CS 726: Homework 1 (Graded)

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web.

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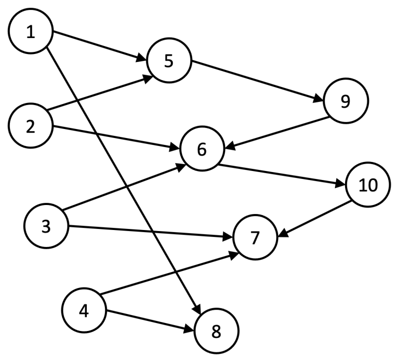
1. For the BN below answer the following questions:



(a) Provide the smallest subset of variables Z so that $x_5 \perp\!\!\!\perp x_4 | Z$

..1

(b) Convert the network to an undirected graphical model that is minimal and correct?

..2

(c) Is the above undirected graphical model triangulated? Justify.

..2

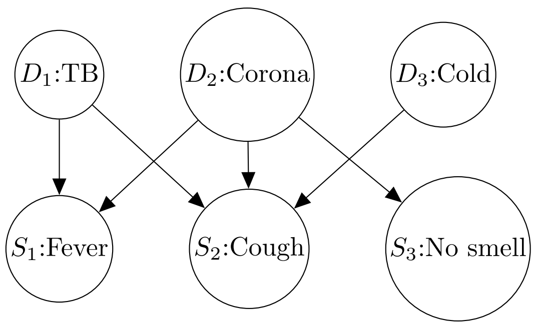
(d) Are there more than one undirected models possible that are both correct and minimal? Justify your answer.

..2

(e) List CIs (if any) that hold in the original BN but not in the graph that you constructed.

..2

2. Consider a special QMR Bayesian network comprising of disease nodes $D_1, \ldots, D_n$ and symptom nodes $S_1, \ldots S_k$ An example appears below.



Assume all variables are binary. Let probability of a disease D_i occurring be denoted as $P(D_i = 1) = \beta_i$ and α_{ij} denote the probability of a patient showing symptom S_j if he only has disease D_i. Also, assume the probability of a symptom not occurring conditioned on its parent nodes is $\Pr(S_j = 0|Pa(S_j)) = \prod_{i \in Pa(S_j)}(1 - \alpha_{ij})^{D_i}$. For example, if someone has D_1, D_2 but not D_3 then the probability of his not showing symptom S_2 is $(1 - \alpha_{12})(1 - \alpha_{22})$

(a) In a study over 1000 random Indians, 20 had Cold, 2 had TB, and 1 had Corona. Doctors have estimated that Corona causes each of these symptoms with probability 0.2, Flu causes cough with probability 0.7, and TB causes fever with probability 0.8, and cough with probability 0.3. Write exact numerical values for the following potentials as a table:

i. $P(D_1)$

..1

ii. $P(S_3|Pa(S_3))$

..2

iii. $P(S_1|Pa(S_1))$

..2

(b) What is the marginal probability $P(S_1 = 1)$ in the above example graphical model? Express only in terms of α_{ij}s and β_is.

..2

(c) If someone has Fever, No sense of smell, and No cough, what is the most likely disease that the person has assuming he has at most one disease? [Give a brief explanation for the answer.] ..2

3. Consider the undirected graphical model below over binary variables.

$$P(x_1, \ldots, x_5) = \frac{1}{Z}\psi_a(x_1, x_3)\psi_b(x_2, x_3)\psi_c(x_3, x_5)\psi_d(x_4, x_5)$$

where the potentials are defined as follows. For example, the first entry says that $\psi_a(x_1 = 1, x_3 = 1) = 2^3$ and $\psi_d(x_4 = 0, x_5 = 1) = 2^2$

$\psi_a(x_1, x_3)$	$\psi_b(x_2, x_3)$	$\psi_c(x_3, x_5)$	$\psi_d(x_4, x_5)$
$\begin{bmatrix} 1 & 2^2 \\ 2 & 2^3 \end{bmatrix}$	$\begin{bmatrix} 2 & 2^2 \\ 2^3 & 1 \end{bmatrix}$	$\begin{bmatrix} 2^4 & 2^3 \\ 2^2 & 2^3 \end{bmatrix}$	$\begin{bmatrix} 2 & 2^2 \\ 2^4 & 1 \end{bmatrix}$

(a) Draw the undirected graphical model corresponding to the above potentials.

..1

(b) Draw a *perfect* Bayesian network for P with variable x_1 as the first variable and an optimal order after that.

..3

(c) Draw another network corresponding to a different order that is also perfect.

..2