Problem Solving

weekly_readings #week-9 #my-notes-755

> ## Parts of a Problem
>
> **Intial State & Givens:**
>
> - Objects, Conditions, and Constraints that affect how the problem is approached and solved
> - Explicit (materials) vs Implicit (experience)
>
> **Obstacles**
>
> - No recall of previous information
> - No clear path
> - No domain knowledge
> - Limited cognitive capacity
>
> **Means: Operations**
>
> - Cognitive: LTM, Attention, working memory
> - Physical: location, weight
>
> **Goal:**
>
> - End-state
> - What is desired

Contents

> Taken from John Anderson - Cognitive Psychology and Its Implications (2015, Freeman_Worth) - libgen.li.pdf
> Ch-8

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The Nature of Problem Solving

A Comparative Perspective on Problem Solving

The larger prefrontal cortex supports the advanced problem solving that only humans are capable of.

Nonetheless, one can find instances of interesting problem solving in other species, particularly in the higher apes such as chimpanzees.

Kohler (1927) - Classic Studies on Chimpazee Problem Solving

- His best participant was a Chimpazee named *Sultan*.
- One problem posed to Sultan was to get some bananas that were outside his cage.
- Sultan had no difficulty, when he was given a "stick" that could reach the bananas. He would simply use the stick to pull the bananas into the cage.
- The problem became harder when Sultan was provided with two poles, neither of which could reach the food.
- After some uncessful attempts and frustration, he suddenly went over to the poles and put one inside the other, creating a pole long enough to reach the bananas.
- Clearly, Sultan had creatively solved the problem.

Essential Features that qualify this episode as an instance of Problem-Solving:

1. **Goal directedness:** The behaviour is organized towards a Goal - (getting the banana)
2. **Subgoal decomposition:**
    - The ape had to decompise the original goal into subtasks, or subgoals (such as: getting the poles, putting them together, pulling the banana, pick it up).
    - If the food was within reach, then that would have been the trivial case.
3. **Operator Application:** .

  - Decomposing the overall goal into subgoals was useful because the ape knows operators that can help him achieve these subgoals.
  - **Operator** refers to an action that will transform the problem state into another problem state.
  - The solution of the overall problem is a sequence of these known operators.

The Problem Solving Process: Problem Space and Search

- **State:**
  - A representation of the problem in some degree of the solution.
  - **Start State:** The initial situation of the problem.
  - **Goal State:** The final desired situation, when the problem is said to have been solved.
  - **Intermediate State:** the situations on the way to goal.
- **State Space:**
  - The various states that the problem solver can achieve define a problem space, also called a state space.
- **Operators:**
  - Problem-solving operators can be thought of as ways to change one state in the problem space into another.
- The *challenge* is to find some possible sequence of operators in the problem space that leads from the start state to the goal state.
- Solving a problem can be described as engaging in a **search**
  - developed by *Allen Newell* and *Herbert Simon*

Example:

Sultan could reach for a stick, stand on his head, sulk, or try other approaches. Suppose he reaches for a stick.
Now he has entered a new state. He can transform it into another state—for example, by letting go of the stick (thereby returning to the earlier state), reaching for the food with the stick, throwing the stick at the food, or reaching for the other stick. Suppose he reaches for the other stick.
Again, he has created a new state. From this state, Sultan can choose to try, say, walking on the sticks, putting them together, or eating them. Suppose he chooses to put the sticks together.
He can then choose to reach for the food, throw the sticks away, or separate them. If he reaches for the food and pulls it into his cage, he will achieve the goal state.

> This search space terminology describes **possible steps** that the problem solver might take.
>
> It leaves two important questions that we need to answer before we can explain the behavior of a particular problem solver.

> 1. What determines the **operators available** to the problem solver?
>   - An answer to the first question *determines the search space* in which the problem solver is working.
> 2. How does the problem solver **select a particular operator** when there are several available?
>   - An answer to the second question *determines which path the problem solver takes*.

Problem Solving Operators

Acquisition of Operators

There are atleast 3 ways to acquire new Problem-solving Operators:

1. Discovery
2. Learning by being Told
3. Observing someone else use them (by Example)

(1) Discovery

Example:

- We might find that a new service station has opened nearby and so learn by discovery a new operator for repairing our car.
- Children might discover that their parents are particularly susceptible to temper tantrums and so learn a new operator for getting what they want.
- A scientist might discover a new drug that kills bacteria and so invent a new operator for combating infections.

> Although operator discovery can involve complex reasoning in humans
>
> It (operator discovery) is the only method that most other creatures have to learn new operators,
> And they certainly do not engage in complex reasoning.

Study by Thorndike 1898

- **Setting & Task:**
    - Placed cats in "puzzle boxes."
    - The boxes could be opened by various nonobvious means.
    - *For instance*, in one box, if the cat hit a loop of wire, the door would fall open. The cats, which were hungry, were rewarded with food when they got out.
- **Observation:**

- Initially, a cat would move about randomly, clawing at the box and behaving ineffectively in other ways until it happened to hit the unlatching device.
- After repeated trials in the same puzzle box, the cats eventually arrived at a point where they would immediately hit the unlatching device and get out.
- **Interpretation:**
  - Animals can learn by discovery
- **Controversy:** A controversy exists to this day over whether:
  - The cats ever really "understood" the new operator they had acquired, OR
  - Just gradually formed a mindless association between being in the box and hitting the unlatching device.
- It has been argued that it **need not** be an **either–or** situation.

Daw, Niv, and Dyan (2005)

- Review evidence that there are two bases for learning such operators from experience:
  1. involves the **basal ganglia** where **simple associations** are gradually reinforced,
  2. involves the **prefrontal cortex**, where a **mental model is built** of how these operators work.
- It is reasonable to suppose that the second system becomes more important in mammals with larger prefrontal cortices.

(2) Learning by being Told ... OR ... (3) By Example

- These are examples of social learning.
- The *first* method (**Being Told**) is a uniquely human accomplishment because it depends on language.
- The *second* (**By Example**) is a capacity thought to be common in primates: "Monkey see, monkey do.

Pyramid Expression - novel to most UG

- **Task:** Students either study:
  - (a) - which gives a semi-formal specification of a pyramid expression
  - (b) - which gives the single example of a pyramid expression

- **TABLE 8.1** Instruction for Pyramid Problems

  (a) Direct Specification

  N$M is a pyramid expression for designating repeated addition where each term in the sum is one less than the previous.
  N, the base, is the first term in the sum.
  M, the height, is the number of terms you add to the base.

  (b) Just an Example

  7$3 is an example of a pyramid expression.

  $$7\$3 = 7 + \underbrace{6 + 5 + 4} = 22$$

  7 is the base

  3 is the height

  - After reading one instruction or the other, they are asked to evaluate pyramid expressions like: `10$2 = ?`
- **Observations:** Carnegie Mellon undergraduates show comparable levels of learning from the single example in part (b) to what they learn from the specification in part (a)

Reed and Bolstad (1991)

- **Task:**
  - Had participants learn to solve problems such as the following:
    - *An expert can complete a technical task in five hours, but a novice requires seven hours to do the same task. When they work together, the novice works two hours more than the expert. How long does the expert work?*
  - Participants received instruction in how to use the following equation to solve the problem:
    - `(rate1 x time1) x (rate2 x time2) = tasks`
  - The participants needed to acquire problem-solving operators for **assigning values to the terms** in this equation.
  - They received either:
    - C1: **Abstract Instructions** - about how to make these assignments.
    - C2: **Saw an Example**: - of how these assignments are made.
    - C3: **Both** abstract instruction and example.
  - They were then tested on later problems
- **Observations:**
  - Solve percentage was: C1: 13%, C2: 28%, C3: 40%

> It has now been shown many times that providing **worked examples is one of the most effective methods of instruction for problem-solving skills** like algebra (for a review, see Lee & Anderson, 2013).
>
> A large number of studies compared learning by worked examples with instructional explanation and without instructional explanation (see Wittwer & Renkl, 2010 for a review).
>
> Sometimes providing instruction in addition to examples actually hurts, sometimes there is no effect, and sometimes it does help, as in the *Reed and Bolstad study* above.

> **Sometimes examples can be obscure and lead to incorrect conclusions without an explanation**.
> A classic example from mathematics involves showing children an example like:
> `3 x 2 + 5 = 6 + 5 = 11`
> and then asking them to solve `4 + 6 x 2 = ?`
> Many children will give 20 as the answer, mistakenly adding 4 and 6 and then multiplying that by 2. Instruction can alert them to the fact that they should always perform multiplication first, rather than perform the first operation in the expression.

Analogy and Imitation

Analogy is the process by which a problem solver extracts the operators used to solve one problem and maps them onto a solution for another problem.

1. Sometimes the analogy can be straightforward
    - a student may take the structure of an example worked out in a section of a mathematics text and map it into the solution for a problem in the exercises at the end of the section.
2. At other times, the transformations can be complex
    - Rutherford, for example, used the solar system as a model for the structure of the atom, in which electrons revolve around the nucleus of the atom in the same way as the planets revolve around the sun

In one study, Christensen and Schunn (2007) found that engineers made 102 analogies in 9 hours of problem solving

Tumor Problem with General-Dictator Analogy - Gick and Holyoak (1980)

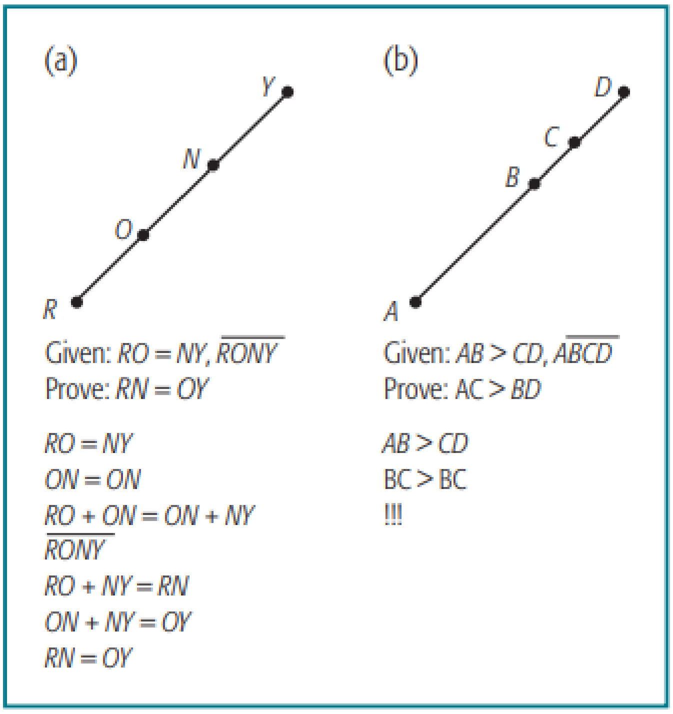
- **Tumor Problem:** *Suppose you are a doctor faced with a patient who has malignant tumor in his stomach. A certain kind of ray can cure it. If used with low intensity it will be ineffective, but if used too much, it can destroy the hea;thy tissues as well. What type of procedure will you use to destroy the tumor and at the same time avoiding destroying the healthy tissues?*

    - This is a very difficult problem and few people are able to solve it.

- **General-Dictator Analogy:** *A small country was ruled by a dictator from a fort. A rebelllious genereal decided to capture the fortress. He knew that if he use his entire army, he can win the fortress. But the dictator had placed mines which gets activated only if there is a large force. The general hence, decided to split his army into groups, which would cross the road and once near the fortress, they would regroup and charge.*

    - Told to use this story as the model for a solution, most partici pants were able to develop an analogous operation to solve the tumor problem

Geometry Problem - solution by analogy didn't work



- The student noted the obvious similarity between the two problems and proceeded to develop the apparent analogy.
  - He thought he could simply substitute points on one line for points on another, and
  - inequality for equality.
- That is, he tried to substitute A for R, B for O, C for N, D for Y, and > for 5.
- With these substitutions, he got the first line correct: Analogous to RO = NY, he wrote AB > CD.
- Then he had to write something analogous to ON = ON, so he wrote BC > BC!
- **This example illustrates how analogy can be used to create operators for problem solving and also shows that it requires some sophistication to use analogy correctly**

Another difficulty with analogy is finding appropriate examples from which to analogize operators

- Often, participants do not notice when an analogy is possible.
- *Gick and Holyoak (1980)* did an experiment in which they read participants the story about the general and the dictator and then gave them *Duncker's (1945) ray problem* (both shown earlier in this section).
- Very few participants spontaneously noticed the relevance of the first story to solving the second.
- To achieve success, participants had to be explicitly told to use the general and dictator story as an analogy for solving the ray problem.

Superficial Similarities between example the problem at hand

When participants do spontaneously use previous examples to solve a problem, they are often guided by superficial similarities in their choice of examples.

B. H. Ross (1984, 1987)

- **Teach by Example:**
  - Taught participants several methods for solving *probability problems*.
  - These methods were taught by reference to specific examples, such as finding the probability that a pair of tossed dice will sum to 7.
- **Test (apply analogy):**
  - Participants were then tested with new problems that were superficially similar to prior examples. The similarity was superficial because both the example and the problem involved the same content (e.g., dice) but not necessarily the same principle of probability.
- **Observations:**
  - Participants tried to solve the new problem by using the operators illustrated in the superficially similar prior example.
  - When that example illustrated the same principle as required in the current problem, participants were able to solve the problem.
  - When it did not, they were unable to solve the current problem.

> ***Analogy involves noticing that a past problem solution is relevant and then mapping the elements from that solution to produce an operator for the current problem.***

Analogy and Imitation from an Evolutionary and Brain Perspective

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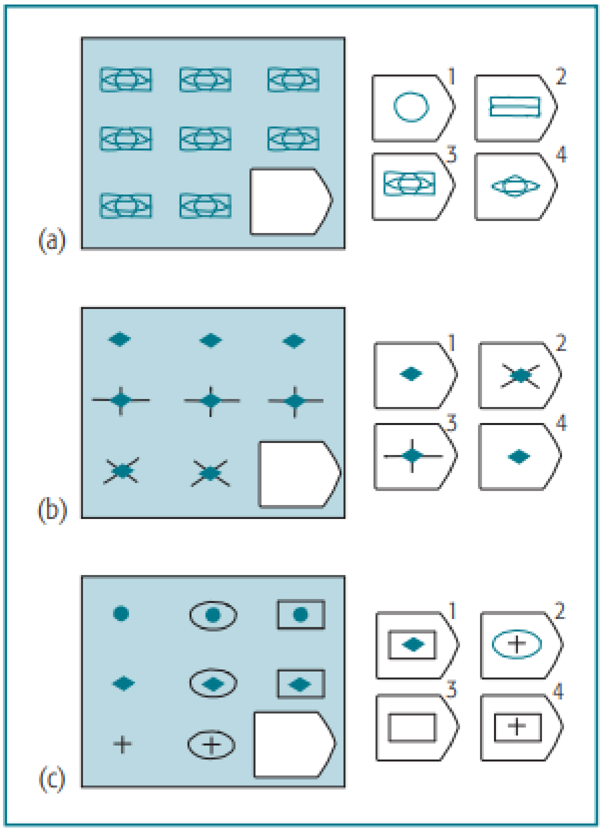
It has been argued that analogical reasoning is a hallmark of human cognition (*Halford, 1992*). The capacity to solve analogical problems is almost uniquely found in humans.

Premack (1976) reported that Sarah, a chimpanzee used in studies of language, was able to solve analogies such as the following:
`Key is to a padlock as what is to a tin can? The answer: can opener.`
In more careful study of Sarah's abilities, however, *Oden et al.* found that although Sarah could solve such problems more often than chance, she was much more prone to error than human participants.

Brain-imaging studies have looked at the cortical regions that are activated in analogical reasoning.

Study by Christoff et al. (2001)

- **Setting:**
  - Used examples of stimuli adapted from **Raven's progressive Matrices Test** (std. test for intelligence)



- Only problems like (c) which require the solver coordinate 2-dimensions, could be said to tap true **analogical reasoning**.
- **Task:**
  - Participants attempted to solve three different types of analogy problem:
  - (a) 0-dimensional; (b) 1-dimensional; and (c) 2-dimensional.
  - The task in each case was to infer the missing figure and select it from among the four alternative choices.
- **Observations:**
  - There is evidence that children under age 5 (in whom the frontal cortex has not yet matured), nonhuman primates, and patients with frontal damage all have special difficulty with problems like the one in (c) and often just cannot solve them.
  - They found that the r**ight anterior prefrontal cortex** was activated **only** when participants had to coordinate **two dimensions**.

Brain-imaging study - Wendelken, O'Hare, Whitaker, Ferrer, and Bunge (2011)

- Found that, in children (unlike adults), activity in this region does not vary appropriately with the difficulty of the task.

> Thus, it seems that one of the things that makes humans such effective problem solvers is that we have special abilities to acquire new problem-solving operators by analogical reasoning.
>
> Analogical problem solving appears to be a capability nearly unique to humans and to depend on the advanced development of the prefrontal cortex.

Operator Selection

> ## Search Strategies:
>
> 1. Algorithms:
>   - Guranteed solution, If steps are followed
> 2. Heuristics:
>   - General Rules and Guidelines
>   - Better (quicker) search
>   - Role of Memory
>
> ### Algorithms
>
> - **Nature of Algorithms**:
>   - Guranteed solution if steps are followed.
>   - Eg- Maths formula / Cooking recipe
> - **Exhaustive Search:**
>   - Checking every possible solution.
>   - Efficiency depends on the size of problem space.
>
> ### Heuristics
>
> - **Nature of Heuristics:**
>   - A cognitive shortcut that relies on previous knowledge (memory)
>   - Does not gurantee a solution
> - **Involves utilization of general knowledge:**
>   - Eg- Where would you look for if you lost your keys

Three criterias that humans use to select problem-solving operators:

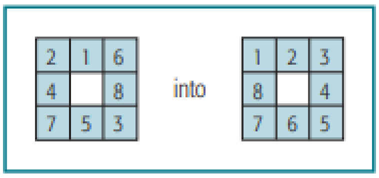
1. **Backup Avoidance:**
    - *Not going back to previous state*
    - For instance, in the eight puzzle, people show great reluctance to take back a step even if this might be necessary to solve the problem.
2. **Difference-Reduction:**
    - *Choosing non-repeating operator that most reduces the difference between current state and goal state*
    - Köhler (1927) described how a chicken will move directly toward desired food and will not go around a fence that is blocking it, coz that might increase it distance from the food.
3. **Means-end Analysis:**
    - *Breaking problem into smaller / simpler problems*
    - By using means-ends analysis, humans and other higher primates can be more resourceful in achieving a goal than they could be if they used only difference

reduction.

Difference-Reduction Method

Human Problem solvers choose operators that transform the current state into a new state that reduces differences and resembles the goal state more closely than the current state.

Example:



- There were four options possible for the first move.
- One possible operator was to move the 1 tile into the empty square, another was to move the 8, a third was to move the 5, and the fourth was to move the 4.
- I chose the last operator. Why? Because it seemed to get me closer to my end goal. I was moving the 4 tile closer to its final destination

Hill Climbing

- One of the strategies that use Difference-reduction
- Each step takes the cutrrent state and moves it closer to the end-state
- Not applicable for all problems
- Leads to local minima / maxima (rather than global)
    - By following it, we might reach the top of some hill that is lower than the highest point of land that is the goal
    - It is myopic in that it considers only whether the next step is an improvement and not whether the larger plan will work.

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Read 2 problems from book (PAGE- Book:193-194, PDF:214-215):
1. Hobbit and Orcs
2. Three Jugs Problem
```

- Hill climbing can also produce suboptimal results when making serious life choices.
- **A classic example,** is someone trapped in a suboptimal job because he or she is unwilling to get the education needed for a better job. The person is unwilling to endure the temporary deviation from the goal (of earning as much as possible) to get the skills to earn a higher salary.

> People experience difficulty in solving a problem at points where the correct solution involves increasing the differences between the current state and the goal state.

Means-End Analysis

This method was extensively studied by *Newell and Simon*, who used it in a computer simulation program (called the **General Problem Solver — GPS**) that models human problem solving.

Means-ends analysis is typified by the following kind of commonsense argument:

I want to take my son to nursery school. What's the difference between what I have and what I want? One of distance. What changes distance? My automobile. My automobile won't work. What is needed to make it work? A new battery. What has new batteries? An auto repair shop. I want the repair shop to put in a new battery; but the shop doesn't know I need one. What is the difficulty? One of communication. What allows communication? A telephone . . . and so on.

This kind of analysis — classifying things in terms of the **functions they serve** and oscillating among **ends**, **functions required**, and **means that perform them** — forms the basic system of GPS. (*Newell & Simon, 1972, p. 416*)

Means-End Analysis as a more Sophisticated version of Difference-Reduction

- (Like difference reduction), it tries to **eliminate the differences between the current state and the goal state**.
    - *For instance, in this example, it tried to reduce the distance between the son and the nursery school.*
- Means-ends analysis will also **identify the biggest difference first** and try to eliminate it.
    - *The focus is on difference in the general location of son and nursery school. The difference between where the car will be parked at the nursery school and the classroom has not been considered yet.*
- (Unlike Difference-Reduction), it will not abandon an operator if that operator cannot be applied immediately.
    - *If the car did not work*:
        - **Difference reduction** would have one "start walking to the nursery school".
        - **Means-ends analysis** focuses on enabling blocked operators.
    - The **"means" temporarily becomes the "end"**. In effect, the problem solver deliberately ignores the real goal and focuses on the goal of enabling the means (which becomes the subgoal).
        - *The problem solver set a subgoal of repairing the automobile, which was the means of achieving the original goal of getting the child to nursery school.*

> **Means-ends analysis involves creating subgoals to eliminate the difference blocking the application of a desired operator (= difference between the current state and some state under which the operator can be applied)**
>
> *Recursive description:*

> 1. Given a current state and some goal (may be a subgoal) state
> 2. Estimate the difference between current and goal (may be a subgoal) state
> 3. You want to apply an operator, that will reduce the difference between current state to (sub)goal state.
> 4. Two cases may arise:
>   - Case-1: The condition under which that operator can be applied, is satisfied by the current state.
>   - Apply the operator.
>   - Case-2: That condition is NOT satisfied by the current state.
>   - There is some state (not the current ofc) in which the operator can be applied.
>   - Make that state the new subgoal
>   - Find the difference between current state and that state.
>   - ... Goto Step (3)

The Tower of Hanoi Problem

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Read from Book (PAGE- Book:196-198, PDF:217-219)
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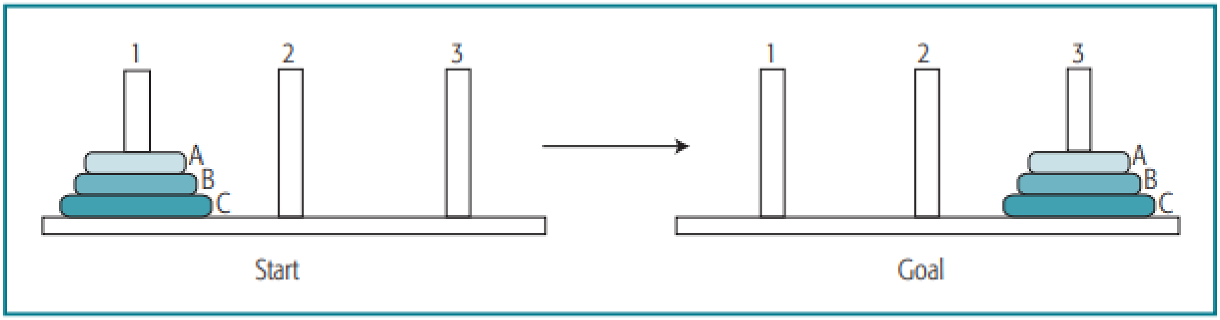


1. Goal: Move *A*, *B*, and *C* to peg 3
2. : Difference is that *C* is not on 3
3. : Subgoal: Make *C* on 3
4. : Operator is to move *C* to 3
5. : Difference is that *A* and *B* are on *C*
6. : Subgoal: Remove *B* from *C*
7. : Operator is to move *B* to 2
8. : Difference is that *A* is on *B*
9. : Subgoal: Remove *A* from *B*
10. : Operator is to move *A* to 3
11. : No difference with operator's condition
12. : Apply operator (move *A* to 3)
13. : Subgoal achieved
14. : No difference with operator's condition
15. : Apply operator (move *B* to 2)
16. : Subgoal achieved
17. : Difference is that *A* is on 3
18. : Subgoal: Remove *A* from 3
19. : Operator is to move *A* to 2
20. : No difference with operator's condition
21. : Apply operator (move *A* to 2)
22. : Subgoal achieved
23. : No difference with operator's condition
24. : Apply operator (move *C* to 3)
25. : Subgoal achieved
26. : Difference is that *B* is not on 3
27. : Subgoal: Make *B* on 3
28. : Operator is to move *B* to 3
29. : Difference is that *A* is on B
30. : Subgoal: Remove *A* from B
31. : Operator is to move *A* to 1
32. : No difference with operator's condition
33. : Apply operator (move *A* to 1)
34. : Subgoal achieved
35. : No difference with operator's condition
36. : Apply operator (move *B* to 3)
37. : Subgoal achieved
38. : Difference is that *A* is not on 3
39. : Subgoal: Make *A* on 3
40. : Operator is to move *A* to 3
41. : No difference with operator's condition
42. : Apply operator (move *A* to 3)
43. : Subgoal achieved
44. : No difference
45. Goal achieved

Goal Structure and the Prefrontal Cortex

It is significant that **complex goal structures**, particularly those involving "**operator subgoaling**", have been observed with any frequency only in *humans* and *higher primates*.

One of the major prerequisites to developing **complex goal structures** is the ability to maintain these goal structures in **working memory**

Goel and Grafman (1995)

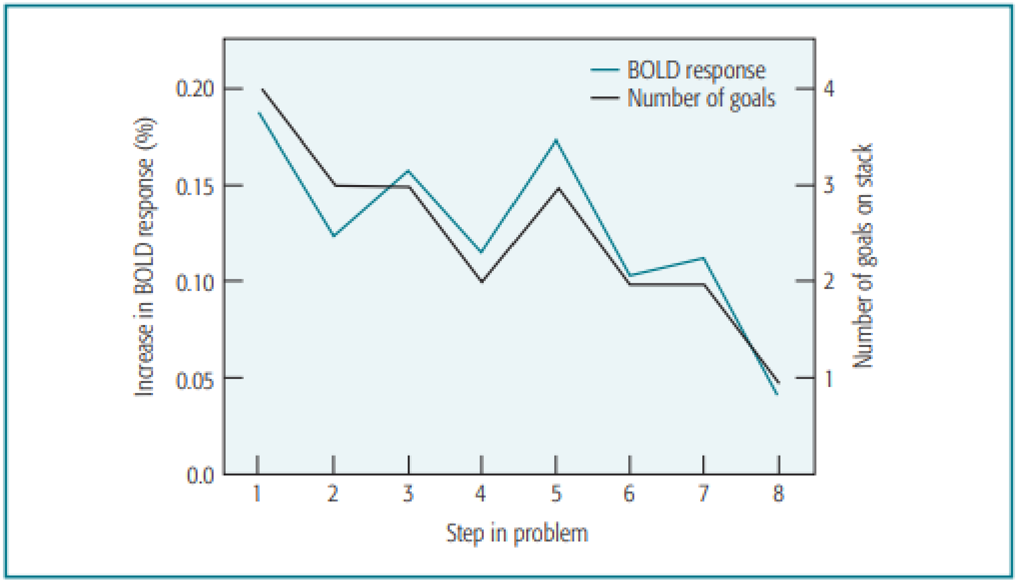
- Looked at how patients with severe prefrontal damage performed in solving the Tower of Hanoi problem.
- Many were veterans from Vietnam War (who had lost large amt of brain tissue due to missile wounds)
- Although they had normal IQs, they showed much worse performance than normal participants on the Tower of Hanoi task.
- One might have a disk at the correct position but have to move it away to enable another disk to be moved to that position. It was exactly at these points where the patients had to move "backward" that they had their problems. Only by **maintaining a set of goals** can one see that a backward move is necessary for a solution.

Gazzaniga, Ivry, & Mangun, 1998

- There is increased activation in the prefrontal cortex during many tasks that involve organizing novel and complex behavior.

Fincham, Carter, van Veen, Stenger, and Anderson (2002)

- **Task:**
    - Had students solve the Towers of Hanoi Problem, but with more complication (eg- 5 disk).
    - It included upto 8 problem solving steps
    - The number of goals being held varied from 1-4
- **Graph Plotting:**
    - The plot contained 2 data:
    - (1) The **fMRI BOLD response** of a region in the **right, anterior, dorsolateral prefrontal cortex**
    - (2) The number of goals at each point.



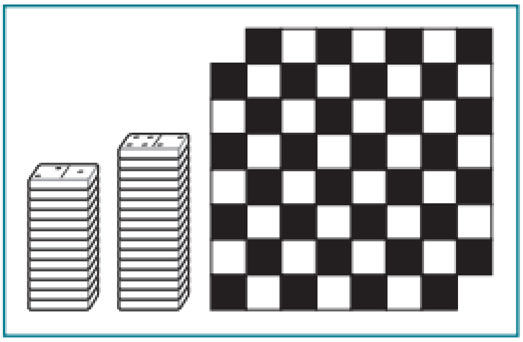
- **Observations:**
  - There seems to be a striking match between the **goal load** and the **magnitude of fMRI response**.
- **Conclusion:**
  - The prefrontal cortex plays a critical role in maintaining goal structures.

Problem Representation

The importance of correct representation

Problems are often represented that don't allow application of appropriate operators.

Mutilated checkboard problem (Kaplan & Simon 1990)



- **Problem Statement:**
  - Suppose we have a checkerboard from which two diagonally opposite corner squares have been cut out, leaving 62 squares. Now suppose that we have 31 dominoes, each of which covers exactly two squares of the board.
  - Can you find some way of arranging these 31 dominoes on the board so that they cover all 62 squares? If it can be done, explain how. If it cannot be done, prove that

it cannot.

- **Observations:**
    - Relatively few people are able to solve it without some hints, and very few see the answer quickly.
- **Answer:**
    - The dominoes cannot cover the checkerboard.
    - The trick to seeing this is to **include in your representation** of the problem the fact that **each domino must cover one black and one white square**, not just any two squares. There is just no way to place a domino on two squares (since we have 32 black squares but only 30 white squares)
- **Contrast this problem with the following “marriage” problem that occurs with many variations in its statement:**
    - In a village in Eastern Europe lived an old marriage broker. He was worried. Tomorrow was St. Valentine’s Day, the village’s traditional betrothal day, and his job was to arrange weddings for all the village’s eligible young people. There were 32 women and 32 young men in the village.
    - This morning he learned that two of the young women had run away to the big city to found a company to build phone apps.
    - Was he going to be able to get all the young folk paired off?
- **Observations:**
    - People almost immediately see that this problem cannot be solved since there are no longer enough women to pair up with the men.

> - Both problems require the same insight of "**matching pairs**
> - Why is the mutilated-checkerboard problem so hard and the marriage problem so easy?
> - The answer is that we ***tend not to*** represent the **checkerboard** in terms of matching *black* and *white* squares, whereas we ***do tend to*** represent **marriages** in terms of matching *brides* and *grooms*.
> - If we use such a matching representation, it **allows the critical operator to apply** (i.e., *checking for parity*).

Bassok (1990) and Bassok and Holyoak (1989)

Inappropriate problem representations often cause students to fail to solve problems even though they have been taught the appropriate knowledge

- Studied high school students who learned to solve problems like:
    - *What is the acceleration (increase in speed each second) of a train, if its speed increases uniformly from 15 m/s at the beginning of the 1st second, to 45 m/s at*

the end of the 12th second?

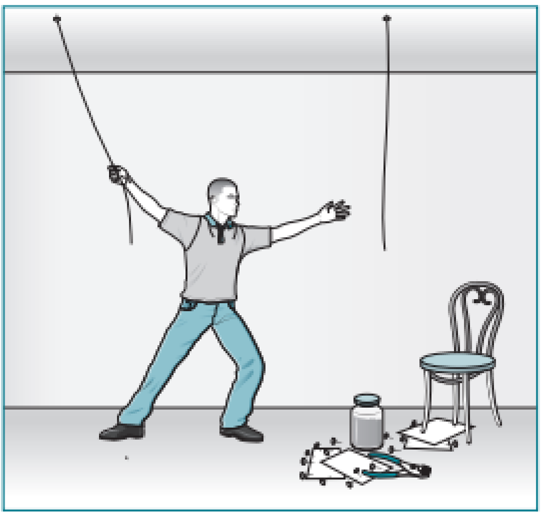
- Students were taught such physics problems and became very effective at solving them.
- However, they had very little success in transferring that knowledge to solving such algebra problems as this one:
    - *Juanita went to work as a teller in a bank at a salary of $12,400 per year and received constant yearly increases, coming up with a $16,000 sal ary during her 13th year of work. What was her yearly salary increase?*
- The students failed to see that their experience with the physics problems was relevant to solving such algebra problems, which actually have the same structure

Functional Fixedness

Functional Fixedness: Tendency to see objects as serving conventional problem-solving functions and thus failing to see possible novel functions.

But sometimes solutions to problems depend on the solver's ability to represent the objects in his or her environment in novel ways.

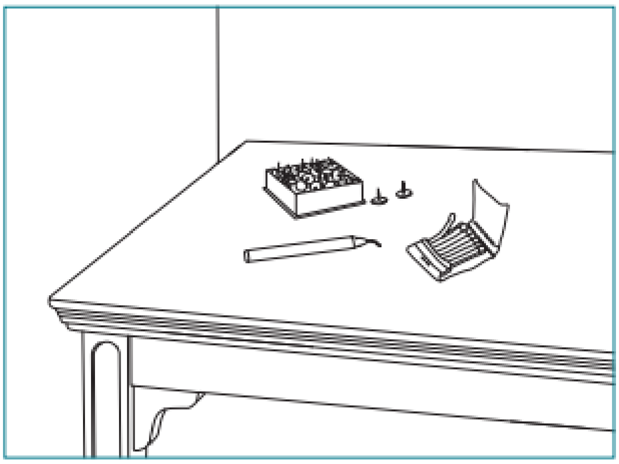
Two-string problem - Maier (1931)



- **Problem Statement:**
    - Two strings hanging from the ceiling are to be tied together, but they are so far apart that the participant cannot grasp both at once. Among the objects in the room are a chair and a pair of pliers.
- **Observations:**
    - Participants try various solutions involving the chair, but these do not work.
    - Only 39% of Maier's par ticipants were able to see this solution within 10 minutes.
- **Solution:**
    - Tie the pliers to one string and set that string swinging like a pendulum; then get the second string, bring it to the center of the room, and wait for the first string with the pliers to swing close enough to catch.

- **Interpretation:**
  - Participants did not perceive the pliers as a weight that could be used as a pendulum.
  - This phenomenon is called **functional fixedness**.

Dunker 1945



- **Problem Statement:**
  - Support a candle on a door, ostensibly for an experiment on vision.
  - Items given: A box of tacks, some matches, and the candle all placed on a table in the room.
- **Solution:**
  - Tack the box to the door and use the box as a platform for the candle.
- **Difficulty with the solution:**
  - This task is difficult because participants see the box as a container, not as a platform.
  - They have greater difficulty with the task if the box is filled with tacks, reinforcing the perception of the box as a container.

Set Effects

People's experiences can bias them to prefer certain operators when solving a problem. Such biasing of the problem solution is referred to as a **set effect**.

Water-Jug problems - Luchins (1942), Luchins & Luchins (1959)

Problem	*Capacity of Jug A*	*Capacity of Jug B*	*Capacity of Jug C*	*Desired Quantity*
1	5 cups	40 cups	18 cups	28 cups
2	21 cups	127 cups	3 cups	100 cups

- **Problem Statement:**

    - Participants are told to imagine that they have a sink so that they can fill jugs from the tap and pour water into the sink or from one jug into another. The jugs start out empty. When filling a jug from the tap, participants must fill the jug to capacity; when pouring the water from a jug, participants must empty the jug completely.
- **Actual Solutions:**
    - Problem-(1): 2A + C (adddition solution)
    - Problem-(2): B - A - 2C (subtraction solution)
- **Observations and Manipulations:**
    - Luchins first gave participants a series of problems that all could be solved by addition, thus creating an "addition set."
    - These participants then solved new addition problems faster, and subtraction problems slower, than control participants who had no practice.

The set effect that Luchins (1942) is most famous for demonstrating is the **Einstellung effect**, or *"mechanization of thought"*

TABLE 8.3 Luchins's Water Jug Problems Used to Illustrate the Set Effect

	Capacity (cups)			
Problem	***Jug A***	***Jug B***	***Jug C***	***Desired Quantity***
1	21	127	3	100
2	14	163	25	99
3	18	43	10	5
4	9	42	6	21
5	20	59	4	31
6	23	49	3	20
7	15	39	3	18
8	28	76	3	25
9	18	48	4	22
10	14	36	8	6

- **Problem Statement:** Task same as before (there are 10 problems)
- **Actual Solutions:**
    - Problems {1, 2, 3, 4, 5, 6, 7, 9, 10} can be solved using `B - 2C - A` method
    - Problems {8} cannot be solved by `B - 2C - A` method, but can be solved by a simpler `A - C` method.
    - For problems {1, 2, 3, 4, 5} `B - 2C - A` is the simplest method
    - For problems {7, 9} the simpler `A + C` also works.
    - For problems {6, 10} the simpler `A - C` also works.
- **Observations:**
    - Of those who received the whole setup of 10 problems:
        - 83% used `B - 2C - A` method on problems {6, 7}
        - 64% failed to solve Problem {8}
        - 79% used the `B - 2C - A` method for problems {9, 10}
    - Of those who received only the last 5 problems (Control Group)
        - < 1% used that `B - 2C - A` solution

        - 5% failed to solve Problem {8}
- **Interpretation:**
    - Thus, the first 5 problems created a powerful bias for a particular solution that hurt the solu   tion of problems 6 through 10.
- **Counter the set-effect:**
    - Luchins found that simply warning participants by saying, “Don’t be blind” after problem 5 allowed more than 50% of them to overcome the set for the `B - 2C - A` solution.

Another kind of set effect in problem solving has to do with the influence of **general semantic factors**

Anagram Solutions - Safren 1962

- **Problem Statement:**
    - Safren presented participants with lists such as the following, in which each set of letters was to be unscrambled and made into a word:
    - *kmli graus teews recma foefce ikrdn* → *milk sugar sweet cream coffee drink*
- **Controls:**
    - The list can either be **organized**, (example the above list in which the individual words are all associated with drinking coffee) or it can be **unorganized**
    - Safren compared **median solution times** for organized lists with times for unorganized lists.
    - Median solution time = (organized list: 12.2 s, unorganized list: 7.4s)
- **Interpretation:**
    - Presumably, the facilitation for the organized lists occurred because the earlier items in the list associatively primed, and so made more available, the later words.
- **This anagram experiment contrasts with the water jug experiment in that no particular procedure was being strengthened. Rather, what was being strengthened was part of the participant’s factual (declarative) knowledge about spellings of associatively related words**

- In general, set effects occur when some knowledge structures become more available than others.

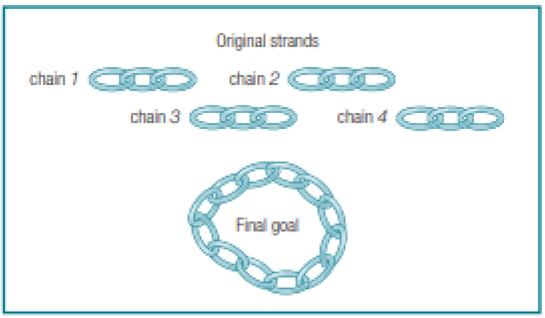
- These structures can be either:
    - **procedures**, (as in the water *jug problem*), or
    - **declarative information**, (as in the *anagram problem*).

- If the available knowledge is what participants need to solve the problem, their problem solving will be facilitated. If the available knowledge is not what is needed, problem solving will be inhibited.

Incubation Effects

> People often report that after trying to solve a problem and getting nowhere, they can put it aside for hours, days, or weeks and then, upon returning to it, can see the solution quickly
>
> The famous French mathematician Henri Poincaré (1929) reported many examples of this pattern, including the following:
> "*Then I turned my attention to the study of some arithmetical questions apparently without much success and without a suspicion of any connection with my preceding researches. Disgusted with my failure, I went to spend a few days at the seaside, and thought of something else. One morning, walking on the bluff, the idea came to me, with just the same characteristics of brevity, suddenness, and immediate certainty, that the arithmetic transformations of indeterminate ternary quadratic forms" were identical with those of non-Euclidean geometry.*"
>
> Such phenomena are called **incubation effects**

Silveira (1971) - Cheap Necklace Problem

- **Problem Statement:**
  - You are given four separate pieces of chain that are each three links in length. It costs 2¢ to open a link and 3¢ to close a link. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15¢.
  - 

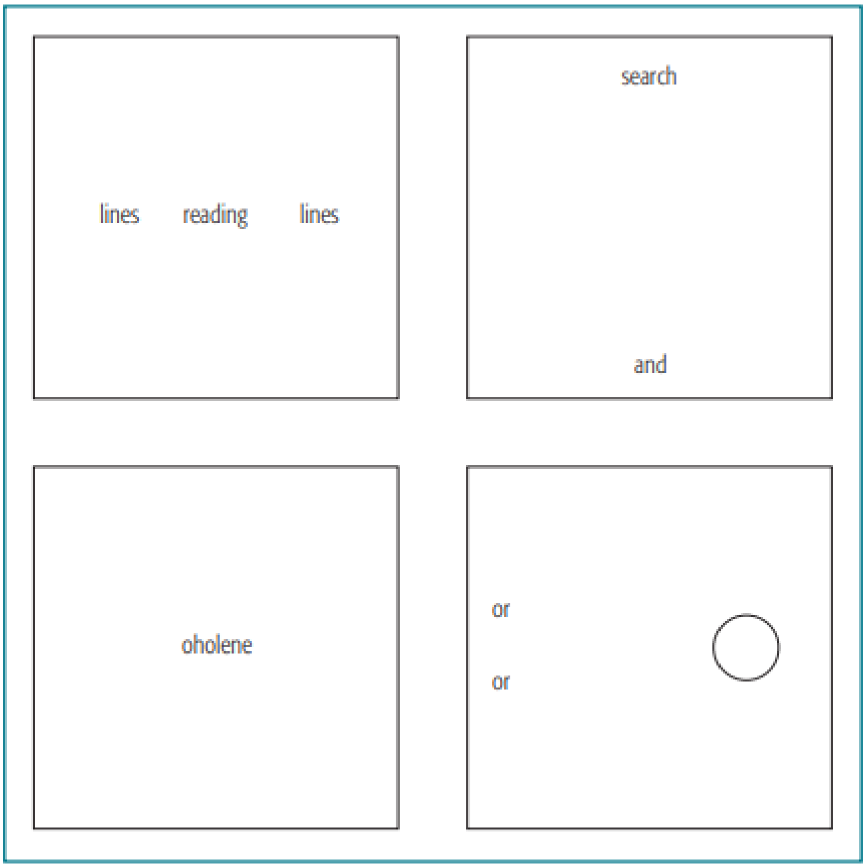
- **Setting:**
  - Participants were divided into 3 groups:
    - G1 (Control Group): worked on the problem for half an hour
    - G2 (1st experimental Group): the half hour spent on the problem was interrupted by a half-hour break in which the participants did other activities.
    - G3 (2nd experimental Group): had a 4-hour break
  - Silveira required her participants to speak aloud as they solved
- **Observations:**

- Percentage solved:
  - G1: 55% solved
  - G2: 64% solved
  - G3: 85% solved
- She found that they did not come back to the problem after a break with solutions completely worked out. Rather, they began by trying to solve the problem much as before.

- **Interpretation:**
  - (obs 2nd point): This result is evidence against a common misconception that people are subconsciously solving the problem during the period that they are away from it.
- **Conclusion:**
  - The best explanation for incubation effects relates them to set effects.
  - During initial attempts to solve a problem, people set themselves to think about the problem in certain ways and bring to bear certain knowledge structures.
    - If this initial set is appropriate, they will solve the problem.
    - If the initial set is not appropriate, however, they will be stuck throughout the session with inappropriate procedures.
    - Going away from the problem allows activation of the inappropriate knowledge structures to dissipate, and people are able to take a fresh approach.

S. M. Smith and Blakenship (1989, 1991)

- **Objective:**
  - A fairly direct test of the hypothesis "***incubation effects occur because people “forget” inappropriate ways of solving problems***"
- **Problem Statement:**
  - Participants had to figure out what familiar phrase was represented by each image.

- 

- For example,
  - the first picture represents the phrase “reading between the lines”;
  - the second, “search high and low”;
  - the third, “a hole in one”; and the fourth, “double or nothing.”
- **Setting:**
  - There were 2 groups (experimental group (fixation group), control group):
    - F (fixation group):
      - They were deliberately provided with incorrect ways to think
      - Example: In Problem 3, they were asked to think of "chemicals".
    - F' (control group):
      - No such clues given.
  - Each group were divided into 2 subgroups (half):
    - C - participants worked on the problems for a continuous period of time,
    - C' - had an incubation period inserted in the middle of their problem solving efforts
  - Therefore 4 types of participants: (FC, FC', F'C, F'C')
- **Observations:**
  1. No. of problems solved: F < F'
  2. The fixation participants showed a greater benefit of the incubation period.
     - When they asked the fixation participants what the misleading clue had been, they found that more of the participants who had an incubation period had **forgotten the inappropriate clue**.
- **Conclusion:**

- Thus, the incubation effect for the fixation participants occurred because they had forgotten the inappropriate way of solving the problem.

Insight

A **common misconception** about learning and problem solving is that there are "magical moments" of insight when everything falls into place and we suddenly see a solution. This is called the “aha” experience, and many of us can report uttering that very exclamation after a long struggle with a problem that we suddenly solve.

The incubation effects just discussed have been used to argue that the subconscious is deriving this insight during the incubation period. As we saw, however, what really happens is that participants simply let go of poor ways of solving problems.

Metcalfe and Wiebe (1987)

- came up with ways to define "insight problems"
- They proposed that problems like the cheap-necklace problem are insight problems, whereas problems requiring multistep solutions, like the Tower of Hanoi problem (see Figure 8.10), are noninsight problems.
- **Objective:** To test the above fact
- **Task:** Asked participants to judge every 15 s how close they felt they were to the solution.
- **Observations:**
    - Fifteen seconds before they actually solved a **noninsight problem**, participants were fairly confident they were close to a solution.
    - In contrast, with **insight problems**, participants had little idea they were close to a solution, even 15 s before they actually solved the problem.

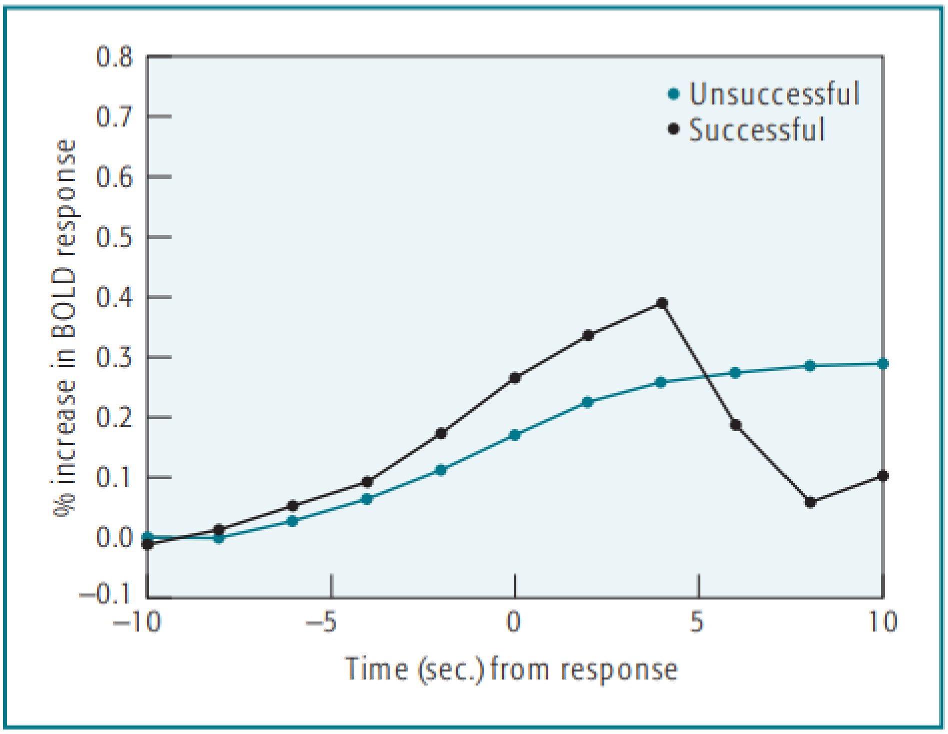
Kaplan and Simon (1990)

- **Task:** They studied participants while they solved the **mutilated-checkerboard problem**, which is another *insight problem*.
- **Observation:**
    - Some participants noticed key features of the solution to the problem—such as that a domino covers one square of each color— early on.
    - Sometimes, though, these participants did not judge those features to be critical and went off and tried other methods of solution; only later did they come back to the key feature.
- **Conclusion:**
    - So, it is not that solutions to insight problems cannot come in pieces,
    - but rather that **participants do not recognize which pieces are key until they see the final solution**.

Mednick 1962

- *one more example of "insight problems"*
- Participants are asked to find some word that can be combined with three words to make a compound word.
- **Example:**
  - Given: {"fox", "man", "peep"}, the solution is "hole"
  - since {"fox-hole", "man-hole", "peep-hole"} all make sense

Jung-Beeman et al., 2004 - Brain Activity Studies

- **Task:**
  - Conducted brain-activity studies while participants solved "insight-problems".
  - Characteristic of insight problems, people often get a sudden feeling of insight when they solve them.
- **Plotting:**
  - Imaging results from our laboratory, which shows activity in the **left prefrontal region** (associated with *retrieval from declarative memory*)
  - The figure compares 2 cases (successfully | unsuccessfully) solved.
  - Time = 0, at the point where the solution was clicked in successful case
- **Observations:**
  - 

  - Both functions are increasing, ⇒ increasing effort as the search progresses.
  - But there is an abrupt drop (time-lagged as we would expect with the BOLD response) after the insight.
- **Interpretation:**
  - In dropping off, the prefrontal cortex is showing a strikingly different response compared to other brain regions and is reflecting the end to the search of memory for the answer.

> ***Insight problems are ones in which solvers cannot recognize when they are getting close to the solution.***