Programming and Data Structures - I

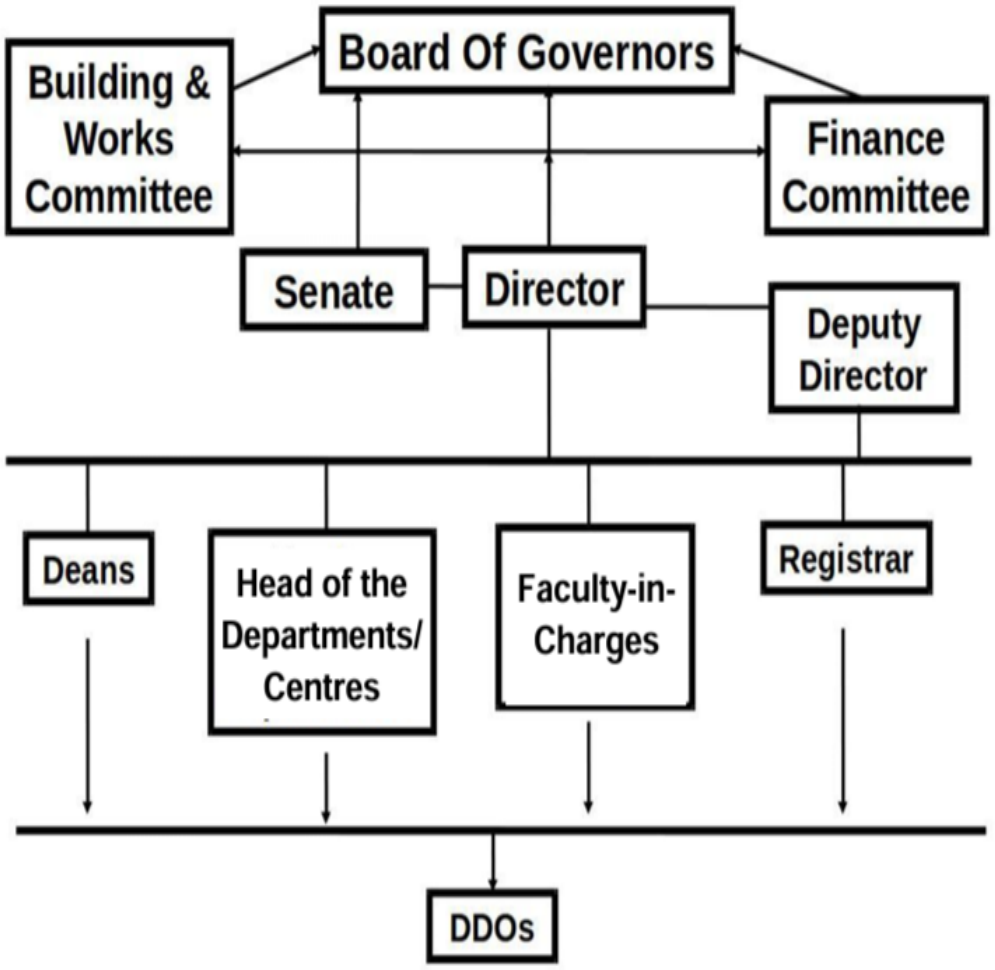
Lecture 11

Kripabandhu Ghosh

CDS, IISER Kolkata

USER-DEFINED FUNCTIONS

IISER Kolkata: Delegation of Authority



User-defined functions

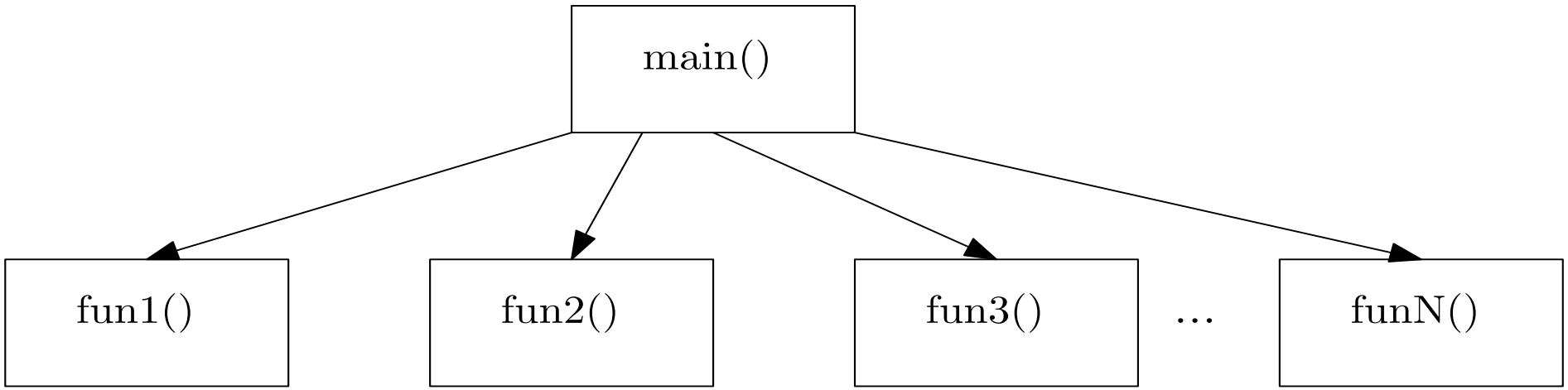
Format

return-type function-name(argument-list)

- **return-type:** Data type returned by the function; default is integer ; can also be void (nothing returned)
- **argument-list:** datatype arg1, datatype arg2,...

- A function may nor may not have any arguments
- A function may or may not return any value (maximum one value)

Functions: utility



Utilities

- Top-down modular programming
- **Decomposition** of a bigger problem into smaller ones, and **recomposition** from the smaller solutions
- Simplification of a complex logic
- Easier debugging

Arguments and return values

Program

```
#include <stdio.h>
void fun1() //No arguments, no return values
{
  printf("Fun1: No arguments, no return values!\n");
}
void fun2(int n) //Has argument(s) but no return values
{
  int i;
  for(i = 1 ; i<=n ; i++)
  {
    printf("Fun2: Has argument(s), but no return values! - %d\n\n", i);
  }
}
int fun3() //No argument(s) but has a returned value
{
  static int a = 0;
  a++;
  printf("Fun3: No argument(s), but has a return value!\n");
  return a;
}
```

Arguments and return values (contd.)

Program

```
int fun4(int n) //Has both argument(s) and a return value
{
   printf("Fun4: Has both argument(s) and a return value!\n");
   return (n + 1);
}

int main()
{
   int n = 4, i;
   fun1();
   fun2(n);
   printf("value = %d\n", fun3());
   printf("value = %d\n", fun4(n));
return 0;
}
```

Arguments and return values (contd.)

Output

```
Fun1: No arguments, no return values!
Fun2: Has argument(s), but no return values! - 1
Fun2: Has argument(s), but no return values! - 2
Fun2: Has argument(s), but no return values! - 3
Fun2: Has argument(s), but no return values! - 4
Fun3: No argument(s), but has a return value!
value = 1
Fun4: Has both argument(s) and a return value!
value = 5
```

Modular programming

Program

```
#include <stdio.h>
#include<math.h>

int factorial(int n) //Calculates factorial
{
   int i, fact = 1;
   for(i = 2 ; i <= n ; i++)
   {
      fact = i*fact;
   }
   return fact;
}

float nCr(int n, int r) //Calculates nCr
{
   float comb;
   comb = factorial(n)/(1.0*factorial(r)*factorial(n-r));
   return comb;
}
```

Modular programming (contd.)

Program

```
float binomialExpansion(int a, int b, int n) //Calculates (a + b)^n using Binomial Expansion
{
   int r;
   float sum = 0;
   for(r = 0 ; r <= n ; r++)
   {
      sum += nCr(n, r)*pow(a, n-r)*pow(b, r);
   }
   return sum;
}

float powerOfSum(int a, int b, int n) //Explicitly calculates (a + b)^n
{
   return (pow(a + b, n));
}
```

Modular programming (contd.)

Program

```
int main()
{
   int a = 4, b = 6, n = 3, r;
   printf("Binomial expansion for (%d + %d)^%d = %f\n", a, b, n,
binomialExpansion(a, b, n));
   printf("Explicit calculation for (%d + %d)^%d = %f\n", a, b, n,
powerOfSum(a, b, n));
   return 0;
}
```

Output

```
Binomial expansion for (4 + 6)^3 = 1000.000000
Explicit calculation for (4 + 6)^3 = 1000.000000
```

Stack

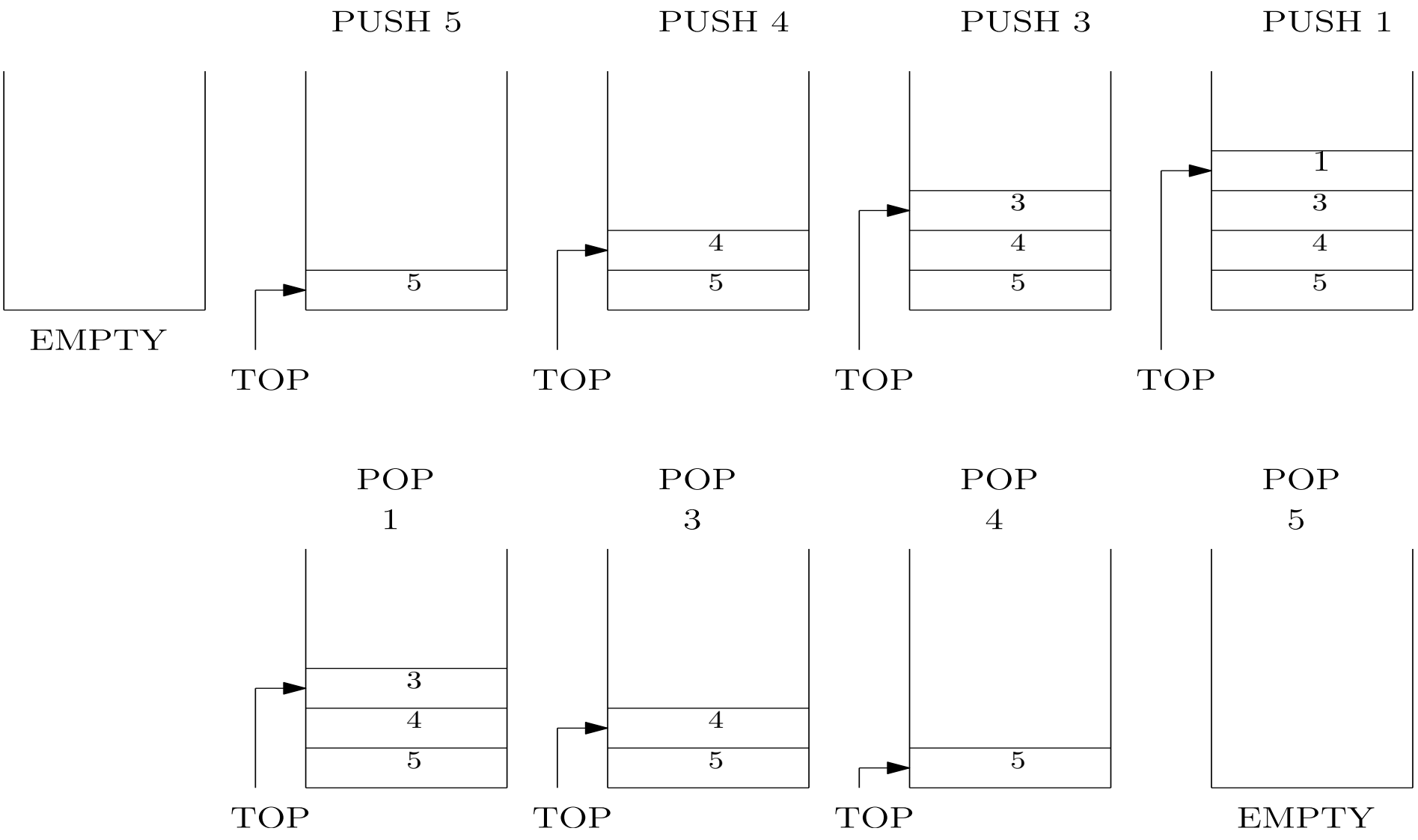
Features

- Dynamic data structure (Abstract Data Type)
- Insertion and deletion at **one end**
- Deletion based on **Last In First Out** (LIFO) policy
- Insertion called **Push** operation
- Deletion called **Pop** operation
- Usually **Top** attribute is used to point to tle last inserted element

Applications

- Function (subroutine) call and return
- Backtracking
- Syntax analysis

Stack example



Origin of Stack

- Jan Lukasiewicz's Polish notation possibly lead to the idea of stack
- Alan Turing used it for subroutines (functions) in 1946
- Konrad Zuse implemented stack in Z4 (computer) in 1945

Stack in function call

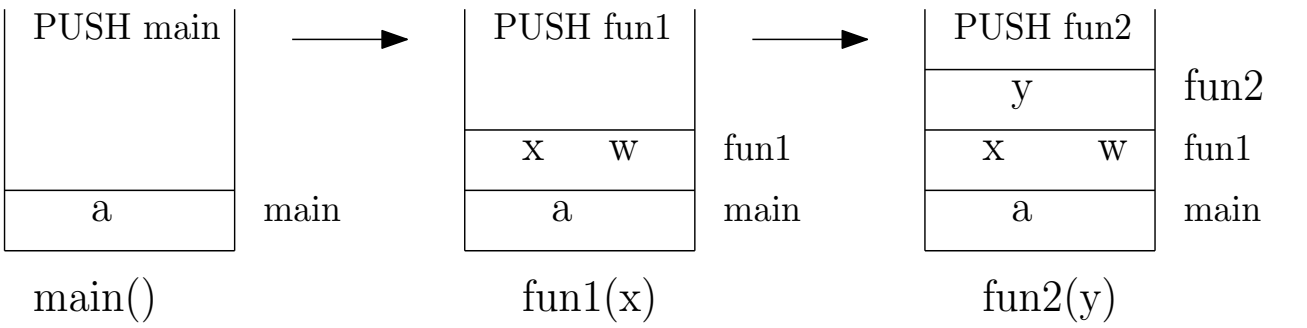
Function elements

- **Code segment:** Machine readable code generated upon the compilation
- **Data segment:** Local variables for a function created upon the compilation

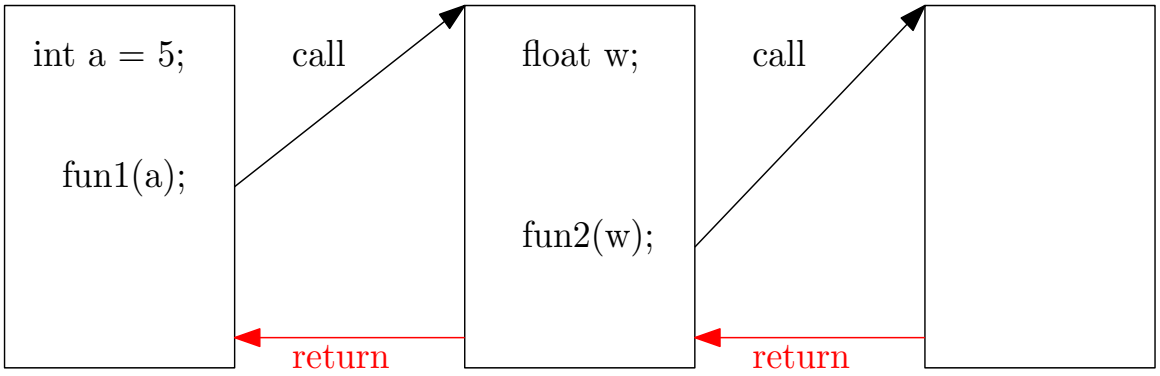
Premise of a function call

- Local data stored in a stack (retrieved on return from the call)
- Return address of the point of return from the function call also stored (retrieved to enable return from the call)

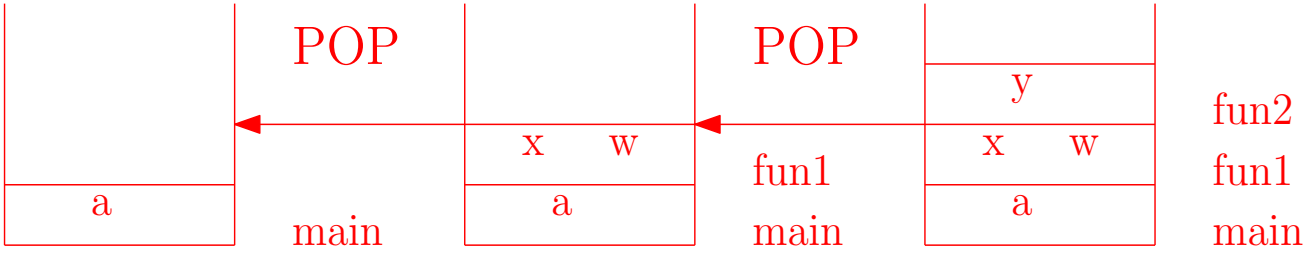
Stack in function call (illustration)



DATA STACK
(return address also Pushed)

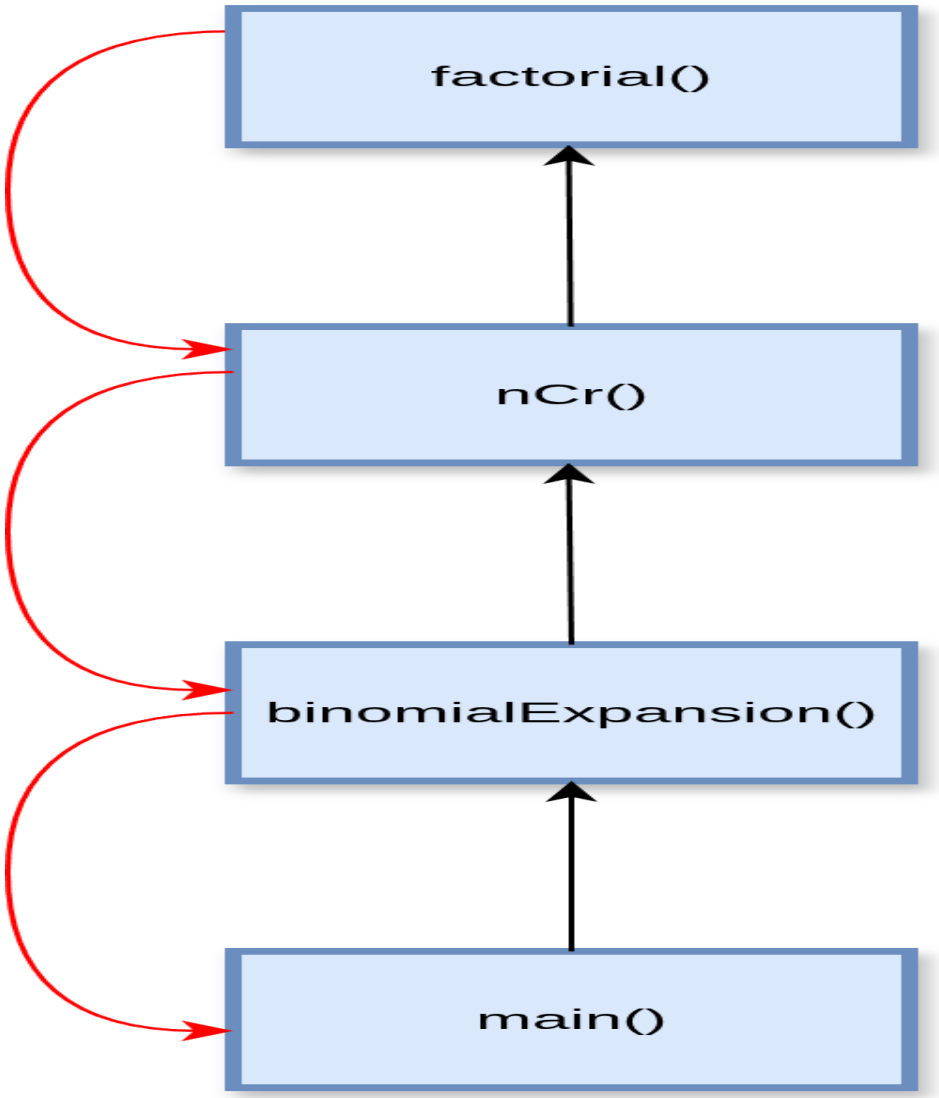


CODE SEGMENT



DATA STACK
(return address also Popped)

Stack in function call (binomial)



Stack trace

```
> gcc -g -o binomial_expansion binomial_expansion.c -lm
```

```
> gdb ./binomial_expansion
```

```
(gdb) b 6
Breakpoint 1 at 0x1174: file binomial_expansion.c, line 6.
(gdb) r
Starting program: /home/kripa/kripa/programs/C/binomial_expansion
```

```
Breakpoint 1, factorial (n=3) at binomial_expansion.c:6
6           int i, fact = 1;
(gdb) bt
#0  factorial (n=3) at binomial_expansion.c:6
#1  0x00005555555551bb in nCr (n=3, r=0) at binomial_expansion.c:21
#2  0x000055555555524f in binomialExpansion (a=4, b=6, n=3) at binomial_expansion.c:33
#3  0x0000555555555365 in main () at binomial_expansion.c:47
```

Figure: Setting breakpoint and printing the stack trace

Customized headers: mathfunc.h

```
// Function declarations
int add(int a, int b);
int subtract(int a, int b);
int multiply(int a, int b);
float divide(int a, int b);
```

Customized headers: mathfunc.c

```c
#include "mathfunc.h"

int add(int a, int b) {
    return a + b;
}

int subtract(int a, int b) {
    return a - b;
}

int multiply(int a, int b) {
    return a * b;
}

float divide(int a, int b) {
    if (b != 0)
        return (float)a / b;
    else
        return 0; // Error: divide by zero
}
```

Customized headers: main program (myprog_withmyheader.c)

```c
#include <stdio.h>
#include "mathfunc.h"    // My header

int main() {
    printf("Add: %d\n", add(5, 3));
    printf("Divide: %.2f\n", divide(10, 2));
    return 0;
}
```

```
> gcc -o myprogout myprog_withmyheader.c mathfunc.c
```

OUTPUT

```
Add: 8
Divide: 5.00
```

RECURSION


Hello Elisa...
...This is Elisa...
Hello, Elisa...
.This is Elisa speaking

Recursion

- Function calls itself
- Solving a large (input size) problem by solving smaller related sub-problems

Conditions

- Base condition (terminating condition)
- Recursive/inductive condition

Recursive definition of factorial

Function elements

$$factorial(n) = \begin{cases} 1 & \text{if } n = 1 \\ n.factorial(n-1) & \text{if } n > 1 \end{cases}$$

Factorial using recursion

factorial(n) = n x factorial(n - 1)
= n x (n - 1) x factorial(n - 2)
...
= n x (n - 1) x...x factorial(1)
= n x (n - 1) x ... x 1

n = 4

factorial(4) = 4 x factorial(3)
= 4 x 3 x factorial(2)
= 4 x 3 x 2 x factorial(1)
= 4 x 3 x 2 x 1
= 24

Factorial using recursion (program)

Program

```
#include <stdio.h>

int fact(int n) //Iterative version
{
  int i, f = 1;
  for(i = 1 ; i <= n; i++)
  {
    f = f*i;
  }
  return f;
}

int fact_recursive(n) //Recursive version
{
  int x;
  if(n == 1)
  {
    printf("Base condition return: n = %d\n", n);
    return 1;
  }
  else
  {
    printf("n = %d: before recursive call!\n", n);
    x = n*fact_recursive(n-1);
    printf("n = %d: after returning from call!\n", n);
    printf("Current value of factorial (x) = %d\n", x);
  return x;
}
```

Factorial using recursion (program) (contd.)

Program

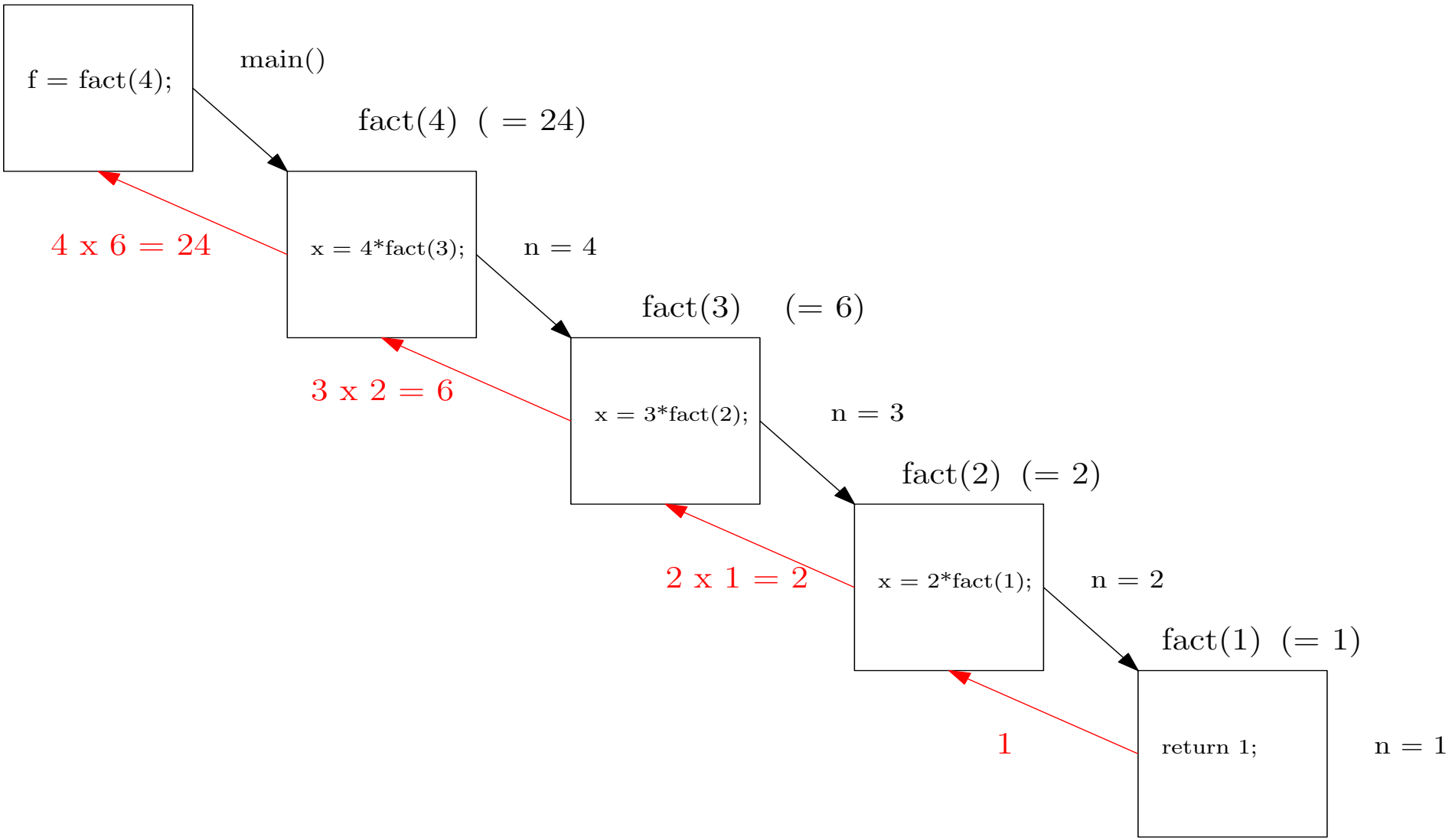
```
int main()
{
   int n = 4, fct, fct_rec;
   printf("Call from main\n");
   fct = fact(n);
   fct_rec = fact_recursive(n);
   printf("Return to main\n");
   printf("Factorial(%d) = %d (recursive)\n", n, fct_rec);
   printf("Factorial(%d) = %d (iterative)\n", n, fct);
   return 0;
}
```

Factorial using recursion (program) (contd.)

Output

```
Call from main
n = 4: before recursive call!
n = 3: before recursive call!
n = 2: before recursive call!
Base condition return: n = 1
n = 2: after returning from call!
Current value of factorial (x) = 2
n = 3: after returning from call!
Current value of factorial (x) = 6
n = 4: after returning from call!
Current value of factorial (x) = 24
Return to main
Factorial(4) = 24 (recursive)
Factorial(4) = 24 (iterative)
```

Factorial using recursion (illustration)



Factorial: decomposition and recomposition

Base condition: n = 1

if n == 1, return value is 1

Inductive condition: n > 1

- Decomposition: fact_recursive(n - 1)
- Recomposition: x = n * fact_recursive(n - 1)

Recursive definition of Fibonacci series

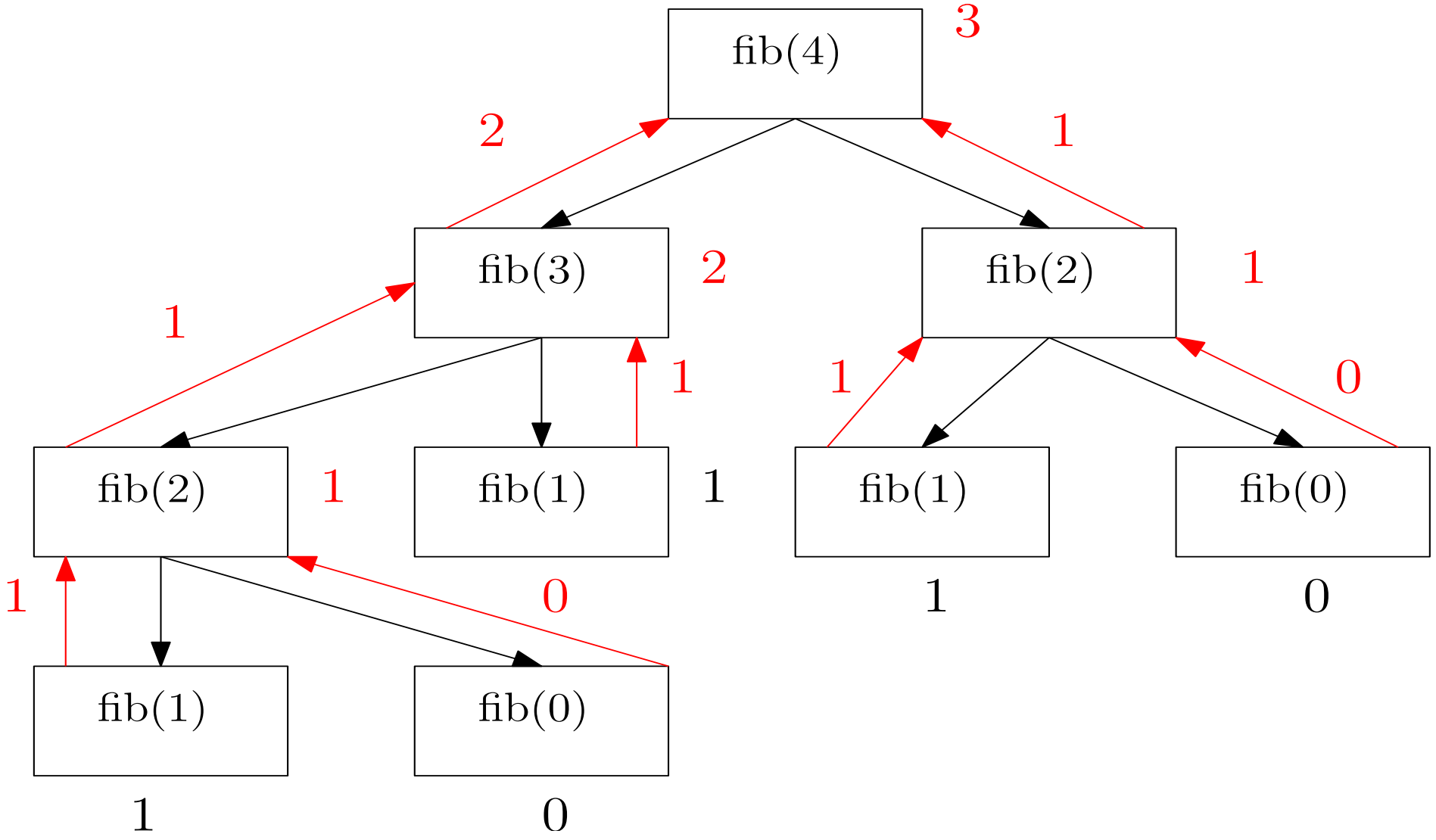
Function elements

$$fib(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ fib(n-1) + fib(n-2) & \text{if } n > 1 \end{cases}$$

Series

0 1 1 2 3 5 ...

Fibonacci series using recursion (illustration)



Fibonacci series using recursion (program)

Program

```
#include <stdio.h>

void fib(int n) //Iterative version
{
  int f0 = 0, f1 = 1, f, i;
  printf("%d %d ", f0, f1);
  for(i = 2 ; i <= n; i++)
  {
    f = f0 + f1;
    printf("%d ", f);
    f0 = f1;
    f1 = f;
  }
  printf("\n");
}

int fib_recursive(n) //Recursive version
{
  int x;
  if(n == 0)
  {
    return 0;
  }
  else
  {
    if(n == 1)
    {
      return 1;
    }
    else
    {
      x = fib_recursive(n-1) + fib_recursive(n - 2);
      return x;
    }
  }
}
```

Fibonacci series using recursion (program) (contd.)

Program

```
int main()
{
   int n = 4, i;
   printf("Recursive: \n");
   for(i = 0 ; i <= n ; i++)
   {
      printf("%d ", fib_recursive(i));
   }
   printf("\n");
   printf("Iterative: \n");
   fib(n);
return 0;
}
```

Output

```
Recursive:
0 1 1 2 3
Iterative:
0 1 1 2 3
```

GCD: Euclid's algorithm

Algorithm: GCD(m, n)

- **Step 1:** Divide m by n and let r be the remainder
- **Step 2:** If $r = 0$, end; n is the answer.
- **Step 3:** Let $m = n$, $n = r$ and go to Step 1.

GCD: Euclid's algorithm

Iterative implementation

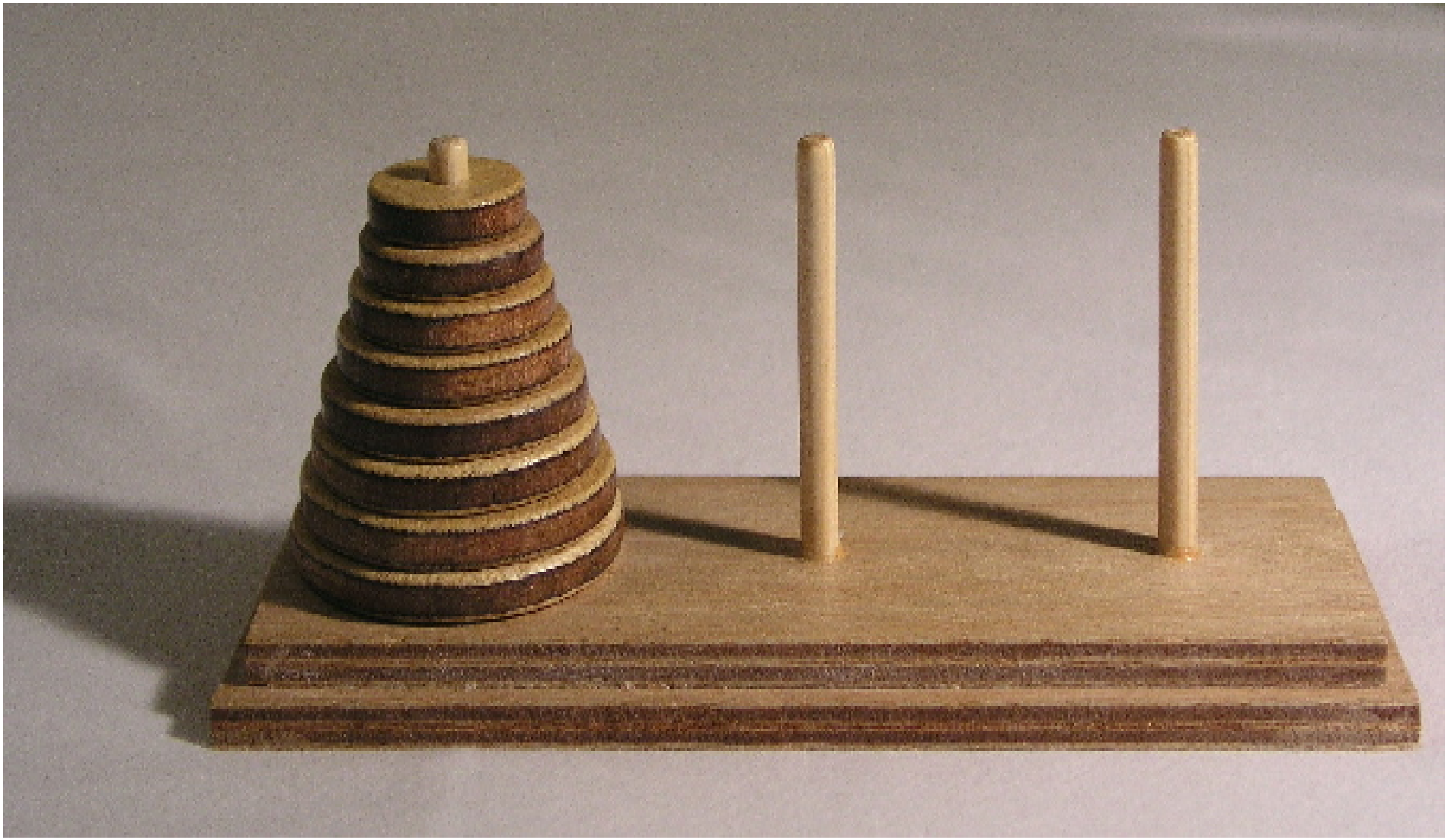
```
int gcd(int a, int b)
{
   int rem;
   while(b != 0) //Step 2
   {
      rem = a%b;//Step 1
      a = b;//Step 3
      b = rem; //Step 3
   }
   return a;
}
```

GCD: Euclid's algorithm

Recursive implementation

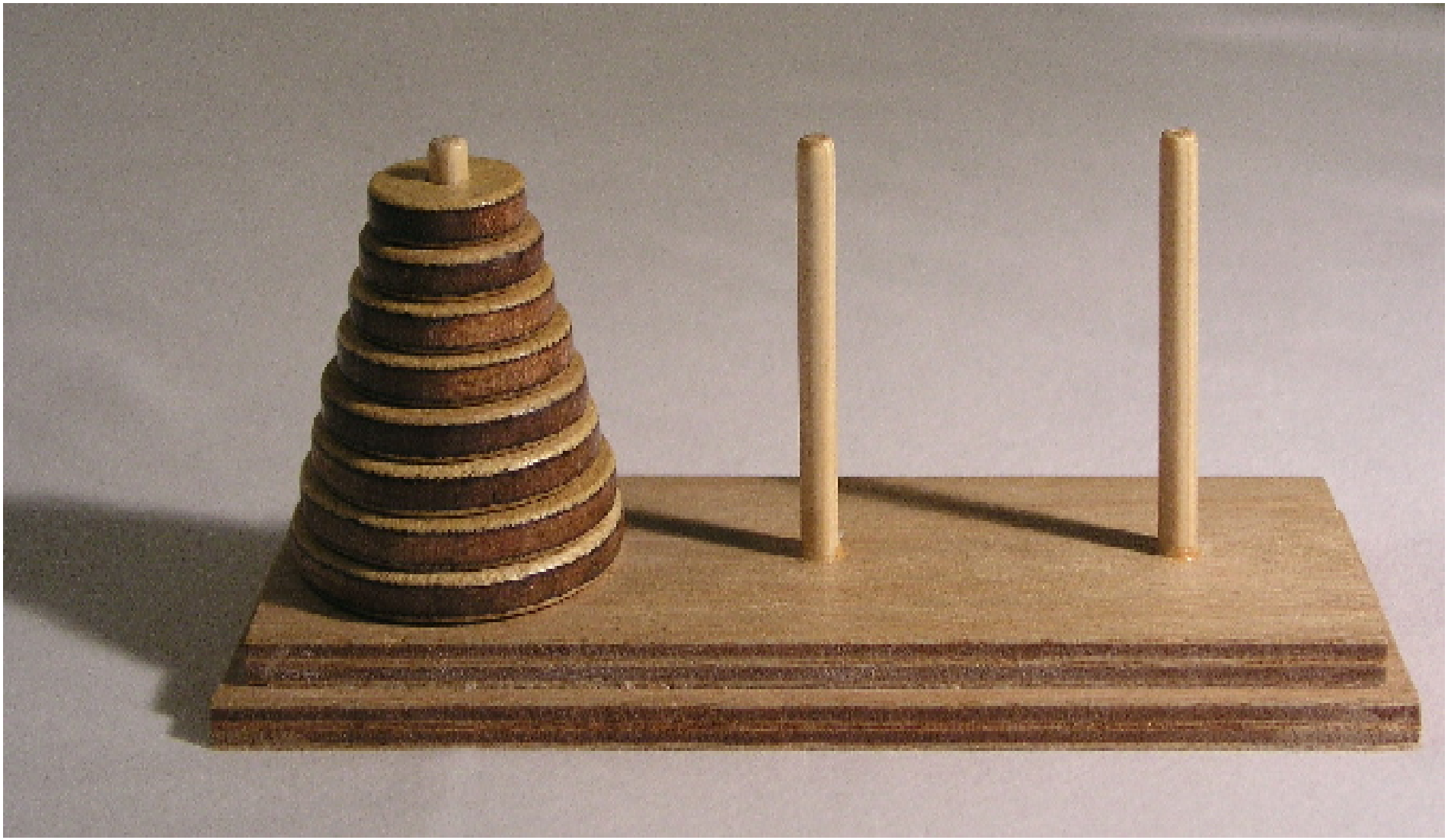
```
int gcd_rec(int a, int b)
{
   if(b == 0)//Step 2
   {
      return a;
   }
   return gcd_rec(b, a%b); //Step 1, 3
}
```

Tower Of Hanoi (Tower of Brahma)[1]



[1] Source: Wikipedia

Tower Of Hanoi (Tower of Brahma)[2]



[2]Source: Wikipedia

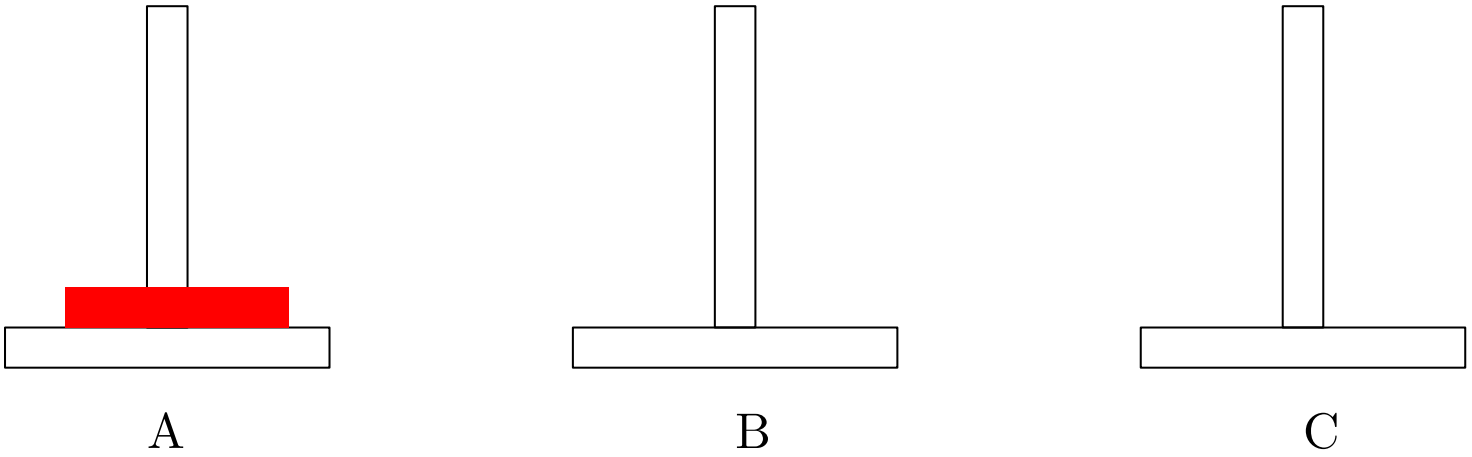
Tower Of Hanoi

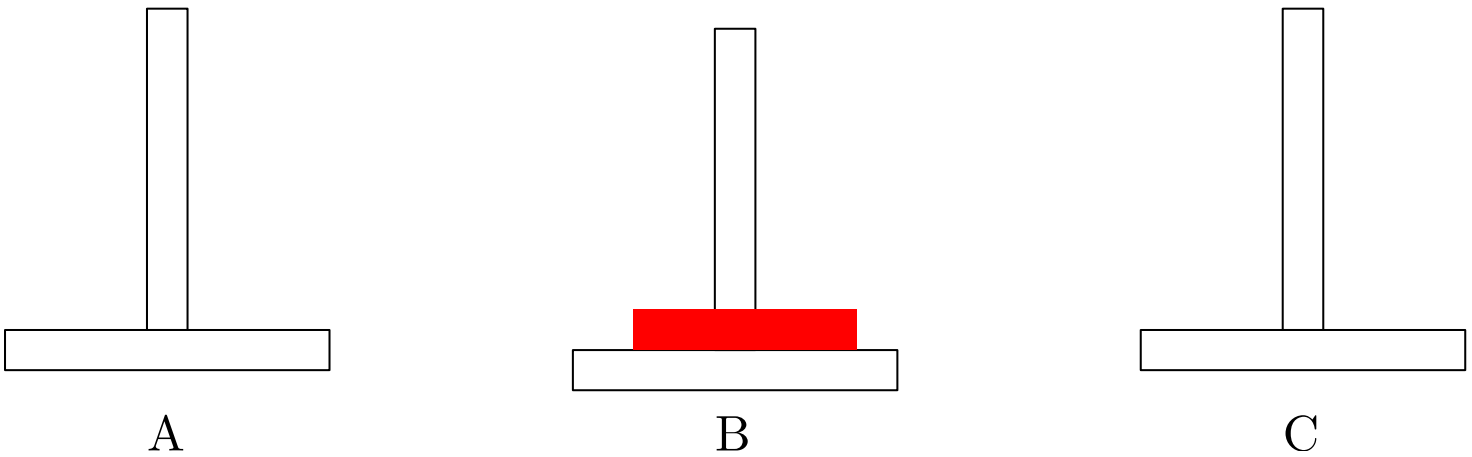
Rules

- Only one disk can be moved at a time
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod
- No larger disk may be placed on top of a smaller disk

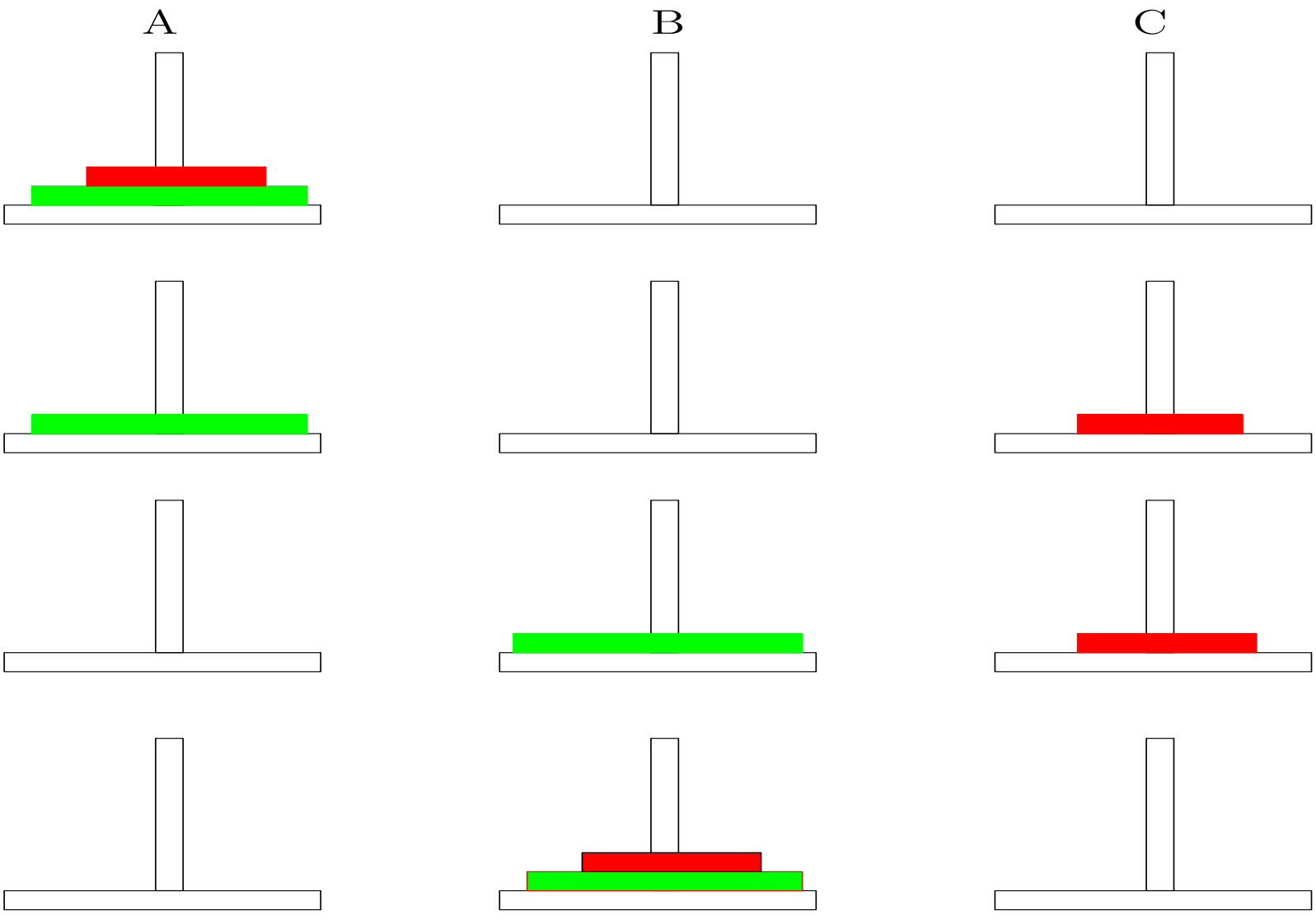
Tower of Hanoi: n = 1 (illustration)

$n = 1$



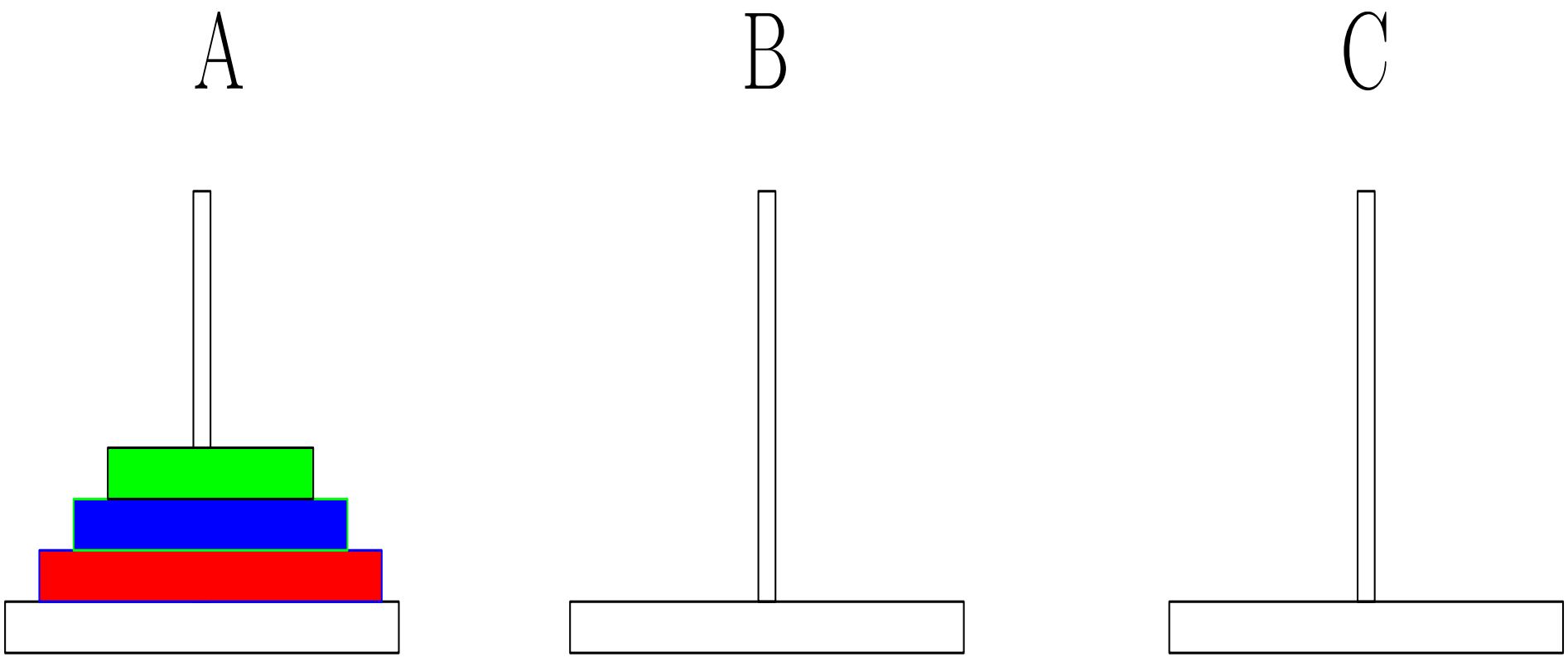


Tower of Hanoi: n = 2 (illustration)

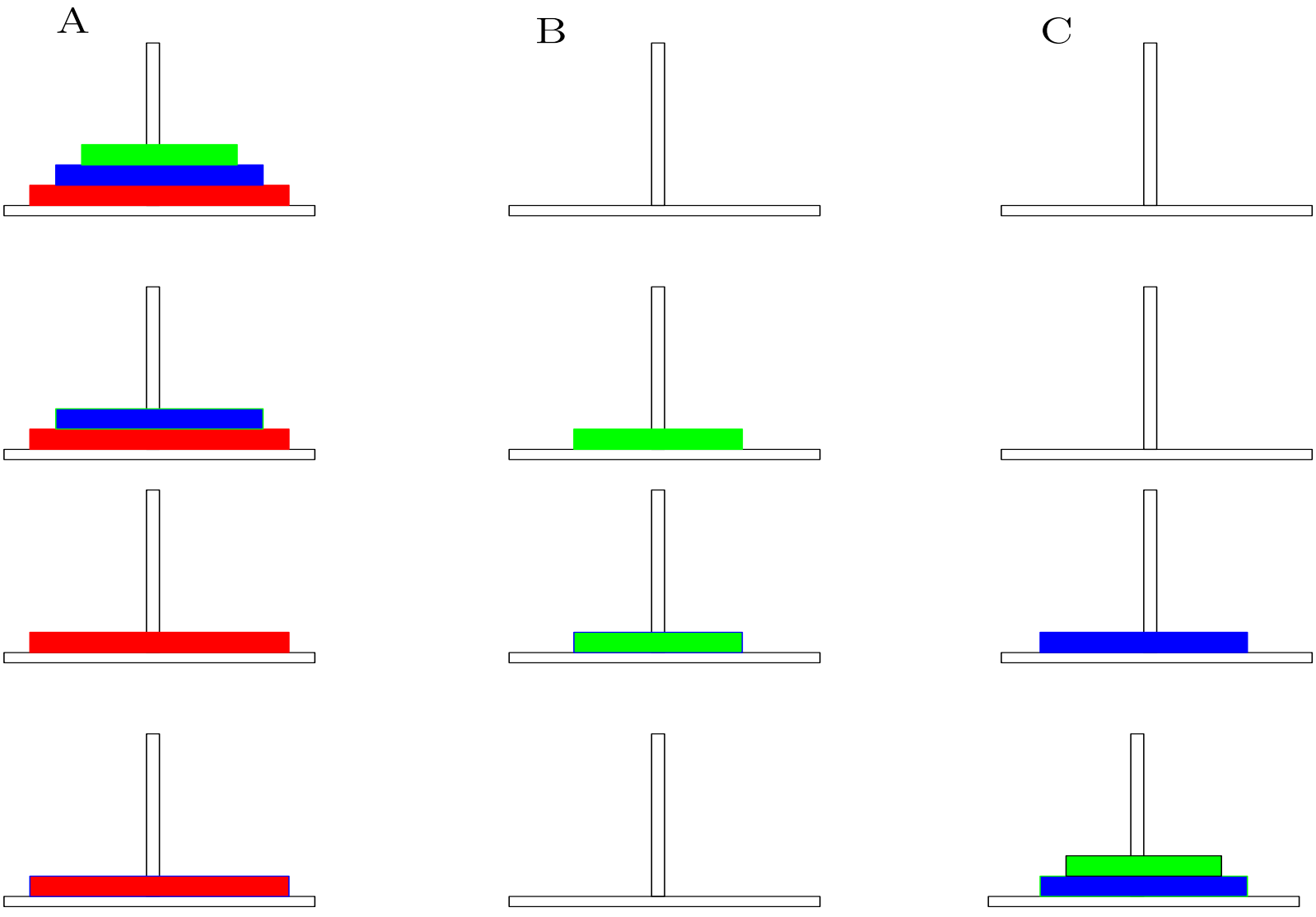


$n = 2$

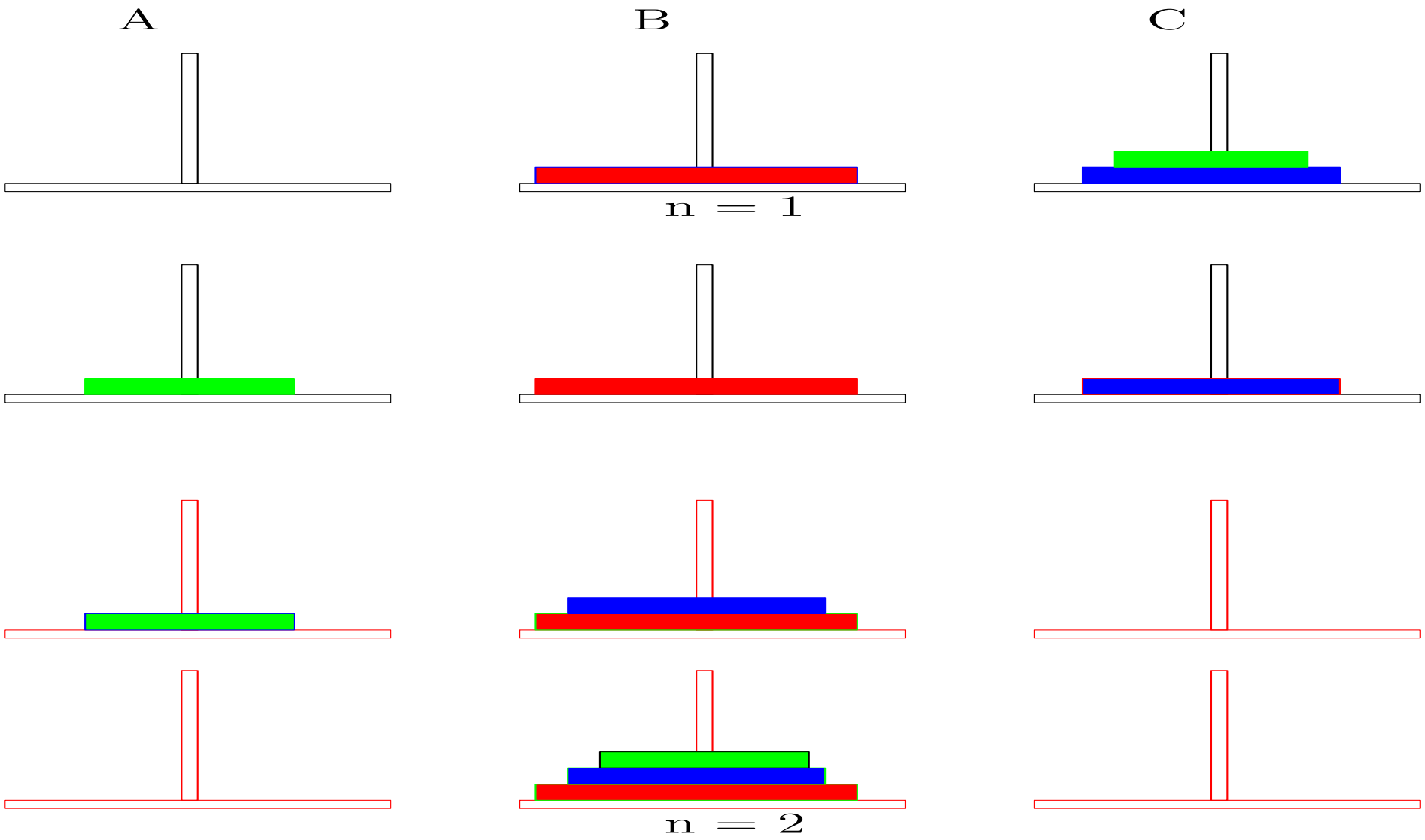
Tower of Hanoi: n = 3 (illustration): zero step



Tower of Hanoi: n = 3 (illustration)



Tower of Hanoi: n = 3 (contd.) (illustration)



Tower Of Hanoi: formal solution

Format

TOWERS(n, from, to, via)

Problem

TOWERS(n, A, B, C)

Tower Of Hanoi: formal solution (contd.)

Base condition

(n = 1): TOWERS(1, A, B, C)

Inductive condition: Decomposition

- L1 = TOWERS(n - 1, A, C, B)
- L2 = TOWERS(1, A, B, C)
- L3 = TOWERS(n - 1, C, B, A)

Inductive condition: Recomposition

L = APPEND(L1, L2, L3)

Tower Of Hanoi: implementation

Program

```
#include<stdio.h>
void tofh(int ndisk, char source, char temp, char dest);
int main(void)
{
   char source = 'A', temp = 'C', dest = 'B';
   int ndisk;
   printf("Enter the number of disks : ");
   scanf("%d", &ndisk );
   printf("Sequence is :\n");
   tofh(ndisk, source, dest, temp);
   return 0;
}
```

Tower Of Hanoi: implementation (contd.)

Program

```
void tofh(int ndisk, char source, char dest, char temp)
{
  if(ndisk==1)
  {
    printf("Move Disk %d from %c->%c\n", ndisk, source, dest);
    return;
  }
  tofh(ndisk-1, source, temp, dest);
  printf("Move Disk %d from %c->%c\n", ndisk, source, dest);
  tofh(ndisk-1, temp, dest, source);
}
```

Tower Of Hanoi: implementation (contd.)

Output

```
Enter the number of disks : 3
Sequence is :
Move Disk 1 from A—>B
Move Disk 2 from A—>C
Move Disk 1 from B—>C
Move Disk 3 from A—>B
Move Disk 1 from C—>A
Move Disk 2 from C—>B
Move Disk 1 from A—>B
```


THANK YOU!