# Carnegie Mellon University

# Architecture Advancement on Transformers

**Large Language Models: Methods and Applications** 

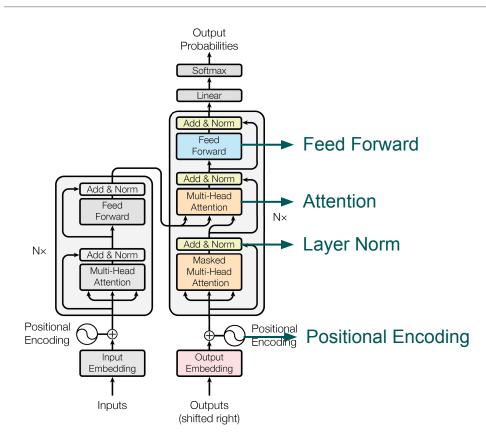
Daphne Ippolito and Chenyan Xiong

# Learning Objectives

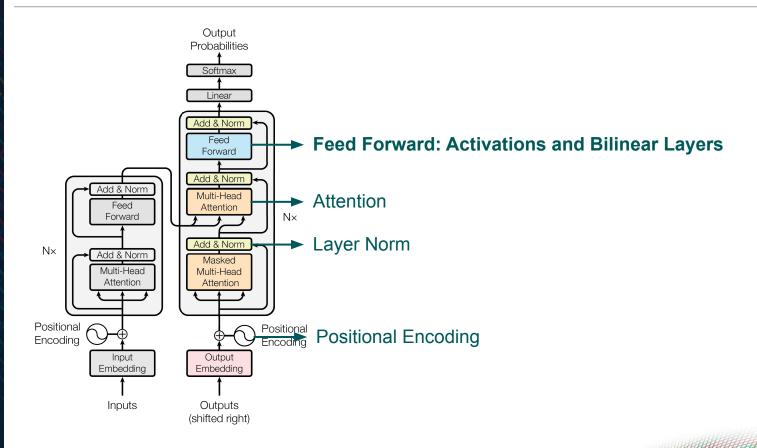
An overview of recent architecture advancements on top of Transformers

- A clear grasp on the details of new architectures
- Understand the motivation and benefits of each architecture upgrades
- Apply the right architecture specifications for target scenarios
- [Optional] Explore new architecture designs in your research

# Places for Improvements



# Places for Improvements



Variants of linear FFN Layers (omitting bias):

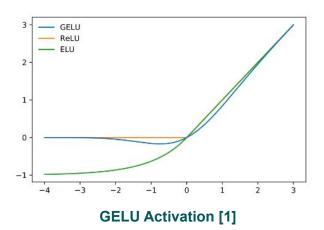
 $FFN_{RELU}(x) = RELU(xW_1)W_2; RELU(xW_1) = \max(0, xW_1)W_2$ 

$$FFN_{RELU}(x) = RELU(xW_1)W_2$$
;  $RELU(xW_1) = max(0, xW_1)W_2$ 

$$FFN_{GELU}(x) = GELU(xW_1)W_2$$
;  $GELU(xW_1) = xP(X < x) = x\Phi(x)$ 

$$FFN_{RELU}(x) = RELU(xW_1)W_2; RELU(xW_1) = \max(0, xW_1)W_2$$

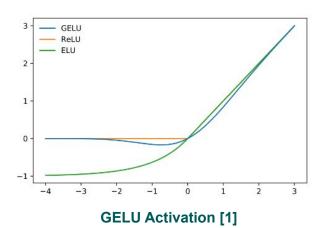
$$FFN_{GELU}(x) = GELU(xW_1)W_2$$
;  $GELU(xW_1) = xP(X < x) = x\Phi(x)$ 



$$FFN_{RELU}(x) = RELU(xW_1)W_2; RELU(xW_1) = \max(0, xW_1)W_2$$

$$FFN_{GELU}(x) = GELU(xW_1)W_2$$
;  $GELU(xW_1) = xP(X < x) = x\Phi(x)$ 

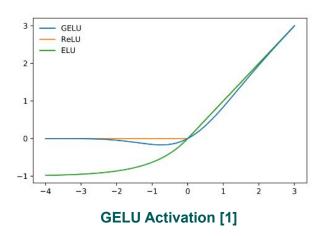
$$FFN_{Switch}(x) = Swish_1(x\mathbf{W}_1)\mathbf{W}_2; Swish_{\beta}(x\mathbf{W}_1) = xSigmod(\beta x)$$

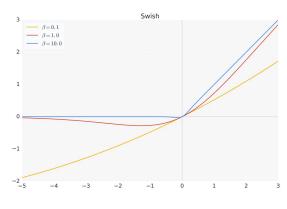


$$FFN_{RELU}(x) = RELU(xW_1)W_2$$
;  $RELU(xW_1) = max(0, xW_1)W_2$ 

$$FFN_{GELU}(x) = GELU(xW_1)W_2$$
;  $GELU(xW_1) = xP(X < x) = x\Phi(x)$ 

$$FFN_{Switch}(x) = Swish_1(x\mathbf{W}_1)\mathbf{W}_2; Swish_\beta(x\mathbf{W}_1) = xSigmod(\beta x)$$





**Switch Activation [2]** 

Bilinear FFNs (Omitting Bias):

 $FFN_{Bilinear}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$ . Two FFN with componentwise product

Bilinear FFNs (Omitting Bias):

```
FFN_{Bilinear}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2. Two FFN with componentwise product
```

$$FFN_{ReGLU}(x) = (RELU(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$$
. Adding RELU activation on one FFN

$$FFN_{GEGLU}(x) = (GELU(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$$
. Adding GELU activation on one FFN

$$FFN_{SwiGLU}(x) = (Swish_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$$
. Adding Swish activation on one FFN

Bilinear FFNs (Omitting Bias):

 $FFN_{Bilinear}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$ . Two FFN with componentwise product

 $FFN_{ReGLU}(x) = (RELU(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$ . Adding RELU activation on one FFN

 $FFN_{GEGLU}(x) = (GELU(xW) \cdot xV)W_2$ . Adding GELU activation on one FFN

 $FFN_{SwiGLU}(x) = (Swish_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$ . Adding Swish activation on one FFN

With reduced hidden dimension of the projections to keep parameter count the same

Training Steps	65,536	524,288	
$\overline{\text{FFN}_{\text{ReLU}}(baseline)}$	1.997 (0.005)	1.677	
$\mathrm{FFN}_{\mathrm{GELU}}$	$1.983\ (0.005)$	1.679	
$\mathrm{FFN}_{\mathrm{Swish}}$	$1.994 \ (0.003)$	1.683	
$\mathrm{FFN}_{\mathrm{GLU}}$	1.982 (0.006)	1.663	
$\mathrm{FFN}_{\mathrm{Bilinear}}$	$1.960 \ (0.005)$	1.648	
$\mathrm{FFN}_{\mathrm{GEGLU}}$	<b>1.942</b> (0.004)	1.633	Improved
$\mathrm{FFN}_{\mathrm{SwiGLU}}$	<b>1.944</b> (0.010)	1.636	speed-quality
$\mathrm{FFN}_{\mathrm{ReGLU}}$	1.953 (0.003)	1.645	

T5 base Perplexity at Pretraining Steps [3]

Bilinear FFNs (Omitting Bias):

 $FFN_{Bilinear}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$ . Two FFN with componentwise product

 $FFN_{ReGLU}(x) = (RELU(xW) \cdot xV)W_2$ . Adding RELU activation on one FFN

 $FFN_{GEGLU}(x) = (GELU(xW) \cdot xV)W_2$ . Adding GELU activation on one FFN

 $FFN_{SwiGLU}(x) = (Swish_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$ . Adding Swish activation on one FFN

With reduced hidden dimension of the projections to keep parameter count the same

Training Steps	65,536	524,288	
$\overline{\text{FFN}_{\text{ReLU}}(baseline)}$	1.997 (0.005)	1.677	_
$\mathrm{FFN}_{\mathrm{GELU}}$	$1.983\ (0.005)$	1.679	
$\mathrm{FFN}_{\mathrm{Swish}}$	$1.994 \ (0.003)$	1.683	
$\mathrm{FFN}_{\mathrm{GLU}}$	1.982 (0.006)	1.663	_
$\mathrm{FFN}_{\mathrm{Bilinear}}$	$1.960 \ (0.005)$	1.648	
$\mathrm{FFN}_{\mathrm{GEGLU}}$	<b>1.942</b> (0.004)	1.633	<b>Improved</b>
$\mathrm{FFN}_{\mathrm{SwiGLU}}$	<b>1.944</b> (0.010)	1.636	speed-quality
$\mathrm{FFN}_{\mathrm{ReGLU}}$	1.953 (0.003)	1.645	

"We offer no explanation as to why these architectures seem to work; we attribute their success, as all else, to divine benevolence"---Noam Shazeer. 2017

T5 base Perplexity at Pretraining Steps [3]

Bilinear FFNs (Omitting Bias):

 $FFN_{Bilinear}(x) = (x\mathbf{W} \cdot x\mathbf{V})\mathbf{W}_2$ . Two FFN with componentwise product

 $FFN_{ReGLU}(x) = (RELU(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$ . Adding RELU activation on one FFN

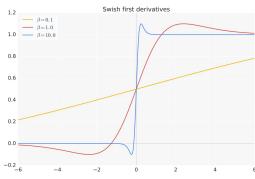
 $FFN_{GEGLU}(x) = (GELU(xW) \cdot xV)W_2$ . Adding GELU activation on one FFN

 $FFN_{SwiGLU}(x) = (Swish_1(x\mathbf{W}) \cdot x\mathbf{V})\mathbf{W}_2$ . Adding Swish activation on one FFN

With reduced hidden dimension of the projections to keep parameter count the same

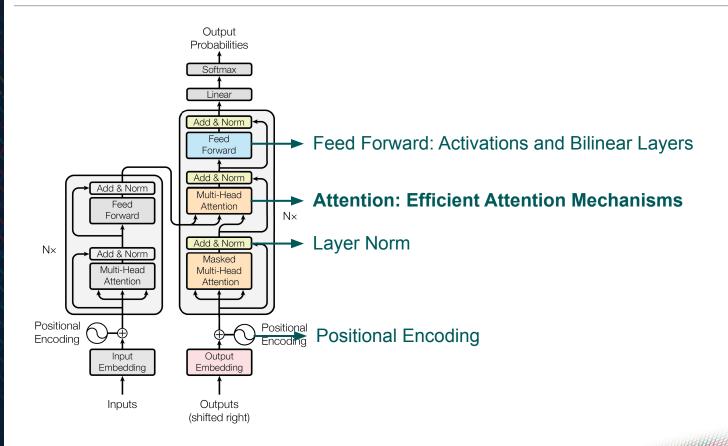
Training Steps	65,536	$524,\!288$	}
$\overline{\text{FFN}_{\text{ReLU}}(baseline)}$	1.997 (0.005)	1.677	_
$\mathrm{FFN}_{\mathrm{GELU}}$	$1.983\ (0.005)$	1.679	
$\mathrm{FFN}_{\mathrm{Swish}}$	$1.994 \ (0.003)$	1.683	
$\mathrm{FFN}_{\mathrm{GLU}}$	1.982 (0.006)	1.663	_
$\mathrm{FFN}_{\mathrm{Bilinear}}$	$1.960 \ (0.005)$	1.648	
$\mathrm{FFN}_{\mathrm{GEGLU}}$	<b>1.942</b> (0.004)	1.633	<b>Improved</b>
$\mathrm{FFN}_{\mathrm{SwiGLU}}$	<b>1.944</b> (0.010)	1.636	speed-quality
$\mathrm{FFN}_{\mathrm{ReGLU}}$	1.953 (0.003)	1.645	





Switch Gradients [2]

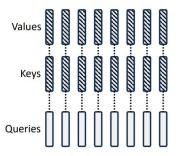
### Places for Improvements



#### Standard Multi-Head Attention

```
\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_H^K, \mathbf{V} \mathbf{W}_H^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}
```

#### Multi-Head Attention (MHA)



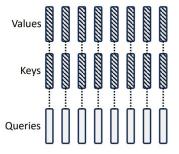
#### Standard Multi-Head Attention

$$\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_H^K, \mathbf{V} \mathbf{W}_H^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}$$

# Inputs and outputs of each layer are the same dimensions:

 $\mathbf{Q} \in \mathbb{R}^{T imes d_{ ext{model}}}$   $\mathbf{K} \in \mathbb{R}^{T imes d_{ ext{model}}}$   $\mathbf{V} \in \mathbb{R}^{T imes d_{ ext{model}}}$ 

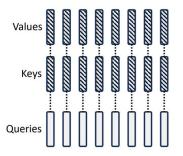
#### Multi-Head Attention (MHA)



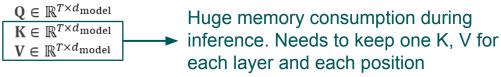
#### Standard Multi-Head Attention

$$\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_H^K, \mathbf{V} \mathbf{W}_H^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}$$

#### Multi-Head Attention (MHA)

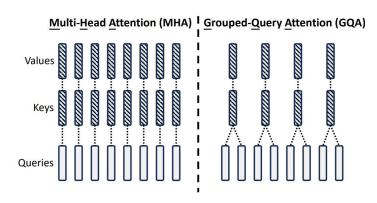


### Inputs and outputs of each layer are the same dimensions:



Grouped-Query Attention: Divide Q in G groups, and share K, V in the same group [4]

```
\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_G^K, \mathbf{V} \mathbf{W}_G^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}
```



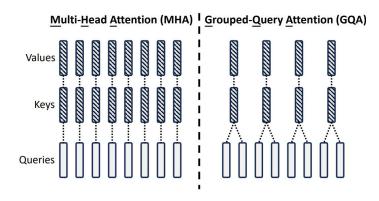
Grouped-Query Attention: Divide Q in G groups, and share K, V in the same group [4]

$$\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_G^K, \mathbf{V} \mathbf{W}_G^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}$$

Inputs and outputs of each layer are the same dimensions:

$$\mathbf{Q} \in \mathbb{R}^{T \times d_{\mathrm{model}}}$$
 $\mathbf{K} \in \mathbb{R}^{T/(\frac{H}{G}) \times d_{\mathrm{model}}}$ 

$$\mathbf{V} \in \mathbb{R}^{T/(\frac{H}{G}) \times d_{\mathrm{model}}}$$

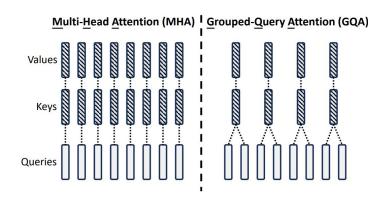


Grouped-Query Attention: Divide Q in G groups, and share K, V in the same group [4]

$$\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_G^K, \mathbf{V} \mathbf{W}_G^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}$$

Inputs and outputs of each layer are the same dimensions:



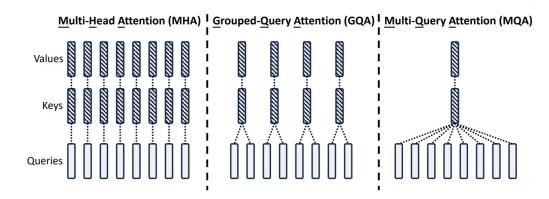


Multi-Query Attention: Single K, V for all Q heads [5]

```
\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}
```

Inputs and outputs of each layer are the same dimensions:

 $\mathbf{Q} \in \mathbb{R}^{T imes d_{ ext{model}}}$   $\mathbf{K} \in \mathbb{R}^{T/H imes d_{ ext{model}}}$   $\mathbf{V} \in \mathbb{R}^{T/H imes d_{ ext{model}}}$ 

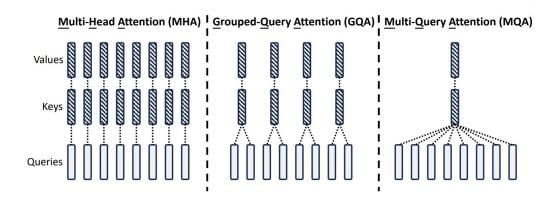


Multi-Query Attention: Single K, V for all Q heads [5]

```
\begin{aligned} \text{head}_1 &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_1^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ & \vdots \\ \text{head}_H &= \text{Attention} \big( \mathbf{Q} \mathbf{W}_H^Q, \mathbf{K} \mathbf{W}_1^K, \mathbf{V} \mathbf{W}_1^V \big) \\ \text{MultiHeadAtt} \big( \mathbf{Q}, \mathbf{K}, \mathbf{V} \big) &= \\ \text{Concat} \big( \text{head}_1, \dots, \text{head}_H \big) \end{aligned}
```

Inputs and outputs of each layer are the same dimensions:



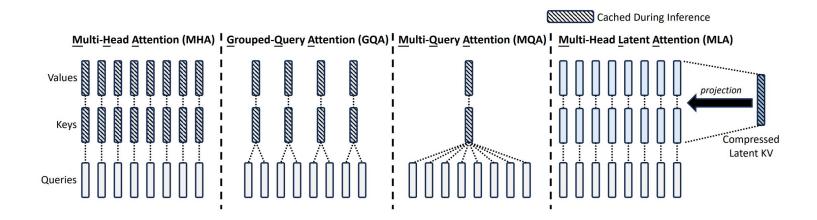


Multi-Head Latent Attention: Project K, V into a lower dimension latent vector [6]

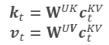
$$k_t = \mathbf{W}^{UK} c_t^{KV} v_t = \mathbf{W}^{UV} c_t^{KV}$$

 $c_t^{KV} = \mathbf{W}^{DKV} \mathbf{h}_t$  Only latent vector to store

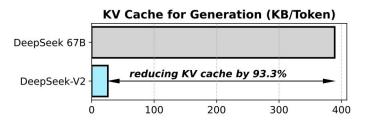
Recovery of k, v can be merged with q in attention layer operations

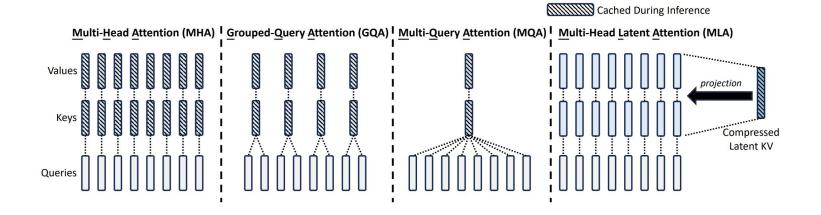


Multi-Head Latent Attention: Project K, V into a lower dimension latent vector [6]



 $c_t^{KV} = \mathbf{W}^{DKV} \mathbf{h}_t$  Only latent vector to store



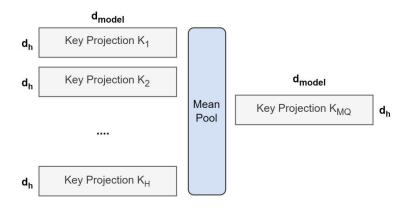


How to use efficient attention mechanisms:

- Pretraining directly with updated architecture
- Or use it as a compression method to speed up a rich multi-head attention model

How to use efficient attention mechanisms:

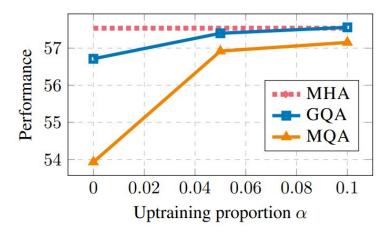
- Pretraining directly with updated architecture
- Or use it as a compression method to speed up a rich multi-head attention model



Mean Pooling Multi-Head Attention to Grouped-Query Attention [4]

#### Performance:

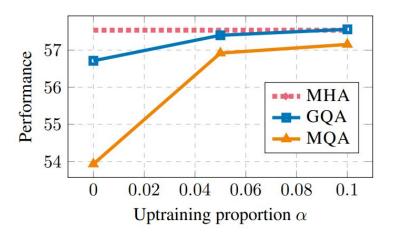
- Recovering similar effectiveness as multi-head attention
- Significantly improve generation speed



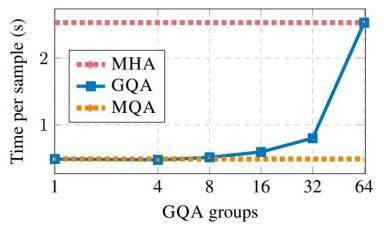
Performance of Grouped-Query Attention Adapted from Multi-Head Attention [4]

#### Performance:

- Recovering similar effectiveness as multi-head attention
- Significantly improve generation speed

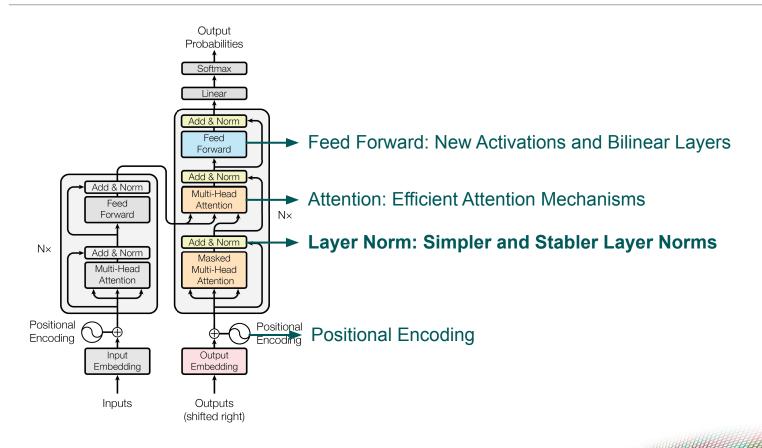


Performance of Grouped-Query Attention Adapted from Multi-Head Attention [4]



Generation Efficiency with Grouped-Query
Attention [4]

### Places for Improvements



The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter started from 1

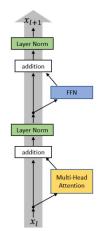
Align the outputs to standard distributions to improve training convergence and stability

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter started from 1

Align the outputs to standard distributions to improve training convergence and stability



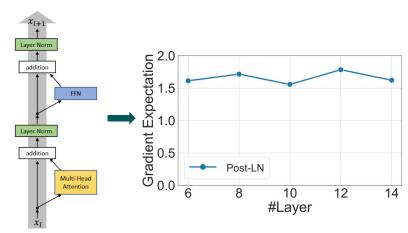
Post-LN: After Residual Sum

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

Learnable parameter started from 1

Align the outputs to standard distributions to improve training convergence and stability



Post-LN: After Residual Sum

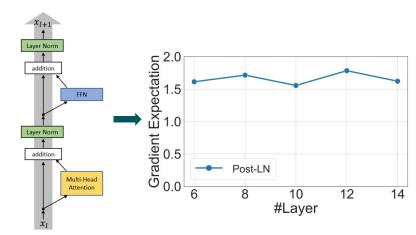
Large gradient L2 norm, leading to unstable training at large scale [6]

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

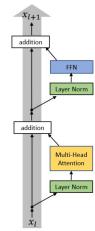
Learnable parameter started from 1

Align the outputs to standard distributions to improve training convergence and stability



Post-LN: After Residual Sum

Large gradient L2 norm, leading to unstable training at large scale [6]



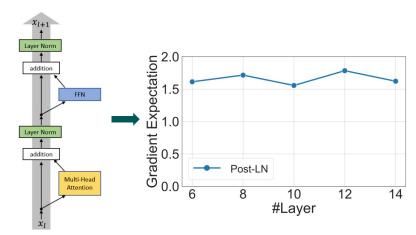
Pre-LN: Before Residual Sum

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

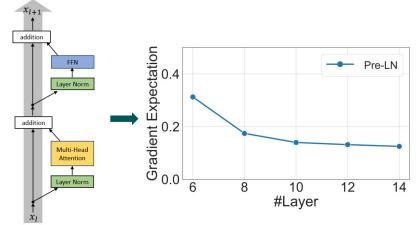
Learnable parameter started from 1

Align the outputs to standard distributions to improve training convergence and stability



Post-LN: After Residual Sum

Large gradient L2 norm, leading to unstable training at large scale [6]



Pre-LN: Before Lower gradient L2 norm, improved Residual Sum training stability [6]

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

## Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

RMSNorm: Only rescaling, no recentering [7]

$$LN(x_i) = \frac{x_i}{RMS(\mathbf{x})} g_i; RMS(\mathbf{x}) = \sqrt{\frac{1}{d} (x_i)^2}$$

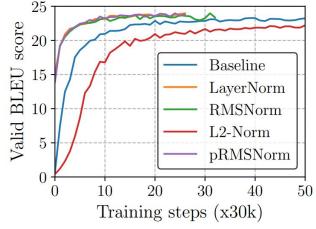
## Layernorm: Simpler and Stabler Layernorms

The Layer Normalization Layer [5]:

$$LN(x_i) = \frac{(x_i - \mu)}{\sigma} g_i; \quad \mu = \frac{1}{d} \sum_i x_i, \sigma = \sqrt{\frac{1}{d} \sum_i (x_i - \mu)^2}$$

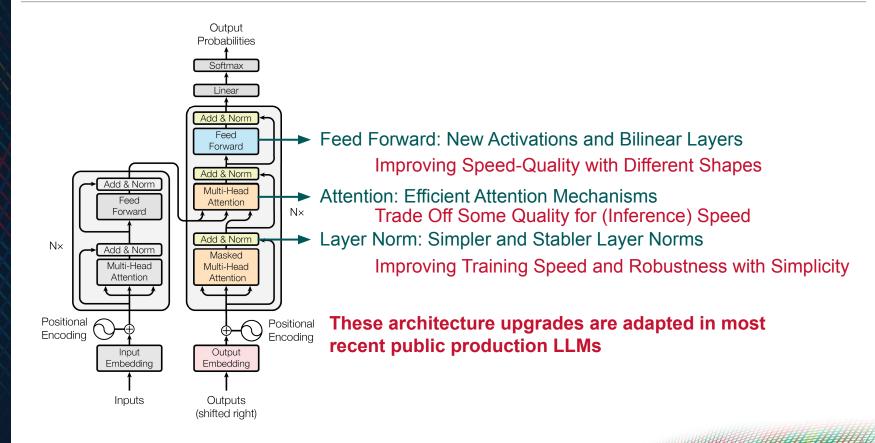
RMSNorm: Only rescaling, no recentering [7]

$$LN(x_i) = \frac{x_i}{RMS(\mathbf{x})} g_i; RMS(\mathbf{x}) = \sqrt{\frac{1}{d} (x_i)^2}$$



Better Convergence Rate on Machine Translation and Many NLP Tasks [7]

### Places for Improvements: Recap



Most Transformer architecture upgrades are for efficiency and large-scale learning stability.

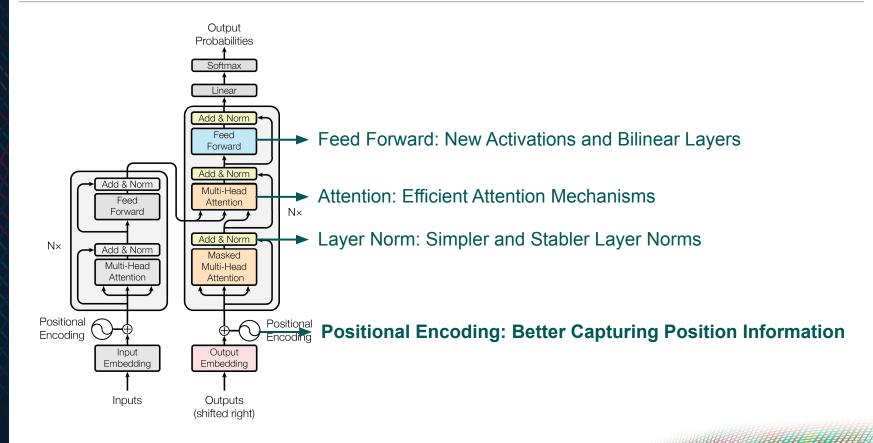
Simplicity is often the winner.

Most Transformer architecture upgrades are for efficiency and large-scale learning stability.

Simplicity is often the winner.

Why?

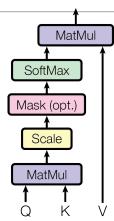
#### Places for Improvements



Transformer itself is position agnostic:

attention output at position 
$$j = \sum_{i=1}^{T} score(\mathbf{q}_{j}, \mathbf{k}_{i}) \cdot \mathbf{v}_{i}$$

$$score(\mathbf{q}_{j}, \mathbf{k}_{i}) = \frac{\mathbf{q}_{j} \cdot \mathbf{k}_{i}}{\sqrt{d_{k}}}$$



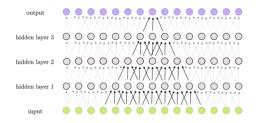
Transformer itself is position agnostic:

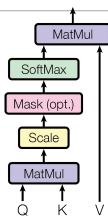
attention output at position 
$$j = \sum_{i=1}^{T} score(\mathbf{q}_{j}, \mathbf{k}_{i}) \cdot \mathbf{v}_{i}$$

$$score(\mathbf{q}_{j}, \mathbf{k}_{i}) = \frac{\mathbf{q}_{j} \cdot \mathbf{k}_{i}}{\sqrt{d_{k}}}$$

In comparison







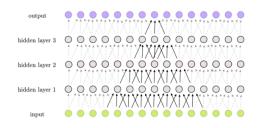
Transformer itself is position agnostic:

attention output at position 
$$j = \sum_{i=1}^{T} score(\mathbf{q}_{j}, \mathbf{k}_{i}) \cdot \mathbf{v}_{i}$$

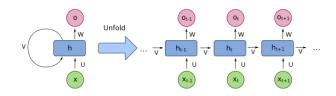
$$score(\mathbf{q}_{j}, \mathbf{k}_{i}) = \frac{\mathbf{q}_{j} \cdot \mathbf{k}_{i}}{\sqrt{d_{k}}}$$

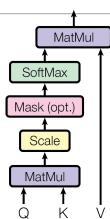
In comparison





#### RNN has sequential prior:



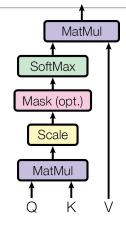


Transformer itself is position agnostic:

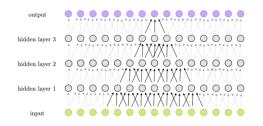
attention output at position 
$$j = \sum_{i=1}^{T} score(\mathbf{q}_{j}, \mathbf{k}_{i}) \cdot \mathbf{v}_{i}$$

$$score(\mathbf{q}_{j}, \mathbf{k}_{i}) = \frac{\mathbf{q}_{j} \cdot \mathbf{k}_{i}}{\sqrt{d_{k}}}$$

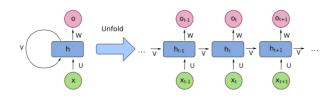
In comparison



#### CNN has locality prior:



#### RNN has sequential prior:



Position encoding adds positional information which is useful for language

Additive Position Encoding: add the positional information in the token embedding layer

$$x' = x + p_{pos}$$

Position Embedding at position pos

Additive Position Encoding: add the positional information in the token embedding layer

$$x' = x + p_{pos}$$

Position Embedding at position pos

Sinusoid position embedding

$$p_{pos,2i} = sin(pos/10000^{2i/d})$$
  

$$p_{pos,2i+1} = cos(pos/10000^{2i/d})$$

Additive Position Encoding: add the positional information in the token embedding layer

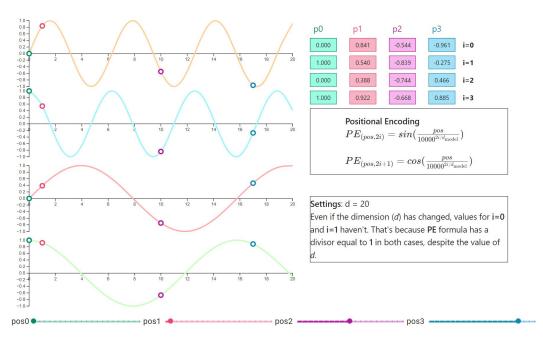
$$x' = x + p_{pos}$$

Position Embedding at position pos

Sinusoid position embedding

$$p_{pos,2i} = sin(pos/10000^{2i/d})$$
  

$$p_{pos,2i+1} = cos(pos/10000^{2i/d})$$



https://erdem.pl/2021/05/understanding-positional-encoding-in-transformers

Additive Position Encoding: add the positional information in the token embedding layer

 $x' = x + p_{pos}$ Adding different values based on positions Position Embedding at position pos p0 Sinusoid position embedding 0.000 0.841 -0.544 -0.961 1.000 -0.839 -0.275  $p_{pos,2i} = \sin(pos/10000^{2i/d})$ 0.000 0.885  $p_{pos,2i+1} = cos(pos/10000^{2i/d})$ 0.6 -**Positional Encoding**  $PE_{(pos,2i)} = sin(rac{pos}{10000^{2i/d_{
m model}}})$ Different wavelength at -0.4 --0.6 --0.8 --1.0 different embedding  $PE_{(pos,2i+1)} = cos(rac{pos}{10000^{2i/d_{
m model}}})$ 1.0 -0.8 -0.6 -0.4 -0.2 dimension to capture different relative positions Settings: d = 20 Even if the dimension (d) has changed, values for i=0and i=1 haven't. That's because PE formula has a divisor equal to 1 in both cases, despite the value of -0.6 --0.8 -

https://erdem.pl/2021/05/understanding-positional-encoding-in-transformers

Additive Position Encoding: add the positional information in the token embedding layer

$$x' = x + p_{pos}$$

Position Embedding at position pos

Fully Learned Embeddings (e.g., in BERT)

$$p_{pos} = Embedding(pos)$$

One embedding vector for each pos

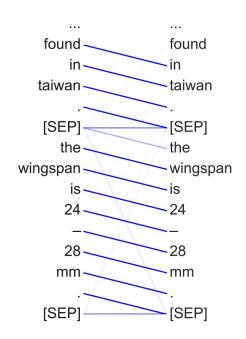
Additive Position Encoding: add the positional information in the token embedding layer

$$x' = x + \underline{p}_{pos}$$
  
Position Embedding at position pos

Fully Learned Embeddings (e.g., in BERT)

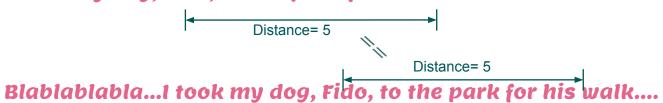
$$p_{pos} = Embedding(pos)$$

One embedding vector for each pos



Learned some strong position-based attention patterns [8]

Language Prior: Only relative positions matters in language I took my dog, Fido, to the park for his walk....



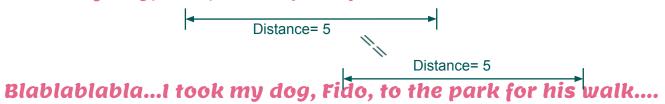
Language Prior: Only relative positions matters in language I took my dog, Fido, to the park for his walk....



Encode relative position information in attention mechanism:

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

Language Prior: Only relative positions matters in language I took my dog, Fido, to the park for his walk....



Encode relative position information in attention mechanism:

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

Using relative position embeddings [9]:

Attention 
$$score(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}} + \underline{b_{i-j}}$$
Trainable Relative Position Bias

56

Language Prior: Only relative positions matters in language I took my dog, Fido, to the park for his walk....

Encode relative position information in attention mechanism:

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

Using relative position embeddings [9]:

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}} + \underline{b_{i-j}}$$

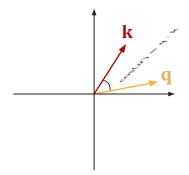
Trainable Relative Position Bias

Model	Position Information	<b>EN-DE BLEU</b>	<b>EN-FR BLEU</b>
Transformer (base)	Absolute Position Representations	26.5	38.2
Transformer (base)	Relative Position Representations	26.8	38.7
Transformer (big)	Absolute Position Representations	27.9	41.2
Transformer (big)	Relative Position Representations	29.2	41.5

Performance of Relative Position Embedding on Machine Translation [10]

Geometry of dot product in attention mechanism

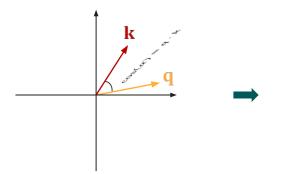
Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$



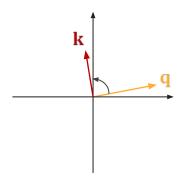
Attention score roughly as cosine of the vectors, assuming unit-lengths.

Geometry of dot product in attention mechanism

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$



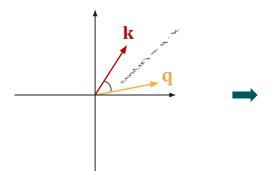




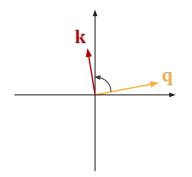
Lower attention importance if positions are far: rotating away

Geometry of dot product in attention mechanism

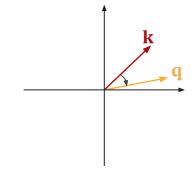
Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = \frac{\mathbf{q}_j \cdot \mathbf{k}_i}{\sqrt{d_k}}$$



Attention score roughly as cosine of the vectors, assuming unit-lengths.



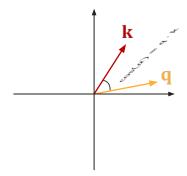
Lower attention importance if positions are far: rotating away



Higher attention importance if positions are close: rotating close

How to make the rotation only depend on relative positions?

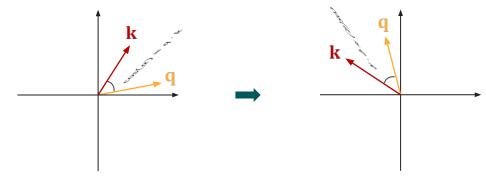
Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$



Attention score roughly as cosine of the vectors, assuming unit-lengths.

How to make the rotation only depend on relative positions?

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$

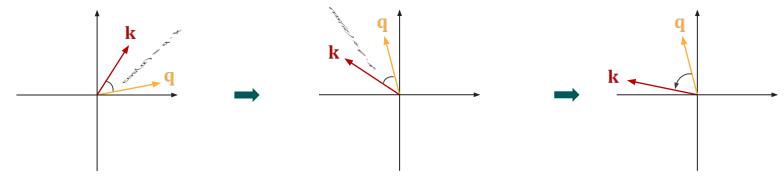


Attention score roughly as cosine of the vectors, assuming unit-lengths.

Rotating vectors together for the same disagrees based on positions changes

How to make the rotation only depend on relative positions?

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = g(\mathbf{q}_j, \mathbf{k}_i, i - j)$$



Attention score roughly as cosine of the vectors, assuming unit-lengths.

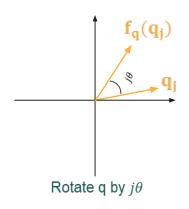
Rotating vectors together for the same disagrees based on positions changes

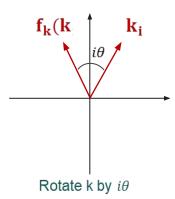
Position based prior holds the same at new absolute positions

Incorporate the vector rotation in the attention mechanism (2d space) [11]:

$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} \begin{pmatrix} q_j^1 \\ q_j^2 \end{pmatrix}$$

$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} \begin{pmatrix} q_j^1 \\ q_i^2 \end{pmatrix} \qquad f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} \begin{pmatrix} k_i^1 \\ k_i^2 \end{pmatrix} \qquad \theta_k = (1/10000^{2(i-1)/d})$$



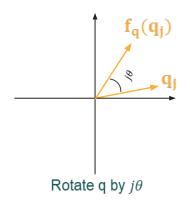


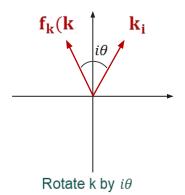
Incorporate the vector rotation in the attention mechanism (2d space) [11]:

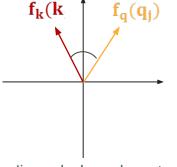
$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} {q_j^1 \choose q_j^2} \qquad f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} {k_i^1 \choose k_i^2} \qquad \theta_k = (1/10000^{2(i-1)/d})$$

$$f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} {k_i^1 \choose k_i^2}$$

$$\theta_k = (1/10000^{2(i-1)/d})$$

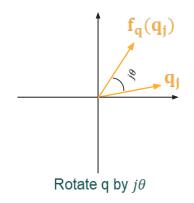


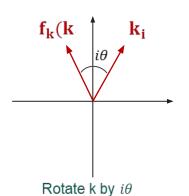


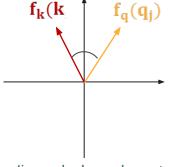


Incorporate the vector rotation in the attention mechanism (2d space) [11]:

$$f_q(q_j) = \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix} {q_j^1 \choose q_i^2} \qquad f_k(k_i) = \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} {k_i^1 \choose k_i^2} \qquad \theta_k = (1/10000^{2(i-1)/d})$$







Attention only depends on i - j

Attention score by the dot prod of rotated vectors:

Attention score 
$$(\mathbf{q}_j, \mathbf{k}_i) = f_q(q_j) \cdot f_k(k_i) = \begin{pmatrix} q_j^1 \\ q_i^2 \end{pmatrix}^T \begin{pmatrix} \cos j\theta & -\sin j\theta \\ \sin j\theta & \cos j\theta \end{pmatrix}^T \begin{pmatrix} \cos i\theta & -\sin i\theta \\ \sin i\theta & \cos i\theta \end{pmatrix} \begin{pmatrix} k_i^1 \\ k_i^2 \end{pmatrix}$$

Full form in the high dimensional space [11]:

$$f_{q,k}(x_i) = \begin{pmatrix} \cos i\theta_1 & -\sin i\theta_1 \\ \sin i\theta_1 & \cos i\theta_1 \end{pmatrix} \quad \cdots \qquad \qquad 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \begin{pmatrix} \cos i\theta_{d/2} & -\sin i\theta_{d/2} \\ \sin i\theta_{d/2} & \cos i\theta_{d/2} \end{pmatrix} \begin{pmatrix} x_i^1 \\ x_i^2 \\ x_i^2 \\ \vdots \\ x_i^{d/2} \\ x_i^{d/2} \end{pmatrix} \quad \text{Partition dimensions into pairs and do the 2d rotation on each pair}$$

With different wavelet lengths:  $\theta_k = (1/10000^{2(k-1)/d})$ 

Full form in the high dimensional space [11]:

$$f_{q,k}(x_i) = \begin{pmatrix} \cos i\theta_1 & -\sin i\theta_1 \\ \sin i\theta_1 & \cos i\theta_1 \end{pmatrix} \dots \qquad 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \begin{pmatrix} \cos i\theta_{d/2} & -\sin i\theta_{d/2} \\ \sin i\theta_{d/2} & \cos i\theta_{d/2} \end{pmatrix} \end{pmatrix} \begin{pmatrix} x_i^1 \\ x_i^2 \\ x_i^2 \\ \vdots \\ x_i^{d/2} \\ x_i^{d/2} \end{pmatrix}$$
 Partition dimensions into pairs and do the 2d rotation on each pair

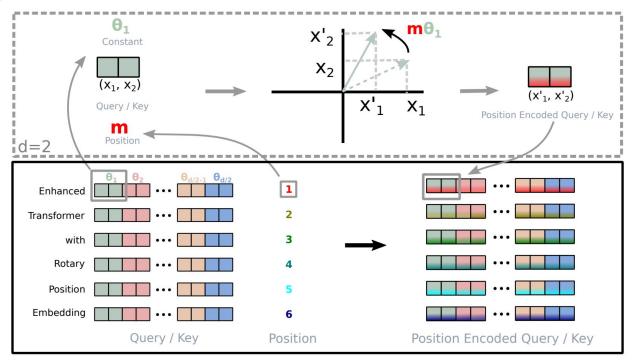
With different wavelet lengths:  $\theta_k = (1/10000^{2(k-1)/d})$ 

Rooted in the sinusoid absolute position embedding, but does multiplication (rotation):

$$x' = x + p_{pos}$$
  $p_{pos,2i} = sin(pos/10000^{2i/d})$   
 $p_{pos,2i+1} = cos(pos/10000^{2i/d})$ 

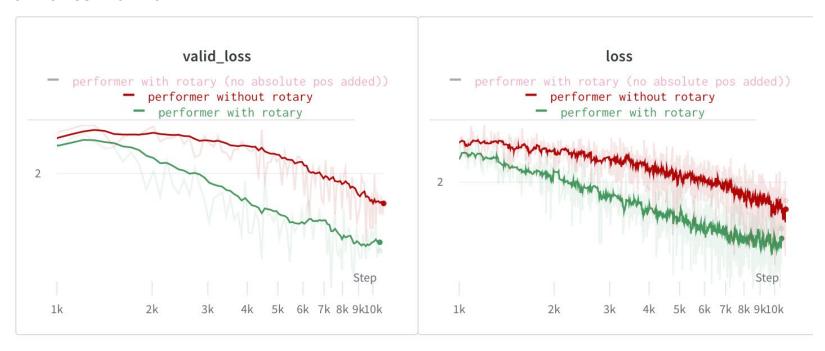
"We chose this function because we hypothesized it would allow the model to easily learn to attend by relative positions"---Transformer Paper

#### Putting it all together:



RoPE Embedding [11]

#### Performance with RoPE



https://blog.eleuther.ai/rotary-embeddings/

#### LLaMA3's Choice

