

Lecture Comprehension, Twists (Chapter 3.3.2, Part 1 of 2)

TOTAL POINTS 3

1. Any instantaneous spatial velocity of a rigid body is equivalent to the motion of the body if it were simultaneously translating along, and rotating about, a **screw** axis $\mathcal{S} = (\mathcal{S}_\omega, \mathcal{S}_v) \in \mathbb{R}^6$. The screw axis is a normalized representation of the direction of motion, and $\dot{\theta}$ represents how fast the body moves in that direction of motion, so that the **twist** is given by $\mathcal{V} = \mathcal{S}\dot{\theta} \in \mathbb{R}^6$. The normalized screw axis for full spatial motions is analogous to the normalized (unit) angular velocity axis for pure rotations.

1 / 1 point

The pitch h of the screw axis is defined as the ratio of the linear speed over the angular speed. Which of the following is true? Select all that apply.

- ☒ If the pitch h is infinite, then $\mathcal{S}_\omega = 0$ and $\|\mathcal{S}_v\| = 1$.

✓ Correct

If the pitch is infinite there is no angular component to the screw axis, so \mathcal{S}_ω must be zero and $\dot{\theta}$ represents the linear speed along the axis, so $\|\mathcal{S}_v\|$ must be 1.

- ☐ If the pitch h is infinite, then $\|\mathcal{S}_\omega\| = 1$ and \mathcal{S}_v is arbitrary.

- ☐ If the pitch h is finite, then $\mathcal{S}_\omega = 0$ and $\|\mathcal{S}_v\| = 1$.

- ☒ If the pitch h is finite, then $\|\mathcal{S}_\omega\| = 1$ and \mathcal{S}_v is arbitrary.

✓ Correct

Since the pitch is finite, there is a nonzero angular component \mathcal{S}_ω to the screw axis. In this case, the speed $\dot{\theta}$ along the screw axis is simply the rotation rate. There are no constraints as to what \mathcal{S}_v could be.

2. You are sitting on a horizontal rotating turntable, like a merry-go-round at an amusement park. It rotates counterclockwise when viewed from above. Your body frame $\{b\}$ has an \hat{x}_b -axis pointing outward (away from the center of the turntable), a \hat{y}_b -axis pointing in the direction the turntable is moving at your location (the direction your eyes are looking), and a \hat{z}_b -axis pointing upward. The turntable is rotating at 0.1 radians per second, and you are sitting 3 meters from the center of the turntable. What is the screw axis $\mathcal{S} = (\mathcal{S}_\omega, \mathcal{S}_v)$ and the twist $\mathcal{V} = (\omega, v)$ expressed in your body frame $\{b\}$? All angular velocities are in radians/second and all linear velocities are in meters/second.

1 / 1 point

- ☐ $\mathcal{S} = (0, 0, 0.1, 0, 0.3, 0)$, $\mathcal{V} = (0, 0, 0.01, 0, 0.03, 0)$
- ☒ $\mathcal{S} = (0, 0, 1, 0, 3, 0)$, $\mathcal{V} = (0, 0, 0.1, 0, 0.3, 0)$
- ☐ $\mathcal{S} = (1, 0, 0, 0, 3, 0)$, $\mathcal{V} = (0.1, 0, 0, 0, 0.3, 0)$

✓ **Correct**

The axis of rotation is aligned with the \hat{z}_b -axis, so \mathcal{S}_ω , the first three elements of \mathcal{S} , must be $(0, 0, 1)$. Rotation about the turntable axis with $\dot{\theta} = 1$ means that the linear motion at $\{b\}$, \mathcal{S}_v , would be 3 units in the \hat{y}_b direction. The twist is just $\mathcal{V} = \mathcal{S}\dot{\theta}$, and $\dot{\theta}$ is equal to 0.1 radians/s.

3. A twist or a screw axis can be represented in any frame. Which of the following statements are true? Select all that apply.

1 / 1 point

☒ A spatial twist is a representation of the twist in the space frame $\{s\}$, and it does not depend on a body frame $\{b\}$.

✓ **Correct**

We only need to define the frame in which the twist (or screw) is being represented. No other frames matter. A "spatial twist" depends on the $\{s\}$ frame.

☒ A body twist is a representation of the twist in the body frame $\{b\}$, and it does not depend on a space frame $\{s\}$.

✓ **Correct**

We only need to define the frame in which the twist (or screw) is being represented. No other frames matter. A "body twist" depends on the $\{b\}$ frame.