AI1103 Assignment 5

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Download all python codes from

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https://https://github.com/Sandeep-L/AI1103_5/
    blob/main/Assignment_5_AI1103.py
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and latex-tikz codes from

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https://https://github.com/Sandeep-L/AI1103_5/
    blob/main/Assignment_5_AI1103.tex
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Question 107

Suppose X follows an exponential distribution with parameter $\lambda > 0$. Fix $a > 0$. Define the random variable Y by
$Y = k, \qquad \text{if } ka \leq X < (k+1)\,a,$
$k = 0, 1, 2 \ldots$
Which of the following statements are correct?

1) $\Pr(4 < Y < 5) = 0$
2) Y follows an Exponential distribution
3) Y follows a Geometric distribution
4) Y follows a Poisson distribution

Solution

Definition. Y takes only the value of positive integers defined by

$$Y = \begin{cases} k & ka \leq X < (k+1)\,a \end{cases} \tag{0.0.1}$$

for $k = 0, 1, 2 \ldots$ and $a > 0$

Definition. X follows an exponential distribution with parameter $\lambda > 0$.Therefore, the P.D.F of X, i.e, $f_X(x)$ is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{0.0.2}$$

Relation between X and Y for $k = 0, 1, 2 \ldots$ and $a > 0$ is given by

$$Y = k \qquad ka \leq X < (k+1)\,a \tag{0.0.3}$$

Lemma 1. The P.M.F of Y, $p_Y(k)$ is given by

$$p_Y(k) = \Pr(Y = k) \tag{0.0.4}$$

From (0.0.1),

$$\Pr(Y = k) = \Pr(ka \leq X < (k+1)\,a) \tag{0.0.5}$$

And Y follows ***Geometric Distribution***, for some p, defined by

$$\Pr(Y = k) = (1-p)^k\,p \qquad k = 0, 1, 2 \ldots \tag{0.0.6}$$

Proof. Let us now prove (0.0.6) from (0.0.5) in *Lemma* 1

$$\Pr(Y = k) = \Pr(ka \leq X < (k+1)\,a) \tag{0.0.7}$$

$$= \int_{ka}^{(k+1)a} f_X(x)\,dx \tag{0.0.8}$$

$$= \int_{ka}^{(k+1)a} \lambda e^{-\lambda x} dx \tag{0.0.9}$$

$$= \left[-e^{-\lambda x}\right]_{ka}^{(k+1)a} \tag{0.0.10}$$

$$\Pr(Y = k) = e^{-a\lambda k}\left(1 - e^{-a\lambda}\right) \tag{0.0.11}$$

Let $p = \left(1 - e^{-a\lambda}\right)$ in the above equation

$$\Pr(Y = k) = \left(e^{-a\lambda}\right)^k \left(1 - e^{-a\lambda}\right) \tag{0.0.12}$$

$$\Pr(Y = k) = \left(1 - \left(1 - e^{-a\lambda}\right)\right)^k \left(1 - e^{-a\lambda}\right) \tag{0.0.13}$$

$$\Pr(Y = k) = (1-p)^k\,p \qquad k = 0, 1, 2 \ldots \tag{0.0.14}$$

□

From (0.0.1), Y doesn't take any value in $(4, 5)$. Therefore, option
1) $\Pr(4 < Y < 5) = 0$ is correct.

From (0.0.14), we can say that Y follows ***Geometric Distribution***.
Therefore, options
2) Y **follows an Exponential distribution** &
4) Y **follows a Poisson distribution** are wrong and option
3) Y **follows a Geometric distribution** is correct.