

AI1103 Assignment 5

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Download latex-tikz codes from

https://github.com/Sandeep-L/AI1103_5/blob/main/Assignment_5_AI1103.tex

QUESTION 107

Suppose X follows an exponential distribution with parameter $\lambda > 0$. Fix $a > 0$. Define the random variable Y by

$$Y = k, \quad \text{if } ka \leq X < (k+1)a, \\ k = 0, 1, 2, \dots$$

Which of the following statements are correct?

- 1) $\Pr(4 < Y < 5) = 0$
- 2) Y follows an Exponential distribution
- 3) Y follows a Geometric distribution
- 4) Y follows a Poisson distribution

SOLUTION

Definition. Y takes only the value of positive integers defined by

$$Y = \begin{cases} k & ka \leq X < (k+1)a \end{cases} \quad (0.0.1)$$

for $k = 0, 1, 2, \dots$ and $a > 0$

Definition. X follows an exponential distribution with parameter $\lambda > 0$. Therefore, the P.D.F of X , i.e, $f_X(x)$ is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (0.0.2)$$

Relation between X and Y for $k = 0, 1, 2, \dots$ and $a > 0$ is given by

$$Y = k \quad ka \leq X < (k+1)a \quad (0.0.3)$$

Proof. The P.M.F of Y is given by

$$\Pr(Y = k) = \Pr(ka \leq X < (k+1)a) \quad (0.0.4)$$

$$= \int_{ka}^{(k+1)a} f_X(x) dx \quad (0.0.5)$$

$$= \int_{ka}^{(k+1)a} \lambda e^{-\lambda x} dx \quad (0.0.6)$$

$$= \left[-e^{-\lambda x} \right]_{ka}^{(k+1)a} \quad (0.0.7)$$

$$\Pr(Y = k) = e^{-a\lambda k} (1 - e^{-a\lambda}) \quad (0.0.8)$$

Let $p = (1 - e^{-a\lambda})$ in the above equation

$$\Pr(Y = k) = (e^{-a\lambda})^k (1 - e^{-a\lambda}) \quad (0.0.9)$$

$$\Pr(Y = k) = (1 - (1 - e^{-a\lambda}))^k (1 - e^{-a\lambda}) \quad (0.0.10)$$

$$\Pr(Y = k) = (1 - p)^k p \quad k = 0, 1, 2, \dots \quad (0.0.11)$$

□

From (0.0.1), Y doesn't take any value in (4,5). Therefore, option

1) $\Pr(4 < Y < 5) = 0$ is correct.

From (0.0.11), we can say that Y follows **Geometric Distribution**.

Therefore, options

2) Y follows an Exponential distribution &

4) Y follows a Poisson distribution are wrong and option

3) Y follows a Geometric distribution is correct.