rate = f_0 /10 = 1.023 Mcps (MHz), 1 ms long
- P code chip rate = f_0 = 10.23 Mcps (MHz), 37 weeks long, each satellite gets a 1 week segment

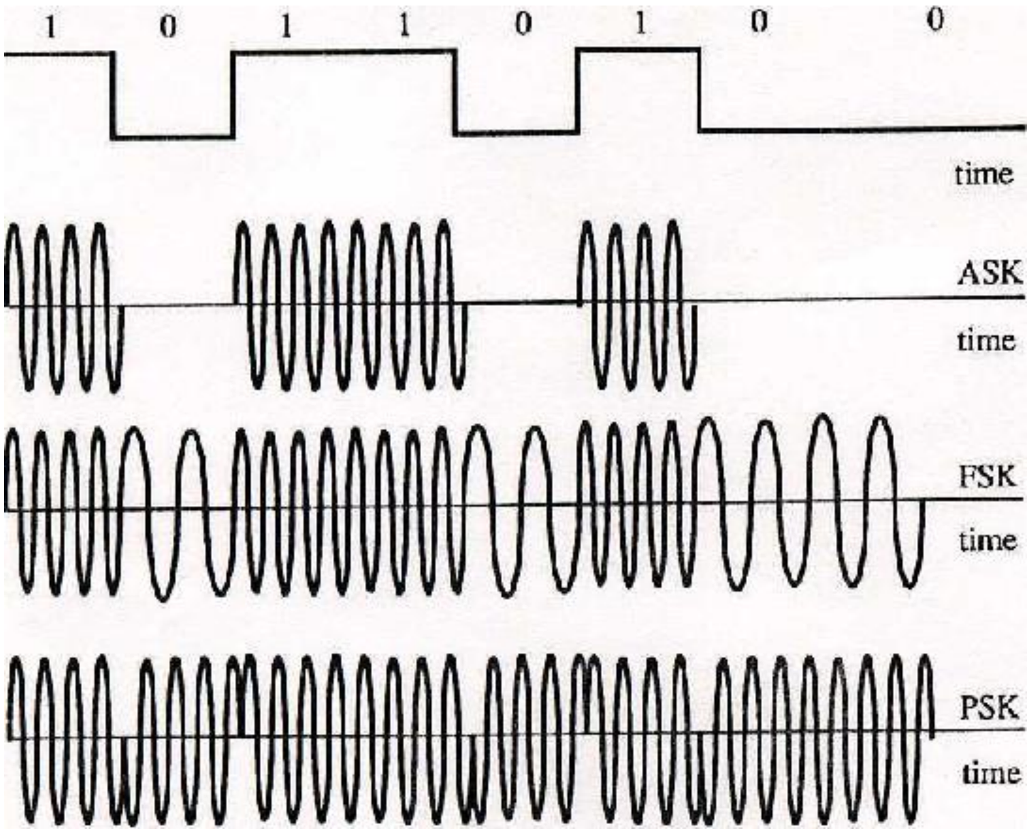
- Navigation Data rate = 50 bps, 20ms per bit, 6 s per frame, 30 s for individual satellite data, 12.5 min for all

PROPERTIES OF CODES

- Protect against interference
- Reduce noise effects
- Permit many signals to be transmitted at the same frequency without interfering (known as : code division multiple access - CDMA)
- Permit high resolution ranging
- Enable communications security (known as : low probability of detection - LPD)
- Key characteristics of codes are their type, length, chip rate.

PRN CODES (pseudo random noise codes)

- In GPS, a pseudo random noise code used to modulate a carrier tone
- Types of modulation: amplitude (AM), frequency (FM), phase (PM)



PRN CODES (pseudo random noise codes)

- GPS uses BPSK - (Biphase phase-shift keying) sequence of "1's" and 0's" that specify when phase of carrier should be reversed (1) or left alone (0).
- Can consider as a sequence of **0's and 1's** or **1 and -1's**
  - In terms of phase-shift keying, 0 corresponds to "don't shift" and 1 corresponds to "shift"
  - In terms of digital amplitude modulation, multiplying by a "1" leaves the carrier alone (i.e don't shift) and multiplying by a "-1" inverts it (i.e. shifts it by 180 deg).
- These codes enable a GPS receiver to measure the transit time of a signal from a GPS satellite to the receiver antenna.
- **Reference -** *Spread Spectrum Systems*, Robert C. Dixon, Wiley 1984.

CORRELATION FUNCTION Rxy(τ)

- Correspondence between a code and a phase-shifted replica of another code
- Cross correlation – correspondence between different codes: Rxy(τ)
- Autocorrelation - correspondence of code w/phase shifted version of itself Rx(τ)

- Simple idea: Compare shifted code replica with the original reference. Number of bits which agree minus the number of bits which disagree is the correlation.

$$R^{j,k}(n) = \sum_{i=1}^{1023} x^{j}(i)x^{k}(i+n), \quad \text{for good cross-correlation} \approx 0, j \neq k$$

$$R(n) = \sum_{i=1}^{1023} x^{j}(i)x^{j}(i+n), \quad \text{for good auto-correlation} \approx 0, n \neq 0$$

CORRELATION CALCULATION - EXAMPLES

1 0 0 1 1 1 1 0 1 1 1 0 1 0 0 0 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 0 1

---

0 0 0 0 1 0 0 0 1 0 1 0 0 1 1 1 0 0 0 0 1 1 1 0 0 1 0 0 1 0 0 0 1

Every time the numbers agree, add 1.

Every time the numbers disagree, subtract 1.

This example: codes for 2 different satellites

1 0 0 1 1 1 1 0 1 1 1 0 1 0 0 0 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1

0 0 0 0 1 0 0 0 1 0 1 0 0 1 1 1 0 0 0 0 1 1 1 0 0 1 0 0 1 0 0 0 1

14 agree

19 disagree

Total score: -5

Perfect agreement would be 33

This example: same satellite codes, but shifted

Not so good - score of -3.

0 1 1 0 0 0 1 0 1 0 1 0 1 1 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 0
1 1 0 0 0 1 0 1 0 1 0 1 1 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 1

But if you recognize they are shifted by 1:

0 1 1 0 0 0 1 0 1 0 1 0 1 1 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0
1 1 0 0 0 1 0 1 0 1 0 1 1 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 1

Agreement is perfect

CALCULATING AUTOCORRELATION(τ)

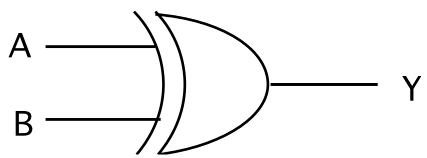
SHIFT	CODE	AGREE	DISAGREE	AUTOCORRELATION
REF	1110010	-	-	-
0	1110010	7	0	7
1	0111001			
2				
3				
4				
5				
6				
7				

CALCULATING AUTOCORRELATION(τ)

SHIFT	CODE	AGREE	DISAGREE	AUTOCORRELATION
REF	1110010	-	-	-
0	1110010	7	0	7
1	0111001	3	4	
2	1011100	3	4	
3	0101110	3	4	
4	0010111	3	4	
5	1001011			
6	1100101			
7	1110010			

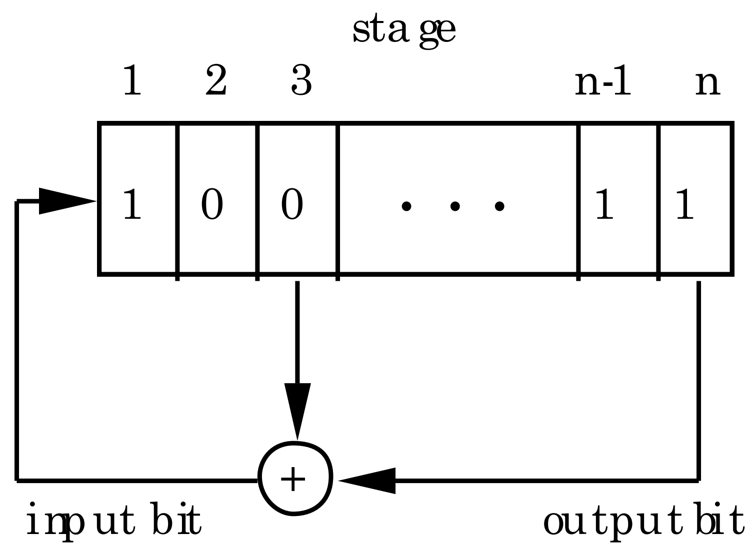
GENERATING PRN CODES

- Tapped feedback shift register
- Epoch state



XOR Logic Table

A	B	Y
0	0	
1	0	
0	1	
1	1	



An n stage shift register PRN generator tapped at [3,n].

GENERATING A CODE

- Example: Generate the PRN code for a 3 stage shift register tapped at [3,2] starting from an all ones state. (The PRN Code is made up of the third stage bit values.)

Shift	Stage			Code
T	1	2	3	
0	1	1	1	
1				
2				
3				
4				
5				
6				
7				

GENERATING A CODE

- Example: Generate the PRN code for a 3 stage shift register tapped at [3,2] starting from an all ones state. (The PRN Code is made up of the third stage bit values.)

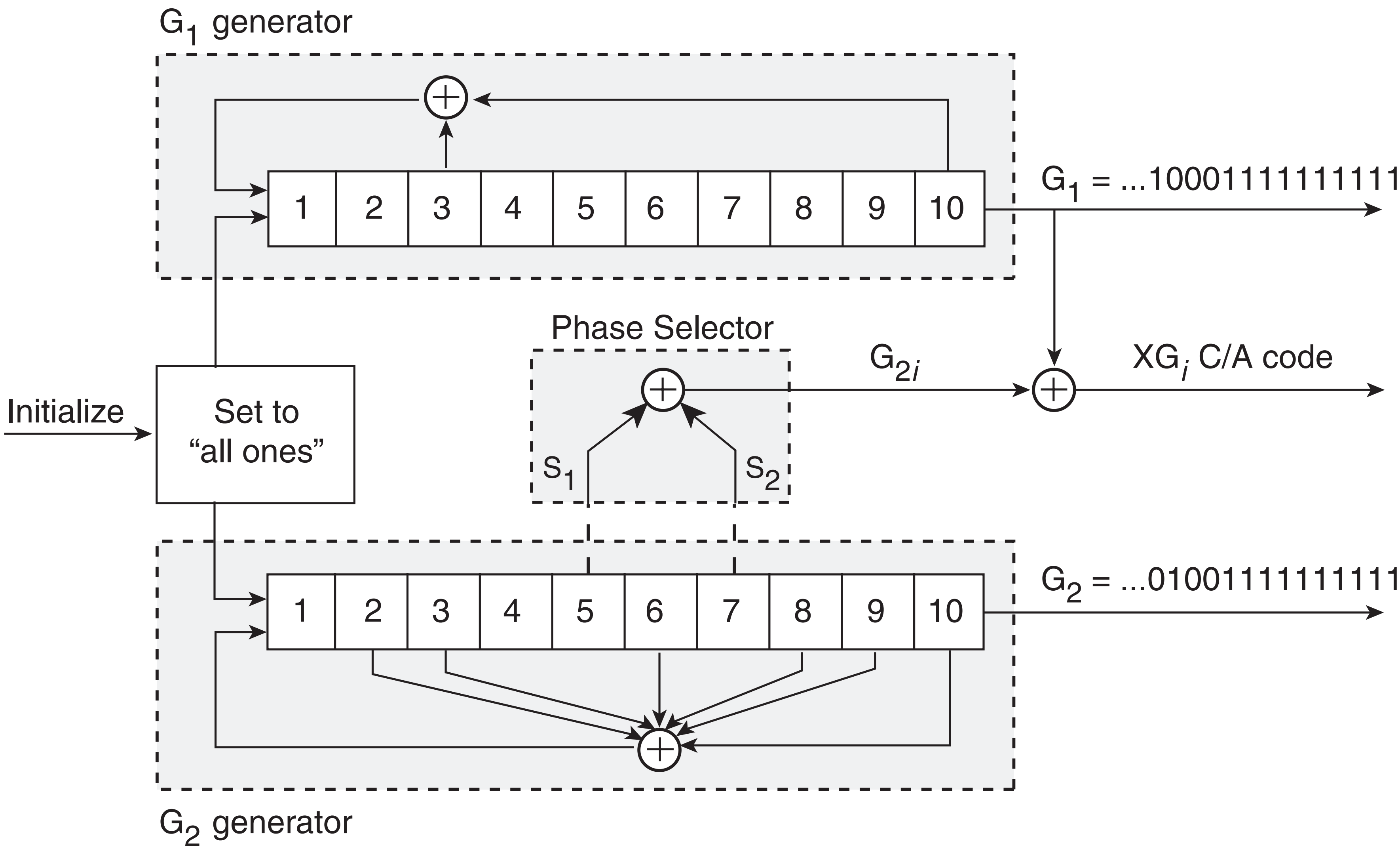
Shift	Stage			Code
T	1	2	3	
0	1	1	1	
1	0	1	1	1
2	0	0	1	1
3	1	0	0	1
4	0	1	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	

MAXIMAL LENGTH CODES

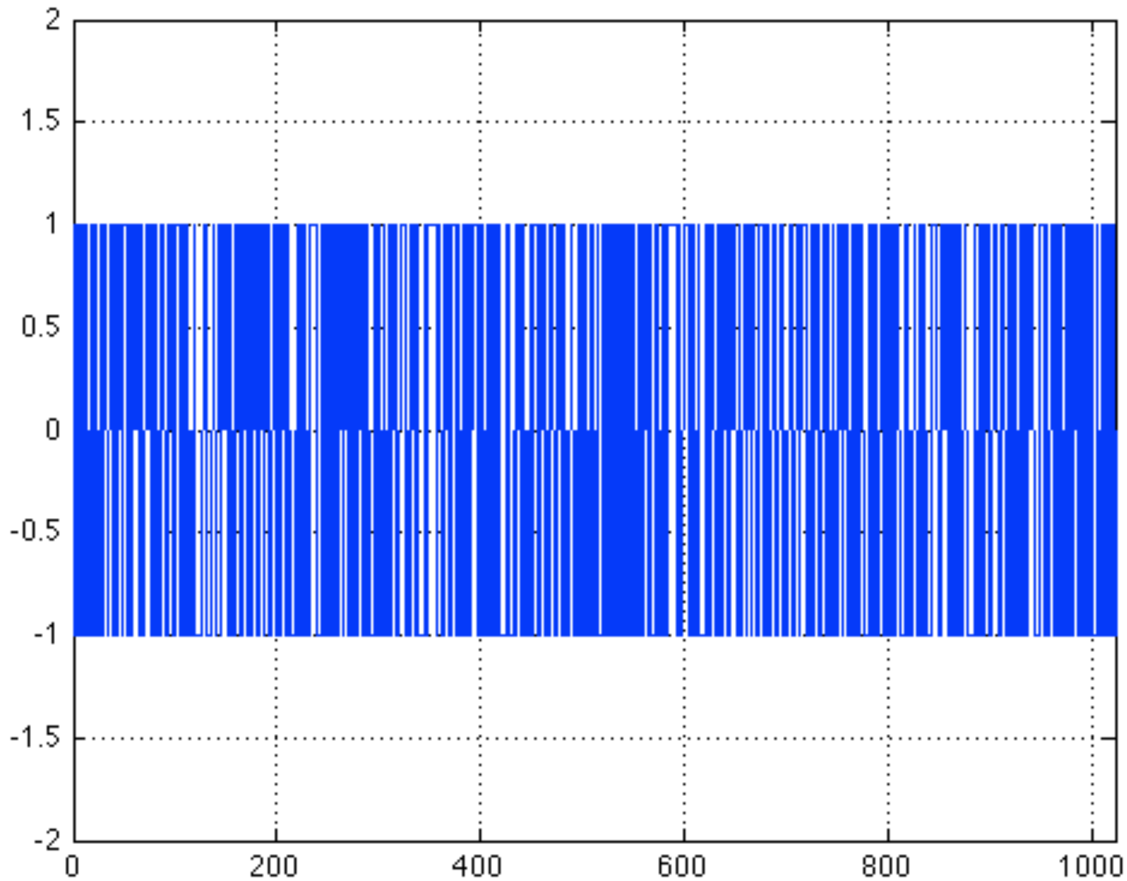
- Maximal length codes are the longest codes that can be generated by a given number of shift register stages. For an *n*-stage shift register, the longest code is $2^n - 1$ chips (i.e. 1's or 0's). Maximal length codes have many interesting properties, especially their autocorrelation function.
- For a maximal length code, the autocorrelation function for a phase shift other than 0 or ($2^n - 1$) is -1. For any code, the autocorrelation for a 0 shift is equal to the number of chips in the code.

Why is maximal length code 2^n -1?

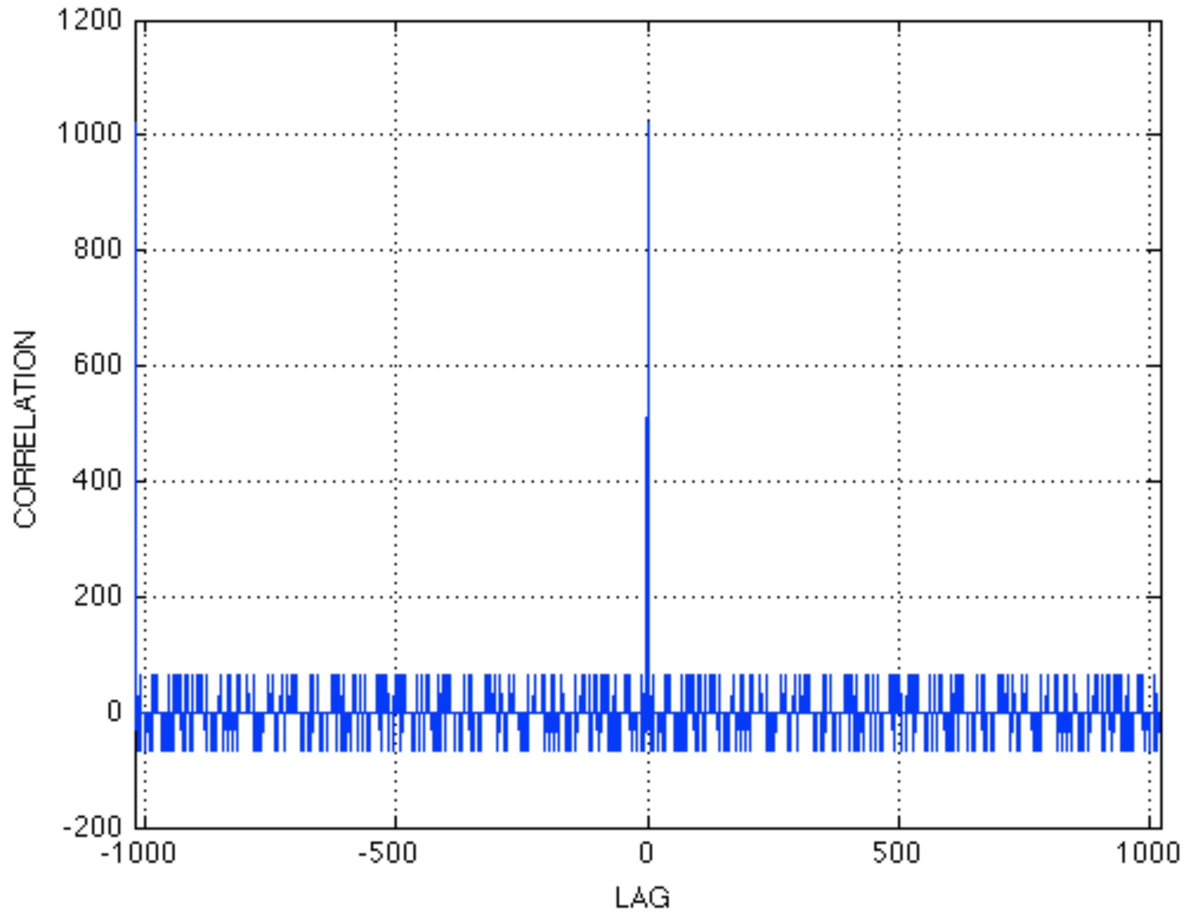
GPS C/A CODES (From Misra & Enge, Problem 2.1)



PRN 5 CODE SEQUENCE



Autocorrelation of PRN 5 C/A Code



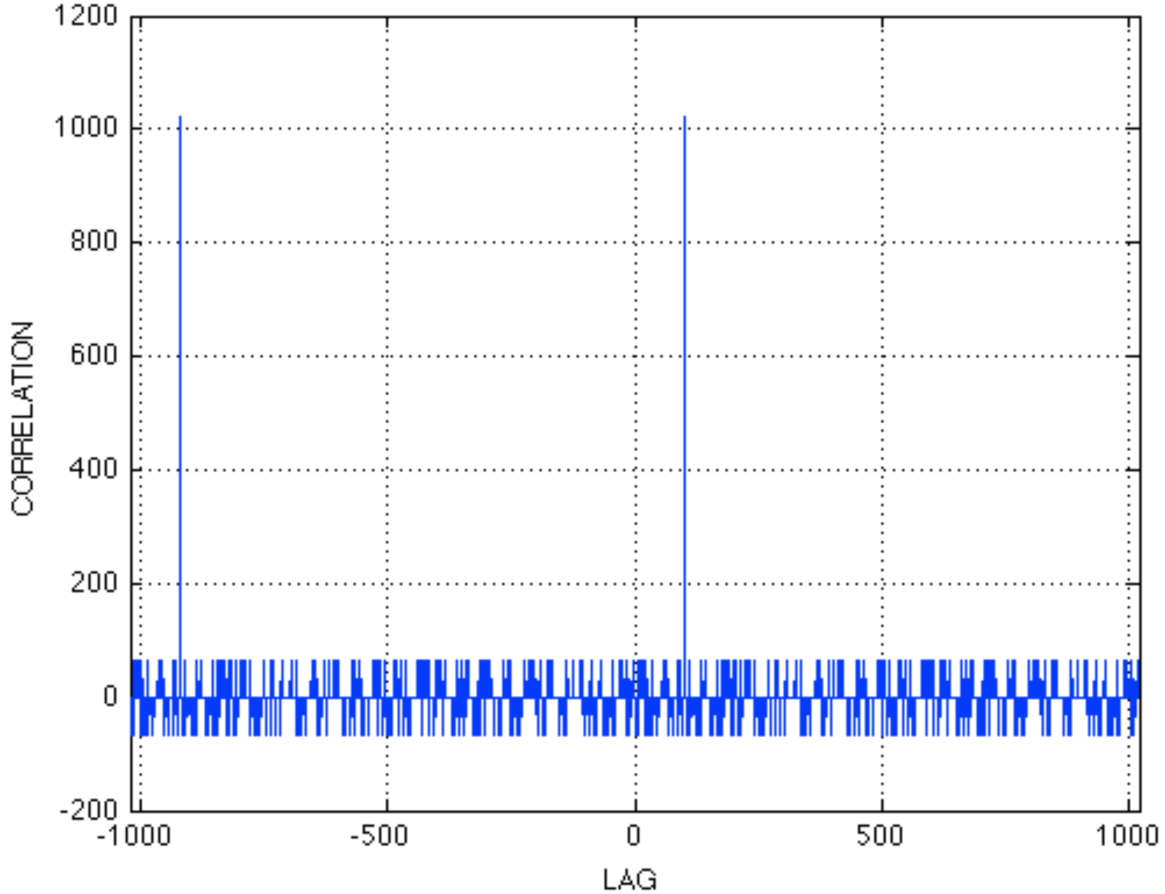
Computing a cyclic correlation

```
function P = cyc_corr(x,y)
% Function to compute the cyclic correlation
% for ASEN 5090
% P. Axelrad, University of Colorado, 9/2009

Y = [y y y]'
n = length(x)
for i = 1:2*n  % sweep the lag from –n to + n
    Rxy(i) = x*Y(i:i+n-1);
    lag(i) = i-1-n;
end
plot(lag,Rxy),grid,xlim([-n n])
xlabel('LAG'),ylabel('CORRELATION')
```

Cross correlation between shifted PRN5 & reference PRN 5

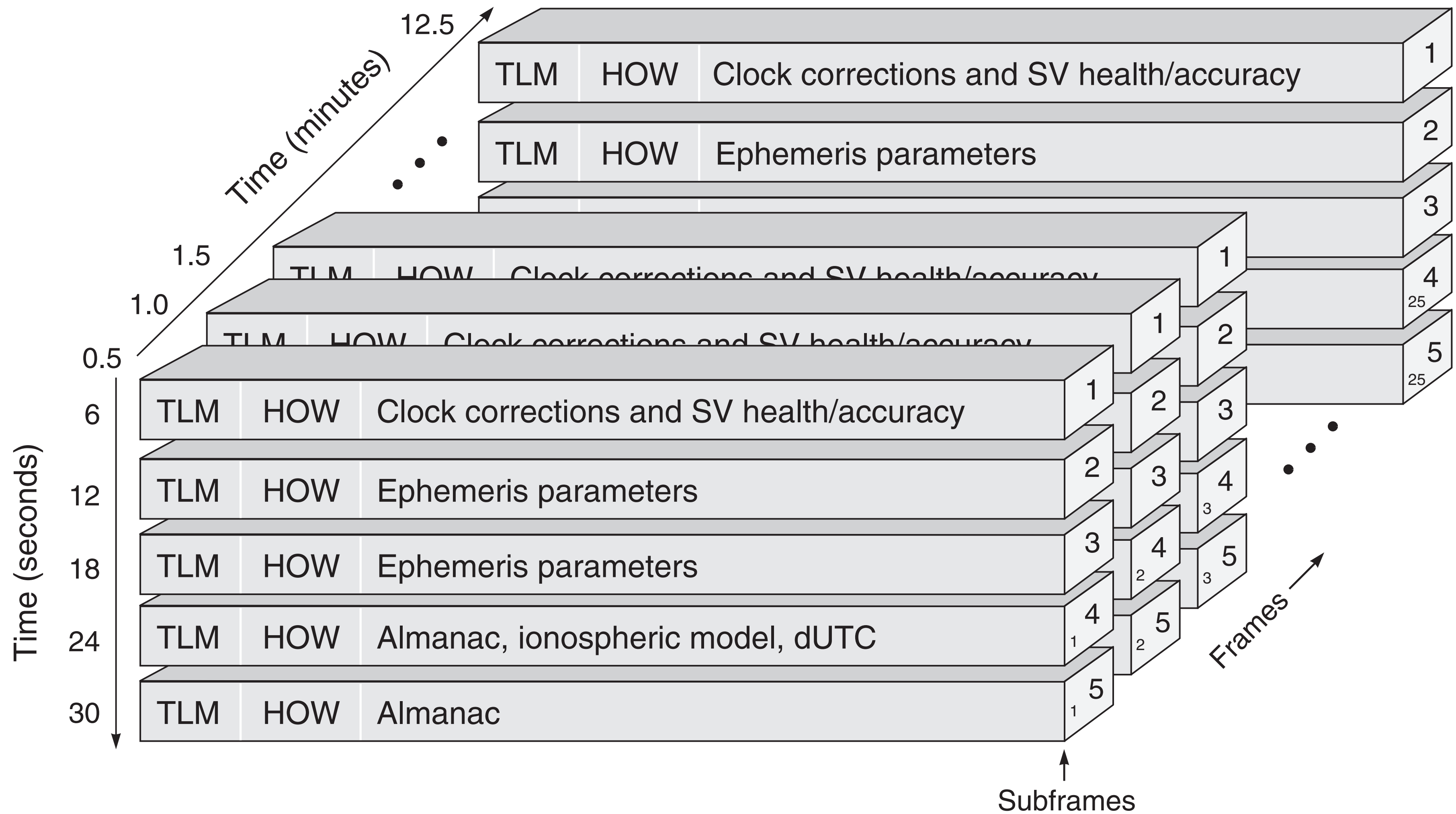
```
yshifted = [y(101:1023) y(1:100)];
P = cyc_corr(yshifted,y);
```



IMPACT OF CORRELATION PROPERTIES

- What makes a useful code?
  - For ranging:
  - For CDMA:
  - For acquisition:

NAVIGATION MESSAGE (from Misra & Enge)

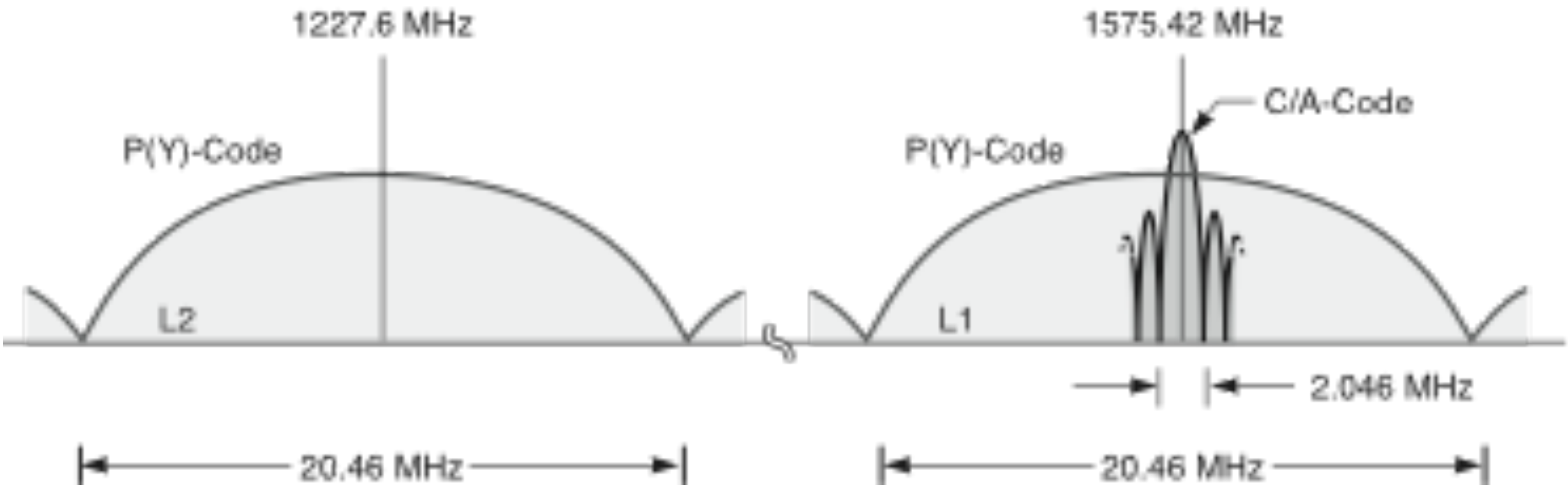


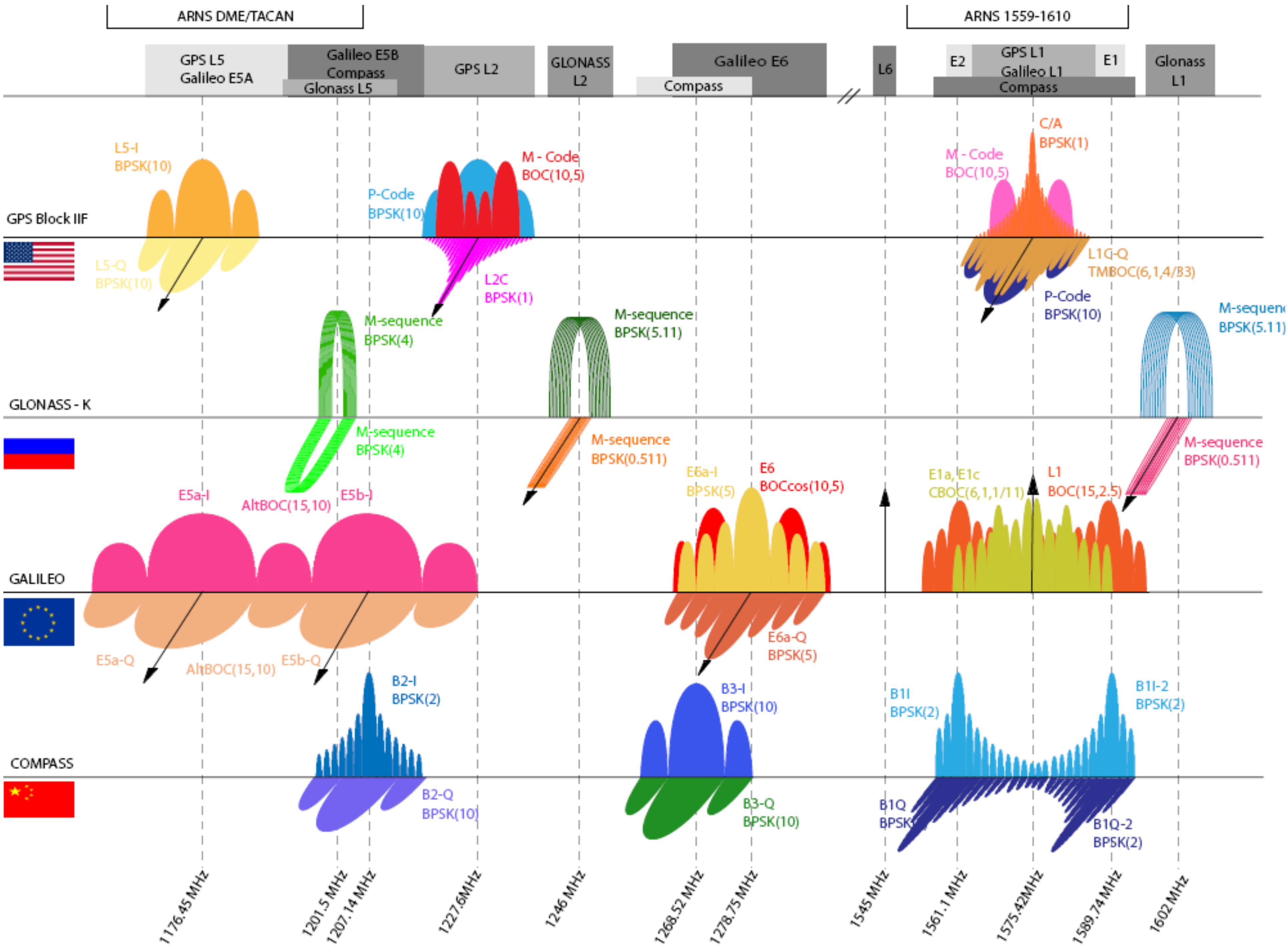
SIGNAL SPECTRUM

- What frequencies are present in the signal

LEGACY GPS SPECTRUM

- PRN codes have a $sinc^2$ spectrum
- Main lobe is 2x chipping rate
- Side lobes are 1x chipping rate
- Power density is very low




ARNS DME/TACAN
ARNS 1559-1610
GPS L5
Galileo E5A
Galileo E5B
Compass
Glonass L5
GPS L2
GLONASS
L2
Galileo E6
Compass
L6
E2
GPS L1
Galileo L1
Compass
E1
Glonass
L1
GPS Block IIF
L5-I
BPSK(10)
L5-Q
BPSK(10)
P-Code
BPSK(10)
M - Code
BOC(10,5)
L2C
BPSK(1)
C/A
BPSK(1)
M - Code
BOC(10,5)
L1C-Q
TMBOC(6,1,4/33)
P-Code
BPSK(10)
GLONASS - K
M-sequence
BPSK(4)
M-sequence
BPSK(4)
M-sequence
BPSK(5.11)
M-sequence
BPSK(0.511)
M-sequen
BPSK(5.11)
M-sequence
BPSK(0.511)
GALILEO
E5a-I
AltBOC(15,10)
E5b-I
E5a-Q
AltBOC(15,10)
E5b-Q
E6a-I
BPSK(5)
E6
BOCcos(10,5)
E6a-Q
BPSK(5)
E1a, E1c
CBOC(6,1,1/11)
L1
BOC(15,2.5)
COMPASS
B2-I
BPSK(2)
B2-Q
BPSK(10)
B3-I
BPSK(10)
B3-Q
BPSK(10)
B1I
BPSK(2)
B1I-2
BPSK(2)
B1Q
BPSK
B1Q-2
BPSK(2)
1176.45 MHz
1201.5 MHz
1207.14 MHz
1227.6MHz
1246 MHz
1268.52 MHz
1278.75 MHz
1545 MHz
1561.1 MHz
1575.42MHz
1589.74 MHz
1602 MHz

SIGNAL POWER

dB

- GPS Satellites transmit 27W toward the earth
- GPS Signal level on the ground:
  - L1 C/A: -158.5 dBW $\sim 10^{-16}$ Watts
  - L1 P(Y): -161.5 dBW
  - L2 P(Y): -164.5 dBW
- Spread spectrum signal is "sub-thermal" or below the noise floor
- After correlation in the receiver the recovered or despread carrier is above the noise (typical SNR's 6-15 dB).
- Processing gain for spread spectrum is signal BW/ data BW

DECIBELS

- RdB = 10 log10 (R)

GPS LEGACY SIGNAL GENERATION