



Machine Learning

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Inductive Classification

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Concept Learning Task

A concept learning task is the problem of **finding a general rule (concept)** that correctly classifies instances.

The goal is to learn a function that maps **instances** \rightarrow **class labels**.

i.e. learn a target concept (function) from a set of training examples.

Therefore,

Definition

Concept learning is the task of inferring a Boolean-valued function from training examples of its inputs and outputs.

Notations

- **Instance Space (X):** All possible examples.

$$X = \{x_1, x_2, \dots, x_n\}$$

- **Target Concept (c):** Unknown function to be learned.

$$c : X \rightarrow \{0, 1\}$$

- **Training Data (D):** Labeled examples.

$$D = \{\langle x_i, c(x_i) \rangle\}$$

- **Hypothesis Space (H):** Set of possible rules.

$$h : X \rightarrow \{0, 1\}$$

Learning Objective

Goal

Find a hypothesis $h \in H$ such that:

$$h(x) \approx c(x), \quad \forall x \in X$$

Hypotheses H : Each hypothesis is a conjunction of constraints on the attributes. Constraints can be:

- ? (any value acceptable),
- ϕ (no value acceptable), or
- A specific value

Concept Learning as a Search Problem

Concept learning can be viewed as a **search problem** where:

- The search space is the **hypothesis space H** .
- Each hypothesis represents a possible concept.
- The goal is to find a hypothesis that is consistent with training data.

Example

- **Attributes:** (Sky, Temp, Humidity, Wind, Water, Forecast)
- **Target concept (c):** “Play Tennis”
- **Training example:**

$\langle (Sunny, Warm, Normal, Strong, Warm, Same), Yes \rangle$

- **Hypothesis (h):**

$$h(x) = (Sky = Sunny) \wedge (Temp = Warm)$$

Problem:

Attribute	Possible Values
Sky	{Sunny, Rainy, Cloudy} $\rightarrow 3$
Temp	{Warm, Cold} $\rightarrow 2$
Humidity	{Normal, High} $\rightarrow 2$
Wind	{Strong, Weak} $\rightarrow 2$
Water	{Warm, Cool} $\rightarrow 2$
Forecast	{Same, Change} $\rightarrow 2$

Each attribute allows one additional option “?” (any value).

Q. Find Instance Space, Total number of concepts, Numbers of Hypothesis.

Number of Concepts

Instance Space Size:

$$|X| = 3 \times 2 \times 2 \times 2 \times 2 \times 2 = 96$$

Definition of a Concept:

$$c : X \rightarrow \{0, 1\}$$

Each instance can be labeled independently as positive or negative.

Total Number of Concepts:

$$\text{Number of concepts} = 2^{|X|} = 2^{96}$$

Number of Hypotheses

For each attribute:

$$\text{Choices} = (\text{number of values}) + 1$$

$$|H| = (3 + 1)(2 + 1)^5$$

$$|H| = 4 \times 3^5 = 4 \times 243$$

$$\boxed{|H| = 972}$$

General-to-Specific Ordering of Hypotheses

Specific hypothesis: only matches very few positive examples.

General hypothesis: matches many positive examples.

Definition

A hypothesis h_j is **more general than or equal to** hypothesis h_k (written $h_j \geq_g h_k$) if:

$$\forall x \in X, (h_k(x) = 1 \implies h_j(x) = 1)$$

Example with Attributes: (*Sky, Temp, Humidity, Wind*)

$$h_1 = (?, \text{Warm}, ?, ?)$$

$$h_2 = (\text{Sunny}, \text{Warm}, \text{High}, ?)$$

So,

$$h_1 \geq_g h_2$$

Concept Learning Task: Find-S Algorithm

Training Examples:

Ex.	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Question:

Using the **Find-S Algorithm**, determine the final hypothesis h after processing all the above examples.

Find-S Algorithm: Step 1 & Step 2

Goal: Find the most specific hypothesis h consistent with all positive training examples.

Step 1: Initialization

- Start with the most specific hypothesis:

$$h \leftarrow \langle \emptyset, \emptyset, \dots, \emptyset \rangle$$

Step 2: For each training example (x, y)

- If y is positive, then for each attribute i :
 - If $h[i] = \emptyset$, set $h[i] \leftarrow x[i]$.
 - If $h[i] \neq x[i]$, set $h[i] \leftarrow ?$.
 - Otherwise leave $h[i]$ unchanged.

Find-S Algorithm: Step 3 & Step 4

Step 3: Processing negative examples

- If y is negative:
 - Do nothing (hypothesis is not updated).

Step 4: Final hypothesis

- After all training examples are processed, return h .

Output: The most specific hypothesis consistent with all positive training examples.

Find-S Steps:

$$S_0 = \langle \phi, \phi, \phi, \phi, \phi, \phi \rangle$$

$$S_1 = \langle \text{Sunny}, \text{Warm}, \text{Normal}, \text{Strong}, \text{Warm}, \text{Same} \rangle \quad (\text{after Example 1})$$

$$S_2 = \langle \text{Sunny}, \text{Warm}, ?, \text{Strong}, \text{Warm}, \text{Same} \rangle \quad (\text{after Example 2})$$

$$S_3 = S_2 \quad (\text{Example 3 is negative, ignored})$$

$$S_{\text{final}} = \langle \text{Sunny}, \text{Warm}, ?, \text{Strong}, ?, ? \rangle \quad (\text{after Example 4})$$

Final Hypothesis:

$$h(x) = \text{EnjoySport if } (Sky = \text{Sunny}) \wedge (AirTemp = \text{Warm}) \wedge (Wind = \text{Strong})$$

Concept Learning Task: Loan Approval

Ex.	Income	Credit	Collateral	Age	Employment	Location	LoanApproval
1	High	Good	Yes	Young	Salaried	Urban	Yes
2	High	Good	No	Young	Salaried	Urban	Yes
3	Low	Bad	Yes	Old	SelfEmp	Rural	No
4	High	Good	Yes	Young	Salaried	Urban	Yes

Q: Using the **Find-S Algorithm**, what is the final hypothesis h after processing all the above examples?

Find-S Solution: Loan Approval (Part 1)

Start with the most specific hypothesis:

$$S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$$

Example 1 (Positive) : (High, Good, Yes, Young, Salaried, Urban)

$$S_1 = \langle \textit{High}, \textit{Good}, \textit{Yes}, \textit{Young}, \textit{Salaried}, \textit{Urban} \rangle$$

Example 2 (Positive) : (High, Good, No, Young, Salaried, Urban)

Collateral differs \rightarrow generalize to ?:

$$S_2 = \langle \textit{High}, \textit{Good}, ?, \textit{Young}, \textit{Salaried}, \textit{Urban} \rangle$$

Find-S Solution: Loan Approval (Part 2)

Example 3 (Negative): (Low, Bad, Yes, Old, SelfEmp, Rural)

Ignore negative example; hypothesis unchanged:

$$S_3 = S_2 = \langle \textit{High}, \textit{Good}, ?, \textit{Young}, \textit{Salaried}, \textit{Urban} \rangle$$

Example 4 (Positive): (High, Good, Yes, Young, Salaried, Urban)

Already consistent, final hypothesis:

$$S = \langle \textit{High}, \textit{Good}, ?, \textit{Young}, \textit{Salaried}, \textit{Urban} \rangle$$

Drawbacks of Find-S Algorithm

- **Ignores Negative Examples:** Updates hypothesis only on positive examples; negative examples are not used.
- **Single Hypothesis Output:** Produces only one final hypothesis, even if multiple consistent hypotheses exist.
- **Dependence on Initial Example:** The first positive example strongly influences the hypothesis, limiting flexibility.

Concept Learning Task: Candidate Elimination Algorithm

Training Examples:

Ex.	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Question: Using the **Candidate Elimination Algorithm**, compute the **final Version Space** $VS_{H,D}$ by updating the S (most specific) and G (most general) boundaries after each example.

Prerequisites for Candidate Elimination Algorithm

- **Hypothesis Space (H):** The set of all possible hypotheses. Each hypothesis is a conjunction of attribute constraints.
- **General-to-Specific Ordering:** A hypothesis h_j is more general than or equal to h_k if:

$$\forall x \in X, (h_k(x) = 1 \implies h_j(x) = 1)$$

- **Boundary Sets:**
 - S — the set of most specific hypotheses consistent with the data.
 - G — the set of most general hypotheses consistent with the data.
- **Version Space:** The set of all hypotheses in H that are consistent with the training data:

$$VS_{H,D} = \{h \in H \mid h(x) = c(x), \forall (x, c(x)) \in D\}$$

Candidate Elimination Algorithm: Step 1 – Initialization

Goal: Learn the version space (all hypotheses consistent with the training data) by maintaining:

- **S** — the set of most specific hypotheses
- **G** — the set of most general hypotheses

Step 1: Initialization

- Start with the most specific hypothesis:

$$S = \langle \emptyset, \emptyset, \dots \rangle$$

- Start with the most general hypothesis:

$$G = \langle ?, ?, \dots \rangle$$

Candidate Elimination: Positive Example Update

For a positive example $(x, y = 1)$:

- ① Remove from G any hypotheses inconsistent with x
- ② For each $s \in S$ inconsistent with x :
 - Generalize s minimally so that $s(x) = 1$
- ③ Remove any hypothesis in S that is more general than another hypothesis in S

Effect: S becomes more general to include positive examples, G stays consistent

Candidate Elimination: Negative Example Update

For a negative example $(x, y = 0)$:

- ① Remove from S any hypotheses inconsistent with x
- ② For each $g \in G$ inconsistent with x :
 - Specialize g minimally so that $g(x) = 0$
- ③ Remove any hypothesis in G that is more specific than another hypothesis in G

Effect: G becomes more specific to exclude negative examples, S stays consistent

Candidate Elimination: Output

After processing all training examples:

$$VS = \{h \mid S \leq_g h \leq_g G\}$$

- All hypotheses in the version space are consistent with all training examples
- S = most specific hypotheses
- G = most general hypotheses
- The version space represents all possible solutions

Candidate Elimination Algorithm: Step-by-Step

Initialization:

- $S_0 = \{\langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle\}$
- $G_0 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

Example 1 (Positive): (Sunny, Warm, Normal, Strong, Warm, Same)

- $S_1 = \{\langle \text{Sunny}, \text{Warm}, \text{Normal}, \text{Strong}, \text{Warm}, \text{Same} \rangle\}$
- $G_1 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

Candidate Elimination Algorithm

Example 2 (Positive): (Sunny, Warm, High, Strong, Warm, Same)

- Generalize S_1 on Humidity $\rightarrow ?$
- $S_2 = \{\langle \text{Sunny}, \text{Warm}, ?, \text{Strong}, \text{Warm}, \text{Same} \rangle\}$
- $G_2 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

Example 3 (Negative): (Rainy, Cold, High, Strong, Warm, Change)

- Specialize G to exclude negative while $\geq S$
- $G_3 = \{\langle \text{Sunny}, ?, ?, ?, ?, ? \rangle, \langle ?, \text{Warm}, ?, ?, ?, ? \rangle, \langle ?, ?, ?, ?, ?, \text{Same} \rangle\}$
- $S_3 = S_2$

Candidate Elimination Algorithm

Example 4 (Positive): (Sunny, Warm, High, Strong, Cool, Change)

- Remove inconsistent hypotheses from G_3
- $G_4 = \{\langle \text{Sunny}, ?, ?, ?, ?, ? \rangle, \langle ?, \text{Warm}, ?, ?, ?, ? \rangle\}$
- Generalize S_3 to cover Water and Forecast differences
- $S_4 = \{\langle \text{Sunny}, \text{Warm}, ?, \text{Strong}, ?, ? \rangle\}$

Final Version Space

Most Specific Boundary S :

$$S = \{\langle \text{Sunny}, \text{Warm}, ?, \text{Strong}, ?, ? \rangle\}$$

Most General Boundary G :

$$G = \{\langle \text{Sunny}, ?, ?, ?, ?, ? \rangle, \langle ?, \text{Warm}, ?, ?, ?, ? \rangle\}$$

Final Version Space:

$$VS_{H,D} = \{h \in H \mid S \preceq h \preceq G\}$$

Concept Learning Task: Car Buying Decision

Training Examples:

Ex.	Make	Doors	Engine	Color	Manual	Buy?
1	Toyota	4	Petrol	Red	Yes	Yes
2	Honda	2	Petrol	Blue	Yes	Yes
3	Ford	4	Diesel	Red	No	No
4	Toyota	2	Petrol	Red	Yes	Yes

Q. Use the **Candidate Elimination Algorithm** to update the S and G boundaries after each example.

Step 0: Initialization

- Most specific hypothesis: $S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$
- Most general hypothesis: $G_0 = \langle ?, ?, ?, ?, ? \rangle$

Step 1: Example 1 (Positive)

- Example 1: $\langle \text{Toyota}, 4, \text{Petrol}, \text{Red}, \text{Yes}, \text{Yes} \rangle$
- Update S to first positive example:

$$S_1 = \langle \text{Toyota}, 4, \text{Petrol}, \text{Red}, \text{Yes} \rangle$$

- G remains maximally general:

$$G_1 = \langle ?, ?, ?, ?, ? \rangle$$

Step 2: Example 2 (Positive)

- Example 2: $\langle \text{Honda}, 2, \text{Petrol}, \text{Blue}, \text{Yes}, \text{Yes} \rangle$
- Generalize S where attributes differ:

$$S_2 = \langle ?, ?, \text{Petrol}, ?, \text{Yes} \rangle$$

- G unchanged:

$$G_2 = \langle ?, ?, ?, ?, ? \rangle$$

Step 3: Example 3 (Negative)

- Example 3: $\langle \text{Ford}, 4, \text{Diesel}, \text{Red}, \text{No}, \text{No} \rangle$
- Specialize G to exclude negative example while including S :

$$G_3 = \langle ?, ?, \text{Petrol}, ?, ? \rangle$$

- S remains unchanged:

$$S_3 = \langle ?, ?, \text{Petrol}, ?, \text{Yes} \rangle$$

Step 4: Example 4 (Positive)

- Example 4: $\langle \text{Toyota}, 2, \text{Petrol}, \text{Red}, \text{Yes}, \text{Yes} \rangle$
- Update S to include this positive example (already covered by S_3):

$$S_4 = \langle ?, ?, \text{Petrol}, ?, \text{Yes} \rangle$$

- G unchanged:

$$G_4 = \langle ?, ?, \text{Petrol}, ?, ? \rangle$$

Final Version Space

- Most Specific Hypothesis:

$$S = \langle ?, ?, \text{Petrol}, ?, \text{Yes} \rangle$$

- Most General Hypothesis:

$$G = \langle ?, ?, \text{Petrol}, ?, ? \rangle$$

- Version space narrowed down to **Engine = Petrol** and **Manual = Yes**.

Concept Learning Task: Fruit Edibility Decision

Training Examples:

Ex.	Color	Size	Shape	Taste	Sweet	Edible
1	Red	Small	Round	Sour	Yes	Yes
2	Yellow	Small	Round	Sweet	Yes	Yes
3	Green	Large	Oval	Bitter	No	No
4	Red	Small	Oval	Sweet	Yes	Yes

Use the Candidate Elimination Algorithm to update the S and G boundaries after each example.

Step 0: Initialization

- Most specific hypothesis: $S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$
- Most general hypothesis: $G_0 = \langle ?, ?, ?, ?, ? \rangle$

Step 1: Example 1 (Positive)

- Example 1: $\langle \text{Red, Small, Round, Sour, Yes, Yes} \rangle$
- Update S to first positive example:

$$S_1 = \langle \text{Red, Small, Round, Sour, Yes} \rangle$$

- G remains maximally general:

$$G_1 = \langle ?, ?, ?, ?, ? \rangle$$

Step 2: Example 2 (Positive)

- Example 2: $\langle \text{Yellow, Small, Round, Sweet, Yes, Yes} \rangle$
- Generalize S where attributes differ:

$$S_2 = \langle ?, \text{Small, Round}, ?, \text{Yes} \rangle$$

- G unchanged:

$$G_2 = \langle ?, ?, ?, ?, ? \rangle$$

Step 3: Example 3 (Negative)

- Example 3: $\langle \text{Green, Large, Oval, Bitter, No, No} \rangle$
- Specialize G to exclude negative example while including S :

$$G_3 = \langle ?, \text{Small}, ?, ?, \text{Yes} \rangle$$

- S remains unchanged:

$$S_3 = \langle ?, \text{Small}, \text{Round}, ?, \text{Yes} \rangle$$

Step 4: Example 4 (Positive)

- Example 4: $\langle \text{Red, Small, Oval, Sweet, Yes, Yes} \rangle$
- Update S to include this positive example (already partially generalized):

$$S_4 = \langle ?, \text{Small}, ?, ?, \text{Yes} \rangle$$

- G remains:

$$G_4 = \langle ?, \text{Small}, ?, ?, \text{Yes} \rangle$$

Final Version Space

- Most Specific Hypothesis:

$$S = \langle ?, \text{Small}, ?, ?, \text{Yes} \rangle$$

- Most General Hypothesis:

$$G = \langle ?, \text{Small}, ?, ?, \text{Yes} \rangle$$

- Version space narrowed down to: **Size = Small** and **Sweet = Yes**.