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Thermomechanical Effects

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§30.1. Introduction

The assumptions invoked in Chapters 2–3 for the example truss result in zero external forces under zero displacements. This is implicit in the linear-homogeneous expression of the master stiffness equation $\mathbf{f} = \mathbf{K}\mathbf{u}$. If $\mathbf{u}$ vanishes, so does $\mathbf{f}$. This behavior does not apply, however, if there are *initial force effects*.[1] If those effects are present, there can be displacements without external forces, and internal forces without displacements.

A common source of initial force effects are temperature changes. Imagine that a plane truss structure is *unloaded* (that is, not subjected to external forces) and is held at a *uniform reference temperature*. External displacements are measured from this environment, which is technically called a *reference state*. Now suppose that the temperature of some members changes with respect to the reference temperature while the applied external forces remain zero. Because the length of members changes on account of thermal expansion or contraction, the joints will displace. If the structure is statically indeterminate those displacements will induce strains and stresses and thus internal forces. These are distinguished from mechanical effects by the qualifier "thermal." For many structures, particularly in aerospace and mechanical engineering, such effects have to be considered in the analysis and design.

There are other physical sources of initial force effects, such as moisture (hygrosteric) effects,[2] member prestress, residual stresses, or lack of fit. For linear structural models *all* such sources may be algebraically treated in the same way as thermal effects. The treatment results in an *initial force* vector that has to be added to the applied mechanical forces. This subject is outlined in §30.3 from a general perspective. However, to describe the main features of the matrix analysis procedure it is sufficient to consider the case of temperature changes.

In this Section we go over the analysis of a plane truss structure whose members undergo temperature changes from a reference state. It is assumed that the disconnection and localization steps of the DSM have been carried out. Therefore we begin with the derivation of the matrix stiffness equations of a generic truss member.

§30.2. Thermomechanical Behavior

Consider the generic plane-truss member shown in Figure 30.1. The member is prismatic and uniform. The temperature T is also uniform. To reduce clutter the member identification subscript will be omitted in the following development until the globalization and assembly steps.

We introduce the concept of *reference temperature* T_{ref}. This is conventionally chosen to be the temperature throughout the structure at which the displacements, strains and stresses are zero if no mechanical forces are applied. In structures such as buildings and bridges T_{ref} is often taken to be the mean temperature during the construction period. Those zero displacements, strains and stresses, together with T_{ref}, define the *thermomechanical reference state* for the structure.

The member temperature variation from that reference state is $\Delta T = T - T_{ref}$. This may be positive or negative. If the member is *disassembled* or *disconnected*, under this variation the member length

[1] Called *initial stress* or *initial strain* effects by many authors. The different names reflect what is viewed as the physical source of initial force effects at the continuum mechanics level.

[2] These are important in composite materials and geomechanics.

is free to change from L to $L + d_T$. If the thermoelastic constitutive behavior is linear[3] then d_T is proportional to L and ΔT:

$$d_T = \alpha L \, \Delta T. \tag{30.1}$$

Here α is the coefficient of thermal expansion, which has physical units of one over temperature. This coefficient will be assumed to be uniform over the generic member. It may, however, vary from member to member.

The *thermal strain* is defined as

$$e_T = d_T/L = \alpha \, \Delta T. \tag{30.2}$$

Now suppose that the member is also subject to mechanical forces, more precisely the applied axial force F shown in Figure 30.1. The member axial stress is $\sigma = F/A$. In response to this stress the length changes by d_M. The *mechanical strain* is $e_M = d_M/L$. The total strain $e = d/L = (d_M + d_T)/L$ is the sum of the mechanical and the thermal strains:

$$\boxed{e = e_M + e_T = \frac{\sigma}{E} + \alpha \Delta T} \tag{30.3}$$

This superposition of deformations is the basic assumption made in the thermomechanical analysis. It is physically obvious for an unconstrained member such as that depicted in Figure 30.1.

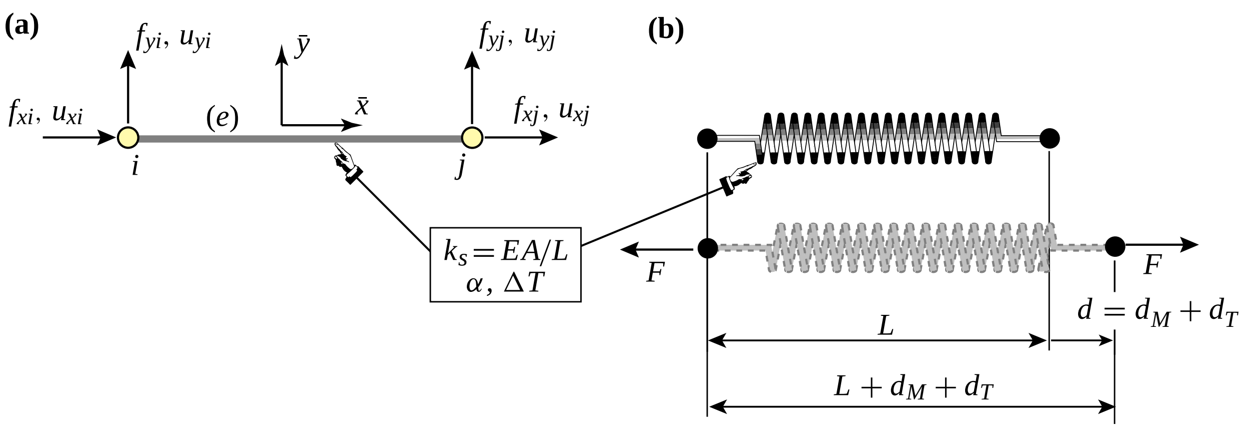

FIGURE 30.1. Generic truss member subjected to mechanical and thermal effects: (a) idealization as bar, (b) idealization as equivalent linear spring.

At the other extreme, suppose that the member is completely blocked against axial elongation; that is, $d = 0$ but $\Delta T \neq 0$. Then $e = 0$ and $e_M = -e_T$. If $\alpha > 0$ and $\Delta T > 0$ the blocked member goes in *compression* because $\sigma = E e_M = -E e_T = -E\alpha \, \Delta T < 0$. This *thermal stress* is further discussed in Remark 30.2.

[3] An assumption justified if the temperature changes are small enough so that α is approximately constant through the range of interest, and no material phase change effects occur.

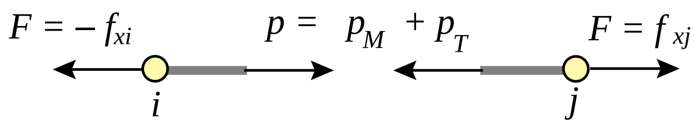

FIGURE 30.2. Equilibrium of truss member under thermomechanical forces.

§30.2.1. Thermomechanical Stiffness Relations

Because $e = d/L$ and $d = \bar{u}_{xj} - \bar{u}_{xi}$, (30.3) can be developed as

$$\frac{\bar{u}_{xj} - \bar{u}_{xi}}{L} = \frac{\sigma}{E} + \alpha\,\Delta T, \tag{30.4}$$

To pass to internal forces (30.4) is multiplied through by EA:

$$\frac{EA}{L}(\bar{u}_{xj} - \bar{u}_{xi}) = A\sigma + EA\alpha\,\Delta T = p_M + p_T = p = F. \tag{30.5}$$

Here $p_M = A\sigma$ denotes the mechanical axial force, and $p_T = EA\alpha\,\Delta T$, which has the dimension of a force, is called (not surprisingly) the *internal thermal force*. The sum $p = p_M + p_T$ is called the *effective internal force*. The last relation in (30.5), $F = p = p_M + p_T$ follows from free-body member equilibrium; see Figure 30.2. Passing to matrix form:

$$F = \frac{EA}{L}\,[\,-1 \quad 0 \quad 1 \quad 0\,] \begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{yi} \\ \bar{u}_{yj} \end{bmatrix}. \tag{30.6}$$

Noting that $F = \bar{f}_{xj} = -\bar{f}_{xi}$ while $\bar{f}_{yi} = \bar{f}_{yj} = 0$, we can relate joint forces to joint displacements as

$$\begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{yi} \\ \bar{f}_{xj} \\ \bar{f}_{yj} \end{bmatrix} = \begin{bmatrix} -F \\ 0 \\ F \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{f}_{Mxi} \\ \bar{f}_{Myi} \\ \bar{f}_{Mxj} \\ \bar{f}_{Myj} \end{bmatrix} + EA\alpha\,\Delta T \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{bmatrix}. \tag{30.7}$$

In compact matrix form this is $\bar{\mathbf{f}} = \bar{\mathbf{f}}_M + \bar{\mathbf{f}}_T = \bar{\mathbf{K}}\bar{\mathbf{u}}$, or

$$\boxed{\bar{\mathbf{K}}\bar{\mathbf{u}} = \bar{\mathbf{f}}_M + \bar{\mathbf{f}}_T.} \tag{30.8}$$

Here $\bar{\mathbf{K}}$ is the same member stiffness matrix derived in Chapter 2. The new ingredient that appears is the vector

$$\bar{\mathbf{f}}_T = EA\alpha\,\Delta T \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \tag{30.9}$$

This is called the vector of *thermal joint forces* in local coordinates. It is an instance of an *initial force* vector at the element level.

Remark 30.1. A useful physical interpretation of (30.8) is as follows. Suppose that the member is precluded from joint (node) motions so that $\bar{\mathbf{u}} = \mathbf{0}$. Then $\bar{\mathbf{f}}_M + \bar{\mathbf{f}}_T = \mathbf{0}$ or $\bar{\mathbf{f}}_M = -\bar{\mathbf{f}}_T$. It follows that $\mathbf{f}_T$ contains the negated joint forces (internal forces) that develop in a heated or cooled bar if joint motions are precluded. Because for most materials $\alpha > 0$, rising the temperature of a blocked bar:$\Delta T > 0$, produces an internal compressive thermal force $p_T = A\sigma_T = -EA\alpha T$, in accordance with the expected physics. The quantity $\sigma_T = -E\alpha\,\Delta T$ is the *thermal stress*. This stress can cause buckling or cracking in severely heated structural members that are not allowed to expand or contract. This motivates the use of expansion joints in pavements, buildings and rails, and roller supports in long bridges.

§30.2.2. Globalization

At this point we restore the member superscript so that the member stiffness equations (30.7) are rewritten as

$$\bar{\mathbf{K}}^e \bar{\mathbf{u}}^e = \bar{\mathbf{f}}^e_M + \bar{\mathbf{f}}^e_T. \tag{30.10}$$

Use of the transformation rules developed in Chapter 2 to change displacements and forces to the global system $\{x, y\}$ yields

$$\mathbf{K}^e \mathbf{u}^e = \mathbf{f}^e_M + \mathbf{f}^e_T, \tag{30.11}$$

where $\mathbf{T}^e$ is the displacement transformation matrix (3.1), and the transformed quantities are

$$\mathbf{K}^e = \left(\mathbf{T}^e\right)^T \bar{\mathbf{K}}^e \mathbf{T}^e, \quad \mathbf{f}^e_M = \left(\mathbf{T}^e\right)^T \bar{\mathbf{f}}^e, \quad \mathbf{f}^e_T = \left(\mathbf{T}^e\right)^T \bar{\mathbf{f}}^e_T. \tag{30.12}$$

These globalized member equations are used to assemble the free-free master stiffness equations by a member merging process.

§30.2.3. Merge

The merge process is based on the same assembly rules stated in §3.1.3 with only one difference: thermal forces are added to the right hand side. The member by member merge is carried out much as described as in §3.1.4, the main difference being that the thermal force vectors $\mathbf{f}^e_T$ are also merged into a master thermal force vector. Force merge can be done by augmentation-and add (for hand work) or via freedom pointers (for computer work). Illustrative examples are provided below. Upon completion of the assembly process we arrive at the free-free master stiffness equations

$$\boxed{\mathbf{K}\mathbf{u} = \mathbf{f}_M + \mathbf{f}_T = \mathbf{f}.} \tag{30.13}$$

§30.2.4. Solution

The master system (30.13) has formally the same configuration as the master stiffness equations (2.3). The only difference is that the *effective* joint force vector $\mathbf{f}$ contains a superposition of mechanical and thermal forces. Displacement boundary conditions can be applied by reduction or modification of these equations, simply by using effective joint forces in the descriptions of Chapters 2–3. Processing the reduced or modified system by a linear equation solver yields the displacement solution $\mathbf{u}$.

§30.2.5. Postprocessing

The postprocessing steps described in Chapter 3 require some modifications because the derived quantities of interest to the structural engineer are *mechanical* reaction forces and internal forces. Effective forces by themselves are of little use in design. Mechanical joint forces including reactions are recovered from

$$\mathbf{f}_M = \mathbf{K}\mathbf{u} - \mathbf{f}_T \tag{30.14}$$

To recover mechanical internal forces in member e, compute p^e by the procedure outlined in Chapter 3, and subtract the thermal component:

$$p^e_M = p^e - E^e A^e \alpha^e \,\Delta T^e. \tag{30.15}$$

This equation comes from solving (30.5) for p_M. The mechanical axial stress is $\sigma^e = p^e_M / A^e$.

§30.2.6. Worked-Out Examples

Example 30.1. The first worked out problem is defined in Figure 30.3. Two truss members are connected in series as shown and fixed at the ends. Properties $E = 1000$, $A = 5$ and $\alpha = 0.0005$ are common to both members. The member lengths are 4 and 6. A mechanical load $P = 90$ acts on the roller node. The temperature of member (1) increases by $\Delta T^{(1)} = 25^\circ$ while that of member (2) drops by $\Delta T^{(2)} = -10^\circ$. Find the stress in both members.

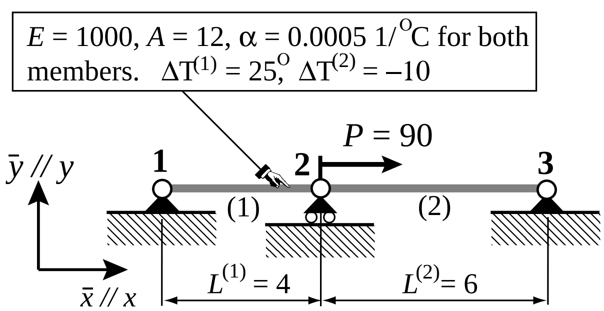

FIGURE 30.3. Structure for worked-out Example 1.

To reduce clutter note that all y motions are suppressed so only the x freedoms are kept: $u_{x1} = u_1$, $u_{x2} = u_2$ and $u_{x3} = u_3$. The corresponding node forces are denoted by $f_{x1} = f_1$, $f_{x2} = f_2$ and $f_{x3} = f_3$. The thermal force vectors, stripped to their $\bar{x} \equiv x$ components, are

$$\bar{\mathbf{f}}^{(1)}_T = \begin{bmatrix} \bar{f}^{(1)}_{T1} \\ \bar{f}^{(1)}_{T2} \end{bmatrix} = E^{(1)}A^{(1)}\alpha^{(1)}\Delta T^{(1)} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -150 \\ 150 \end{bmatrix},\ \bar{\mathbf{f}}^{(2)}_T = \begin{bmatrix} \bar{f}^{(2)}_{T2} \\ \bar{f}^{(2)}_{T3} \end{bmatrix} = E^{(2)}A^{(2)}\alpha^{(2)}\Delta T^{(2)} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 60 \\ -60 \end{bmatrix}. \tag{30.16}$$

The element stiffness equations are:

$$3000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{u}^{(1)}_1 \\ \bar{u}^{(1)}_2 \end{bmatrix} = \begin{bmatrix} \bar{f}^{(1)}_{M1} \\ \bar{f}^{(1)}_{M2} \end{bmatrix} + \begin{bmatrix} -150 \\ 150 \end{bmatrix},\quad 2000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{u}^{(2)}_2 \\ \bar{u}^{(2)}_3 \end{bmatrix} = \begin{bmatrix} \bar{f}^{(2)}_{M2} \\ \bar{f}^{(2)}_{M3} \end{bmatrix} + \begin{bmatrix} 60 \\ -60 \end{bmatrix}, \tag{30.17}$$

No globalization is needed because the equations are already in the global system, and thus we can get rid of the localization marker symbols: $\bar{f} \rightarrow f$, $\bar{u} \rightarrow u$. Assembling by any method yields

$$1000 \begin{bmatrix} 3 & -3 & 0 \\ -3 & 5 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_{M1} \\ f_{M2} \\ f_{M3} \end{bmatrix} + \begin{bmatrix} -150 \\ 150+60 \\ -60 \end{bmatrix} = \begin{bmatrix} f_{M1} \\ f_{M2} \\ f_{M3} \end{bmatrix} + \begin{bmatrix} -150 \\ 210 \\ -60 \end{bmatrix}. \tag{30.18}$$

The displacement boundary conditions are $u_1 = u_3 = 0$. The mechanical force boundary condition is $f_{M2} = 90$. On removing the first and third equations, the reduced system is $5000\,u_2 = f_{M2} + 210 = 90 + 210 = 300$, which yields $u_2 = 300/5000 = +0.06$. The mechanical internal forces in the members are recovered from

$$
\begin{aligned}
p_M^{(1)} &= \frac{E^{(1)}A^{(1)}}{L^{(1)}}(u_2 - u_1) - E^{(1)}A^{(1)}\alpha^{(1)}\Delta T^{(1)} = 3000 \times 0.06 - 12000 \times 0.0005 \times 25 = 60,\\
p_M^{(2)} &= \frac{E^{(2)}A^{(2)}}{L^{(2)}}(u_3 - u_2) - E^{(2)}A^{(2)}\alpha^{(2)}\Delta T^{(2)} = 2000 \times (-0.06) - 12000 \times 0.0005 \times (-10) = -72,
\end{aligned}
\tag{30.19}
$$

whence the stresses are $\sigma^{(1)} = 60/12 = 5$ and $\sigma^{(2)} = -72/12 = -6$. Member (1) is in tension and member (2) in compression.

Example 30.2. The second example concerns the example truss of Chapters 2-3. The truss is mecanically *unloaded*, that is, $f_{Mx2} = f_{Mx3} = f_{My3} = 0$. However the temperature of members (1) (2) and (3) changes by ΔT, $-\Delta T$ and $3\Delta T$, respectively, with respect to T_{ref}. The thermal expansion coefficient of all three members is assumed to be α. We will perform the analysis keeping α and ΔT as variables.

The thermal forces for each member in global coordinates are obtained by using (30.10) and the third of (30.12):

$$
\begin{aligned}
\mathbf{f}_T^{(1)} &= E^{(1)}A^{(1)}\alpha^{(1)}\,\Delta T^{(1)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 100\alpha\,\Delta T \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix},\\
\mathbf{f}_T^{(2)} &= E^{(2)}A^{(2)}\alpha^{(2)}\,\Delta T^{(2)} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 50\alpha\,\Delta T \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix},\\
\mathbf{f}_T^{(3)} &= E^{(3)}A^{(3)}\alpha^{(3)}\,\Delta T^{(3)} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 200\alpha\,\Delta T \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}.
\end{aligned}
\tag{30.20}
$$

Merging the contribution of these 3 members gives the master thermal force vector

$$
\mathbf{f}_T = \alpha\,\Delta T \begin{bmatrix} -100 + 0 - 200 \\ 0 + 0 - 200 \\ 100 + 0 + 0 \\ 0 - 50 + 0 \\ 0 + 0 + 200 \\ 0 + 50 + 200 \end{bmatrix} = \alpha\,\Delta T \begin{bmatrix} -300 \\ -200 \\ 100 \\ -50 \\ 200 \\ 250 \end{bmatrix}
\tag{30.21}
$$

The master stiffness matrix **K** does not change. Consequently the master stiffness equations are

$$
\begin{bmatrix} 20 & 10 & -10 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & -10 & -10 \\ -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 \\ -10 & -10 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & -5 & 10 & 15 \end{bmatrix} \begin{bmatrix} u_{x1} = 0 \\ u_{y1} = 0 \\ u_{x2} \\ u_{y2} = 0 \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} f_{Mx1} \\ f_{My1} \\ f_{Mx2} = 0 \\ f_{My2} \\ f_{Mx3} = 0 \\ f_{My3} = 0 \end{bmatrix} + \alpha\,\Delta T \begin{bmatrix} -300 \\ -200 \\ 100 \\ -50 \\ 200 \\ 250 \end{bmatrix}
\tag{30.22}
$$

in which f_{Mx1}, f_{My1} and f_{My2} are the unknown mechanical reaction forces, and the known forces and displacements have been marked. Since the prescribed displacements are zero, the reduced system is simply

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 10 \\ 0 & 10 & 15 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \alpha\,\Delta T \begin{bmatrix} 100 \\ 200 \\ 250 \end{bmatrix} = \alpha\,\Delta T \begin{bmatrix} 100 \\ 200 \\ 250 \end{bmatrix}. \tag{30.23}$$

Solving (30.23) gives $u_{x2} = u_{x3} = u_{y3} = 10\alpha\,\Delta T$. Completing $\mathbf{u}$ with the prescribed zero displacements and premultiplying by $\mathbf{K}$ gives the complete effective force vector:

$$\mathbf{f} = \mathbf{K}\mathbf{u} = \begin{bmatrix} 20 & 10 & -10 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & -10 & -10 \\ -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 \\ -10 & -10 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & -5 & 10 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \\ 10 \\ 10 \end{bmatrix} \alpha\,\Delta T = \alpha\,\Delta T \begin{bmatrix} -300 \\ -200 \\ 100 \\ -50 \\ 200 \\ 250 \end{bmatrix}. \tag{30.24}$$

But this vector is exactly $\mathbf{f}_T$. Consequently

$$\mathbf{f}_M = \mathbf{K}\mathbf{u} - \mathbf{f}_T = \mathbf{0}. \tag{30.25}$$

All mechanical joint forces, including reactions, vanish, and so do the internal mechanical forces. This is a consequence of the example frame being statically determinate.[4]

Initial Force Effects

As previously noted, a wide spectrum of mechanical and non-mechanical effects can be acommodated under the umbrella of the *initial force* concept. The stiffness equations at the local (member) level are

$$\boxed{\bar{\mathbf{K}}^e \bar{\mathbf{u}}^e = \bar{\mathbf{f}}^e_M + \bar{\mathbf{f}}^e_I = \bar{\mathbf{f}}^e,} \tag{30.26}$$

and at the global (assembled structure) level:

$$\boxed{\mathbf{K}\mathbf{u} = \mathbf{f}_M + \mathbf{f}_I = \mathbf{f}.} \tag{30.27}$$

In these equations subscripts M and I identify mechanical and initial node forces, respectively. The sum of the two: $\bar{\mathbf{f}}$ at the local member level and $\mathbf{f}$ at the global structure level, are called *effective* forces.

A physical interpretation of (30.26) can be obtained by considering that the structure is blocked against all motions: $\mathbf{u} = \mathbf{0}$. Then $\mathbf{f}_M = -\mathbf{f}_I$, and the undeformed structure experiences mechanical forces. These translate into internal forces and stresses. Engineers also call these *prestresses*. ‘prestress

Local effects that lead to initial forces at the member level are: temperature changes (studied in §4.2, in which $\mathbf{f}_I \equiv \mathbf{f}_T$), moisture diffusion, residual stresses, lack of fit in fabrication, and in-member prestressing. Global effects include prescribed nonzero joint displacements (studied in §4.1) and multimember prestressing (for example, by cable pretensioning of concrete structures).

As can be seen there is a wide variety of physical effects, whether natural or artificial, that lead to nonzero initial forces. The good news is that once the member equations (30.25) are formulated, the remaining DSM steps (globalization, merge and solution) are identical. This nice property extends to the general Finite Element Method.

[4] For the definition of static determinacy, see any textbook on Mechanics of Materials. Such structures *do not develop thermal stresses under any combination of temperature changes.*

Pseudo Thermal Inputs

Some commercial FEM programs do not have a way to handle directly effects such as moisture, lack of fit, or prestress. But all of them can handle temperature variation inputs. Since in linear analysis all such effects can be treated as initial forces, it is possible (at least for bar elements) to model them as fictitious thermomechanical effects, by inputting phony temperature changes. The following example indicate that this is done for a bar element.

Suppose that a prestress force F_P is present in a bar. The total elongation is $d = d_M + d_P$ where $d_P = F_P L/(EA)$ is due to prestress. Equate to a thermal elongation: $d_T = \alpha \Delta T_P\, L$ and solve for $\Delta T_P = F_P/(EA\alpha)$. This is input to the program as a fictitious temperature change. If in addition there is a real temperature change ΔT one would of course specify $\Delta T + \Delta T_P$.

If this device is used, care should be exercised in interpreting results for internal forces and stresses given by the program. The trick is not necessary for personal or open-source codes over which you have full control.

Notes and Bibliography

The additional DSM topics treated in this Chapter are covered in virtually all books on Matrix Structural Analysis, such as the often quoted one by Przemieniecki [575]. Several recent FEM books ignore these topics as too elementary.

The physics of thermomechanics and the analysis of thermal stresses is covered adequatedly in textbooks such as Boley and Wiener [95], or manuals such as the widely used by Roark et. al. [620].

For the separate problems of heat conduction and heat transfer, the book by Özişik [516] provides a comprehensive classic treatment. There is a vast literature on prestressed structures; search under “prestress” in `http://www3.addall.com`.

The concepts of static determinacy and its counterpart: static indeterminacy, are important in skeletal structures such as trusses and frameworks. The pertinent design tradeoff is: insensitivity to initial force effects versus redundant safety. A discussion of this topic is beyond the scope of the book. Once going past skeletal structural systems, however, indeterminacy is the rule.

References

Referenced items have been moved to Appendix R.

Homework Exercises for Chapter 30

Thermomechanical Effects

EXERCISE 30.1 [N:20] Use the same data of Exercise 3.7, except that $P = 0$ and consequently there are no applied mechanical forces. Both members have the same dilatation coefficient $\alpha = 10^{-6}$ 1/°F. Find the crown displacements u_{x2} and u_{y2} and the member stresses $\sigma^{(1)}$ and $\sigma^{(2)}$ if the temperature of member (1) rises by $\Delta T = 120\,^\circ$F above T_{ref}, whereas member (2) stays at T_{ref}.

Shortcut: the element stiffnesses and master stiffness matrix are the same as in Exercise 3.7, so if that Exercise has been previously assigned no stiffness recomputations are necessary.

EXERCISE 30.2 [A:15] Consider the generic truss member of §2.4, reproduced in Figure E30.1 for convenience.

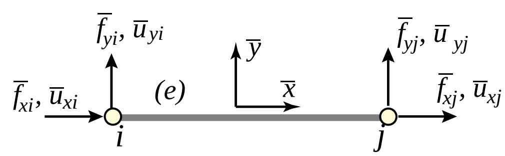

FIGURE E30.1. Generic truss member.

The disconnected member was supposed to have length L, but because of lack of quality control it was fabricated with length $L + \delta$, where δ is called the "lack of fit." Determine the initial force vector $\bar{\mathbf{f}}_I$ to be used in (30.26). *Hint*: find the mechanical forces that would compensate for δ and restore the desired length.

EXERCISE 30.3 [A:10] Show that the lack of fit of the foregoing exercise can be viewed as equivalent to a prestress force of $-(EA/L)\delta$.

EXERCISE 30.4 [A:20] Show that prescribed nonzero displacements can, albeit somewhat artificially, be placed under the umbrella of initial force effects. Work this out for the example of §30.1.1. Hint: split node displacements into $\mathbf{u} = \mathbf{u}_H + \mathbf{u}_N$, where $\mathbf{u}_N$ (the "nonhomogeneous" or "particular" part of the solution) carries the nonzero displacement values.

EXERCISE 30.5 [A:35]. (Research paper level). Prove that any statically determinate truss structure is free of thermal stresses.