Relegation-Based League Championship Algorithm

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Sandarbh Yadav
ABV–IIITM Gwalior
Gwalior, India
apexpacesandy@gmail.com

Pramod Kumar Singh
ABV–IIITM Gwalior
Gwalior, India
pksingh@iiitm.ac.in

***Abstract*—Classical optimization techniques often prove insufficient in case of large scale combinatorial problems and non-linear problems. Consequently, heuristic based optimization techniques have been introduced. Recently, a novel metaheuristic named league championship algorithm (LCA) has been proposed for global optimization in continuous search space. LCA is a population based algorithm which mimics a sports league with a fixed number of teams. These teams, denoting solutions, compete against each other according to a predetermined schedule and the winner is determined on the basis of playing strength of the teams. The formations of the teams keep on improving throughout the season and at the end an optimal solution is obtained. Since, its inception, LCA has been employed for solving many optimization problems. However, there are few limitations of LCA in form of premature convergence and slow convergence rate. This paper attempts to overcome the shortcomings of LCA by proposing relegation based LCA which incorporates the concept of relegation into the original LCA. Comparative experiments have been performed on 10 different test functions and promising results are obtained. Additionally, a study is also done to investigate the impact of control parameters on the performance of relegation based LCA.**

Keywords—optimization, metaheuristic, league championship algorithm, relegation

I. Introduction

Metaheuristics are a special class of algorithms which are used to solve different optimization problems by mimicking natural or artificial phenomena. For example, genetic algorithm [1] models the evolutionary process of natural selection; particle swarm optimization [2] imitates the flocking behaviour of birds; ant colony optimization [3] mimics the communication process of ants and simulated annealing (SA) [4] simulates the annealing process of metallurgy. Occasionally a new metaheuristic is introduced which solves optimization problems by using a novel metaphor as guide. League championship algorithm (LCA), proposed by Ali Kashan [5] in 2009, is one such metaheuristic which models sports leagues for global optimization of a problem in continuous search space. Here, a fixed number of teams compete against each other and at the end of every season, the best team is crowned as winner.

Since its introduction, LCA has been used successfully to solve many optimization problems. For example, Kashan [6] used LCA for optimizing mechanical engineering design; Pourali et al. [7] used LCA for industrial optimization; Sun et al. [8] use LCA for a resource allocation scheme; Lenin et al. [9] employed LCA to solve reactive power dispatch problem; Jalili et al. [10] used LCA for optimum design of pin jointed structures; Gade et al. [11] proposed adaptive league championship algorithm (ALCA) with improved learning rate and applied it for independent task scheduling in cloud computing. Although, researchers have used LCA extensively to solve different optimization problems, little effort has been made to improve the LCA itself. LCA suffers from issues of premature convergence and slow convergence rates. Exploration and exploitation are imbalanced which results in LCA getting stuck in local optima. In 2012, Kashan et al. [12] revamped LCA by incorporating halftime analysis. The modified version, termed realistic league championship algorithm (RLCA), was compared with LCA and PSO on 5 benchmark functions. Although RLCA performed better than PSO on all 5 benchmark functions, the difference between performances of RLCA and LCA was not very significant. Bingol et al. [13] introduced chaotic league championship algorithm (CLCA) by incorporating concepts of chaos theory. 6 different versions of CLCA were proposed based on usage of chaotic maps instead of random number arrays. Although, out of 6 versions, 2 provided better performance than LCA, there was no analytical result which guaranteed improvement in performance of LCA. Therefore, there is scope for improvement in LCA; hence, relegation based LCA is proposed.

The remainder of the paper is organized as follows. Section II provides a brief overview of league championship algorithm. Section III introduces the concept of relegation and describes the proposed relegation based league championship algorithm. Section IV presents a comparative study of relegation based LCA and original LCA on 10 benchmark functions. It also describes the impact of parameter values on performance of relegation based LCA. Finally, Section V concludes the paper.

II. The League Championship Algorithm (LCA)

League championship algorithm (LCA) is a metaheuristic which models artificial sports leagues where a fixed number of teams compete against each other and at the end of every season, the best team is crowned as winner. This section presents a brief overview of LCA.

A. Analogy between Sports Leagues and Metaheuristics

As LCA mimics sports leagues, the analogy between the concepts of sports leagues and metaheuristics is shown in Table I.

B. Parameters of LCA

The parameters of LCA are described in Table II.

C. Working of LCA

Being a population based metaheuristic, LCA employs a population of solutions. All the solutions are represented by n dimensional vectors. Formation of team i at week w is denoted by $X_i^w = (x_{i1}^w, x_{i2}^w, \dots, x_{in}^w)$. The fitness value of X_i^w is denoted by $f(X_i^w)$ where f denotes the fitness function. $B_i^w = (b_{i1}^w, b_{i2}^w, \dots, b_{in}^w)$ denotes the best formation of team i encountered until week w. B_i^w is determined by performing a greedy selection between $f(X_i^w)$ and $f(B_i^{w-1})$ at each iteration: if $f(X_i^w)$ is better than $f(B_i^{w-1})$, B_i^w is replaced by X_i^w; else by B_i^{w-1}. Fig. 1 describes the basic steps of LCA.

TABLE I. ANALOGY BETWEEN SPORTS LEAGUES AND METAHEURISTICS

Sports Term	Metaheuristic Term	Description
League	Population	As league refers to a group of teams, a population consists of a set of solutions.
Week	Iteration	As season comprises of many weeks, a metaheuristic consists of many iterations.
Team	Solution	A team in a league is equivalent to a solution in the population. As the team formation changes over the weeks, the solutions change over the iterations.
Playing Strength	Fitness Value	As winning team is decided by playing strength, a better solution is decided by its fitness value.
Match Analysis	Updation	A match analysis is performed to improve the team. Updation of solutions after each iteration is the analogous concept.

TABLE II. PARAMETERS OF LCA

Parameter	Description
League size (L)	It refers to the number of teams in the league. In metaheuristic terminology, it is equivalent to size of the population.
Number of seasons (S)	It denotes the number of seasons for which league championship is held. It provides a termination condition in form of maximum number of iterations equalling $S(L-1)$.
Number of players in a team (n)	It denotes the number of players in a team. It is analogous to the dimensionality of optimization problem/search space. The solution vectors have n dimensions.
Scale coefficients (c_1 and c_2)	They determine how much strength and weakness component contribute to the updating.
Control Parameter (p_c)	It controls how many changes are done to a team's formation during updating process.

In LCA, a league tournament is organized in which each team faces every other team exactly once. In a league comprising of L teams, each team participates in $L-1$ matches scattered over $L-1$ weeks. Thus, the total number of matches in a season are $L(L-1)/2$ as $L/2$ matches are held in each of the $L-1$ weeks. The league schedule is generated in a round robin manner. The league tournament is organized for S seasons resulting in $S(L-1)$ iterations.

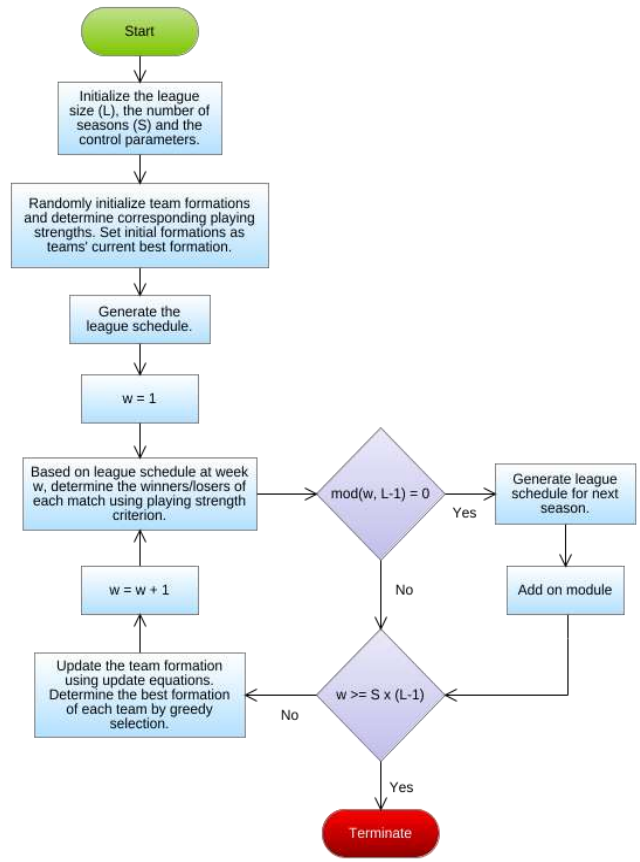

Fig. 1. Flow diagram of league championship algorithm.

Winners of matches are determined by binary tournament selection based on fitness values of teams. SWOT analysis is performed to update the formations of the teams. SWOT stands for strength, weakness, opportunity and threat. Suppose team i won (lost) against team j at week w. The win (loss) is a direct result of its own strength (weakness) or an indirect result of weakness (strength) of team j. Suppose, team i faces team l in week $w+1$ and team l won (lost) against team k at week w. The formation behind this win (loss) can serve as threat (opportunity) for team i. By concentrating on strength (weakness) of team l, team i can avoid threats (take advantage of opportunities). Indirectly, team i can concentrate on weakness (strength) of team k.

Gap analysis is performed to derive the update equations. $X_k^w - X_i^w$ denotes the gap between formations of team k and team i determined by concentrating on strengths of team k. Suppose, team k defeated team l at week w. Then a formation similar to team k can be beneficial for team i. Similarly, $X_i^w - X_k^w$ denotes the gap between formations of team i and team k determined by concentrating on weaknesses of team k. Suppose team l defeated team k at week w. Then a formation similar to team k might prove harmful for team i. Formations of team i at week $w+1$ can be derived using update equations given below.

If team i and team l were winners,

$$x_{id}^{w+1} = b_{id}^{w} + y_{id}^{w}\left(c_1 r_1 (x_{id}^{w} - x_{kd}^{w}) + c_1 r_2 (x_{id}^{w} - x_{jd}^{w})\right) \#(1)$$

If team i and team k were winners,

$$x_{id}^{w+1} = b_{id}^{w} + y_{id}^{w}\left(c_2 r_1 (x_{kd}^{w} - x_{id}^{w}) + c_1 r_2 (x_{id}^{w} - x_{jd}^{w})\right) \#(2)$$

If team j and team l were winners,

$$x_{id}^{w+1} = b_{id}^{w} + y_{id}^{w}\left(c_1 r_2 (x_{id}^{w} - x_{kd}^{w}) + c_2 r_1 (x_{jd}^{w} - x_{id}^{w})\right) \#(3)$$

If team j and team k were winners,

$$x_{id}^{w+1} = b_{id}^{w} + y_{id}^{w}\left(c_2 r_2 (x_{kd}^{w} - x_{id}^{w}) + c_2 r_1 (x_{jd}^{w} - x_{id}^{w})\right) \#(4)$$

Here, d denotes the dimension of solution vector and can take integer values from 1 to n. The c_1 and c_2 are scale coefficients which determine the contribution of strength and weakness components respectively to the updating process. r_1 and r_2 are random numbers generated in the range $[0, 1]$. y_{id}^{w} is a binary variable indicating whether d^{th} element will be changed or not. A value of 1 means that d^{th} element will be changed while a value of 0 means that there will be no change. Let $Y_i^w = (y_{i1}^{w}, y_{i2}^{w}, \dots, y_{in}^{w})$ be the binary change vector with number of ones equal to q_i^w. Generally, the number of changes done by the coach is less. Hence, q_i^w should take a small value. The value of q_i^w is simulated using a truncated geometric distribution given by following equation.

$$q_i^w = \left\lceil \frac{\ln(1 - (1 - (1 - p_c)^n) r)}{\ln(1 - p_c)} \right\rceil \#(5)$$

Here, r is a random number generated in the range [0,1] while p_c is a control parameter in range (0,1) which controls the value of q_i^w. The larger the value of p_c, smaller the value of q_i^w. After calculating the number of changes by above equation, q_i^w elements are randomly selected from B_i^w and their values are changed according to the update equations.

III. Relegation Based League Championship Algorithm

This section introduces the concept of relegation and describes the proposed relegation based league championship algorithm.

A. Concept of Relegation

In established leagues, there are multiple divisions in a hierarchical manner. A fixed number of teams, generally 20, compete at each division for a season. After the end of every season, based on rankings of the teams, the worst 3 teams from each division (except the lowest division) are relegated to the below division and best 3 teams from each division (except the highest division) are promoted to the above division. This concept of relegation can be integrated in original LCA where worst R (suppose) solutions after every season are discarded and replaced by newly generated solutions. The proposed algorithm is named relegation based league championship algorithm.

B. Parameters of Relegation Based LCA

Relegation based LCA incorporates all the parameters of original LCA which are described in Table II. Additionally, a new parameter R is introduced which represents the number of teams which are relegated at the end of each season. In sports leagues, around 15% of teams are relegated at the end of the season. Thus, in a 20 team league, 3 teams are relegated at the end of each season. However, the value of R is not fixed. An experimental study is provided in section IV describing the impact of R on performance of relegation based LCA.

C. Proposed Algorithm

Algorithm 1 describes the basic steps of relegation based LCA. Lines 14-16 show the proposed improvement in LCA by incorporation of concept of relegation.

ALGORITHM 1: RELEGATION BASED LEAGUE CHAMPIONSHIP ALGORITHM

INPUT: Algorithm parameters L, S, R, n, c_1, c_2 & p_c

OUTPUT: Optimal value of objective function

1 Initialize team formations by randomly generating population of L solutions;
2 **for** $i = 1 \ to \ L$ **do**
3   Evaluate playing strength;
4   Set the current formation as best formation;
5 end
6 Generate schedule of league matches;
7 $w = 1$;
8 **while** $w \leq S(L-1)$ **do**
9   Determine the results of each match according to league schedule;
10   **for** $i = 1 \ to \ L$ **do**
11   Update the team formations using (1) to (4);
12   Determine the best formation by greedy selection;
13   end
14   **if** $mod(w, L-1) = 0$ **then**
15   Relegate R worst performing teams by replacing them with newly generated random solutions;
16   end
17     $w = w + 1$;
18 end

IV. EXPERIMENTS AND RESULTS

This section discusses the test functions used and the experimental results obtained. It also provides an experimental study describing impact of R on algorithm.

TABLE III. DESCRIPTION OF TEST FUNCTIONS

Test Function	Properties	Formula
Ackley Function	Non-convex, differentiable, Multimodal, and non-separable	$f(\mathbf{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$
Alpine Function	Non-convex, non-differentiable, Multimodal, and separable	$f(\mathbf{x}) = \sum_{i=1}^{n} \lvert x_i \sin(x_i) + 0.1x_i \rvert$
Griewank Function	Non-convex, differentiable, Multimodal, and non-separable	$f(\mathbf{x}) = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right)$
Rastrigin Function	Non-convex, differentiable, Multimodal, and separable	$f(\mathbf{x}) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$
Rosenbrock Function	Non-convex, differentiable, Unimodal, and non-separable	$f(\mathbf{x}) = \sum_{i}^{n-1} (100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2)$
Schwefel Function	Non-convex, non-differentiable, Multimodal, and non-separable	$f(\mathbf{x}) = 418.9829n - \sum_{i=1}^{n} x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$
Sphere Function	Convex, differentiable, Unimodal, and separable	$f(\mathbf{x}) = \sum_{i=1}^{n} x_i^2$
Sum of Different Powers Function	Convex, non-differentiable, Unimodal, and separable	$f(\mathbf{x}) = \sum_{i=1}^{n} \lvert x_i \rvert^{i+1}$
Sum Squares Function	Convex, differentiable, Unimodal, and separable	$f(\mathbf{x}) = \sum_{i=1}^{n} i x_i$
Zakharov Function	Convex, differentiable Unimodal, and non-separable	$f(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} 0.5 i x_i\right)^2 + \left(\sum_{i=1}^{n} 0.5 i x_i\right)^4$

A. Test Functions

Test functions are functions which are used to evaluate the effectiveness of optimization algorithms. There are a huge number of test functions available in literature. In order to maintain a rich diversity, 10 different test functions are used. Table III provides a detailed description of all 10 test functions. All these functions are n-dimensional and have global optima value as 0.

B. Results

This section presents a comparison done between original LCA and relegation based LCA. The algorithmic parameters are as follows: $L = 20$; $S = 50$; $R = 3$; $n = 10$; $c_1 = 0.2$; $c_2 = 1$; $p_c = 0.3$. Both algorithms are run on same parametric values for a fair comparison between them in similar environment. We experiment with a range of reasonable values of parameters and find that the above parametric values obtain the best result.

30 different runs of both algorithms are performed for each of the 10 test functions and a comparative study is done on their performances. Fig. 2 presents a visual representation of convergence of both algorithms for Ackley function during the first of the 30 runs. Table IV summarizes the comparative results between original LCA and relegation based LCA, obtained during the 30 different runs.

C. *Verification of Results*

As observed from Table IV, relegation based LCA results in better function values compared to original LCA.

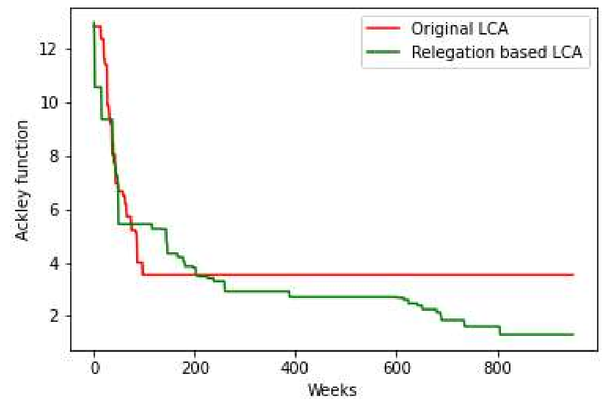

Fig. 2. Convergence graph for Ackley function during the first run.

TABLE IV. EXPERIMENTAL RESULTS OBTAINED DURING 30 DIFFERENT RUNS

Test Function	Original LCA			Relegation based LCA		
	Mean	Standard Deviation	Best	Mean	Standard Deviation	Best
Ackley Function	4.110765	1.514951	1.014089	1.858368	0.900234	0.182607
Alpine Function	2.553196	1.616669	0.121907	1.636277	0.898516	0.012698
Griewank Function	0.621392	0.216047	0.071970	0.458808	0.169806	0.064532
Rastrigin Function	47.065508	13.150035	17.660459	38.748249	8.333133	15.967540
Rosenbrock Function	921.046607	780.905291	63.136866	128.369312	96.091490	7.160220
Schwefel Function	4130.866378	20.002376	4063.200173	4099.816674	28.852650	3994.254356
Sphere Function	9.112696	9.131957	0.243558	0.096699	0.374367	0.000454
Sum of Different Powers Function	99.829557	140.792658	0.000198	0.003878	0.014757	0.000000
Sum Squares Function	33.099359	30.191493	2.386982	0.357874	1.031760	0.005929
Zakharov Function	75.045158	44.809268	27.451777	30.775599	23.465757	1.272265

Moreover, the fluctuations in values are also less as signified by relatively lower values of standard deviations. Statistical test based on t-statistic is performed to verify the results. For level of significance $\alpha = 0.01$, the critical value of t-statistic is 2.392. The calculated values of t-statistic for each test function are given in Table V. For each test function, the calculated value of t-statistic is greater than critical value. Hence it is verified with 99% confidence that relegation based LCA performs better than original LCA.

TABLE V. VALUES OF t-STATISTIC FOR EACH TEST FUNCTION

Test Function	$\boldsymbol{t}$-statistic
Ackley Function	7.001
Alpine Function	2.715
Griewank Function	3.241
Rastrigin Function	2.926
Rosenbrock Function	5.518
Schwefel Function	4.844
Sphere Function	5.403
Sum of Different Powers Function	3.883
Sum Squares Function	5.936
Zakharov Function	4.794

D. *Effect of R on Performance of Relegation Based LCA*

To study the impact of R on performance of relegation based LCA, 5 different values of R are considered. Experiments are conducted on Ackley function for $R = 1, 2, 3, 4, and\ 5$ while keeping the values of other parameters same as before. Fig. 3 shows the convergence graph obtained.

The convergence graph suggests the choice of $R = 3$ as it results in best performance of relegation based LCA. Lower and higher values of R result in relatively poor performance. However, the value of R is not fixed and should be determined empirically depending upon the problem.

V. CONCLUSION

LCA is a metaheuristic which mimics artificial sports leagues and performs global optimisation in continuous search space. Although LCA has been used to solve many optimisation problems successfully, there is still scope for improvement in it as it suffers from issues related to premature convergence and slow convergence rate. In this paper, LCA has been improved by integrating concept of relegation. Experiments have been conducted on 10 different test functions and the results indicate that relegation based LCA performs better than original LCA. The results have been verified statistically using hypothesis test based on t-statistic. An experimental study is also done to understand the impact of number of relegated teams on performance of relegation based LCA. Relegation based LCA takes little more computation time compared to the original LCA owing to incorporation of an extra relegation module; however, it is insignificant compared to the improvement made in the performance of the algorithm.

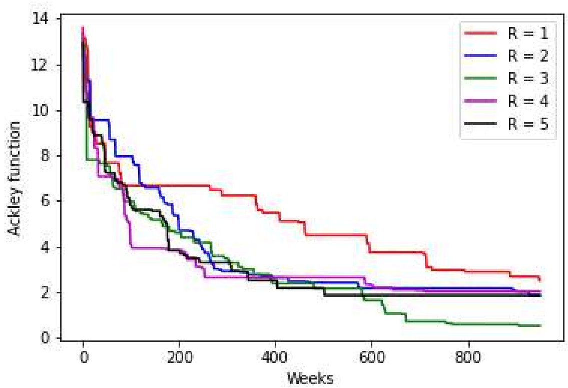

Fig. 3. Convergence of relegation based LCA for different values of R.

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