Lecture Comprehension, Exponential Coordinates of Rotation (Chapter 3.2.3, Part 2 of 2)

TOTAL POINTS 3

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1. The solution to the differential equation $\dot{p}(t) = \hat{\omega} \times p(t) = [\hat{\omega}]p(t)$ is $p(t) = e^{[\hat{\omega}\theta]}p(0)$, where $p(0)$ is the initial vector and $p(t)$ is the vector after it has been rotated at the angular velocity $\hat{\omega}$ for time $t = \theta$ (where $\hat{\omega}\theta$ are the exponential coordinates). You can think of $R = e^{[\hat{\omega}\theta]}$ as the rotation operation that moves $p(0)$ to $p(t) = p(\theta)$.

1 / 1 point

Which of the following statements is correct? Select all that apply.

- [x] $R_{sb'} = R_{sb}e^{[\hat{\omega}\theta]}$ represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by θ about an axis w represented in the {b} frame as $\hat{\omega}$.

Correct
Multiplication of the rotation operation on the right corresponds to ω being interpreted in the frame of the second subscript, {b}.

- [ ] $R_{sb'} = R_{sb}e^{[\hat{\omega}\theta]}$ represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by θ about an axis w represented in the {s} frame as $\hat{\omega}$.

- [ ] $R_{sb'} = e^{[\hat{\omega}\theta]}R_{sb}$ represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by θ about an axis w represented in the {b} frame as $\hat{\omega}$.

- [x] $R_{sb'} = e^{[\hat{\omega}\theta]}R_{sb}$ represents the orientation of a new frame {b'} relative to {s} after the frame {b} has been rotated by θ about an axis w represented in the {s} frame as $\hat{\omega}$.

Correct
Multiplication of the rotation operation on the left corresponds to ω being interpreted in the frame of the first subscript, {s}.

2. The simple closed-form solution to the infinite series for the matrix exponential when the matrix is an element of $so(3)$ (a skew-symmetric 3x3 matrix) is called what?

1 / 1 point

- ( ) Ramirez's formula.
- (•) Rodrigues' formula.

2. The simple closed-form solution to the infinite series for the matrix exponential when the matrix is an element of $so(3)$ (a skew-symmetric 3x3 matrix) is called what?

1 / 1 point

- ( ) Ramirez's formula.
- (•) Rodrigues' formula.
- ( ) Robertson's formula.

Correct

3. The matrix exponential and the matrix log relate a rotation matrix (an element of $SO(3)$) and the skew-symmetric representation of the exponential coordinates (elements of $so(3)$), which can also be thought of as the $so(3)$ representation of the angular velocity followed for unit time. Which of the following statements is correct? Select all that apply.

1 / 1 point

- [x] exp: $so(3) \to SO(3)$

Correct

The matrix exponential "integrates" the skew-symmetric $so(3)$ representation of an angular velocity for unit time to yield the rotation matrix describing the orientation achieved after rotating from an initial orientation described by the identity matrix.

- [ ] exp: $SO(3) \to so(3)$
- [ ] log: $so(3) \to SO(3)$
- [x] log: $SO(3) \to so(3)$

Correct

The matrix logarithm of a rotation matrix R gives the angular velocity that must be followed for unit time, starting from a frame represented as the identity matrix, to rotate to R. It "differentiates" the net rotational displacement to find the angular velocity that must be followed for unit time.