ASEN 5070
Exam No. 2
November 5, 1997
Open book and notes

1. (40%) Given the joint density function

$$f(x,y) = \frac{x}{4}(1+3y^2) \qquad 0 \le x \le 2$$
$$0 \le y \le 1$$
$$= 0 \qquad elsewhere$$

Find:
a. $p(0 \le x \le 1, 0 \le y \le 1/2)$
b. $p(0 \le x \le 1)$
c. $p(0 \le y \le 1/2 \ / 0 \le x \le 1)$
d. Are x and y independent? Why or why not?

2. (40%) Given *a priori* information

$$\bar{P} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \bar{X} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \Phi(t, t_0) = I$$

and an observation-state relation

$$y = 2x_1 + x_2 + \varepsilon$$
$$where \quad \varepsilon \approx N[0, 2]$$

Assume an observation y = 4 is taken. Using the sequential algorithm, find $\hat{X}$ and P, the best estimate of the state and its associated covariance matrix based on processing the observation and including the *a priori* information.

3. (20%) Given that x is an n x1 vector of random variables with mean $\bar{x}$ and covariance P, where $P = E[(x-\bar{x})(x-\bar{x})^T]$. Let y be an mx1 vector of random variables related to x by

$$y = Hx + \varepsilon$$

where ε is zero mean with covariance R and is independent of x.

a. Find the mean of y.
b. Show that the variance-covariance of y is given by

$$P_y \equiv E[(y-\bar{y})(y-\bar{y})^T] = HPH^T + R$$