Lecture 4: Return and Autocorrelation

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    - Simple return
    - Log return
2. Probability Review
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    - Covariance and Correlation
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4. Date and Time

Return

Why use return series instead of price series.

- Return is a complete and scale-free summary of an asset.
- Return has more attractive statistical properties. (weakly stationary)

Return

Simple Return

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

$$\therefore R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Log Return (countinuously compounded return)

$$r_t = log(1 + R_t) = log(P_t/P_{t-1})$$

$$= log(P_t) - log(P_{t-1})$$

Return

For multiple time intervals, say K:

Simple

$$1 + R_{t[k]} = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{t-2}} \dots \frac{P_{t-k+1}}{P_{t-k}}$$

$$= (1 + R_t)(1 + R_{t-1})(1 + R_{t-2})$$

Log

$$r_{t[k]} = log(1 + R_{t[k]}) = log[\frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{t-2}} \dots \frac{P_{t-k+1}}{P_{t-k}}]$$

$$= log(\frac{P_t}{P_{t-1}}) \dots log(\frac{P_{t-k+1}}{P_{t-k}}) = \prod_{j=1}^{k-1} r_j$$

Continuously Compounding

one year: $\lim_{m\to\infty}(1+\frac{r}{m})^m = e^r$

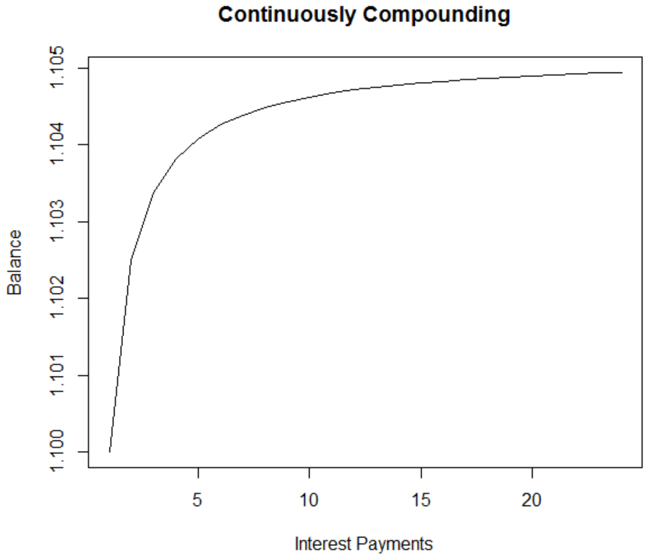
t years: $\lim_{m\to\infty}((1+\frac{r}{m})^m)^t = e^{rt}$

Continuously Compounding

Example

```
> result <- NULL
> x <- 1:24
> for (m in x)
+ {
+  result <- c(result, contConpound(0.1, m))
+ }
> plot.ts(result, main = "Continuously Compounding",
          xlab = "Interest Payments", ylab = "Balance")
```

Continuously Compounding



Probability Review

Covariance is a measure of how much two random variable change together.

Covariance

$$Cov(X, Y) = E[(X - \mu_x)(Y - \mu_y)] = E[XY] - E[X]E[Y]$$

Covariance

Thus we have: $E[XY] = E[X]E[Y] + Cov(X, Y)$

Correlation Coefficient

$$\rho = Corr(X, Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

Autocorrelation

What is the relationship between the return of today and of yesterday?

The correlation coefficient of r_t and r_{t-l} is called *lag-l* autocorrelation.

Autocorrelation

$$\text{lag 1: } \rho_1 = Corr(r_t, r_{t-1}) = \frac{Cov(r_t, r_{t-1})}{\sigma_t \sigma_{t-1}} = \frac{Cov(r_t, r_{t-1})}{Var(r_t)}$$

$$\text{lag 2: } \rho_2 = Corr(r_t, r_{t-2}) = \frac{Cov(r_t, r_{t-2})}{\sigma_t \sigma_{t-2}} = \frac{Cov(r_t, r_{t-2})}{Var(r_t)}$$

$$\text{lag l: } \rho_l = Corr(r_t, r_{t-l}) = \frac{Cov(r_t, r_{t-l})}{\sigma_t \sigma_{t-l}} = \frac{Cov(r_t, r_{t-l})}{Var(r_t)}$$

We use γ_l represent lag-l covariance of r_t, apparently:

$$\rho_l = \gamma_l / \gamma_0$$

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