Probability

Numerical Rep^n of likelihood of an event.

| ML, DL } uses prob.
| — o —
| Prob., stat. ✓
| P

★ N.R. : Chances → Numerically

$0 \leq P \leq 1$ ✓

0% chance ✓

0.7 ✓

100% chance ✓

0.9 ↑

0.1 ↓

★ Likelihood : Chance

★ Event : outcome of Random Experiment
✓

unbiased experiment that has certain o/c associated with it.

eg:- Tossing a coin → getting a Head $= E_1$, " " Tail $= E_2$

R.E. ✓

Event ✗

events

Rolling a dice — 1, 2, 3, 4, 5, 6 → possible events

⇓

R.E.

1. What is Prob. of occurrence of 2 when I roll a dice?

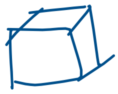
— $$P(\text{Rolling a 2}) = \frac{\cancel{2}\ 1}{6}$$

2. What is Prob. of occurrence of even nos. in Rolling of a dice?

— $$P(\text{Rolling an even}) = \frac{n(\text{Event})}{n(\text{Total})} = \frac{3}{6} = \frac{1}{2}$$

✓ $\{2, 4, 6\}$ → Set of event

✓ $\{1, 2, 3, 4, 5, 6\}$ → Sample Space (Set)

 = $\boxed{\{1, 2, 3, 4, 5, 6\}}$

Sample space

$P(\text{less than } 3)$

$P(\text{even})$

$P(\text{odd})$

$P(\leq 4)$

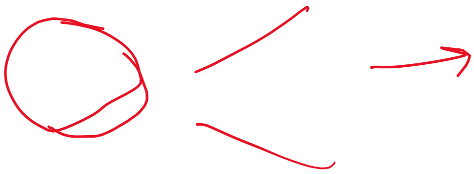
$P(\geq 5)$

$P(1)$

$P(2)$

$P(2 \,\&\, 5)$

⋮



H H H | $\boxed{\frac{1}{2}}$

4th Head;

H T H | $\boxed{1/2}$

T T T | $\boxed{1/2}$

Theorem

1. Additive Thm
2. Multiplicative Theorem
3. Bayes' Theorem

Additive

(2)

30 students → 17 B, 13 G

Grade A → 4 B, 5 G

9

what is the probability of choosing a girl or an A student?

grade A

E_1 = choosing a girl → $P(E_1) = \frac{13}{30}$

E_2 = choosing a grade A student → $P(E_2) = \frac{9}{30}$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$$

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30}$$

= 17/30

dependent event

$E_1 =$ $\frac{1}{52}$ 6♡

$E_2 =$ $\cancel{\frac{1}{52}} \Rightarrow \frac{1}{51}$ 7♢

$\frac{1}{52}$ → card out

52 → $\frac{1}{51}$

51

50

$P(E_1) =$

$P(E_2)$ ✗ → $P(E_2 | E_1)$

$P(E_2 | E_1)$

$$\boxed{P(E_2 | E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)}}$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

$P(B | A) \neq P(A | B)$

A & B

B & A

$$= \frac{\text{Common}}{\text{Already-Happened}}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \qquad P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

$$P(A \text{ and } B) = P(A|B) \times P(B) \quad \text{— (I)}$$

$$P(B \text{ and } A) = P(B|A) \times P(A) \quad \text{— (II)}$$

from (I) & (II)

$$P(A|B)\, P(B) = P(B|A)\, P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}}$$

Spam filtering → Real use case

$$P(\text{spam}|\text{words}) = \boxed{\frac{P(\text{words}|\text{spam})\, P(\text{spam})}{P(\text{words})}}$$

0.9 → sp~

0.4 ← not.

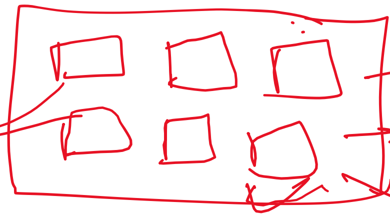
Set of words.

0.2

Prior knowledge.

1

spam



Free

$

Won.

— 0 —

NLP, N.B.

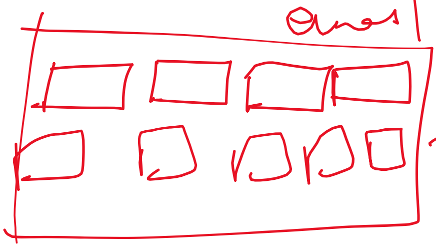
Q.

$P(\text{spam} \mid \text{words}) \Leftrightarrow P(\text{words} \mid \text{spam})$

0.8 → < spam

Building a model to detect spam.

email



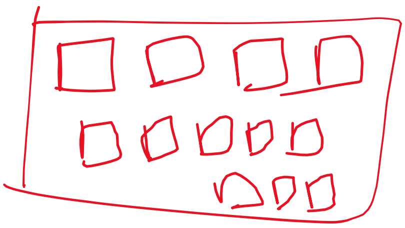
→ spam

P($) → spam

Won

win → spam

— b —

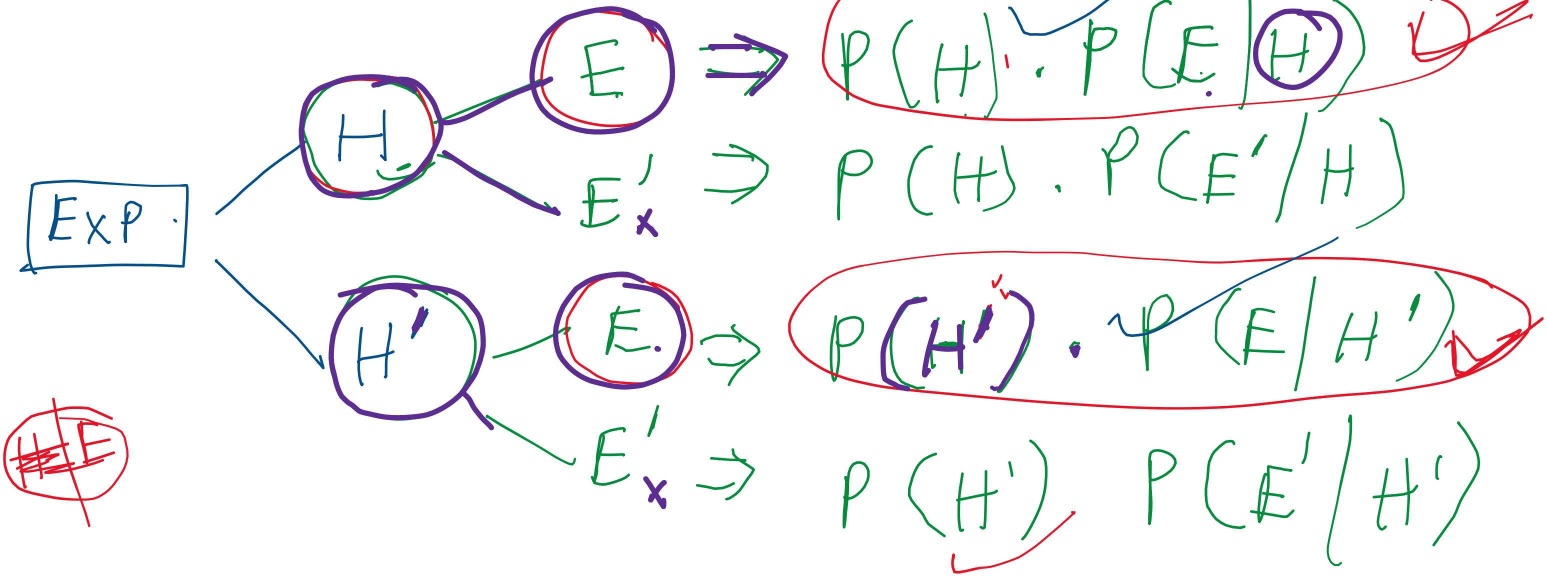


new email

X ✓

$H \Rightarrow$ denotes H has happened

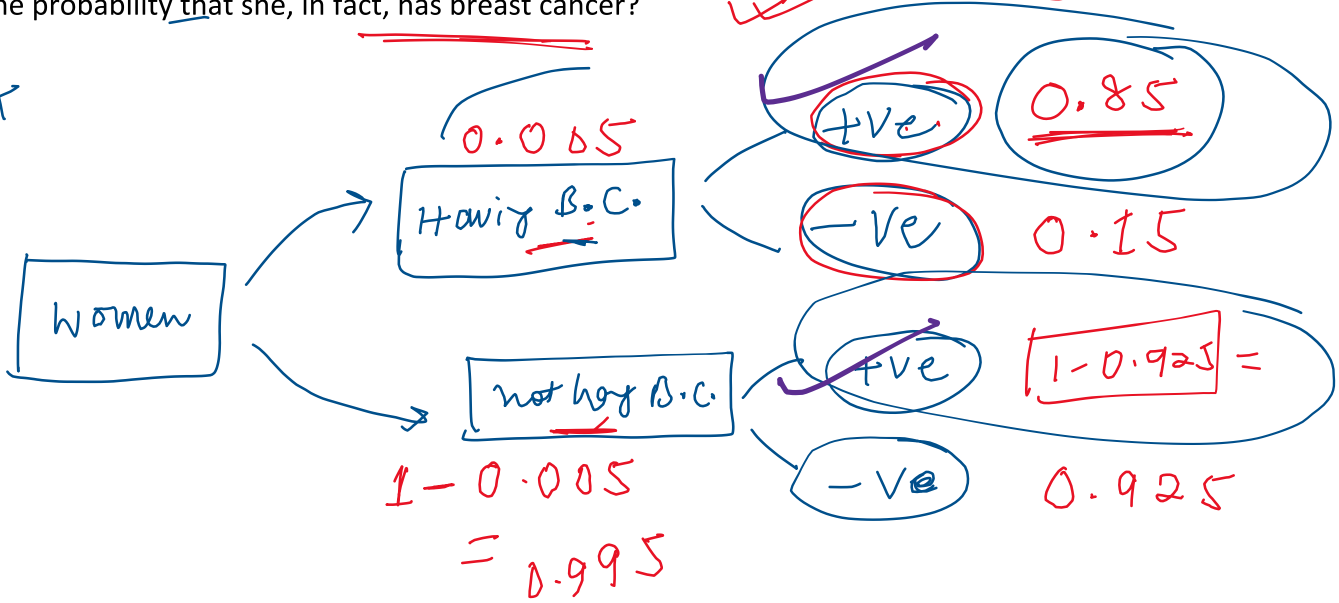
$H' \Rightarrow$ " H has not happened



Tree diagram.

Q: Epidemiologists claim that the probability of breast cancer among Caucasian women in their mid 50s is 0.005. An established test identified people who had breast cancer and those that were healthy. A new mammography test in clinical trials has a probability of 0.85 for detecting cancer correctly. In women without breast cancer, it has a chance of 0.925 for a negative result. If a 55-year-old Caucasian woman tests positive for breast cancer, what is the probability that she, in fact, has breast cancer?

COVID +



$$P(C|P) = \frac{P(P|C) \times P(C)}{P(P)}$$

$$= \frac{P(P|C) \times P(C)}{\boxed{P(C) \cdot P(P|C) + P(C') \cdot P(P|C')}}$$

$$= \frac{0.85 \times 0.005}{0.85 \times 0.005 + 0.075 \times 0.995}$$

$$= \boxed{}$$

95%

D - III

I, II