Institute of Infrastructure, Technology, Research And Management

Enrollment no. : ˜ Date: 24/04/2024

B. Tech. Semester *IV* ˜ **End-semester Examination**

Course Name: Probability and Random Processes˜ **Full Marks:** 30

Course Code: MA 192003˜ **Maximum Time:** 2 **hours**

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1. State whether the following statements are true or false: [4]

   (i) For a bivariate Gaussian random variable (X, Y), if the correlation coefficient of X and Y is zero then X and Y are independent.

   (ii) Suppose Z and W are two independent continuous random variables. Then the joint probability density function of (Z, W) is the product of the marginal density function of Z and the marginal density function of W.

   (iii) A wide-sense stationary random process is always strict-sense stationary.

   (iv) Suppose U is a random variable which is uniformly distributed over the interval $(2, 5)$. Then the value of the cumulative distribution function of U is constant throught the interval $(2, 5)$.

2. Suppose the probability that an item produced by a certain machine will be defective is 0.1. Assuming that the quality of successive items is independent, find the probability that a sample of 10 items will contain at most one defective item. [3]

3. Let X and Y be two random variables with variances σ_X^2 and σ_Y^2 respectively. Also let $\rho_{X,Y}$ be the correlation coefficient between them. Then show that [4]

$$-1 \leq \rho_{X,Y} \leq 1.$$

$\left[\text{Hint: Use Var}(\frac{X}{\sigma_X} + \frac{Y}{\sigma_Y}) \geq 0 \text{ and Var}(\frac{X}{\sigma_X} - \frac{Y}{\sigma_Y}) \geq 0\ \right]$

4. Suppose X and Y are continuous random variables which are independent. Also, let F_X, F_Y denote the cumulative distribution functions of X and Y respectively, and f_X, f_Y denote the probability density functions of X and Y respectively. Then show that

   (i) $\mathrm{P}(X + Y \leq b) = \int\limits_{-\infty}^{\infty} F_X(b - y) f_Y(y) dy$ [3]

   (ii) $\mathrm{P}(X \leq Y) = \int\limits_{-\infty}^{\infty} F_X(y) f_Y(y) dy$ [2]

5. From past experiences a professor knows that the test score of a student taking her final examination is a random variable with a mean of 75. Then,

   (i) find an upper bound for the probability that a student's test score will exceed 85. [2]

   Suppose, in addition, the professor knows that the variance of a student's test score is equal to 25.

   (ii) What can be said about the probability that a student will score between 65 and 85? [2]

6. (i) What is a wide-sense stationary random process (state its defining conditions)? Give an example of a wide-sense stationary process by showing that the defining conditions are valid for the example. [1+2]

   (ii) Let the random process $\underline{X} = \{X(t), t \in [0, \infty)\}$ be defined by

   $$X(t) = Y + tZ, \ t \in [0, \infty),$$

   where Y and Z both are standard normal random variable, and they are independent. Let A be the random variable given by:

   $$A = X(1) + X(2).$$

   Then what kind of random variable is A (answer with proper justification)? Find its probability density function (in case it is continuous) or its probability mass function (in case it is discrete). Find the autocorrelation function for the random process $\underline{X}$. [2+3+2]