

2D Matrix $[M \times N]$

A

		0	1	2	3	4	5	6	
Top left	0							0,6	Top right
	1			1,2			1,5		
	2								
	3								
	4			4,2			4,5		
Bottom left	5								Bottom right

1D array \longrightarrow subarray? $l \leq r$
 $A[l:r]$
 Continuous part of array

2D arrays / Matrix \longrightarrow submatrix
 Continuous part of matrix

TL and BR
 (x_1, y_1) (x_2, y_2)

$$x_1 \leq x_2 \quad y_1 \leq y_2$$

Q1) Given a matrix of size $M \times N$. For each query, find sum of given submatrix

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

For each query
iterate on the submatrix
and find sum

Queries

(T, L)

(B, R)

$(2, 1)$

$(4, 2)$

26

$(1, 1)$

$(3, 3)$

33

TC: $O(M \times N \times Q)$

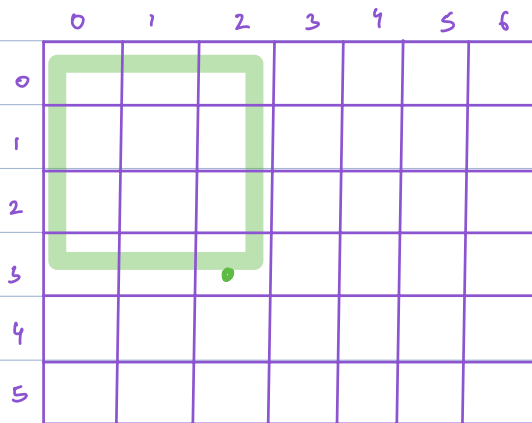
	0	1	2	3
0				
1				
2				
3				
4				

$$pf[i] = \text{sum}(A[0:i])$$

1D array

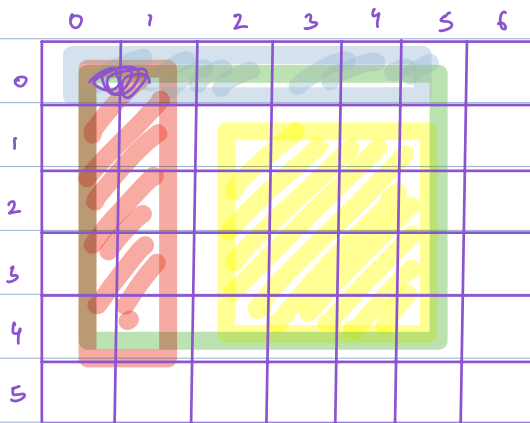
$$pf[i][j] = \text{sum}(A[0,0] : A[i,j])$$

pf matrix



$$pf[3,2] = \text{sum}(A[0,0] - A[3][2])$$

pf matrix

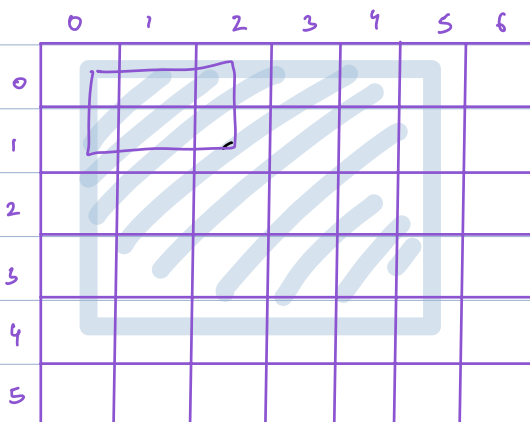


TL
(1,2)

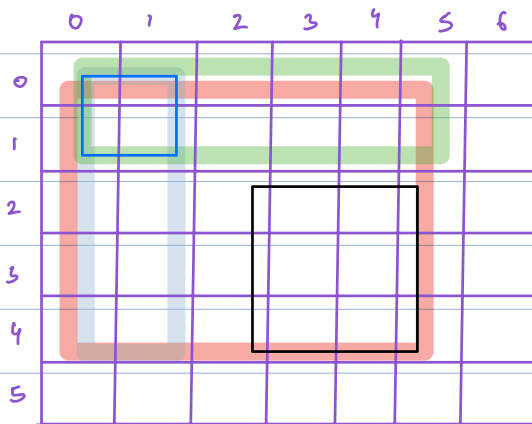
BR
(4,5)

$$pf[4,5] - pf[1,2]$$

\propto



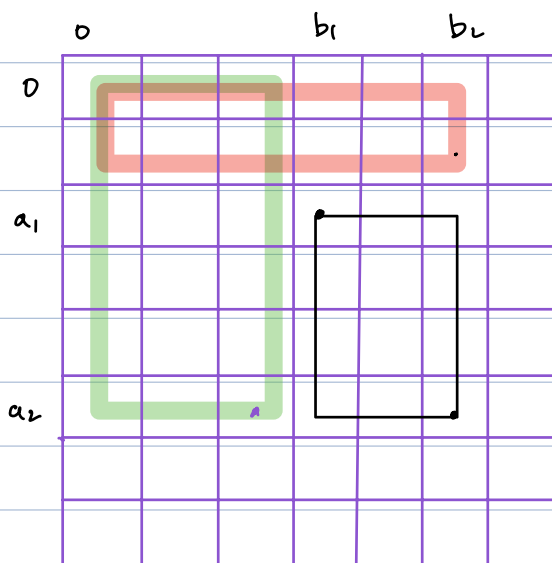
$$pf[4,5] - pf[4,1] - pf[0,5] + pf[0,1]$$



TL
2, 2

BR
4, 5

$$pf[4,5] - pf[4,1] - pf[1,5] + pf[1,1]$$



TL
(a₁, b₁)

BR
(a₂, b₂)

$$pf(a_2, b_2) - pf(a_2, b_1 - 1) - pf(a_1 - 1, b_2) + pf(a_1 - 1, b_1 - 1)$$

$b_1 > 0$
 $a_1 > 0$
 $a_1 > 0 \text{ \& } b_1 > 0$

A

$$pf[i][j] = a_0 + a_1 + b_0 + b_1$$

a_0	b_0	c_0
a_1	b_1	c_1
a_2	b_2	c_2

find row wise and column wise prefix sum

row wise
↓
↓

→	a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$	Col wise →	a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
→	a_1	$a_1 + b_1$	$a_1 + b_1 + c_1$		$a_0 + a_1$	$a_0 + b_0$ $a_1 + b_1$	$a_0 + b_0 + c_0$ $a_1 + b_1 + c_1$
	a_2	$a_2 + b_2$	$a_2 + b_2 + c_2$		$a_0 + a_1 + a_2$	$a_0 + b_0$ $a_1 + b_1$ $a_2 + b_2$	$a_0 + b_0 + c_0$ $a_1 + b_1 + c_1$ $a_2 + b_2 + c_2$

$$pf[i][j] = \text{sum}(A[0][j] - A[i][j])$$

$$A[i] += A[i-1]$$

pf sum matrix

Row wise pf sum

```
for (i=0; i < rows; i++) {
    for (j=1; j < cols; j++) {
        A[i][j] = A[i][j] + A[i][j-1]
    }
}
```

RxC

RxC

col wise pf

```
for (i=0; i < cols; i++) {
    for (j=1; j < rows; j++) {
        A[j][i] += A[j-1][i]
    }
}
```

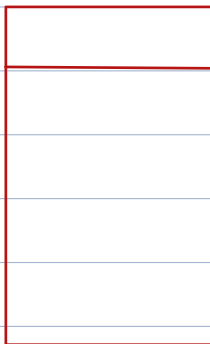
TL

BR

a, b

a, b

a₁ b₁ a₂ b₂



```
for (i=0; i < len(Q); i++) {
```

a₁ = Q[i][0] a₂ = Q[i][2]

b₁ = Q[i][1] b₂ = Q[i][3]

prin(

pf(a₂, b₂) - pf(a₂, b₁-1) - pf(a₁-1, b₂) + pf(a₁-1, b₁-1)

b₁ > 0

a₁ > 0

a₁ > 0 & b₁ > 0

TC: O(Q + nN)

SC: O(1)

Q2) Given a matrix, find sum of all submatrix sum

3 ⁶	1 ⁶
-1 ⁸	-2
2	4

$$\begin{bmatrix} 3 \end{bmatrix} \quad \begin{bmatrix} 3 \ 1 \end{bmatrix} \quad \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 \\ -1 & -2 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 4 \end{bmatrix} \quad \begin{bmatrix} -2 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 2 & 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \end{bmatrix} \quad \begin{bmatrix} 2 \end{bmatrix}$$

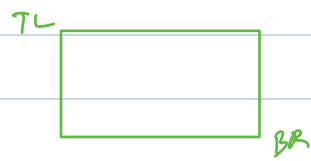
$$\boxed{\text{sum: } 36}$$

$$\frac{(M^2 N^2)}{2} \quad \text{no. of submatrix}$$

$$TC: O(M^2 N^2)$$

$$\sum_{i,j} A[i][j] \times \text{contribution}(A[i][j])$$

	0	1	2	3	4	5	6
0	TL	TL	TL	TL			
1	TL	TL	TL	TL			
2	TL	TL	TL	TL			
3	TL	TL	TL	TL BR	BR	BR	BR
4				BR	BR	BR	BR
5				BR	BR	BR	BR



22 16 72

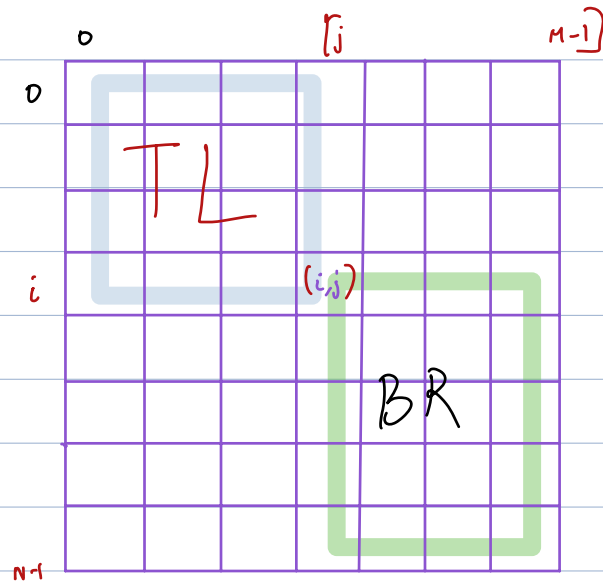
21
 4 × 4 × 3 × 7

BR → 12 × 16 =
 TL

3	1
-1	-2
2	4

TL → 4 Ans: 8
 BR → 2

$[0, i)$



$$TL \rightarrow (j+1) * (i+1)$$

$$BR \rightarrow (m-j) * (n-i)$$

$$\text{freq of occurance} = TL * BR$$

$$\text{contribution} = \text{freq} * A[i][j]$$

Back (10:46 - 10:56)

Q3) Maximum submatrix sum for row wise & col wise sorted matrix

	0	1	2	3
0	-10	-6	-2	1
1	-5	-4	4	5
2	-3	0	6	20
3	2	5	10	25

sorted \rightarrow

-2	-1	3	7	10
----	----	---	---	----

17 19 20 17 10



suffix sum

$\text{sum}(A[i:N-1])$

	0	1	2	3
0	-10	-6	-2	1
1	-5	-4	4	5
2	-3	0	6	20
3	2	5	10	25

suffix sum of matrix

$A[(i,j) (M-1, N-1)]$

	0	1	2	3
0			69	51
1			70	50
2		61	61	45
3		40	35	25

	0	1	2	3
0	-10	-6	-2	1
1	-5	-4	4	5
2	-3	0	6	20
3	2	5	10	25

	0	1	2	3
0	-17	-7	-1	1
1	0	5	9	5
2	23	26	26	20
3	42	40	35	25



	0	1	2	3
0	47	64	69	51
1	65	71	70	50
2	65	66	61	45
3	42	40	35	25

TL: $O(mn)$

Q4) Given a matrix where every row & column is sorted find an element k (No repetition in single row/col)

α

	0	1	2	3	4	5
0	-1	2	4	5	9	11
1	1	4	7	8	10	11
2	3	7	9	10	12	18
3	6	10	12	14	16	20
4	9	13	16	19	22	26
5	11	15	19	21	24	27
6	14	20	25	29	31	39

α

Search k

$k = 25$ $k = 32$

Idea 1) iterate & search the entire matrix

TC: $O(M \cdot N)$

Idea 2) Apply BS on each row

TC: $N \times \log M$

while ($i < \text{row} \ \& \ j \geq 0$) {

if ($A[i][j] == k$) {

1 return true

2

else if ($A[i][j] < k$) {

1 i++

3

else ($A[i][j] > k$) {

1 j--

3

}

return false

TC: $O(M + N)$