AI1103 Assignment 4

Sandeep L – CS20BTECH11044

Download all python codes from

https://https://github.com/Sandeep−L/AI1103_4/blob/main/Assignment_4_AI1103.py

and latex-tikz codes from

https://https://github.com/Sandeep−L/AI1103_4/blob/main/Assignment_4_AI1103.tex

110 Question

Suppose X has density $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}$, $x > 0$ where $\theta > 0$ is unknown. Define Y as follows:
$Y = k$ if $k \le X < k+1$, $k = 0, 1, 2\ldots$
Then the distribution of Y is

1) Normal
2) Binomial
3) Poisson
4) Geometric

Solution

Relation between X and Y for $k = 0, 1, 2\ldots$ is given by

$$Y = k \qquad k \le X < k+1 \tag{0.0.1}$$

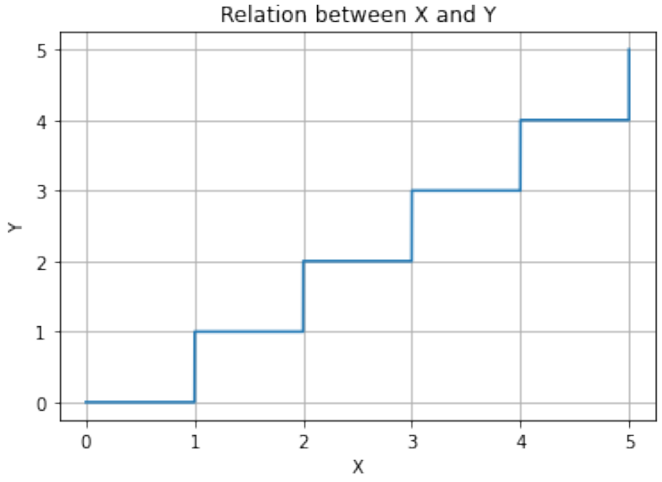

Fig. 4: Relation between X and Y

The P.M.F of Y is given by

$$\Pr(Y = k) = \Pr(k \le X < k+1) \tag{0.0.2}$$
$$= \int_k^{k+1} f(x|\theta)\,dx \tag{0.0.3}$$
$$= \int_k^{k+1} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx \tag{0.0.4}$$
$$= \left[-e^{-\frac{x}{\theta}}\right]_k^{k+1} \tag{0.0.5}$$
$$\Pr(Y = k) = e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{1}{\theta}}\right) \tag{0.0.6}$$

Let $p = 1 - e^{-\frac{1}{\theta}}$, from the above equation,

$$\Pr(Y = y) = \left(e^{-\frac{1}{\theta}}\right)^y\left(1 - e^{-\frac{1}{\theta}}\right) \tag{0.0.7}$$
$$\Pr(Y = y) = \left(1 - \left(1 - e^{-\frac{1}{\theta}}\right)\right)^y\left(1 - e^{-\frac{1}{\theta}}\right) \tag{0.0.8}$$
$$\Pr(Y = y) = (1-p)^y\,p \qquad y = 0, 1, 2\ldots \tag{0.0.9}$$

Therefore, the distribution of Y is 4) Geometric.