Name: Sandeep N Kundalwal
Roll No: T22051
Network Theory - Assignment 1

Page
Date

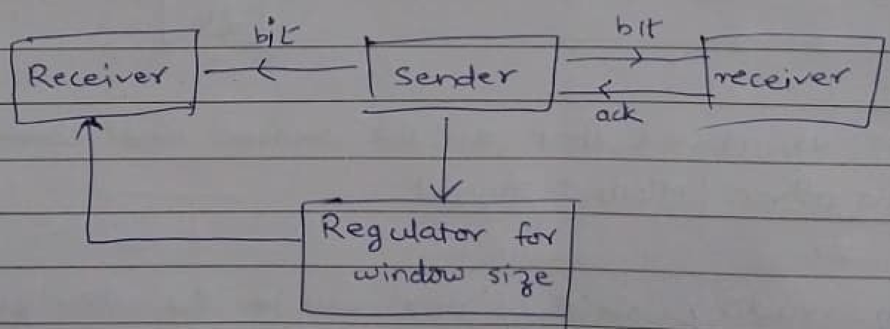
Ans 1. Powerline Networks:

Powerline or power distribution networks are most commonly seen network in our daily life. In this network, first of all, power is generated in a power-plant. Then it is supplied to power grids and then it reaches to our homes or offices.

In this network, there are multiple towers having powerline connected to each other for longer range supply. But these towers are connected to each other forming a cluster and then these clusters are connected to grid. That means it has inter-cluster connection through grid.

The most interesting thing or we can say problem in this network is, as these are human-made networks that might be involved in random failures as well as target attacks. Failures may have cascading effects i.e. the failure of one node may recursively provoke the failure of connected node

Ans 2. a)



b) The probability that a packet is dropped before it reaches destination is 'p', so the probability that the packet reaches is '1-p'

Let assume that a packet reaches n times without

Page
Date

dropping, then the window size (s) will increase by 1 every time So, new window size (s') will be given by,

$$s' = (1-p)^n [s+n] \quad — \text{I}$$

And if the packet drops $\overset{k \text{ times}}{\uparrow}$ before reaching, then the new window size (s'') will be given by,

$$s'' = p^k \left[\frac{s}{2^k} \right] \quad — \text{II}$$

c) The sliding window will remain constant when the ~~pro~~ window size of I & II remains constant i.e.

$$s = s' = s''$$

$$\therefore (1-p)^n [s+n] = p^k \left[\frac{s}{2^k} \right]$$

Ans 3. Let us assume that all the sensors are connected to each other (Mesh Network).

So, reading each sensor will be the average of reading of all other sensors For e.g, the first reading i.e. for k = 1, will be,

$$s_1 = \frac{1}{10} (s_1 + s_2 + s_3 + \dots + s_{10})$$

Similarly for all 10 sensors, their first reading gets updated in the same way as that for first sensor. So, the generalized formula can be,

$$\sum_{k=1}^{10}\sum_{i=1}^{10} s_i^{k+} = \underbrace{\frac{1}{10}\left[\sum_{j=1}^{k-1}\sum_{i=1}^{10} s_i^{j+}\right]}_{\text{LAS operation}}$$

Hence,

$$[s_i^k] = C.[s_i]$$

OR

$$\sum_{k=1}^{10}\sum_{i=1}^{10} s_i^{k+} = \sum_{j=1}^{k-1}\frac{1}{10^k}\left(\sum_{i=1}^{10} s_i^{+}\right)$$

Ans 5. $A = \begin{bmatrix} -2 & -1 & -3 \\ 4 & 3 & 3 \\ 2 & 1 & 1 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & -1 & -3 \\ 4 & 3-\lambda & 3 \\ -2 & 1 & -1-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)\left((3-\lambda)(-1-\lambda) - 3\right) + 1\left(4(-1-\lambda) - 3(-2)\right)$$
$$- 3\left(4 - (-2(3-\lambda))\right) = 0$$

$$(-2-\lambda)(-3-3\lambda+\lambda+\lambda^2-3) + 1(-4-4\lambda+6) - 3(4+6-2\lambda) = 0$$
$$(-2-\lambda)(\lambda^2-2\lambda-6) - 4\lambda+2 - 3(10-2\lambda) = 0$$
$$-\cancel{2\lambda^2} + \cancel{4\lambda} + 12 - \lambda^3 + \cancel{2\lambda^2} + 6\lambda - \cancel{4\lambda} + 2 - 30 + 6\lambda = 0$$

Page
Date

Characteristic Equation

$-\lambda^3 + 12\lambda - 16 = 0$

for $\lambda = 2$, satisfies the above characteristic equation

$$\lambda^2(\lambda-2) + 2\lambda(\lambda-2) - 8(\lambda-2) = 0$$
$$(\lambda-2)(\lambda^2+2\lambda-8) = 0$$
$$\lambda^2 + 2\lambda - 8 = 0$$
$$\lambda^2 + 4\lambda - 2\lambda - 8 = 0$$
$$\lambda(\lambda+4) - 2(\lambda+4) = 0$$
$$(\lambda-2)(\lambda+4)$$

Eigen Values :-

$\lambda_1 = 2$

$\lambda_2 = 2$

$\lambda_3 = -4$

For $\lambda = -4$,

$$A + 4I = \begin{bmatrix} -2+4 & -1 & -3 \\ 4 & 3+4 & 3 \\ 2 & 1 & -1+4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & -3 \\ 4 & 7 & 3 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \frac{x_1}{\begin{vmatrix} 7 & 3 \\ 1 & 3 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 4 & 3 \\ -2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & 7 \\ -2 & 1 \end{vmatrix}}$$

$$\frac{x_1}{21-3} = \frac{-x_2}{12+6} = \frac{x_3}{4+14}$$

$$= 18 = 18 = 18$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Eigen Vector for $\lambda = -4$

Similarly for $\lambda = 2$, we find the Eigen Vector as,

$$A - 2I = \begin{bmatrix} -2-2 & -1 & -3 \\ 4 & 3-2 & 3 \\ -2 & 1 & -1-2 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & -1 & -3 \\ 4 & 1 & 3 \\ -2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore x_1 = \begin{vmatrix} 1 & 3 \\ 1 & -3 \end{vmatrix} = -3-3 = -6$$

$$-x_2 = \begin{vmatrix} 4 & 3 \\ -2 & -3 \end{vmatrix} = -12+6 = -6$$

$$x_3 = \begin{vmatrix} 4 & 1 \\ -2 & 1 \end{vmatrix} = 4+2 = 6$$

$\therefore$ Eigen Vector for $\lambda = 2 \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$\therefore$ Eigen Values	AM	GM
2	2	1
-4	1	1

Jordon Normal Form :-

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -4 \end{bmatrix}$$

For $\lambda = 1 \rightarrow \begin{bmatrix} -1 \\ +1 \\ +1 \end{bmatrix}$, norm $= |\lambda| = \sqrt{(-1)^2 + (1)^2 + (1)^2}$

$= \sqrt{3}$

Page
Date

For $\lambda = -4 \rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $|\lambda| = \sqrt{3}$

Q.4 The given system is,

$$x(k+1) = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} x(k)$$

$$A = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix}$$

Finding Eigen Values of A,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1/2 \\ 1/2 & 1/3 - \lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1/3 - \lambda) - 1/4 = 0$$

$$\lambda^2 - 4/3\,\lambda + 1/12 = 0$$

$$\therefore \lambda = \frac{1}{6}(4 \pm \sqrt{13})$$

Page ____ Date ____

Finding eigen vectors for the ~~same~~, $\lambda = \frac{1}{6}(4+\sqrt{13})$

$$(A-\lambda I)\,x = 0$$

$$\begin{bmatrix} 1-\left(\frac{4}{6}+\frac{\sqrt{13}}{6}\right) & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3}-\left(\frac{4}{6}+\frac{\sqrt{13}}{6}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2-\sqrt{13}}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-2-\sqrt{13}}{6} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving the above.

$$v_1 = \begin{bmatrix} \frac{1}{3}(2+\sqrt{13}) \\ 1 \end{bmatrix}$$

Similarly, eigen vector for $\lambda = \frac{1}{6}(4-\sqrt{13})$

$$v_2 = \begin{bmatrix} \frac{1}{3}(2-\sqrt{13}) \\ 1 \end{bmatrix}$$

Now using Trajectory Equation of Modal Decomposition

$$x(k) = \lambda_1^{k}\left(v_1^{T}x(0)\right)v_1 + \lambda_2^{k}\left(v_2^{T}x(0)\right)v_2$$

$$\therefore x(k) = \left(\frac{1}{6}\left(4+\sqrt{13}\right)\right)^k \begin{bmatrix} \frac{1}{3}(2+\sqrt{13}) & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \frac{1}{3}(2+\sqrt{13}) \\ 1 \end{bmatrix}$$

$$+ \left(\frac{1}{6}\left(4-\sqrt{13}\right)\right)^k \begin{bmatrix} \frac{1}{3}(2-\sqrt{13}) & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \frac{1}{3}(2-\sqrt{13}) \\ 1 \end{bmatrix}$$