Lecture Comprehension, Exponential Coordinates of Rigid-Body Motion (Chapter 3.3.3)

TOTAL POINTS 6

1. Although we use six numbers to represent a screw $\mathcal{S} = (\mathcal{S}_\omega, \mathcal{S}_v)$, the space of all screws is only 5-dimensional. Why? **1 / 1 point**

- ○ $\mathcal{S}_\omega$ must be unit length.
- ○ $\mathcal{S}_v$ must be unit length.
- ◉ Either $\mathcal{S}_\omega$ or $\mathcal{S}_v$ must be unit length.

✓ **Correct**
If both the angular and linear components of the screw are nonzero, then the screw is defined so that $\|\mathcal{S}_\omega\| = 1$.

2. A transformation matrix T_{ab}, representing {b} relative to {a}, can be represented using the 6-vector exponential coordinates $\mathcal{S}\theta$, where $\mathcal{S}$ is a screw axis (represented in {a} coordinates) and θ is the distance followed along the screw axis that displaces {a} to {b}. Which of the following is correct? Select all that apply. **0 / 1 point**

- ☐ $T_{ab} = e^{\mathcal{S}\theta}$
- ☑ $T_{ab} = e^{[\mathcal{S}]\theta}$

✓ **Correct**
θ is just a scalar, so $[\mathcal{S}]\theta = [\mathcal{S}\theta]$.

- ☐ $T_{ab} = e^{[\mathcal{S}\theta]}$
- ☐ $T_{ab} = e^{\mathcal{S}[\theta]}$

You didn't select all the correct answers

3. The matrix representation of the exponential coordinates $\mathcal{S}\theta \in \mathbb{R}^6$ is $[\mathcal{S}\theta]$. What space does $[\mathcal{S}\theta]$ belong to? 1 / 1 point

- $SO(3)$
- $so(3)$
- $SE(3)$
- $se(3)$

Correct
This is the space of matrix representations of twists (and exponential coordinates).

4. $T_{ab'} = T_{ab}e^{[\mathcal{S}\theta]}$ is a representation of the new frame {b'} (relative to {a}) achieved after {b} has followed 1 / 1 point

- the screw axis $\mathcal{S}$, expressed in {b} coordinates, a distance θ.
- the screw axis $\mathcal{S}$, expressed in {a} coordinates, a distance θ.

Correct
Multiplying the matrix exponential on the right means that $\mathcal{S}$ is interpreted as being represented in the frame {b} (the second subscript of T_{ab}).

5. $T_{ab'} = e^{[\mathcal{S}\theta]}T_{ab}$ is a representation of the new frame {b'} (relative to {a}) achieved after {b} has followed 1 / 1 point

- the screw axis $\mathcal{S}$, expressed in {b} coordinates, a distance θ.
- the screw axis $\mathcal{S}$, expressed in {a} coordinates, a distance θ.

Correct
Multiplying the matrix exponential on the left means that $\mathcal{S}$ is interpreted as being represented in the frame {a} (the first subscript of T_{ab}).

6. Which of the following statements is true? Select all that apply.

1 / 1 point

- [x] The matrix exponential maps $[\mathcal{S}\theta] \in se(3)$ to a transformation matrix $T \in SE(3)$, where T is the representation of the frame (relative to {s}) that is achieved by following the screw $\mathcal{S}$ (expressed in {s}) a distance θ from the identity configuration (i.e., a frame initially coincident with {s}).

  ✓ **Correct**

- [x] The matrix exponential maps $[\mathcal{V}] \in se(3)$ to a transformation matrix $T \in SE(3)$, where T is the representation of the frame (relative to {s}) that is achieved by following the twist $\mathcal{V}$ (expressed in {s}) for unit time from the identity configuration (i.e., a frame initially coincident with {s}).

  ✓ **Correct**

  If we choose $\mathcal{V} = \mathcal{S}\theta$, then following the twist $\mathcal{V}$ for unit time is equivalent to following the screw axis $\mathcal{S}$ a distance θ.

- [ ] The matrix log maps an element of $se(3)$ to an element of $SE(3)$.

- [x] The matrix log maps an element of $SE(3)$ to an element of $se(3)$.

  ✓ **Correct**

- [x] There is a one-to-one mapping between twists and elements of $se(3)$.

  ✓ **Correct**