Week9
Simple Linear Regression
Gradient Descent

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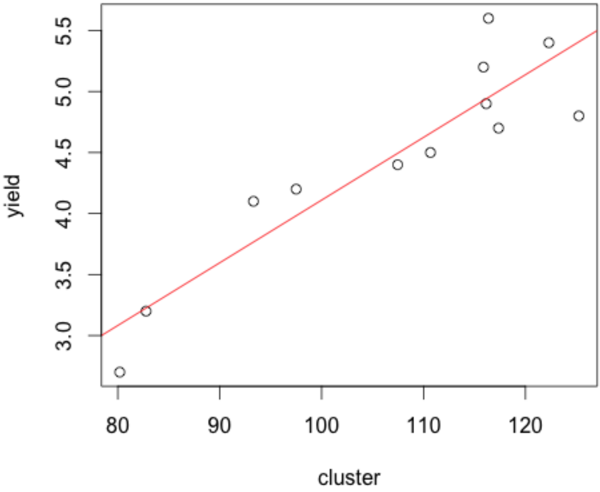
Outline

1. Simple Linear Regression
2. Gradient Descent
3. Project Ideas

Simple Linear Regression

Simple Linear Regression



Simple Linear Regression

Objective

Describe the relationship between two variables, say X and Y as a straight line, that is, Y is modeled as a linear function of X.

Variables

X: explanatory variable

Y: response variable

After data collection, we have pairs of observations:

$$(x_1, y_1), ..., (x_n, y_n)$$

Simple Linear Regression

Model

Then we fit our data set into this simple linear model.

$$Y = \alpha + \beta X + \epsilon, \text{ where } i = 1, 2, ..., n$$

- Residuals: $\epsilon_i \sim N(0, \sigma^2)$, independent
- Estimate y_j by the value of x_j, $\hat{y}_j = \alpha + \beta x_j = E(y_j | x_j)$

Parameters

We need to figure out a way to estimate these parameters:

- α (Intercept): point in which the line intercepts the y-axis
- β (Slope): increase in Y per unit change in X

Simple Linear Regression

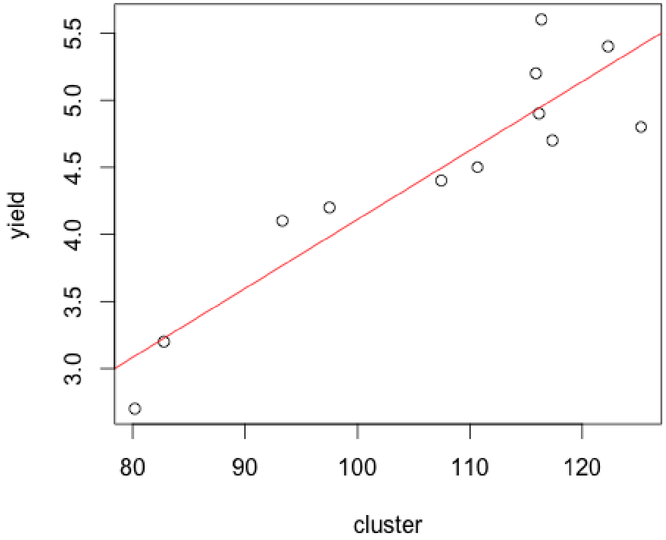
Example

```
> yield <- data$yield
> cluster <- data$cluster.count
> plot(yield ~ cluster)
> lm(yield ~ cluster)

Call:
lm(formula = yield ~ cluster)

Coefficients:
(Intercept)      cluster
   -1.02790      0.05138

> lm.r <- lm(yield ~ cluster)
> abline(lm.r, col='red')
```


yield
5.5
5.0
4.5
4.0
3.5
3.0
80
90
100
110
120
cluster

Example

```
> summary(lm.r)
Call:
lm(formula = yield ~ cluster)

Residuals:
     Min       1Q   Median       3Q      Max
-0.60700 -0.19471 -0.03241  0.23220  0.64874

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.02790    0.78355  -1.312    0.219
cluster      0.05138    0.00725   7.087 3.35e-05 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.3641 on 10 degrees of freedom
Multiple R-squared:  0.834, Adjusted R-squared:  0.8174
F-statistic: 50.23 on 1 and 10 DF,  p-value: 3.347e-05
```

Different Types of Linear Models in R

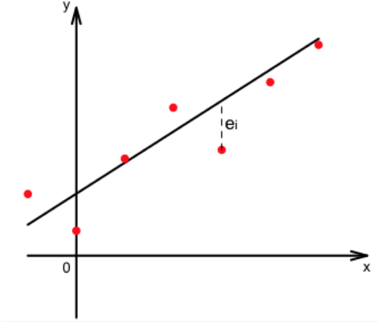
Syntax	Model
$Y \sim X$	$Y = \alpha + \beta X$
$Y \sim -1 + X$	$Y = \beta X$
$Y \sim X + I(X^2)$	$Y = \alpha + \beta_1 X + \beta_2 X^2$
$Y \sim X_1 + X_2$	$Y = \alpha + \beta_1 X_1 + \beta_2 X_2$
$Y \sim X_1 : X_2$	$Y = \alpha + \beta X_1 X_2$
$Y \sim X_1 * X_2$	$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$

Least Squares

We want to find the equation of the line that best fits the data. It means finding α and β such that the fitted values of y_j , given by

$$\hat{y}_j = \alpha + \beta x_j$$

are as close as possible to the observed values y_i, for all $i = 1, 2, ..., n$.



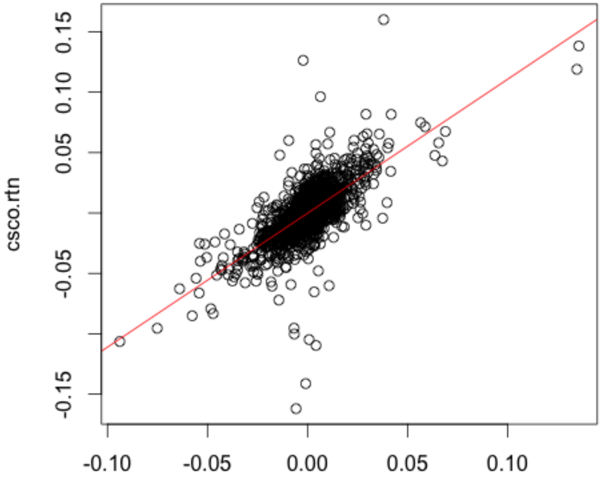
residuals given by:
$\epsilon_i = y_i - \hat{y}_i$

Example on Stock Returns

Example

```
> getSymbols("CSCO")
> getSymbols("DIA")
> csco <- data.frame(CSCO)
> dia <- data.frame(DIA)
> # get last price
> csco.price <- csco$CSCO.Adjusted
> dia.price <- dia$DIA.Adjusted
> # get returns
> csco.rtn<-diff(csco.price, lag = 1)/
          csco.price[-length(csco.price)]
> dia.rtn<-diff(dia.price, lag = 1)/
          dia.price[-length(dia.price)]
> plot(csco.rtn ~ dia.rtn)
> lm1 <- lm(csco.rtn ~ dia.rtn)
> abline(lm1, col = 'red')
```

Example on Stock Returns



Least Squares

Estimation of Parameters

A usual way of calculating α and β is based on the minimization of the sum of the squared residuals, or residual sum of squares (RSS):

$$RSS = \Sigma_{i=1}^{n} \epsilon_i^2$$

$$= \Sigma_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \Sigma_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$$

Least Squares

- In fact, we can derive the formula for α and β given certain data set. It is called normal equation:
$$\theta = (X^T X)^{-1} X^T Y$$
where $\theta = (\alpha, \beta)$

- Another way is to use numerical algorithms calculating the values of α and β: Gradient Descent, Newton's Method.

Gradient Descent

Gradient

Definition

If we have a bivariate function $f(x, y)$, then the partial derivatives, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are the rate of change of f with respect to x and y.

We put them together in a vector, and call it *Gradient of f*:

$$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$$

Gradient

- Of course, if we specify a point P_0 and we can calculate the gradient on that point:

$$\nabla f|_{P_0} = \langle \frac{\partial f}{\partial x}|_{P_0}, \frac{\partial f}{\partial y}|_{P_0} \rangle$$

- For functions with only one variable, the gradient equals to the derivative.
- Similarly, for higher dimensional functions, for example $f(x_1, ..., x_n)$, we have:

$$\nabla f = \langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, ..., \frac{\partial f}{\partial x_n} \rangle$$

Gradient Descent Algorithm

Algorithm: Single Variate Function

```
loop {
```

$$x := x - \alpha \frac{df}{dx}$$

```
    # until converge
}
```

Algorithm: Multi Variate Function

```
loop {
```

$$\mathbf{x} := \mathbf{x} - \alpha \nabla f$$

```
    # until converge
}
```

where α is named “step size” or “learning rate”.

Gradient Descent Algorithm

Analysis:

- if $df/dx > 0$, which means $x_0 > x_{min}$, $x - \alpha\frac{df}{dx} \downarrow$
- if $df/dx < 0$, which means $x_0 < x_{min}$, $x - \alpha\frac{df}{dx} \uparrow$
- x_0 will converge to the extrema in no matter which case.
- The step size α also determines the speed of convergence.

Gradient Descent in R

Let's see how to implement this algorithm in R.

Example (pseudo code)

```
# set initial
# set convergence condition
# loop:
while(1)
{
    # algorithm
    # ...
    # if (converge)
    # {
    #    break
    # }
}
```

Gradient Descent in R

First define the function to be optimized: $f(x) = x^2 - 10x + 3$

Example

```
> # function f(x) = x^2 - 10x + 3
> # x.min = -b/2a = 5
> fx <- function(x) {
+     y = x^2 - 10 * x + 3
+     return (y)
+ }
> # derivative of f(x)
> # df = x^2 - 10
> df <- function(x) {
+     y = x*2 - 10
+     return (y)
+ }
```

Gradient Descent in R

Example

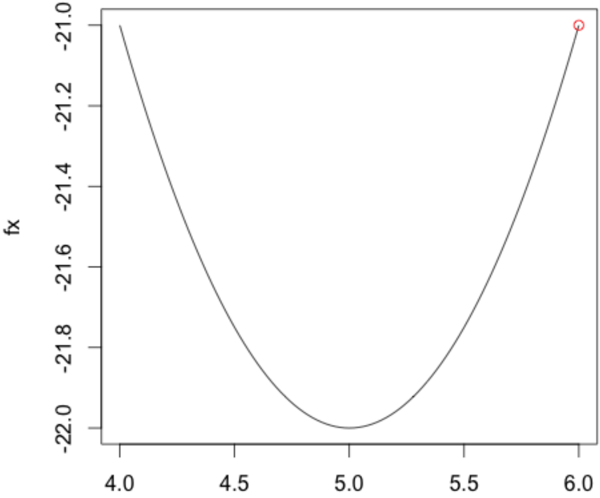
```
> plot(fx, xlim = c(4, 6))

> x0 = 6        # initial value
> points(x0, fx(x0), col = "red")

> alpha = 0.2   # step length

> epsilon = 0.0001 # condition to terminate the algorithm
> # step count
> step = 1
```

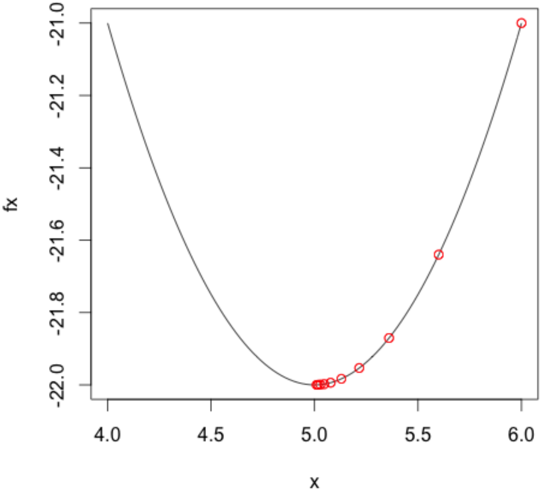
Gradient Descent in R



Example

```
> while(1)
+ {
+     cat("Calculating, step ", step, '\n', sep = "")
+     x1 = x0 - alpha * df(x0)    # update x
+
+     # check convergence
+     if (abs(fx(x1) - fx(x0)) < epsilon)
+     {
+         cat("x = ", x1, '\n', sep='')
+         cat("Final step: ", step, '\n', sep='')
+         break
+     }
+     points(x1, fx(x1), col = "red")
+     x0 = x1
+     step = step + 1
+     Sys.sleep(1.2) # suspend for a while
+ }
```

Gradient Descent in R



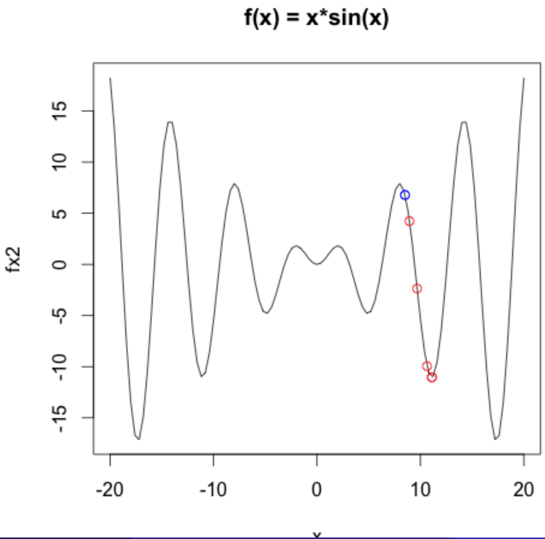
Gradient Descent in R

Two potential issue with this algorithm

- The size of α
- Local extrema.

Local Extrema

Local extrema, see details in the code



Gradient Descent and Linear Regression

Recall that we use *least squares* method to determine the parameters in linear regression.

$$RSS = \Sigma_{i=1}^{n} \epsilon_i^2$$

$$= \Sigma_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \Sigma_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

Thus our target is to minimize:

$$J(\beta) = \frac{1}{2} \Sigma_i (y_i - \beta^T X)^2, \text{ where } \beta = (\beta_0, \beta_1)^T$$

Gradient Descent and Linear Regression

Keep in mind that our algorithm is: $\beta := \beta - \alpha \frac{dJ}{d\beta}$, thus we need to calculate the gradient (derivative) w.r.t β.

Example

```
> x0 <- c(1,1,1,1,1) # column of 1's
> x1 <- c(1,2,3,4,5) # original x-values

> x <- cbind(x0,x1)
> y <- as.matrix(c(3,7,5,11,14))

> # number of elements
> m <- nrow(y)
```

Example

```
# define the gradient function dJ/d_beta:
# 1/m * (y^hat-y))*x where y^hat = x*beta
# in matrix form this is as follows:
> grad <- function(x, y, beta) {
+     gradient <- (1/m) * (t(x) %*% ((x %*% t(beta)) - y))
+     return(t(gradient))
+ }
# define gradient descent update algorithm
> grad.descent <- function(x, maxit){
+     beta <- matrix(c(0, 0), nrow=1) # Initialize
+
+     alpha = .05 # set learning rate
+     for (i in 1:maxit) {
+         beta <- beta - alpha  * grad(x, y, beta)
+     }
+     return(beta)
+ }
```

Project Ideas

R Packages

Packages Covered:

- lubridate
- fBasics
- quantmod
- TRTH
- Rmarkdown

Suggested Packages

- plyr: Tools for splitting, applying and combining data
- car: Applied Regression.
- ggplot2: An implementation of the grammar of graphics in R.
- ...

Connect to Bloomberg

- Package: Rbbg
    - Have to use on a Bloomberg Terminal
    - Dependencies: Java, rJava, Bloomberg Java API V3

Example

```
# sample 1
> conn <- blpConnect()
> bdp(conn, "AMZN US Equity", "NAME")

# sample 2
> securities <- c("AMZN US Equity", "OCN US Equity")
> fields <- c("NAME", "PX_LAST", "TIME", "SETTLE_DT",
"HAS_CONVERTIBLES")
> bdp(conn, securities, fields)
```

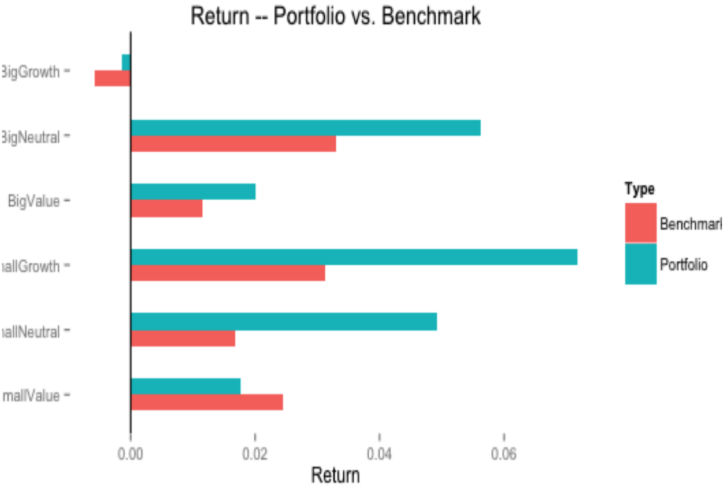
Connect to Database

- Package: RPostgreSQL
  - A database interface and PostgreSQL driver for R
  - It could let you connect to database and execute database command through R
  - Structured Query Language, PostgreSQL, MySQL, SQL Server

Portfolio Perfomance Analysis

- Package: pa
- Objective:
  - Create a portforlio based on the Fama & Frech style factors.
  - Apply Brinson model which is build in the package "pa" to analyze the Brinson.

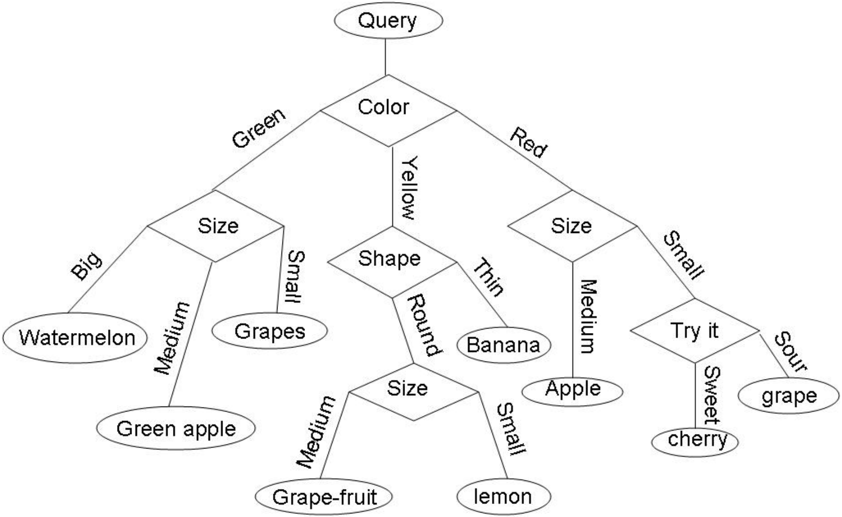
Portfolio Perfomance Analysis



Decision Trees

- Package: Party
  - Decision Trees are used as a classification tool in machine learning.
  - Data sets consist of large number of data points, where each data point is defined based on the set of the attributes and a label.
  - Decision Trees are a predictive method based on a branching series of Boolean tests

Decision Trees



Trading Indicators

- Package: TTR
  - TTR package serves as an excellent platform for developing and back-testing technical indicators.
  - Functions and data to construct technical trading rules with R
  - R topics documented: adjRatios, ADX, aroon, ATR, Bbands, CCI, chaikinAD, chaikinVolatility, CLV, CMF, CMO, DonchianChannel, DPO, DVI, EMV, GMMA, KST, lags, MACD, MFI, OBV, PBands, ROC, rollSFM, RSI, runPrecentRank, runSum, SAR, SMA, stoch, stockSymbols, TDI, TRIX, TTR, ttrc, VHF, volatility, williamsAD, WPR, ZigZag