Lecture Comprehension, Exponential Coordinates of Rotation (Chapter 3.2.3, Part 1 of 2)

TOTAL POINTS 3

1. The orientation of a frame {d} relative to a frame {c} can be represented by a unit rotation axis $\hat{\omega}$ and the distance θ rotated about the axis. If we rotate the frame {c} by θ about the axis $\hat{\omega}$ expressed in the {c} frame, we end up at {d}. The vector $\hat{\omega}$ has 3 numbers and θ is 1 number, but we only need 3 numbers, the exponential coordinates $\hat{\omega}\theta$, to represent {d} relative to {c}, because

1 / 1 point

- (selected) though we use 3 numbers to represent $\hat{\omega}$, $\hat{\omega}$ actually only represents a point in a 2-dimensional space, the 2-dimensional sphere of unit 3-vectors.
- the choice of θ is not independent of $\hat{\omega}$.

✓ **Correct**

2. One reason we use 3x3 rotation matrices (an implicit representation) to represent orientation is because it is a good global representation: there is a unique orientation for each rotation matrix, and vice-versa, and there are no singularities in the representation. In what way does the 3-vector of exponential coordinates fail these conditions? Select all that apply.

1 / 1 point

- [x] There could be more than one set of exponential coordinates representing the same orientation.

✓ **Correct**

If $\hat{\omega}\theta$ is a representation of the orientation, then we could change θ by any integral multiple of 2π and get a different set of exponential coordinates representing the same orientation. If we restrict the exponential coordinate vector to have a magnitude of π or less (a solid sphere in 3-space), then opposite points on the outer surface of the sphere correspond to the same orientation (one corresponding to rotation about an axis by π, the other corresponding to rotation about the negative of the axis by π).

- [ ] Some orientations cannot be represented by exponential coordinates.

3. The vector linear differential equation $\dot{x}(t) = Bx(t)$, where x is a vector and B is a constant square matrix, is solved as $x(t) = e^{Bt}x(0)$, where the matrix exponential e^{Bt} is defined as

1 / 1 point

- (selected) the sum of an infinite series of matrices of the form $(Bt)^0 + Bt + (Bt)^2/2! + (Bt)^3/3! \ldots$.
- the sum of an infinite series of matrices of the form $Bt + Bt/2 + Bt/3 + \ldots$.

✓ **Correct**