

AI1103 Assignment 4

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Download all python codes from

https://github.com/Sandeep-L/AI1103_4/blob/main/Assignment_4_AI1103.py

and latex-tikz codes from

https://github.com/Sandeep-L/AI1103_4/blob/main/Assignment_4_AI1103.tex

110 QUESTION

Suppose X has density $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}$, $x > 0$ where $\theta > 0$ is unknown. Define Y as follows:

$Y = k$ if $k \leq X < k + 1$, $k = 0, 1, 2, \dots$

Then the distribution of Y is

- | | |
|-------------|--------------|
| 1) Normal | 3) Poisson |
| 2) Binomial | 4) Geometric |

SOLUTION

Relation between X and Y for $k = 0, 1, 2, \dots$ is given by

$$Y = k \quad k \leq X < k + 1 \quad (0.0.1)$$

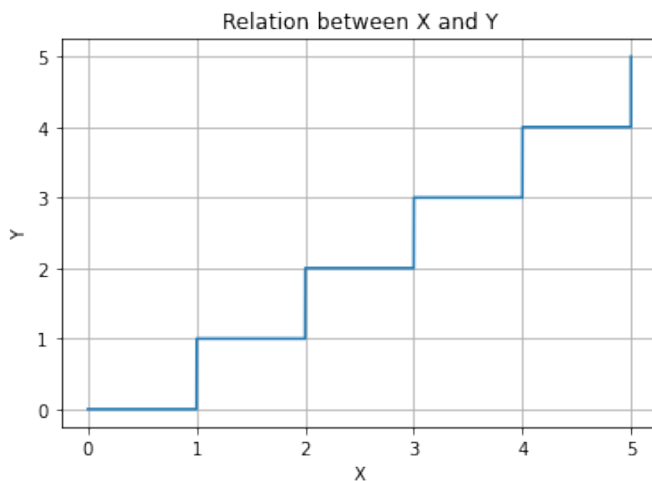


Fig. 4: Relation between X and Y

The P.M.F of Y is given by

$$\Pr(Y = k) = \Pr(k \leq X < k + 1) \quad (0.0.2)$$

$$= \int_k^{k+1} f(x|\theta) dx \quad (0.0.3)$$

$$= \int_k^{k+1} \frac{1}{\theta} e^{-x/\theta} dx \quad (0.0.4)$$

$$= \left[-e^{-x/\theta} \right]_k^{k+1} \quad (0.0.5)$$

$$\Pr(Y = k) = e^{-k/\theta} (1 - e^{-1/\theta}) \quad (0.0.6)$$

Let $p = 1 - e^{-1/\theta}$, from the above equation,

$$\Pr(Y = k) = \left(e^{-1/\theta} \right)^k (1 - e^{-1/\theta}) \quad (0.0.7)$$

$$\Pr(Y = k) = \left(1 - (1 - e^{-1/\theta}) \right)^k (1 - e^{-1/\theta}) \quad (0.0.8)$$

$$\Pr(Y = k) = (1 - p)^k p \quad k = 0, 1, 2, \dots \quad (0.0.9)$$

Therefore, the distribution of Y is 4) Geometric.