Starling Flock Mathematical Model

COP290 design project

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1 Introduction

Flocking birds can be considered as a group of birds flying with following certain rules and hence forming some patterns in group which amazes us in a certain way. A this kind of simulation for Flocking starlings can be done using this mathematical model.

2 A Boid's Model

2.1 Boid structure

For representing a boid in 3D coordinates we need some information in coordinates system, basic information about a boid are:

- Position(x, y, z coordinates)
- Velocity(v, v, v)
- Mass: mass of the boid.
- Color(for representational purpose)
- other constant properties like size or boundary bounds can also be defined as per the software package requirements.

boids are restricted in knowledge of field of view and distance of other boids or obstacles which affect the trajectory and behaviour of the boid.

3 Constraint rules for starling simulation

A Starling simulation can be considered as a bunch of boids flying around with some ruling constraints between them. There is also priorities are give. The basic rules for a simulation can be described as

3.1 Alignment

Alignment is the tendency of aligning the velocity of a boid with the direction of average velocity of the fellow neighbour's.

Alignment deciding force for boid can be calculated as a function of angle of average velocity of neighbouring boids and velocity of the boid. A simple representation of direction of this kind of force can be seen be seen in the direction of the vector $V_(alignment) = V_(average)$ - $V_(boid)$

3.2 Cohesion

Cohesion is the tendency of a boid to remain with its neighbours by moving toward the average position of the group of neighbours. Effect of cohesion on velocity of boid can be derived as a component of force towards the centre of position of group as a function of distance of boid from the centre of group of its neighbours.

$F_c \ \alpha \ 1/(R_c)$ F_c is cohesive force, R_c is distance from centre of group. Determination of force acting for maintaining cohesion property can be seen as direction of boids velocity from the direction of centre of neighbour from boid.

3.3 Separation

It can be described as a repulsive force in opposite direction from a body to boid. A body can be another boid or an obstacle. It can be calculated as a function of inverse of the distance between a body and the boid.

$F_s \ \alpha \ 1/(r^2)$ F_s is seperation force, r is distance from another boid/obstacle.

Another effects of conditions like Obstacle avoidance, Goal seeking, predator approaching can also be applied with proper dependencies of factors for applying related force.

4 Application of constraints

Applying these constraints to boids can be a simulation defining factor. Each of the factor is considered with a priority in certain conditions to give a reality effect to the flocking birds. For example in case of an obstacle is there in front, the boid may try to avoid the obstacle as first priority then seek for other factors like being in group. For this situation every factor responsible for a force on boid in some conditions is provided with priorities in accordance with situations which can be determined by data during the simulation. For example in case of two boids are about to collide, priority of separation force will be high.

Now given the all the effecting factors and characteristics of boid flocking simulation can be generated by calculating boids characteristics at t+dt

1. **Position:-** Position at time t+dt can be calculated by using position and velocity at t as.

$$x_{t+dt} = x_t + vx_t * dt$$
$$y_{t+dt} = y_t + vy_t * dt$$
$$z_{t+dt} = z_t + zy_t * dt$$

2. **Neighbours:-** Neighbours after a change can be found by finding all the boids within a distance of neighbour radius r.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} < r$$

3. **Velocity:-** Velocity of a boid can be determined by considering all the factors into account by applying corresponding force of each factor.

$$V_f = Fc * V_c + Fa * V_a + Fs * V_s$$

Here Vc, Va and Vs are force factors due to cohesion, alignment and separation respectively and Fc, Fa, Fs are corresponding forces. Vf is the final effective force with a direction.

4. **Boundaries :-**
We have assumed that the birds will be confined within a certain volume (a cuboid). So whenever, a bird reaches crosses a boundary , it will change its directions back to inside of the cuboid.

5 Energy and Force calculations

1. **Average energy of each bird :-** Two methods can be used to calculate average energy of each bird.

   (a) We can calculate the average energy of the entire flock at a time t, by using the velocity at time t.

   $$E_{avg} = (m_1(v_1)^2 + m_2(v_2)^2 + ... + m_n(v_n)^2)/2n$$

   (b) We can integrate the energy of a bird until time T to calculate average energy spent by a boid and then take average over it to calculate average energy at time T.

   $$E_{avg} = \int_0^T m(v^2)dt/T$$

2. **Average angular momentum of a bird :-** Integrate angular momentum(about origin) of the bird untill time T and then take average over it.

$$\vec{L}_{avg} = \int_0^T m(\vec{v} * \vec{r})dt/T$$

3. **Average force experienced by a bird :-** Since average force has to be calculated, we can simply write it as

$$\vec{F}_{avg} = \int_0^T m(\frac{dv}{dt})dt/T = \int_0^T m(dv)/T = m(v_T - v_0)/T$$