

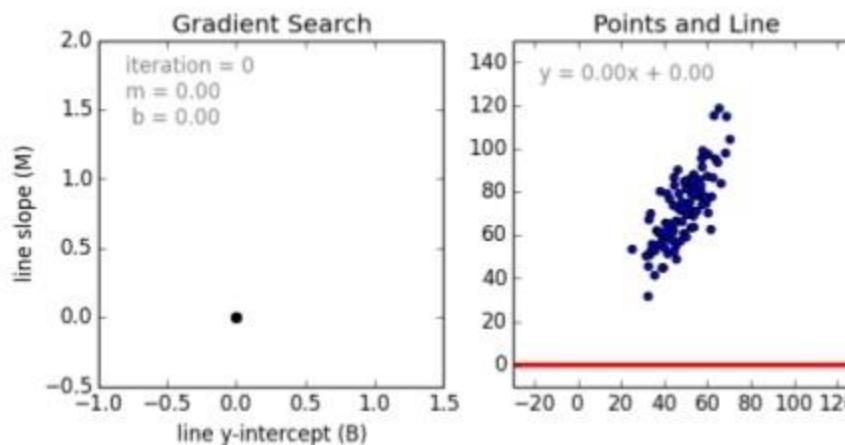
# Linear Regression



# Linear Regression Algorithm

# Agenda for Today's Session

- What is Regression?
- Regression Use-case
- Types of Regression – Linear vs Logistic Regression
- What is Linear Regression?
- Finding best fit regression line using Least Square Method
- Checking goodness of fit using R squared Method
- Implementation of Linear Regression using Python
  - Linear Regression Algorithm using Python from scratch
  - Linear Regression Algorithm using Python (scikit lib)



# What is Regression?

“Regression analysis is a form of predictive modelling technique which investigates the relationship between a dependent and independent variable”



# Uses of Regression

Three major uses for regression analysis are

- Determining the strength of predictors
- Forecasting an effect, and
- Trend forecasting



# Linear vs Logistic Regression

Basis	Linear Regression	Logistic Regression
Core Concept	The data is modelled using a straight line	The probability of some obtained event is represented as a linear function of a combination predictor variables.
Used with	Continuous Variable	Categorical Variable
Output/Prediction	Value of the variable	Probability of occurrence event
Accuracy and Goodness of fit	measured by loss, R squared, Adjusted R squared etc.	Accuracy, Precision, Recall, F1 score, ROC curve, Confusion Matrix, etc

# What is Linear Regression?

“Linear Regression is a method to predict dependent variable based on values of independent variables (X). It can be used for cases where we want to predict some continuous quantity.”



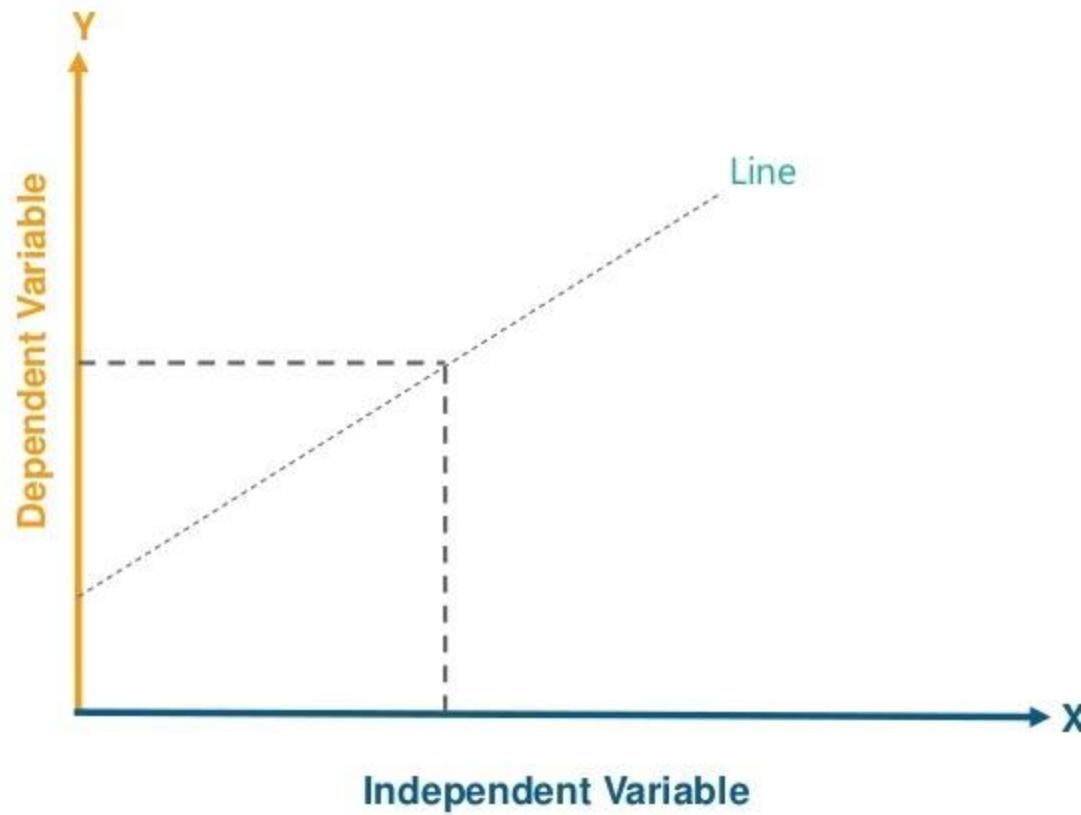
# Linear Regression Selection Criteria

- Classification and Regression Capabilities
- Data Quality
- Computational Complexity
- Comprehensible and Transparent

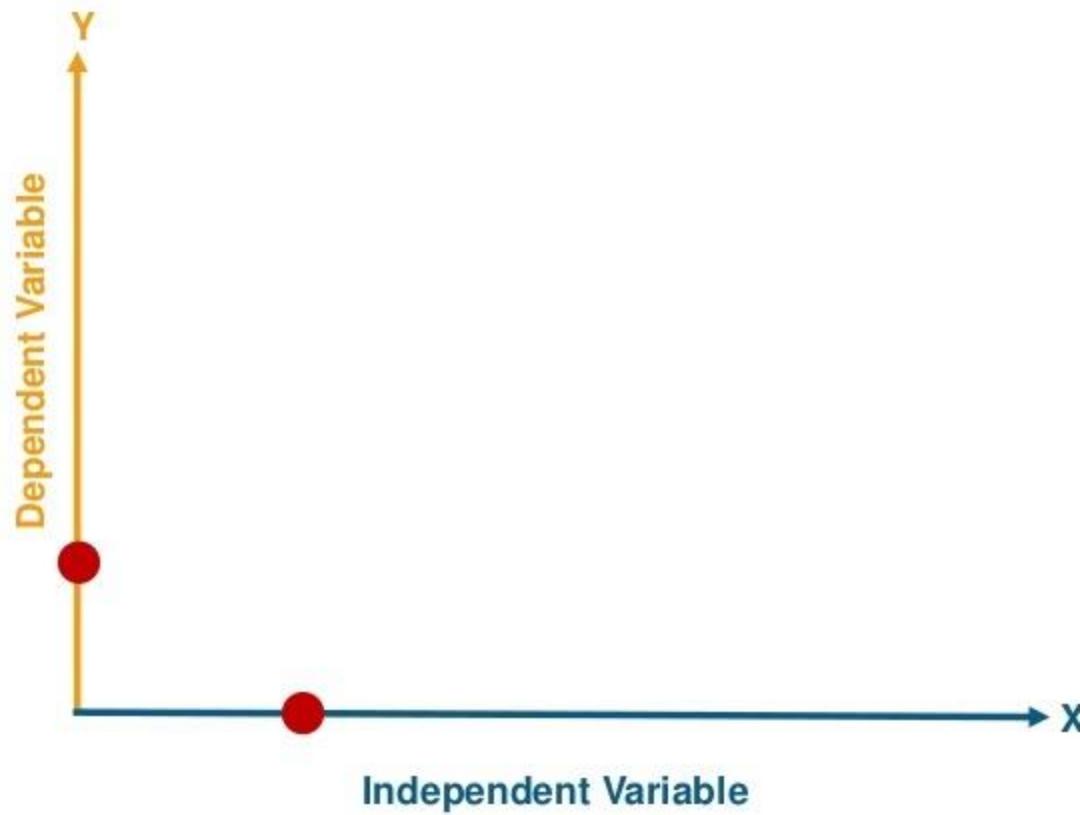
# Where is Linear Regression used?

- Evaluating Trends and Sales Estimates
- Analyzing the Impact of Price Changes
- Assessment of risk in financial services and insurance domain

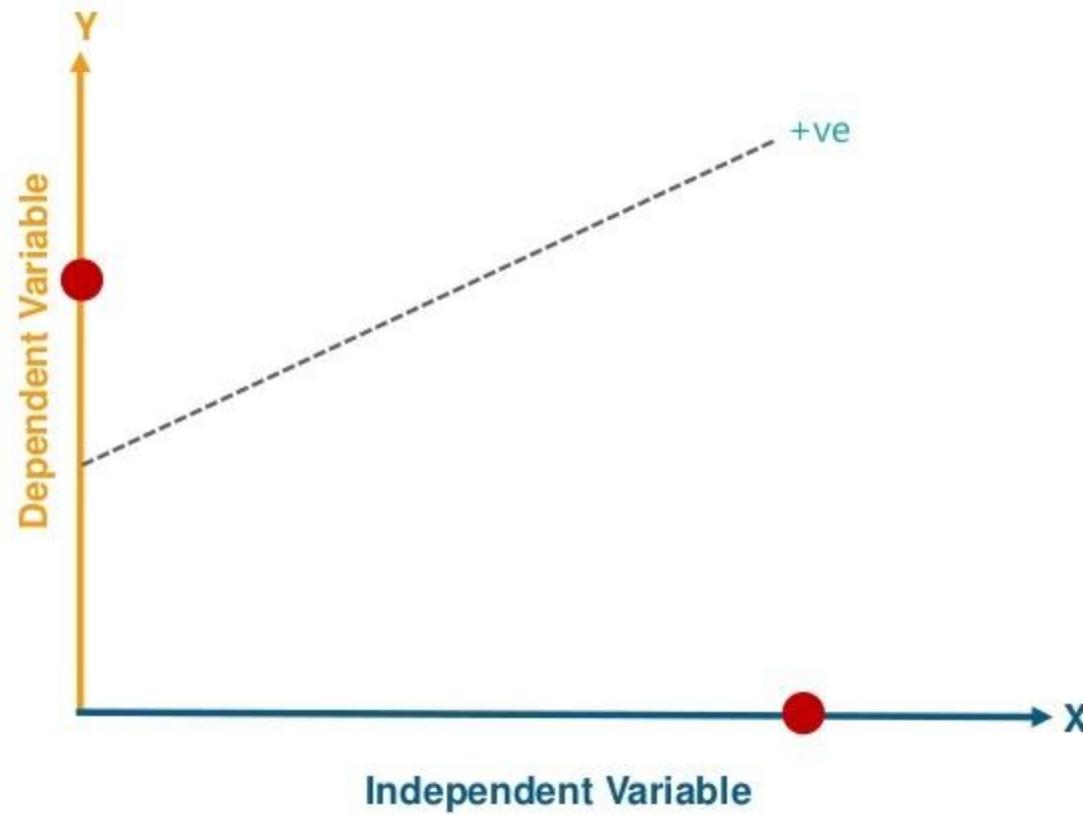
# Understanding Linear Regression Algorithm



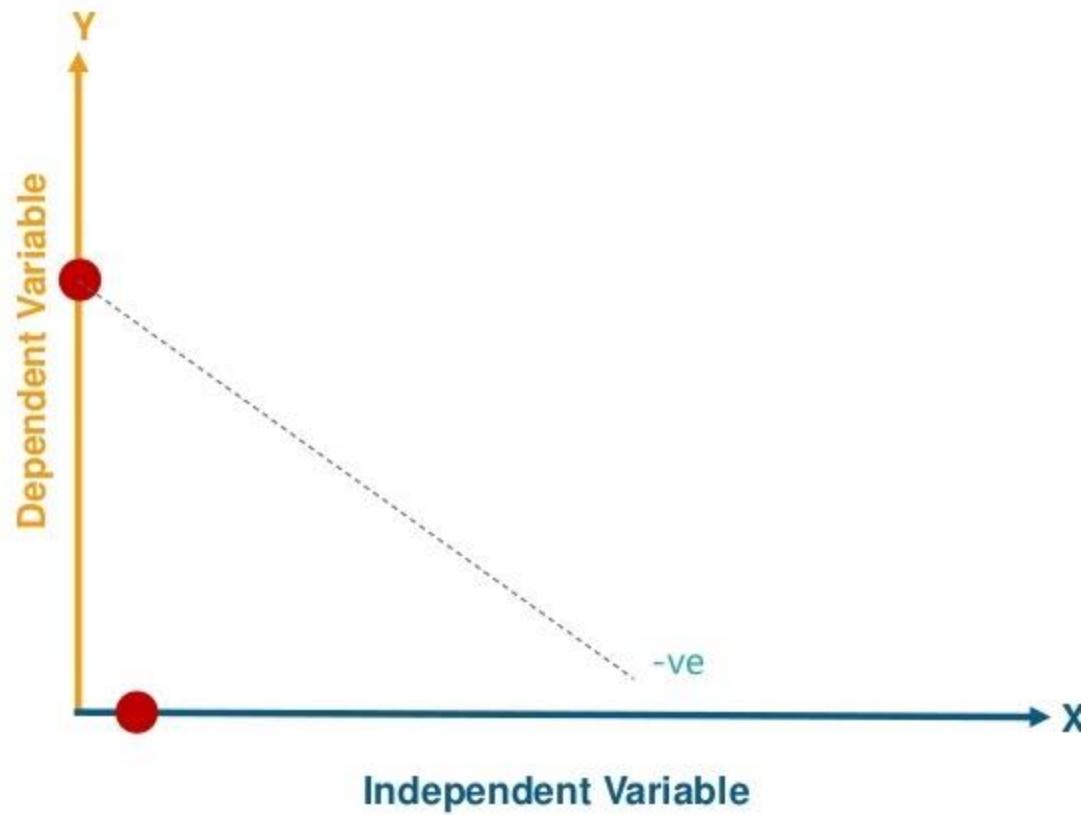
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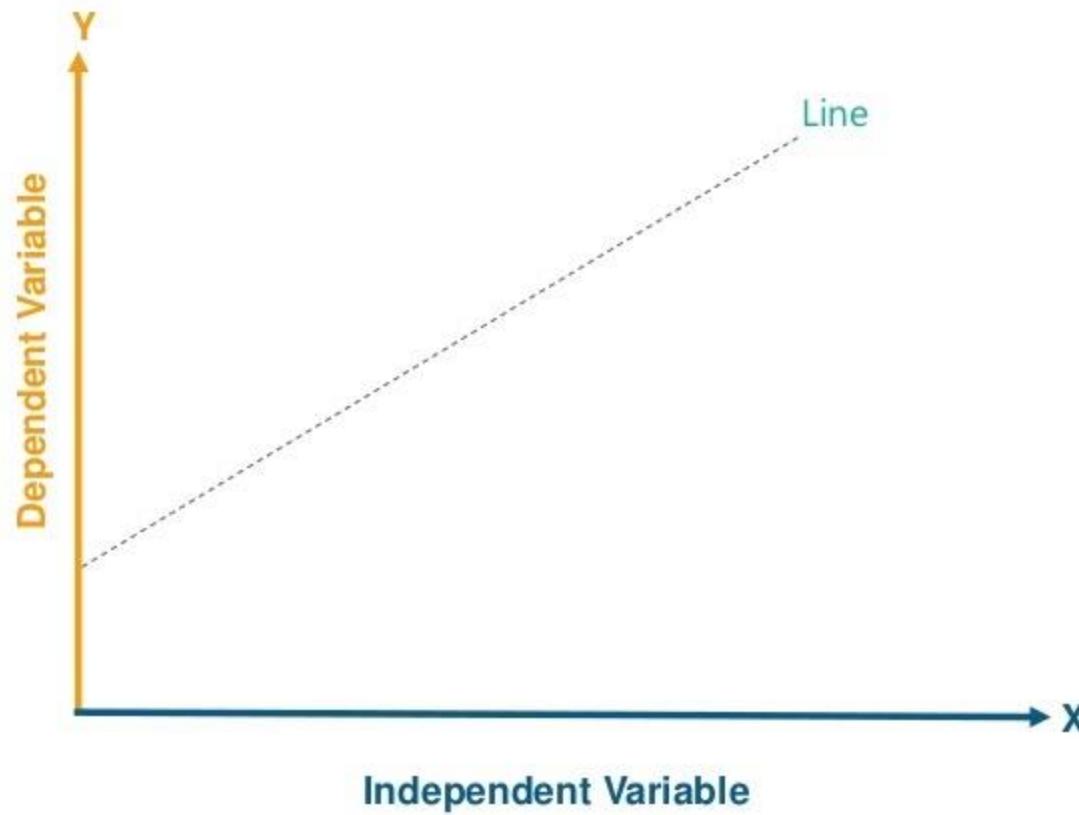
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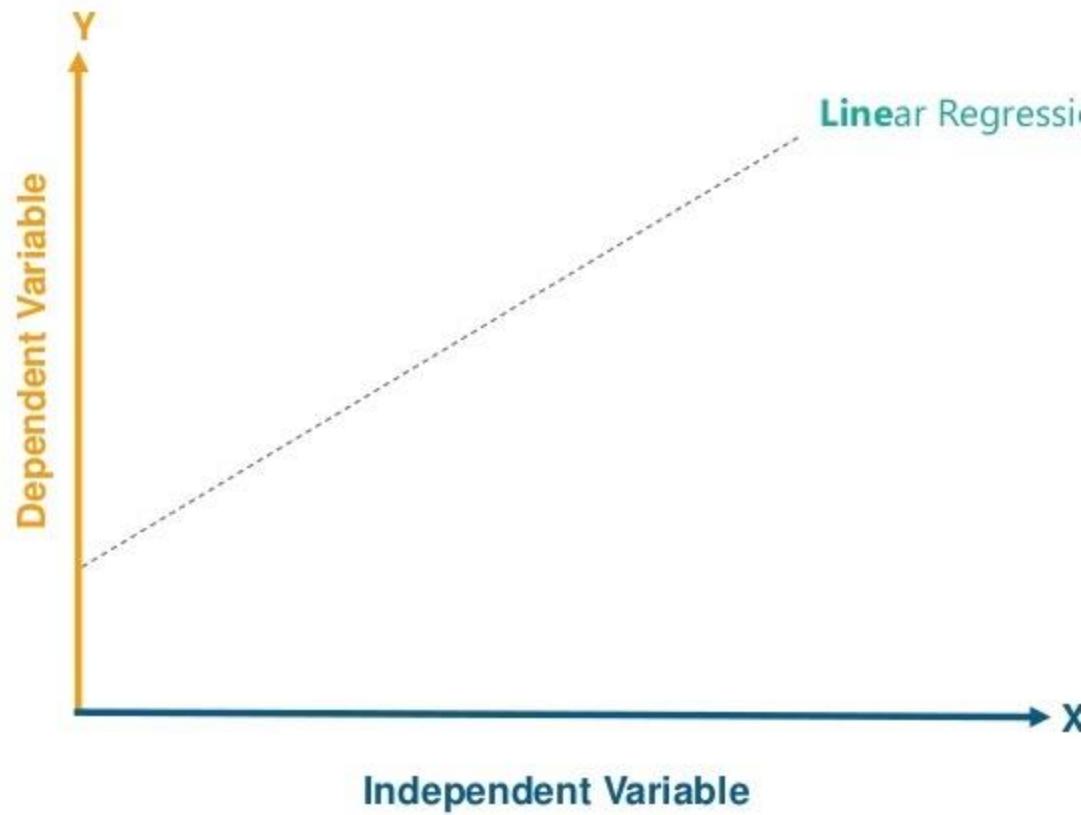
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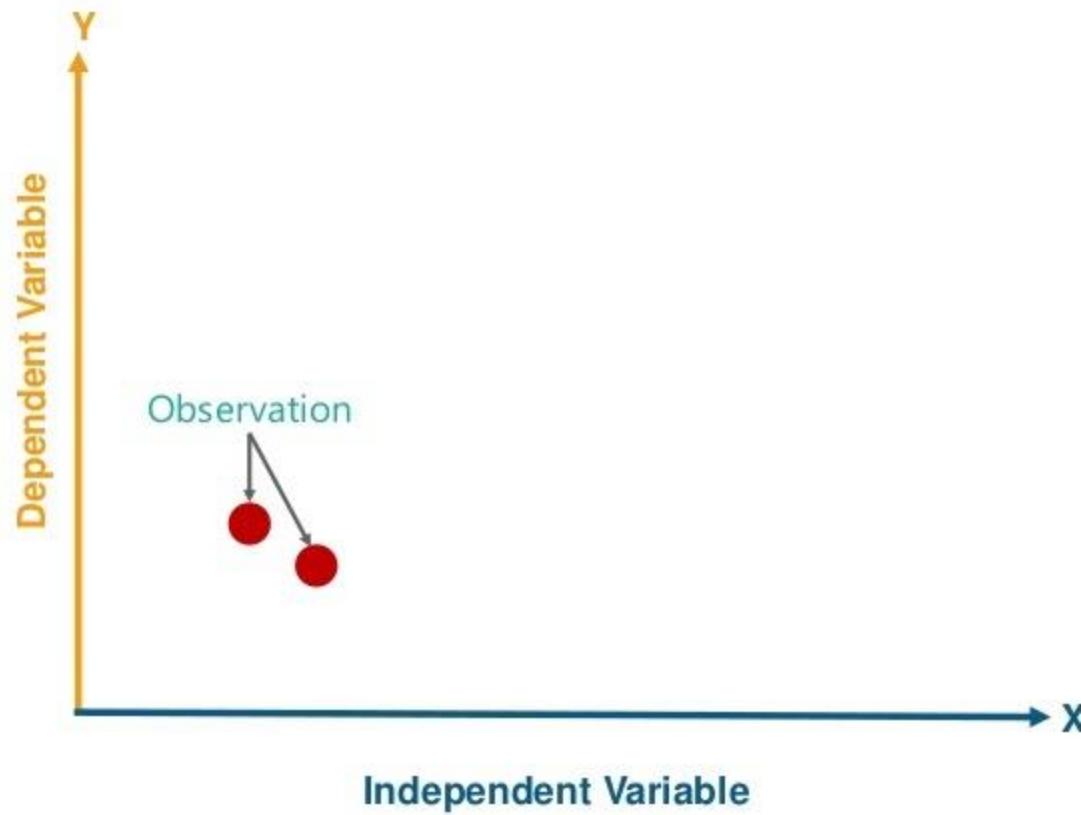
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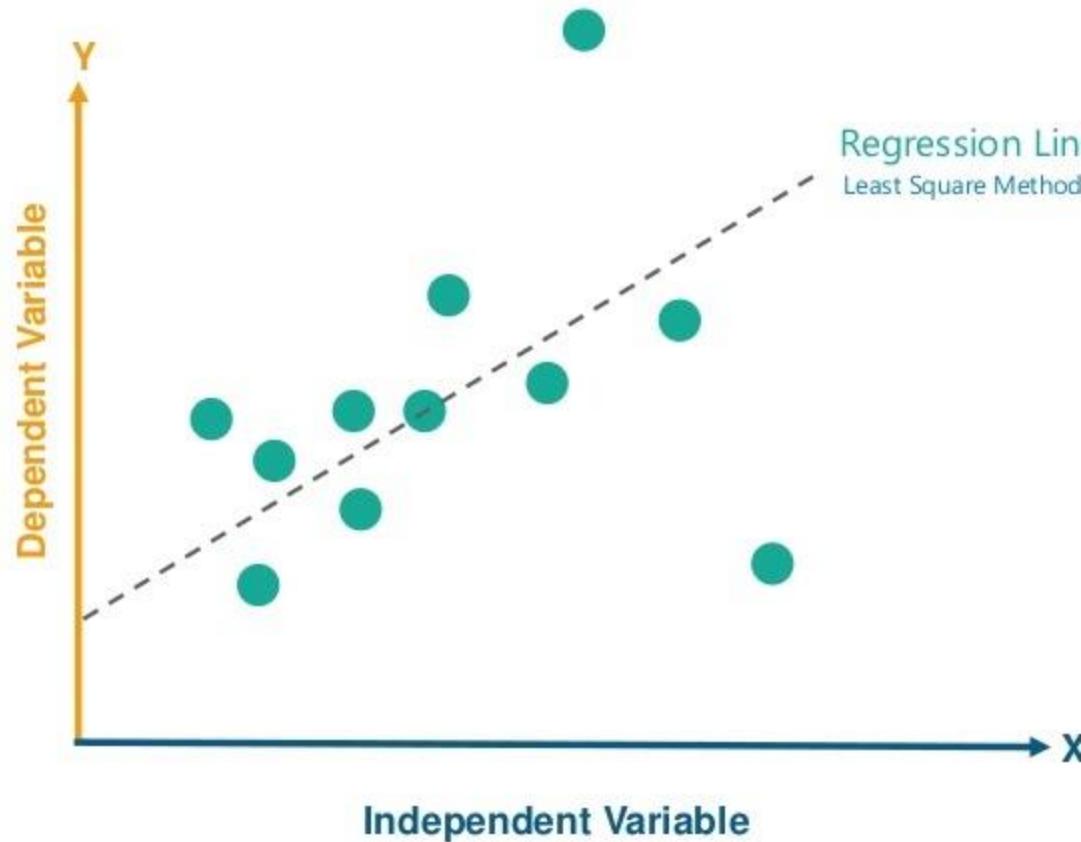
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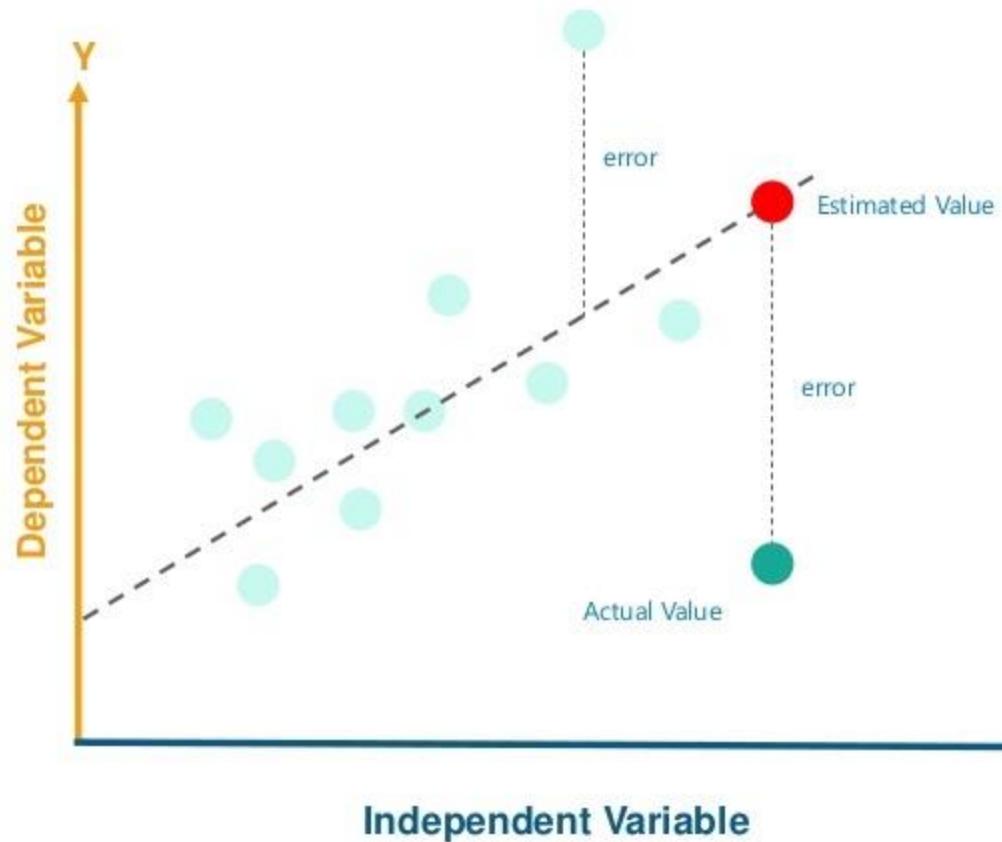
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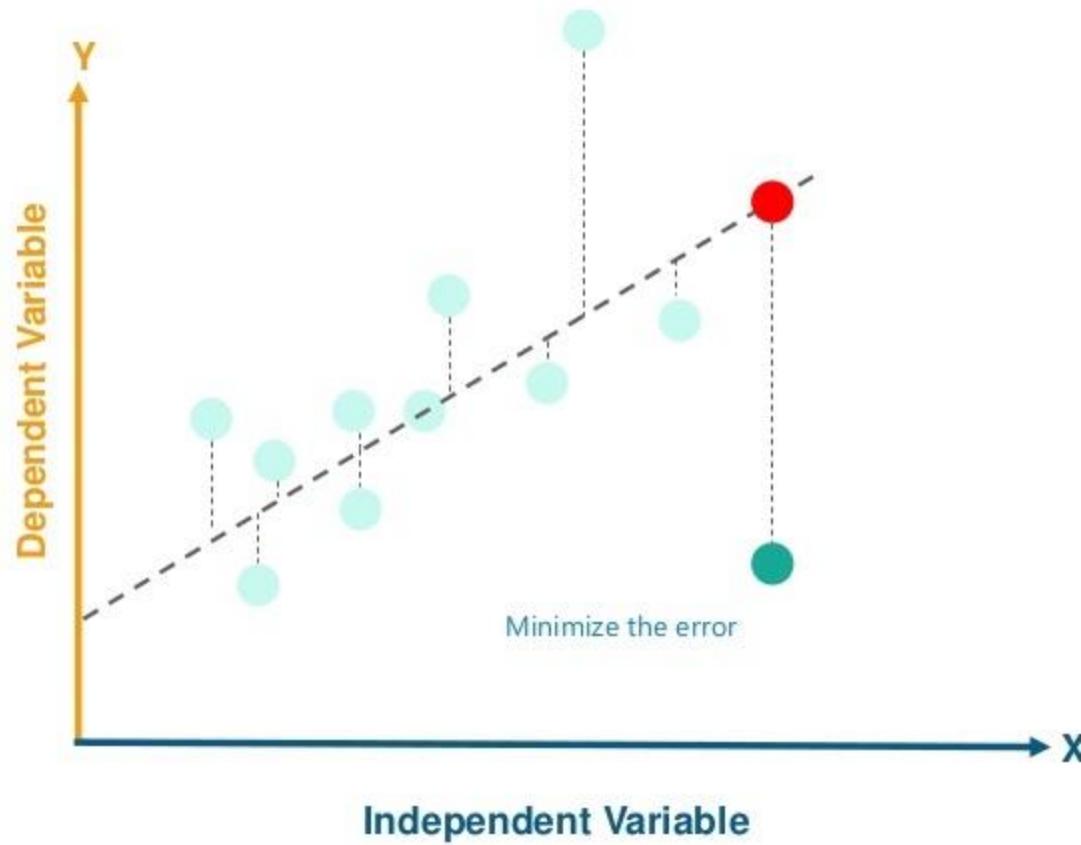
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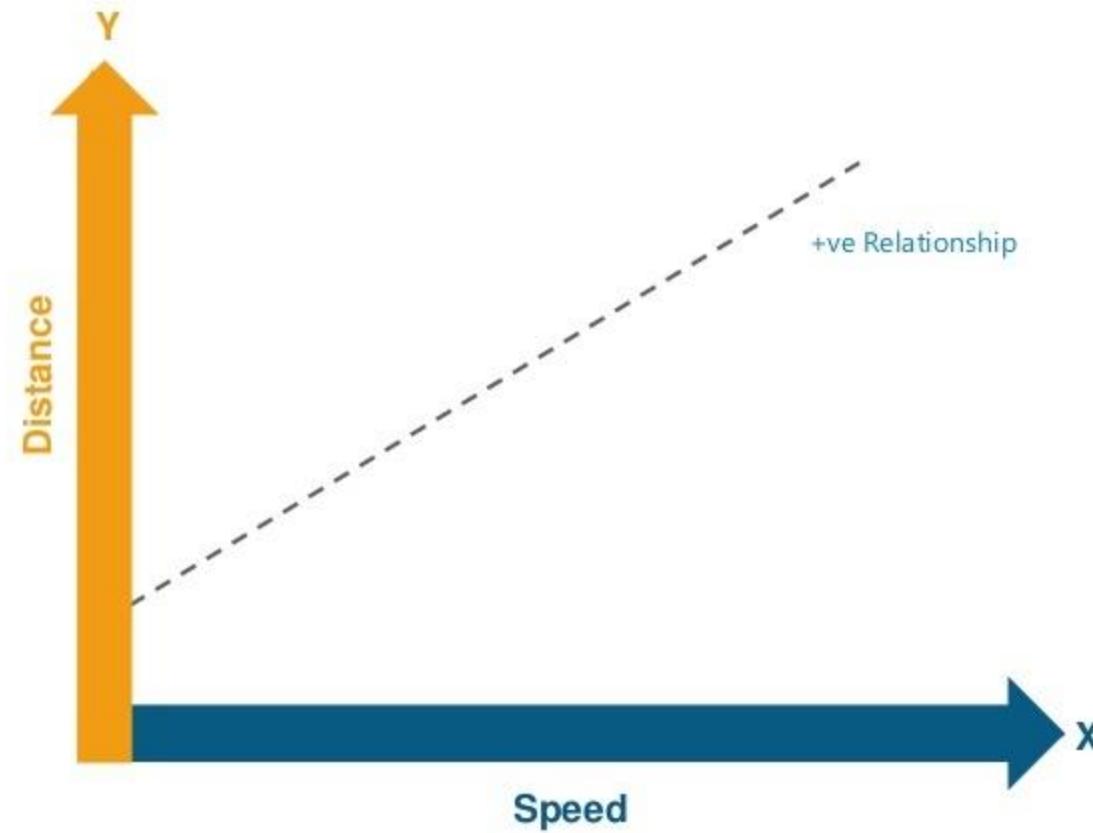
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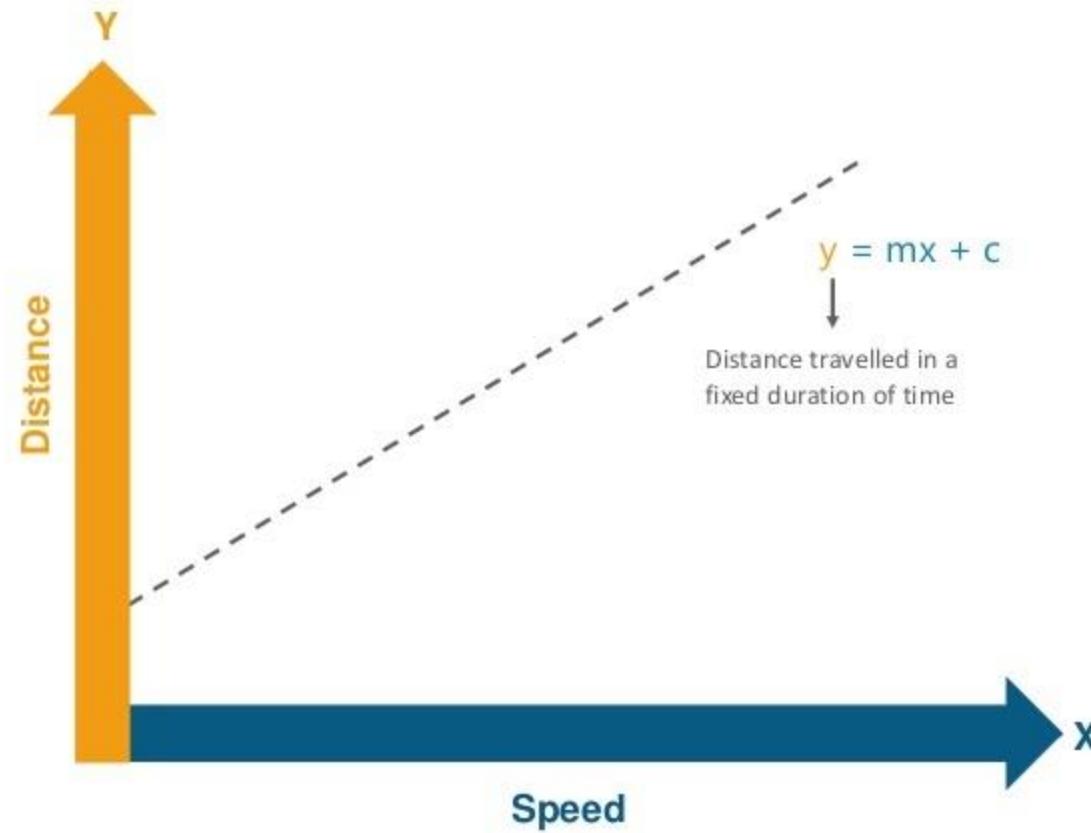
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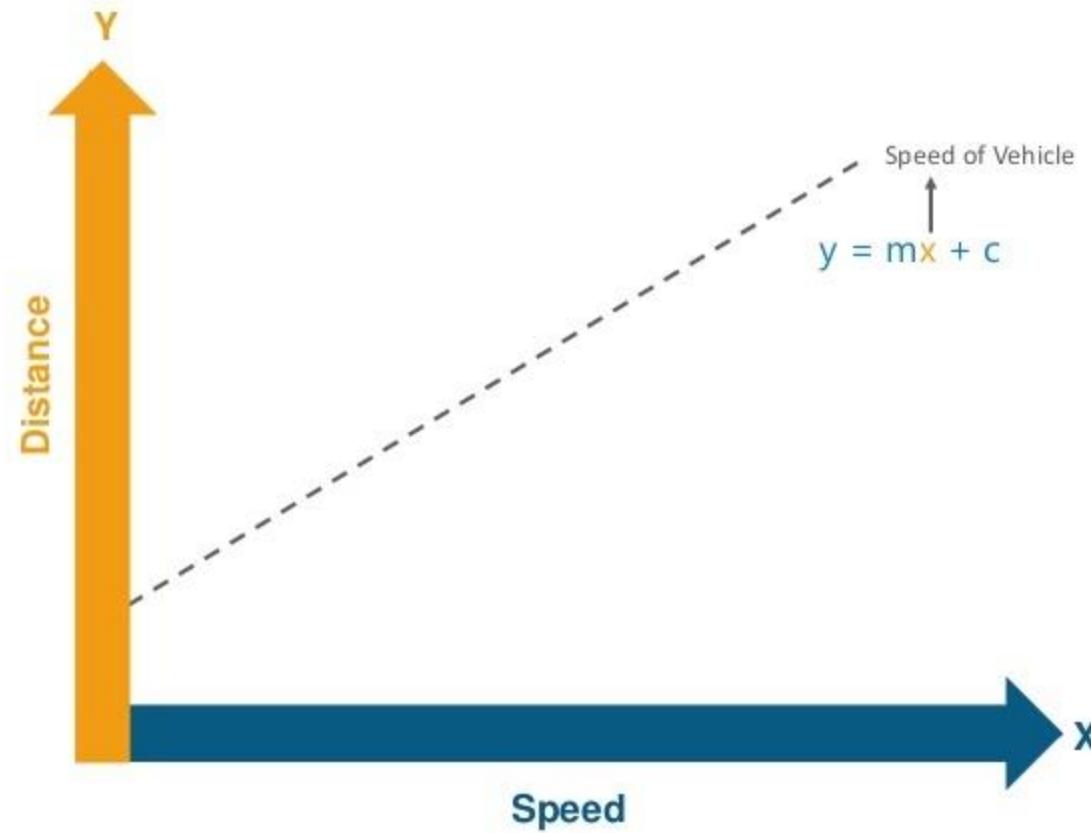
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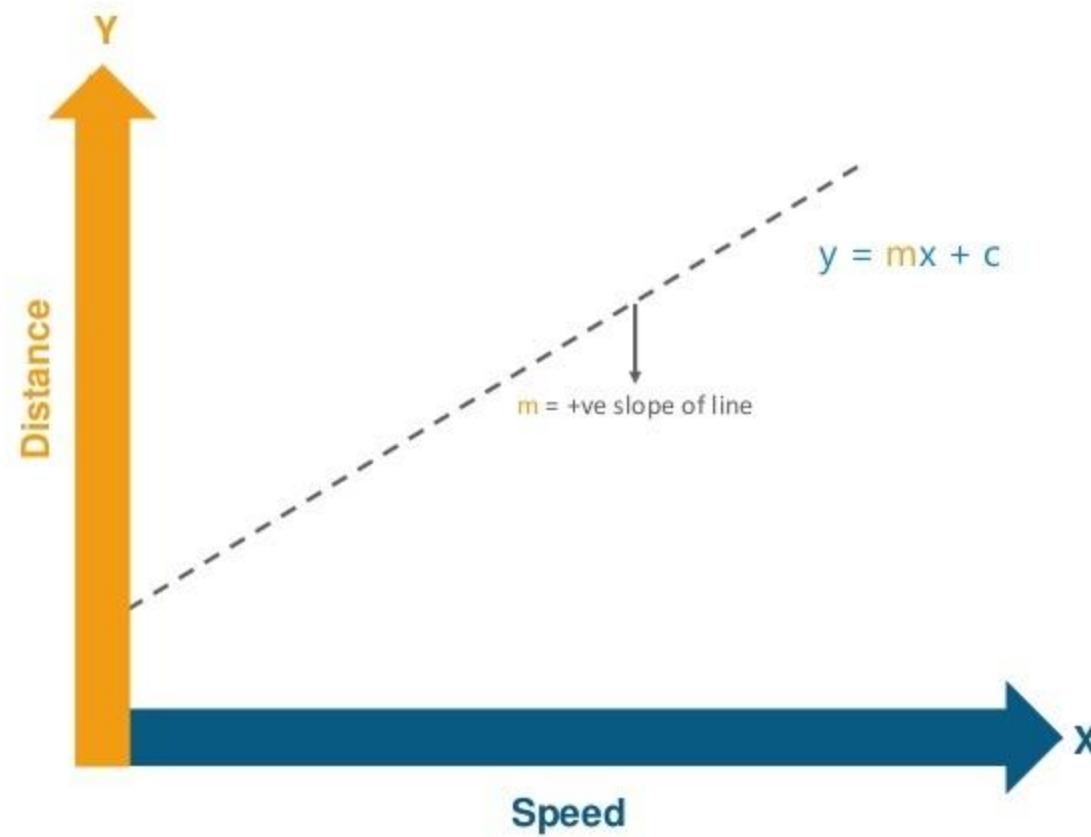
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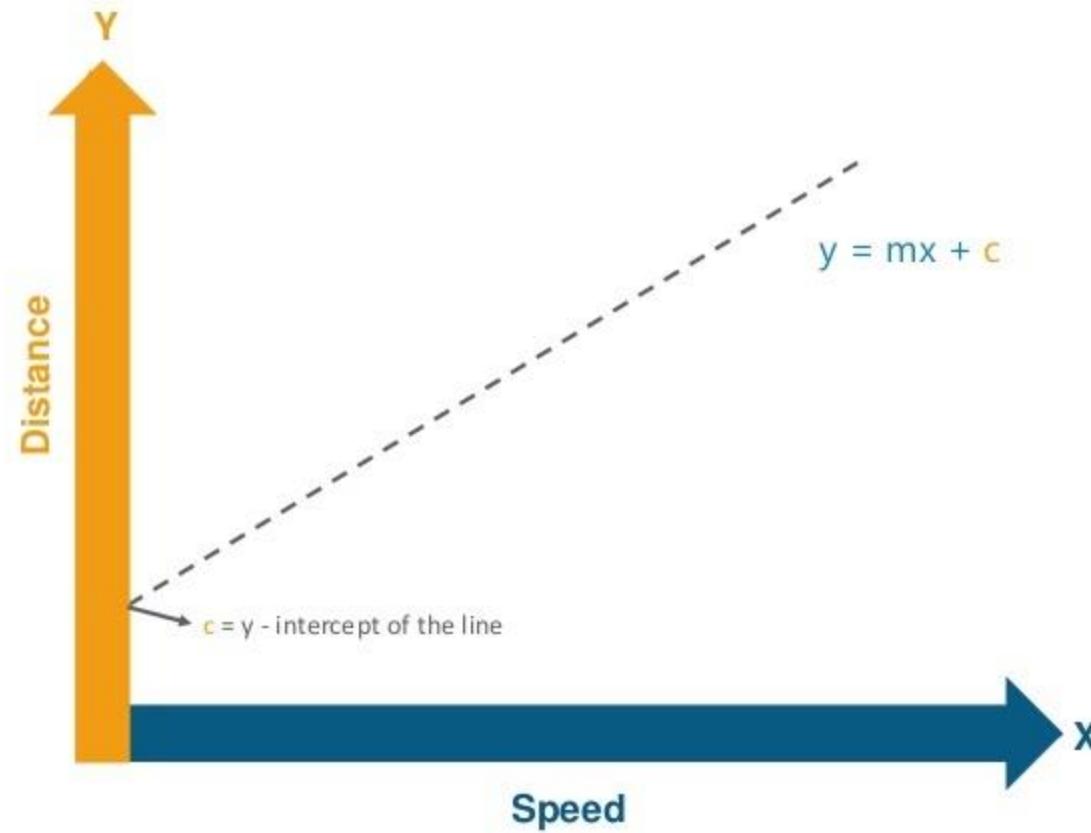
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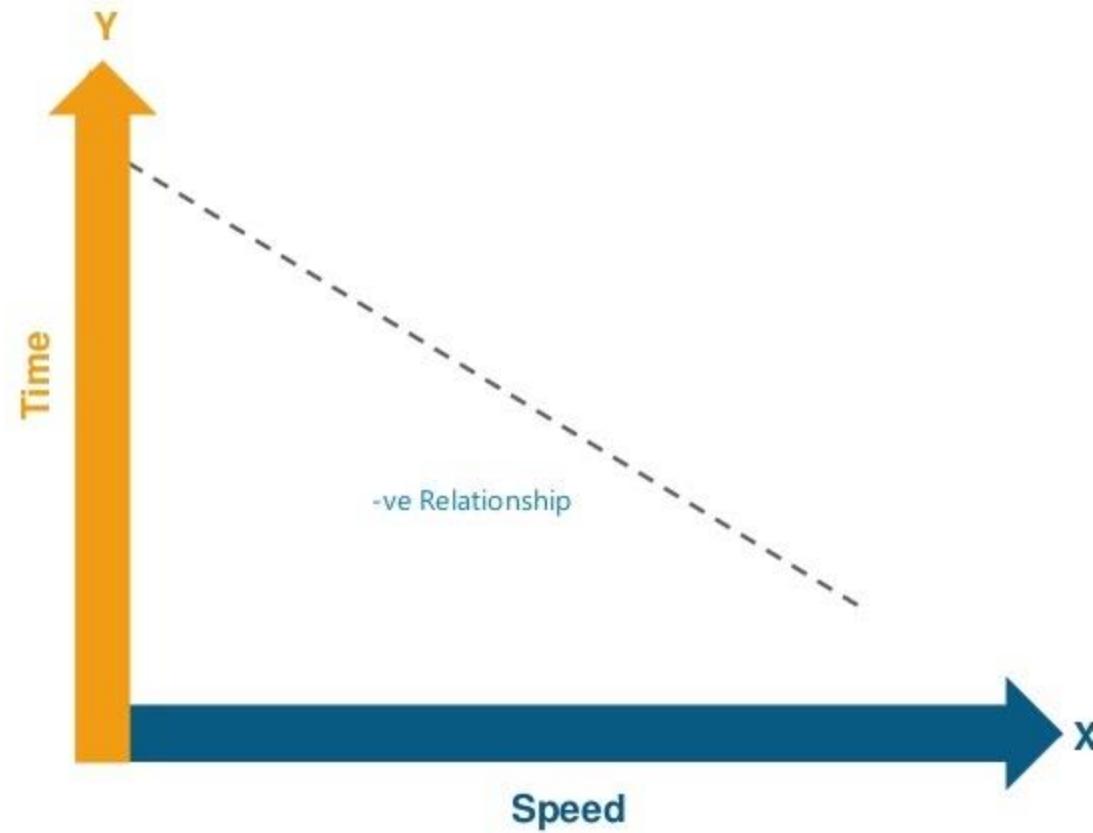
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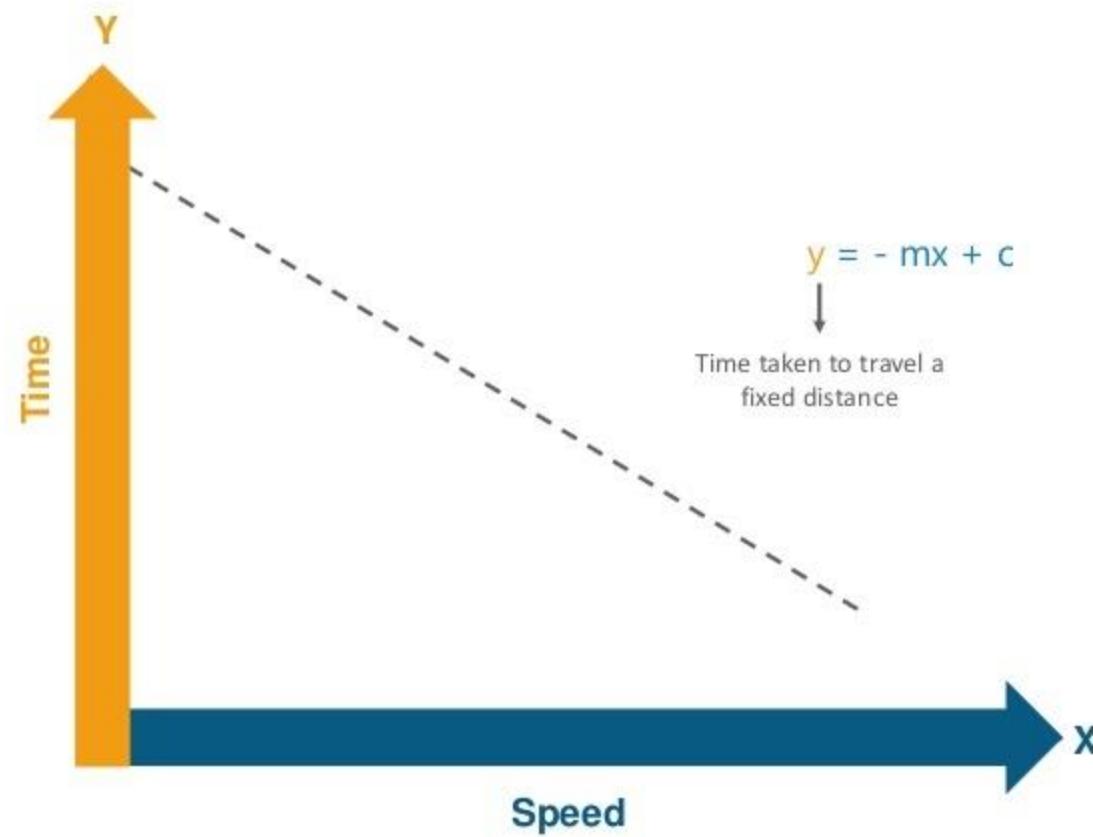
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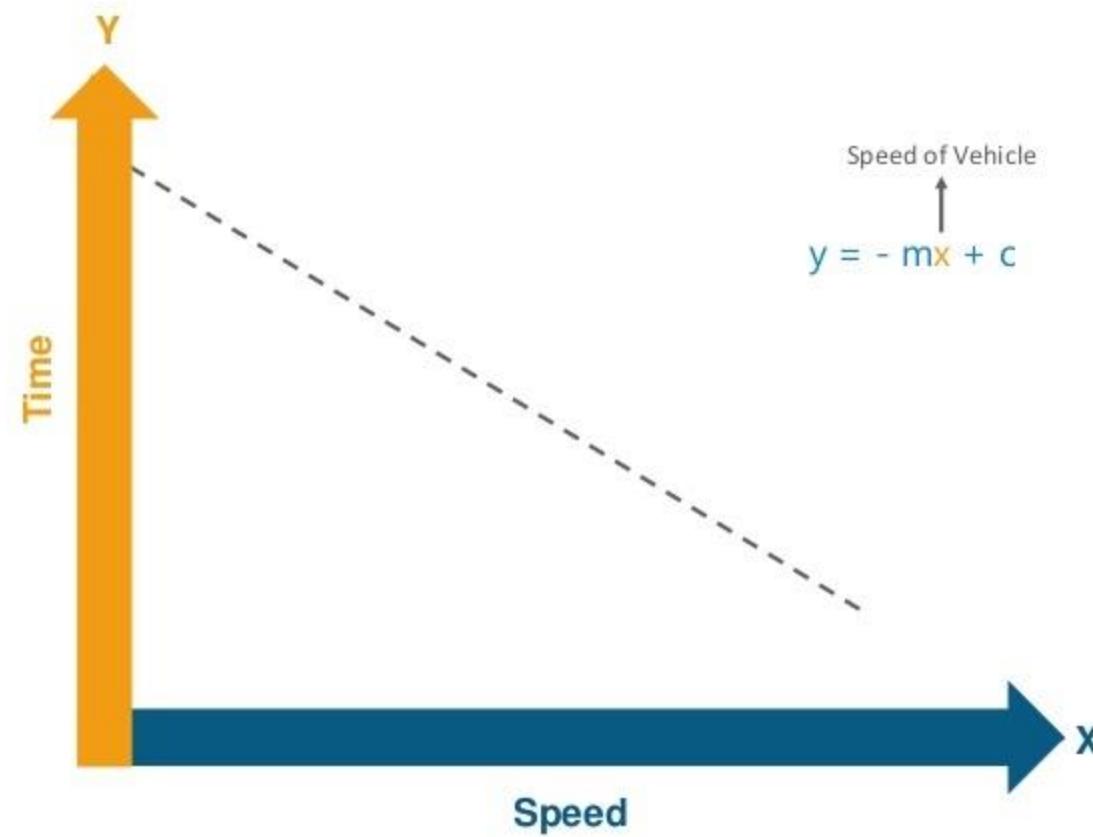
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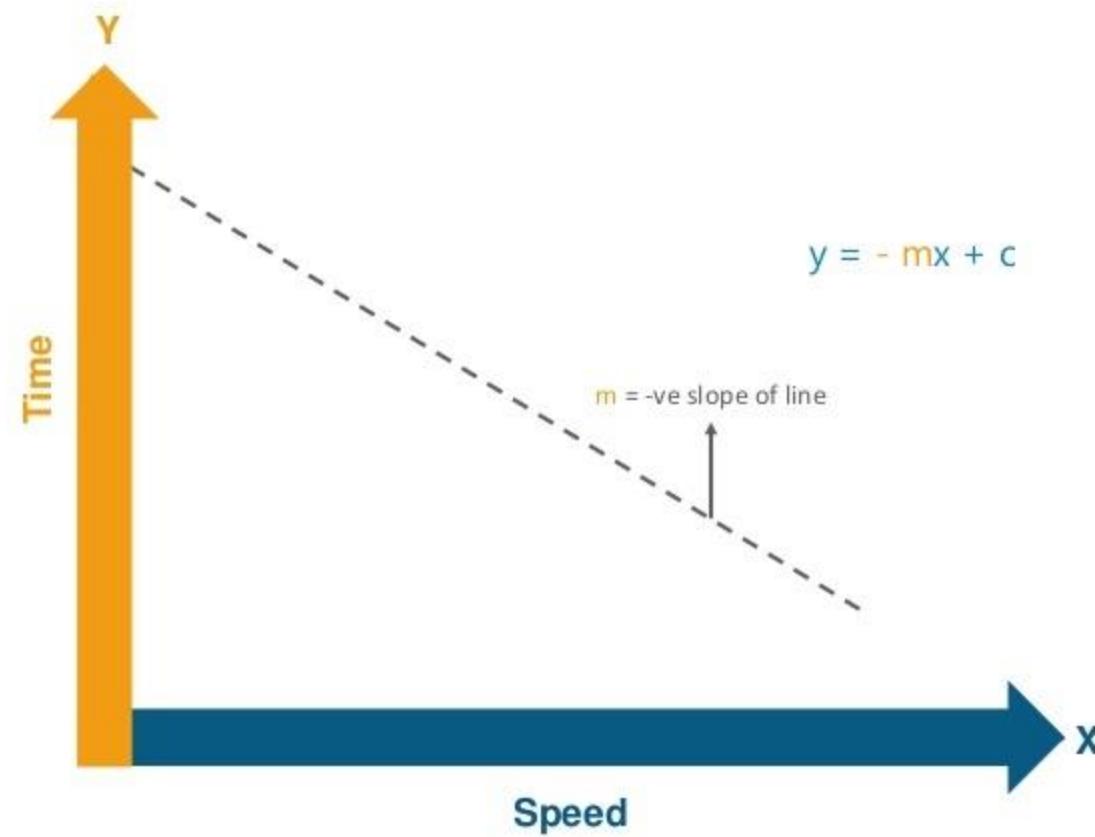
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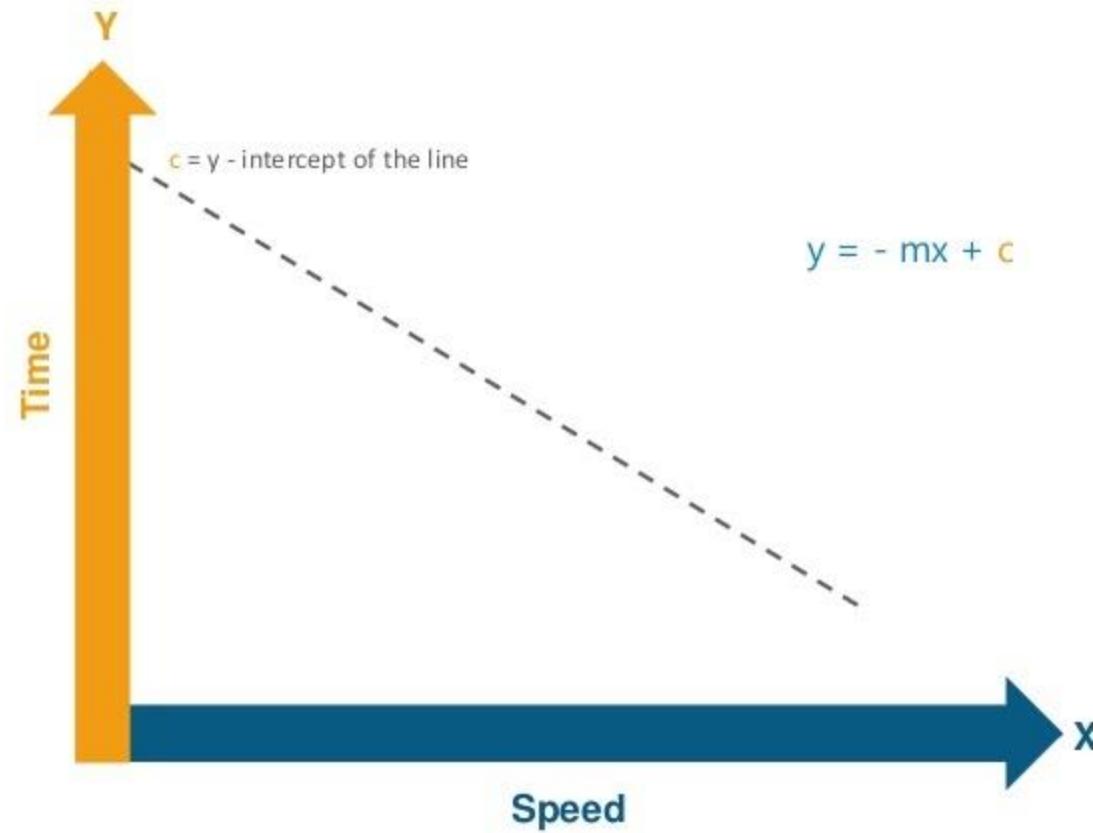
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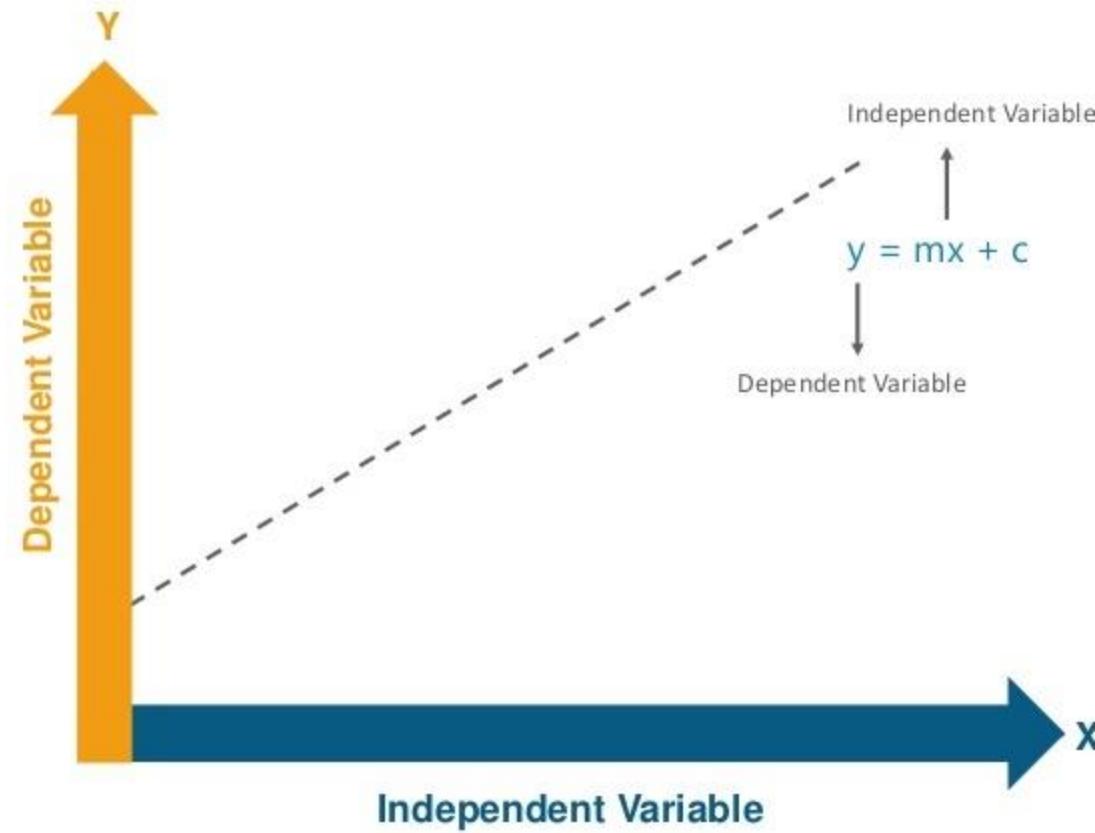
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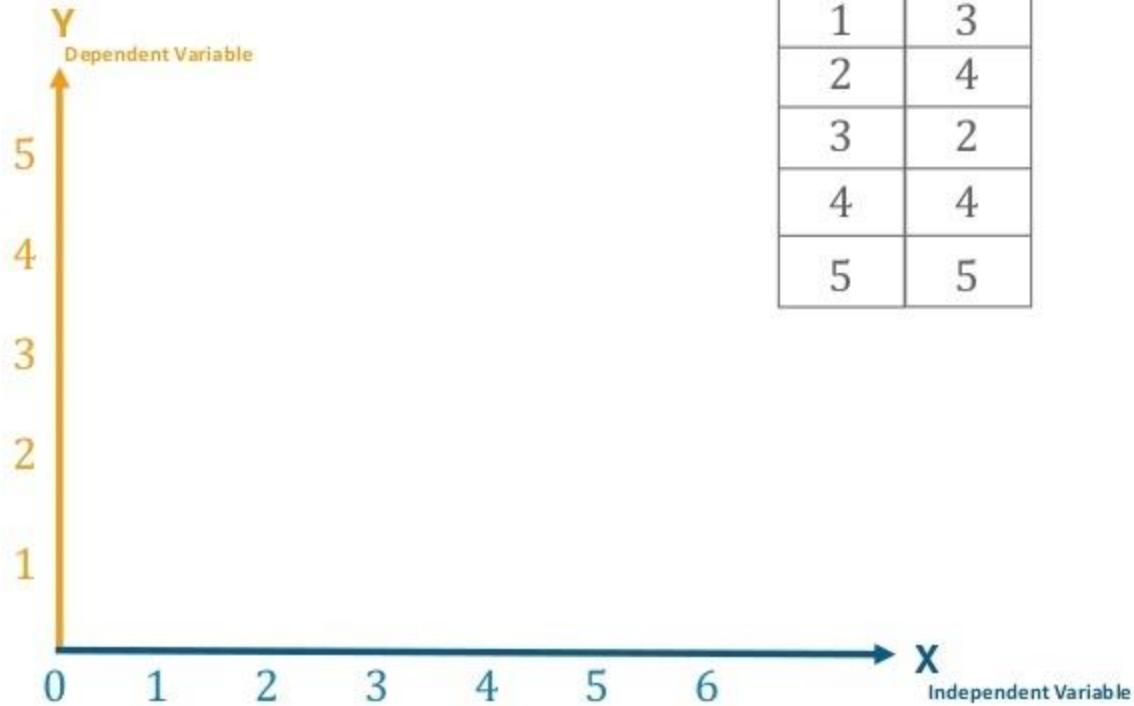
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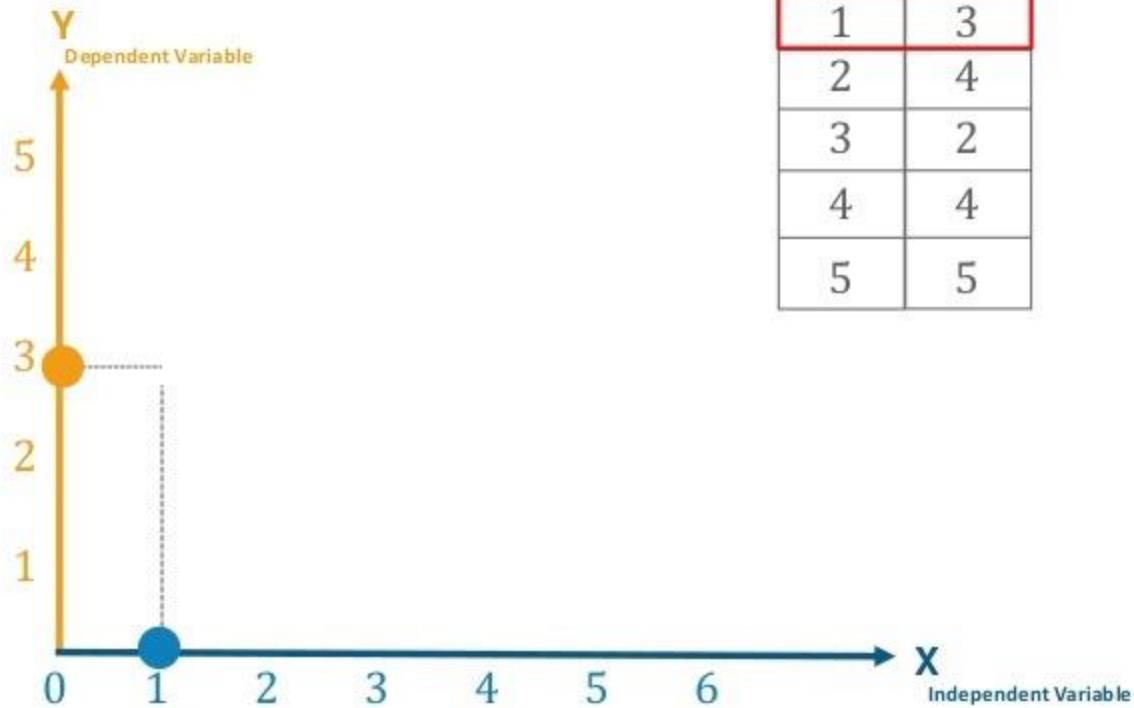
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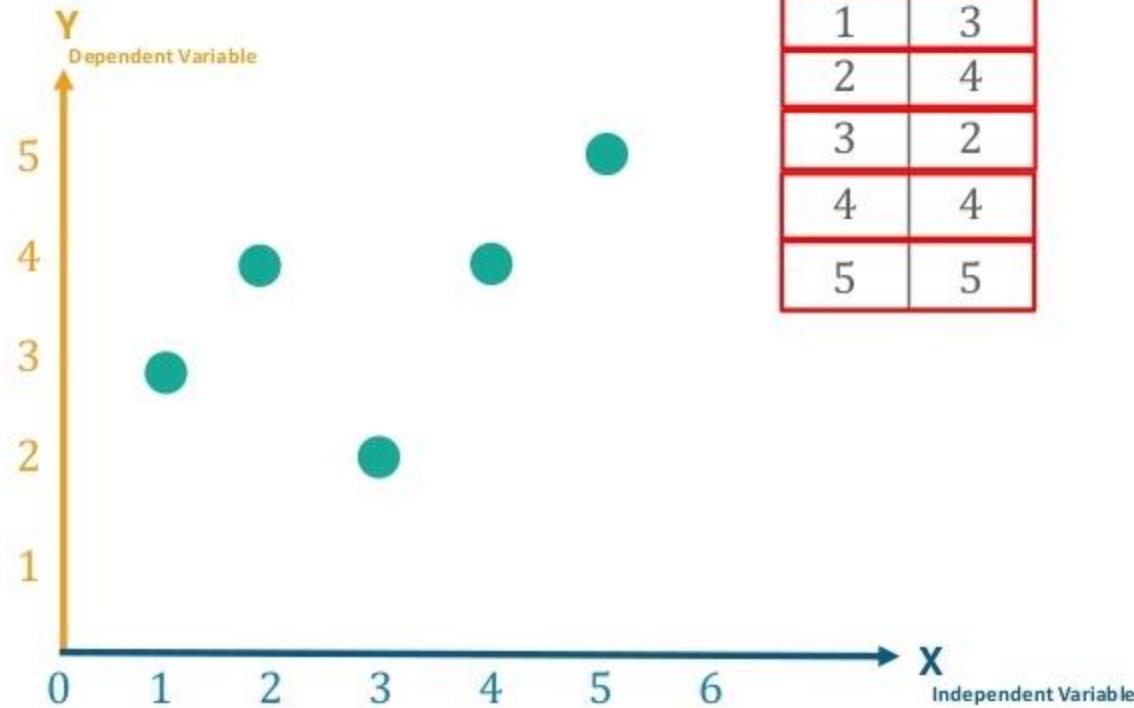
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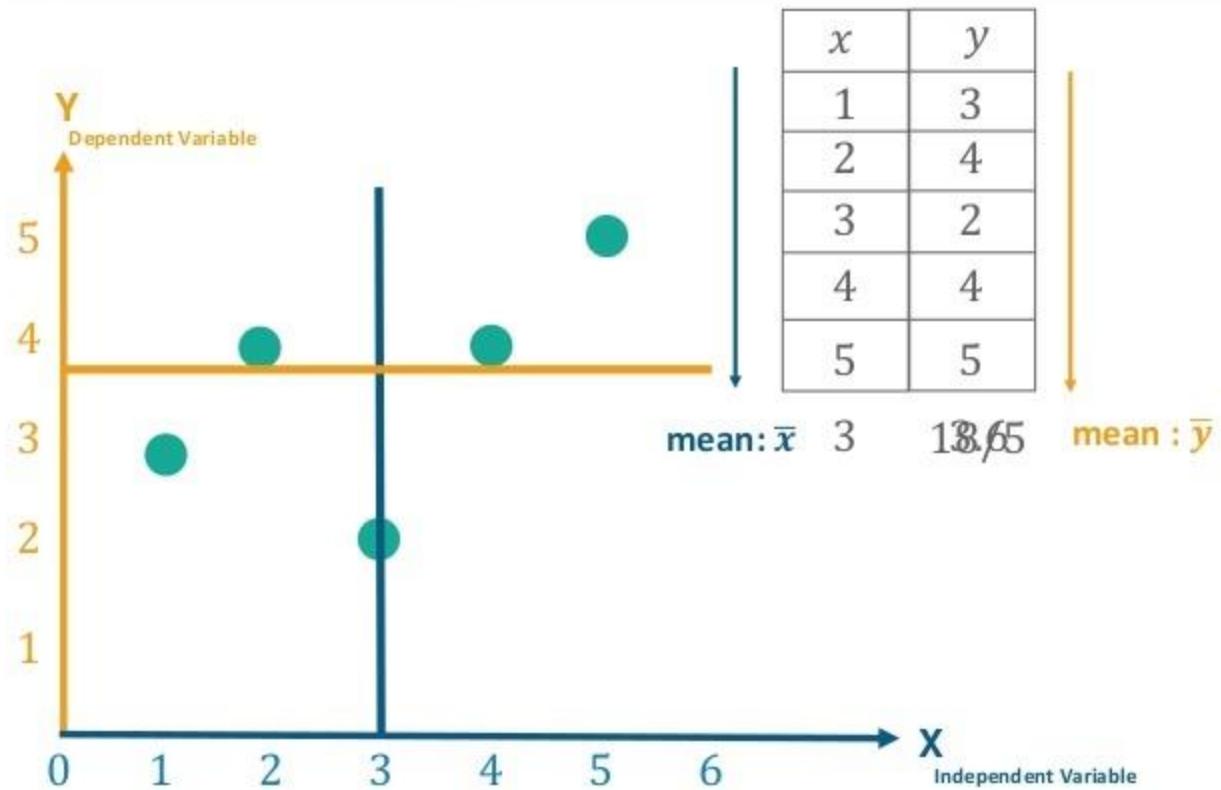
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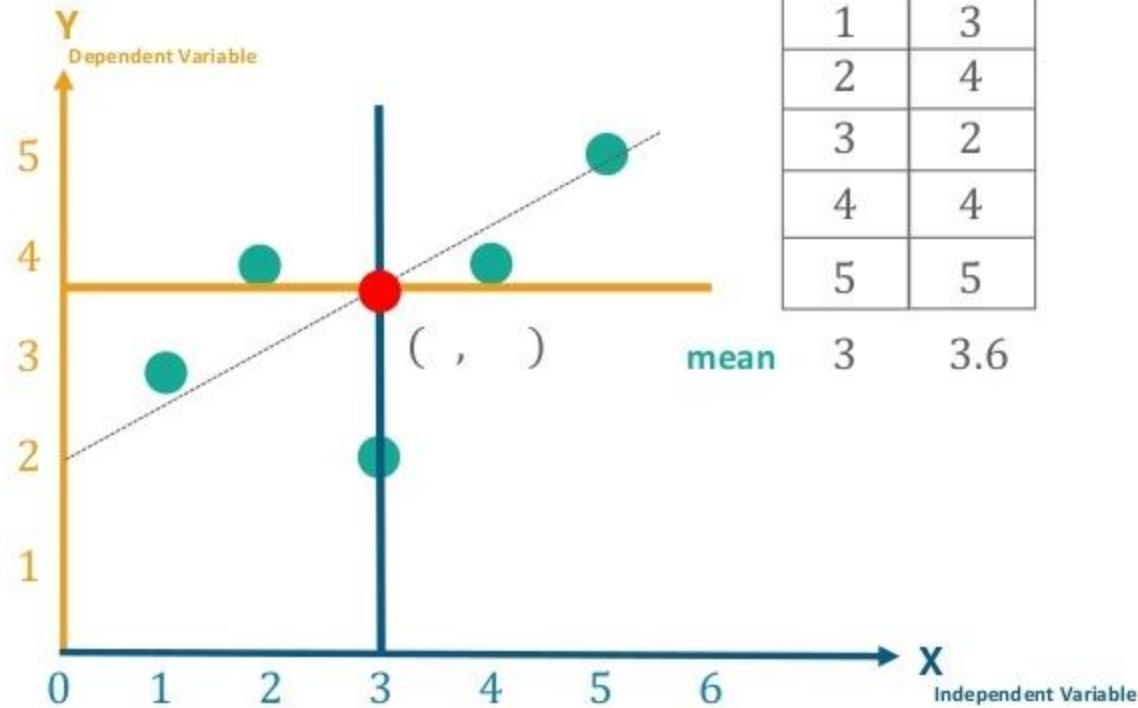
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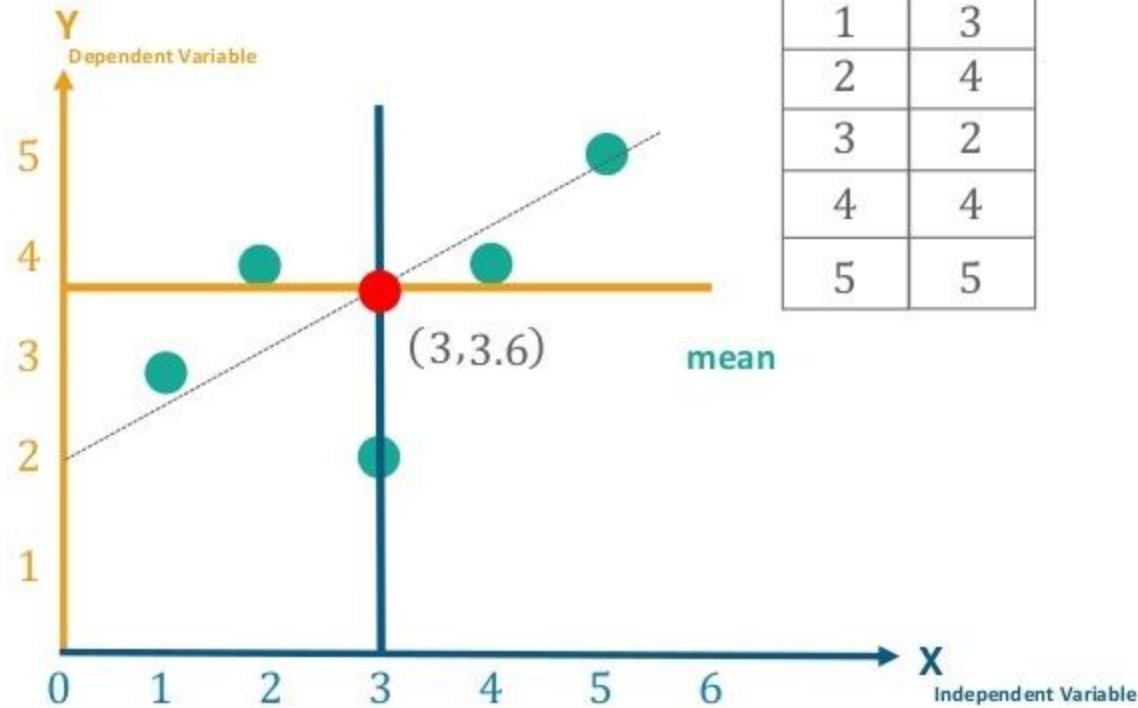
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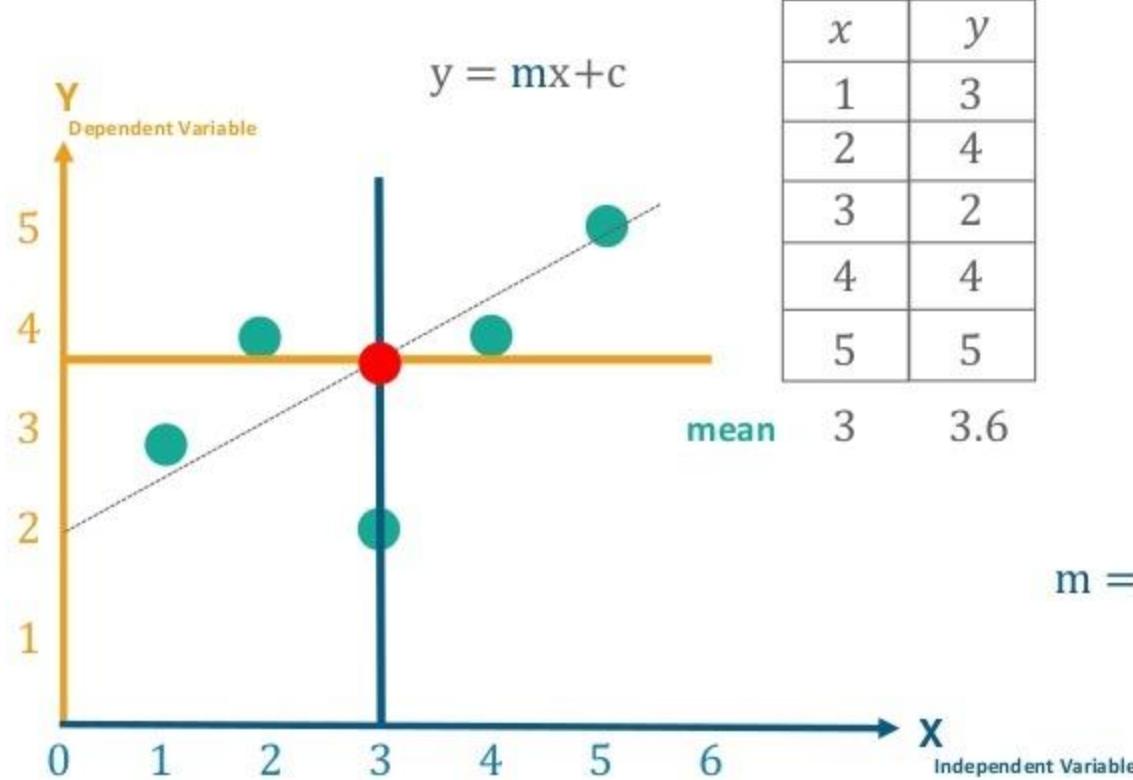
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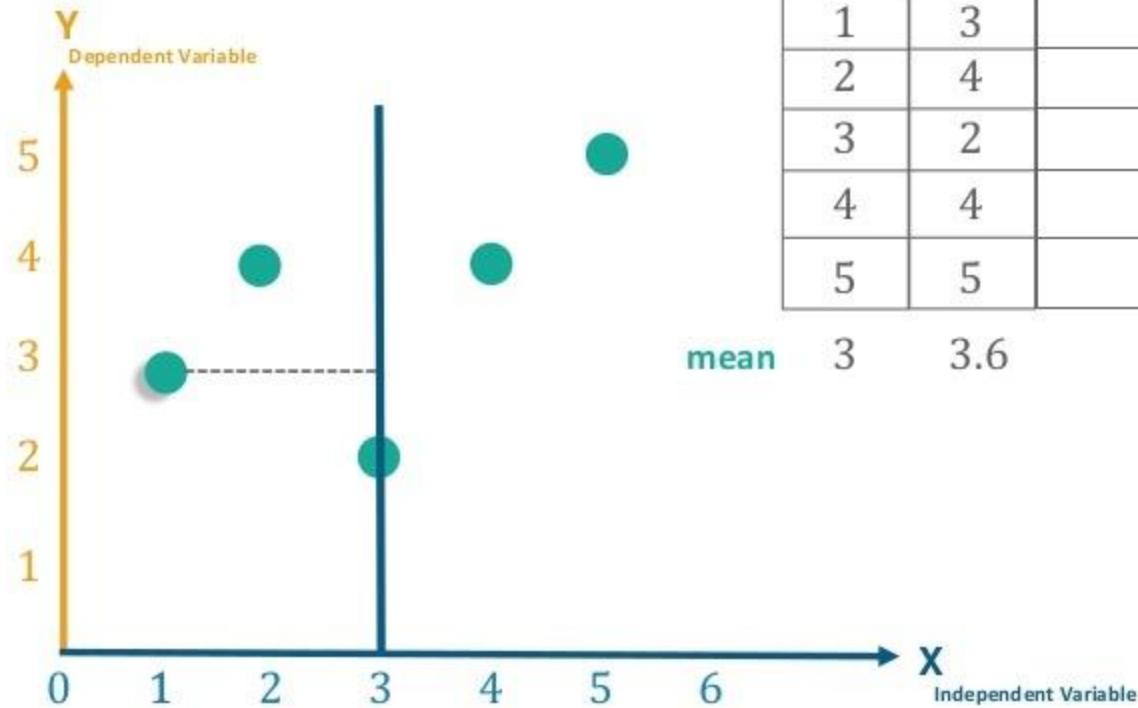


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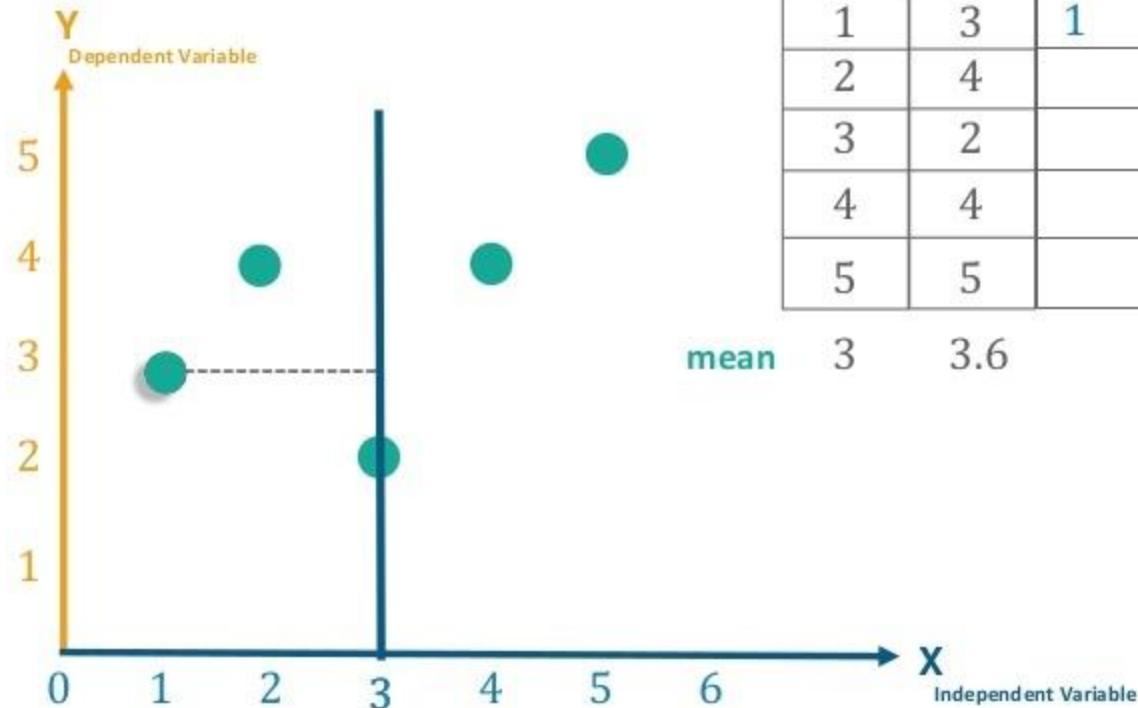


$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

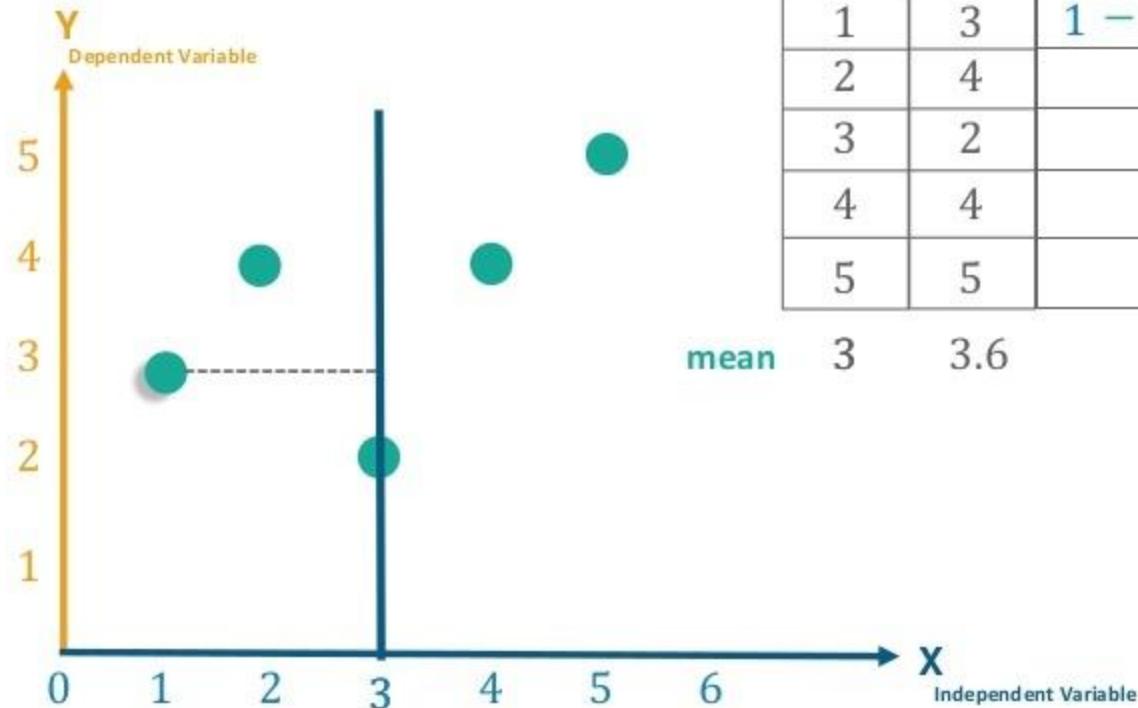
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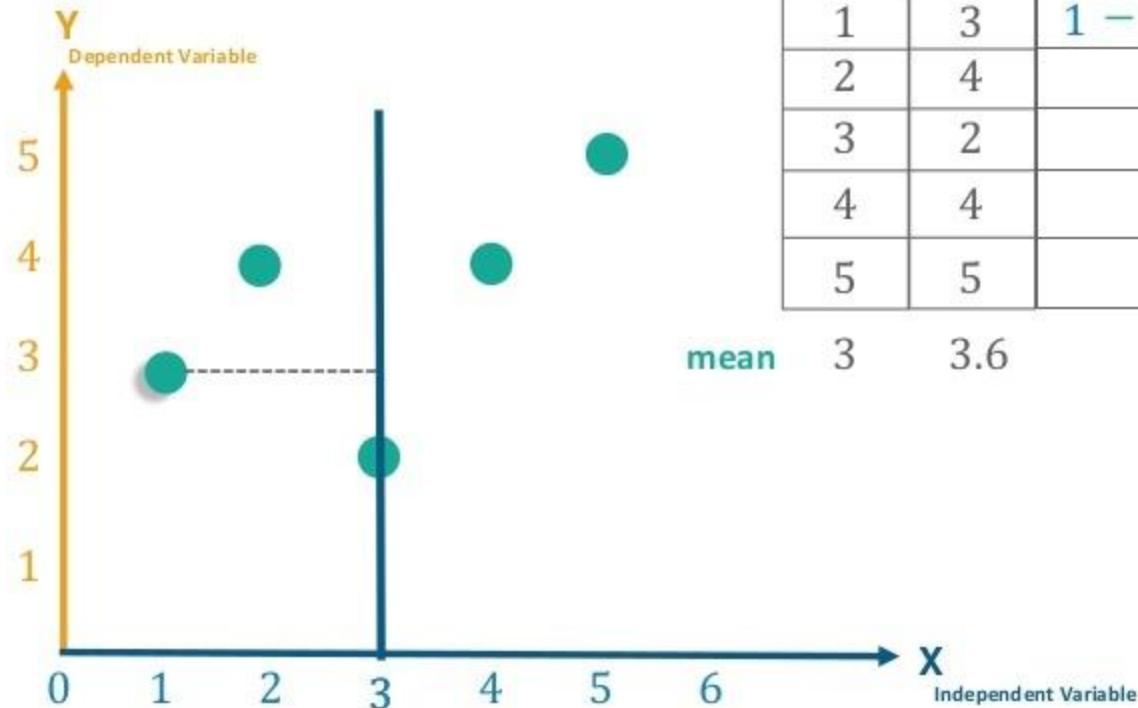
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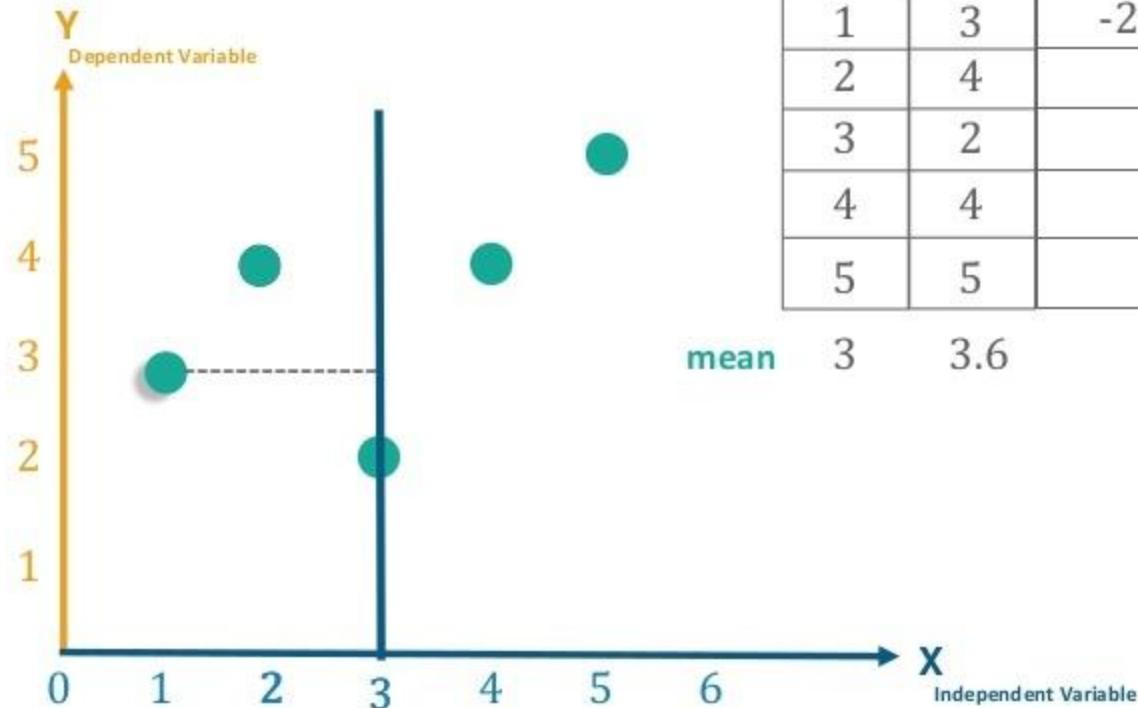
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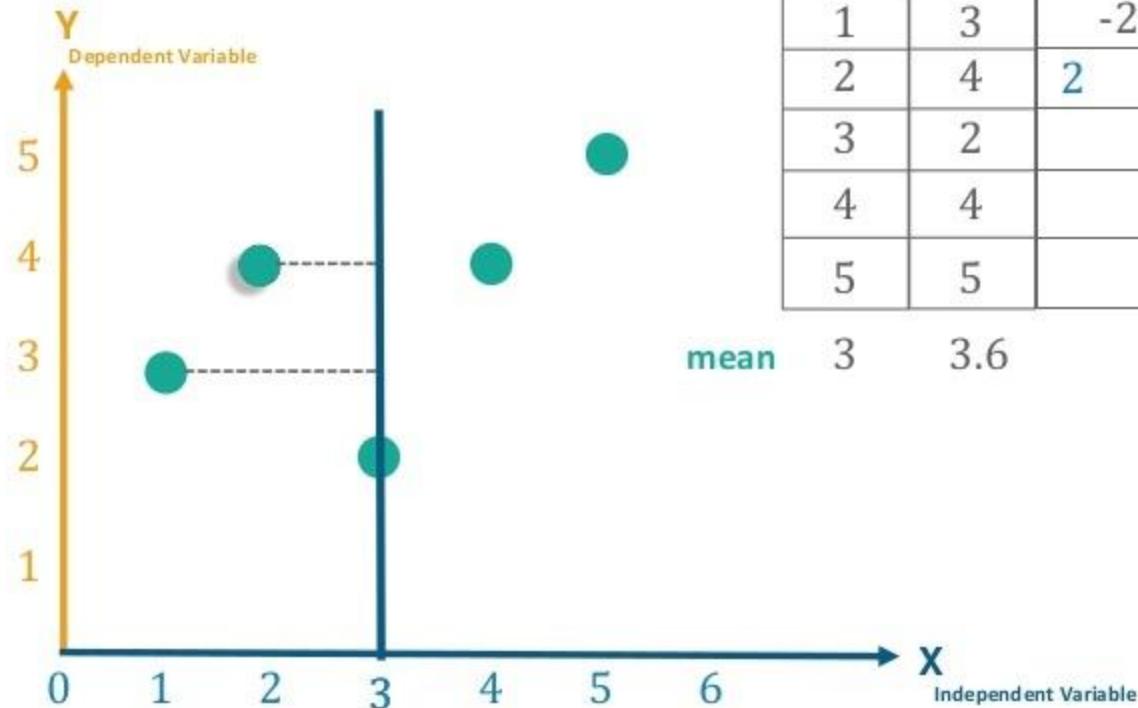
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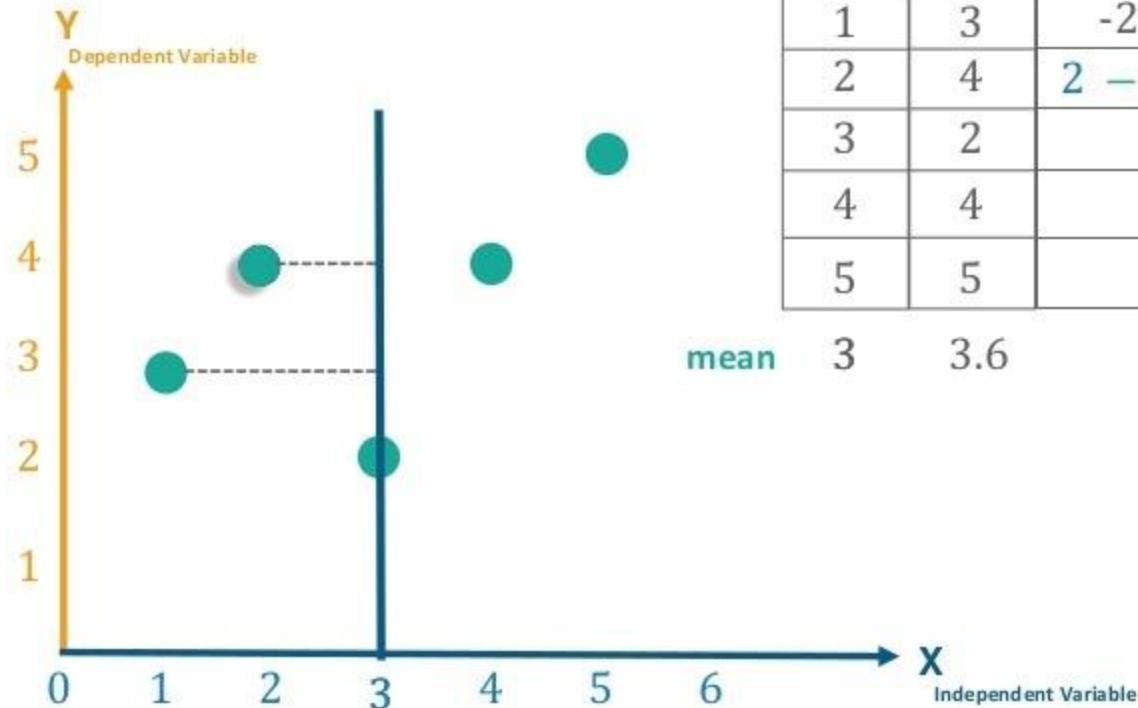
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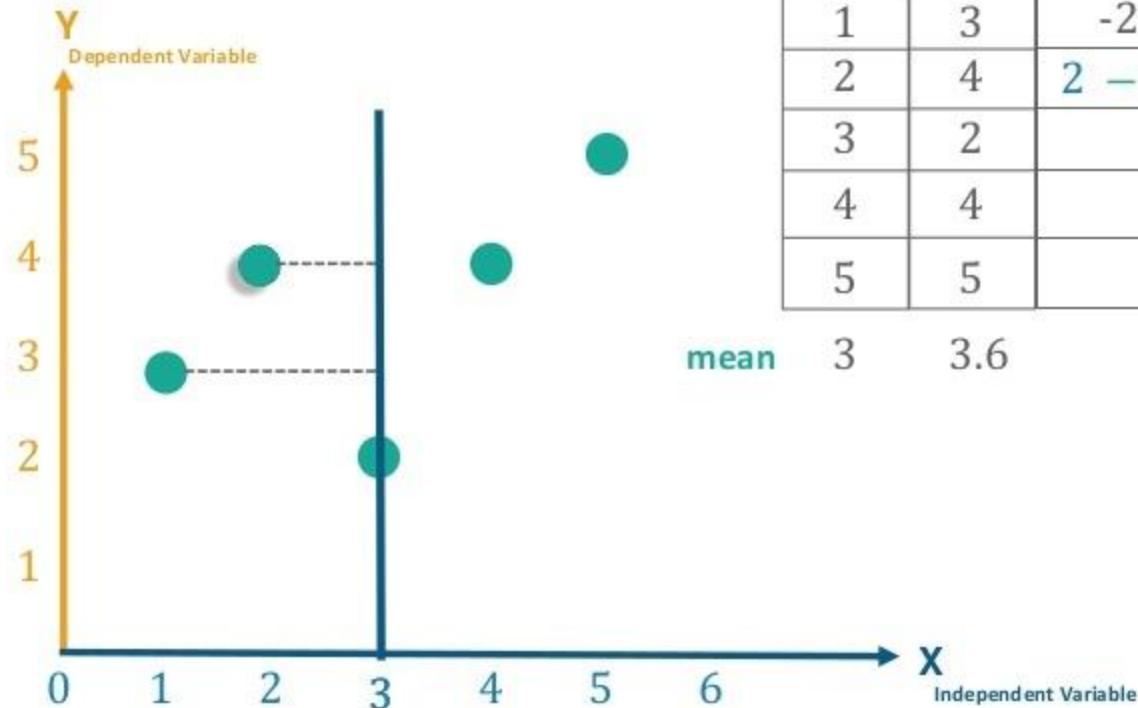
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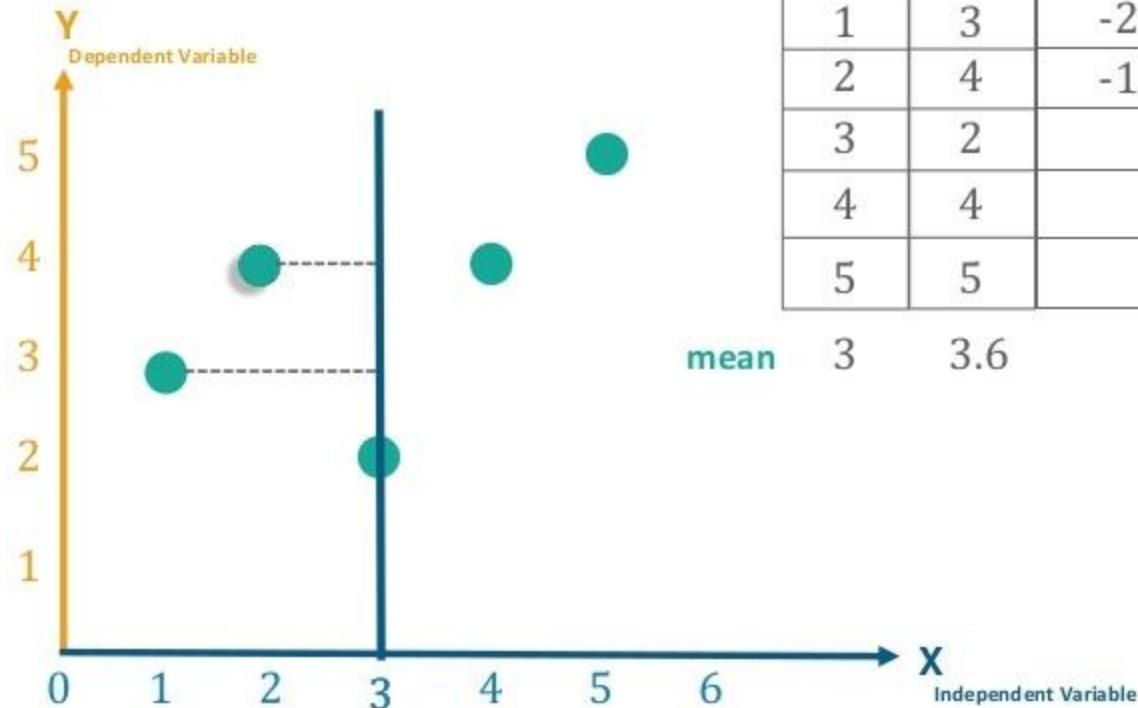
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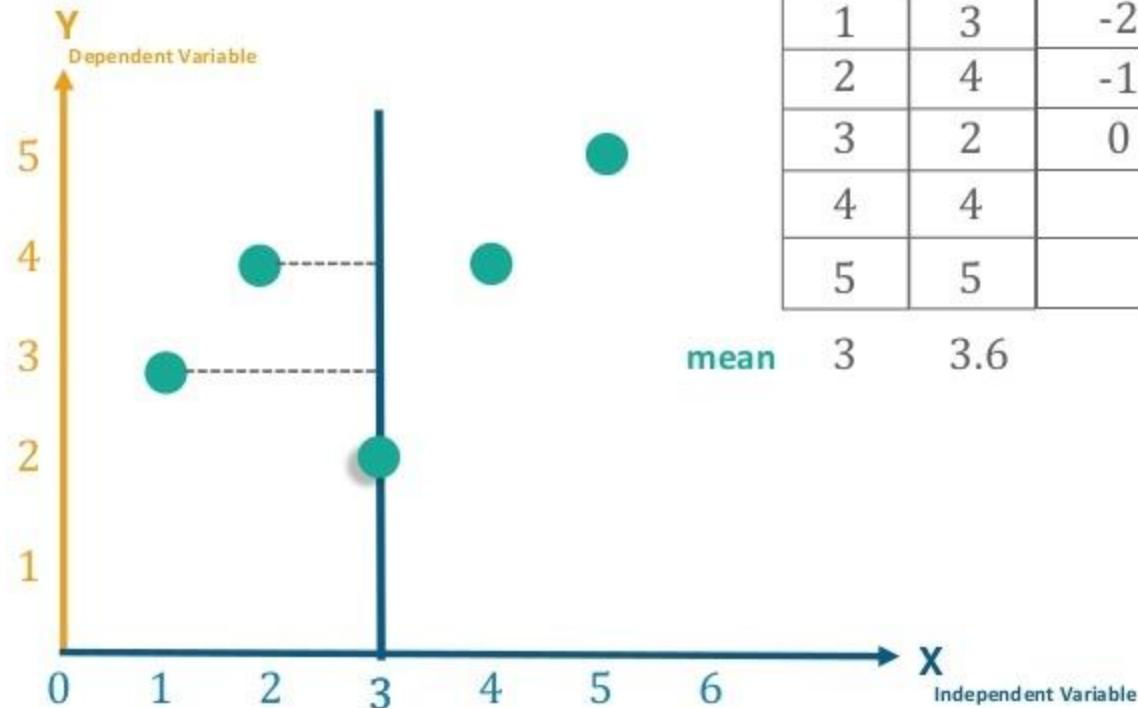
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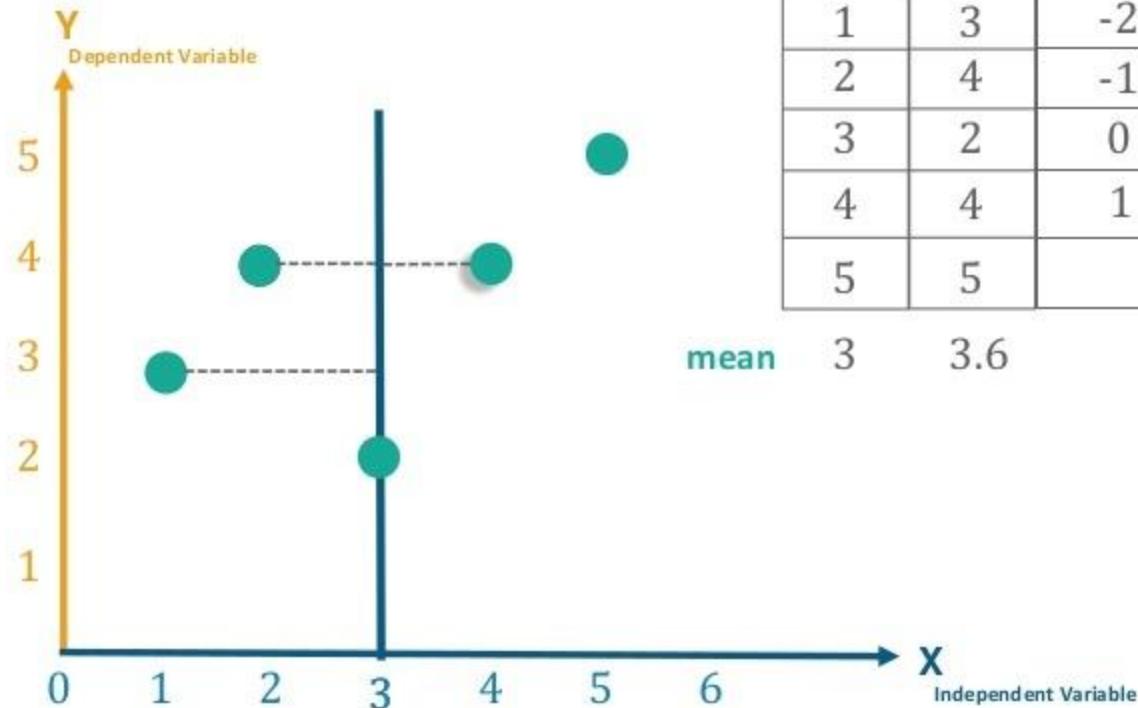
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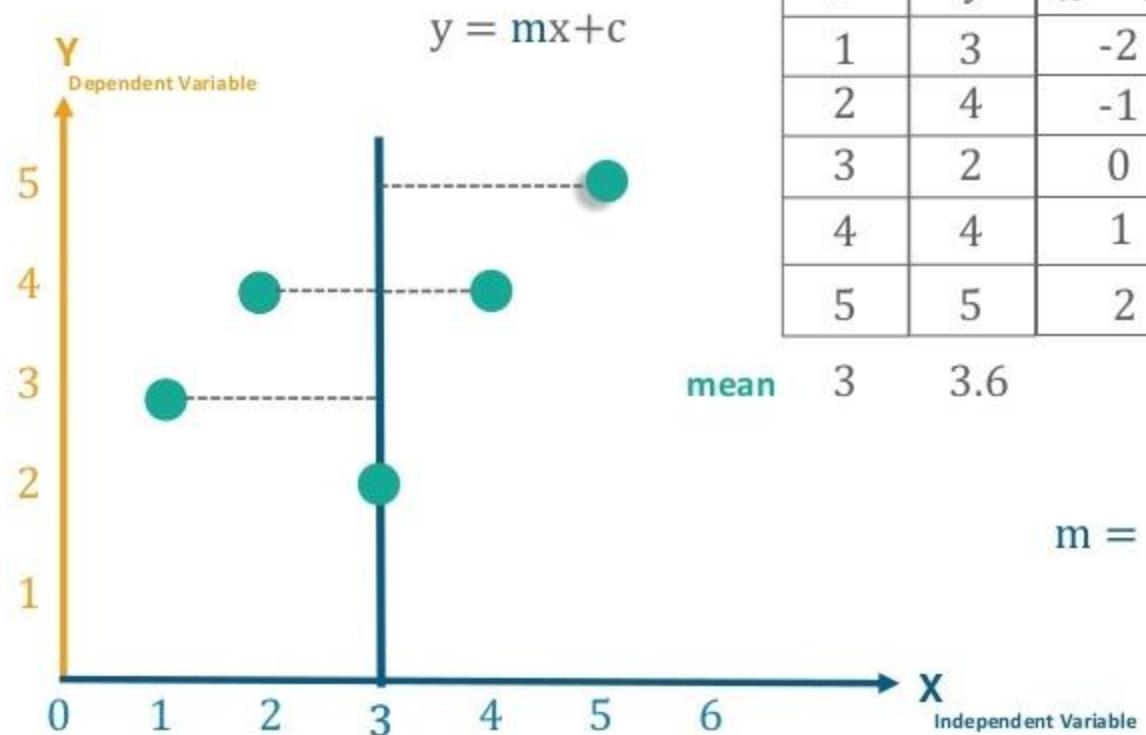
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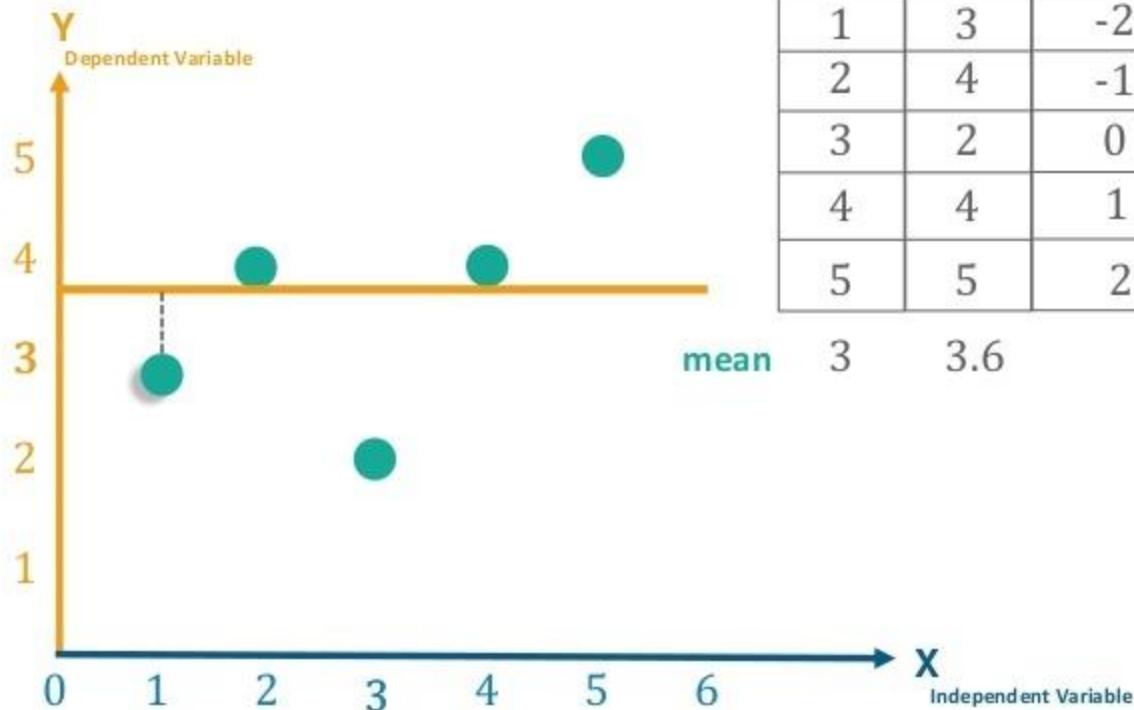


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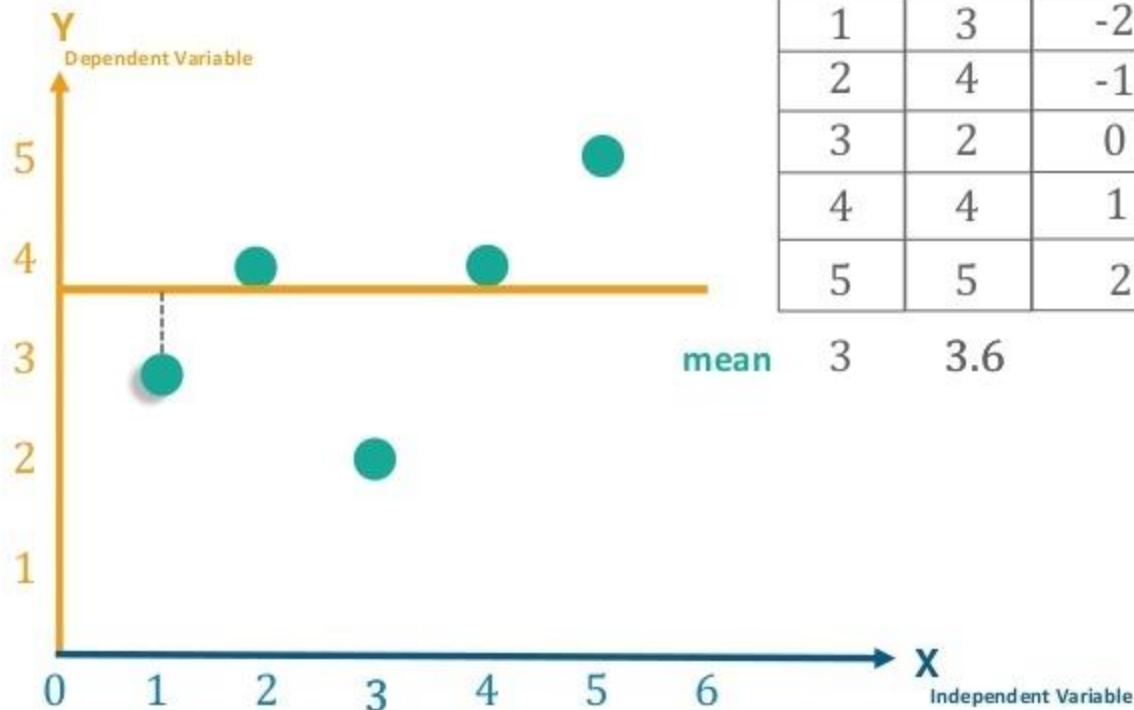
$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

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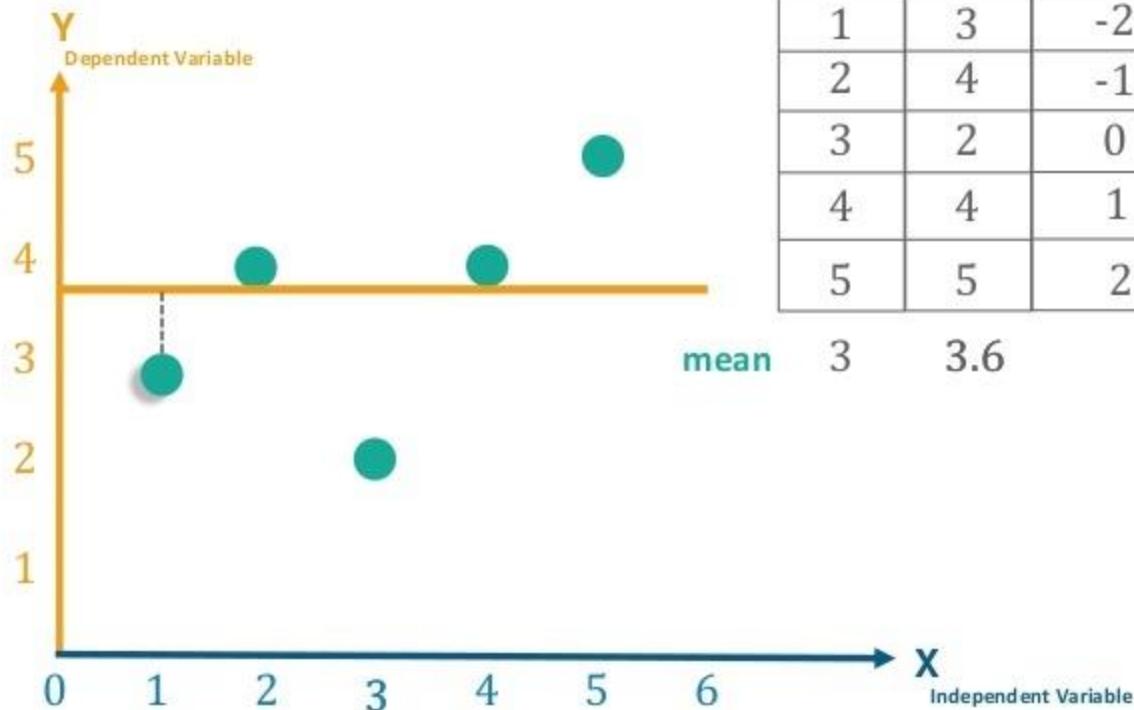
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	
2	4	-1	
3	2	0	
4	4	1	
5	5	2	

# Understanding Linear Regression Algorithm



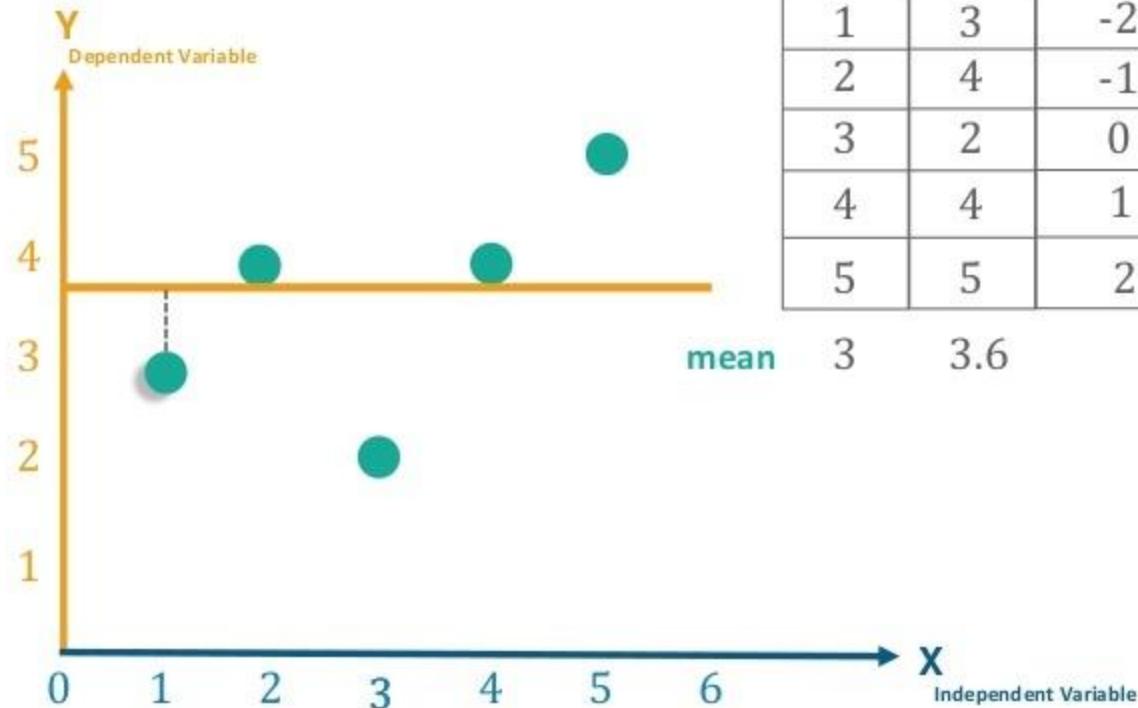
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	3
2	4	-1	
3	2	0	
4	4	1	
5	5	2	

# Understanding Linear Regression Algorithm



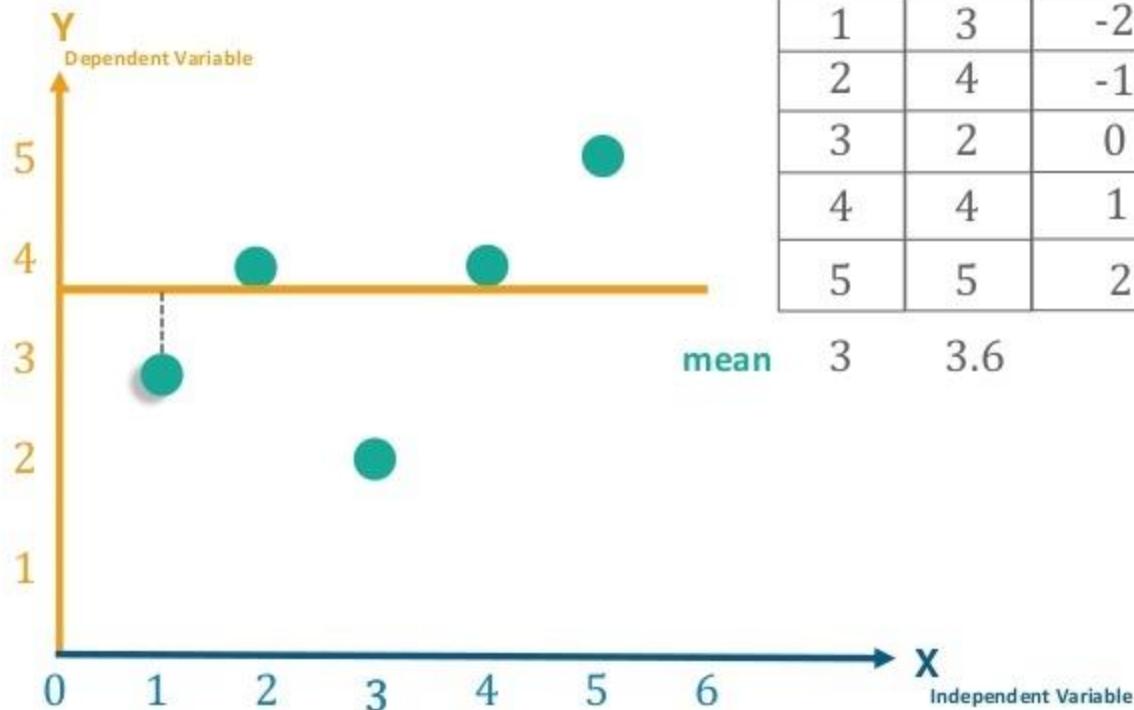
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	3 -
2	4	-1	
3	2	0	
4	4	1	
5	5	2	

# Understanding Linear Regression Algorithm

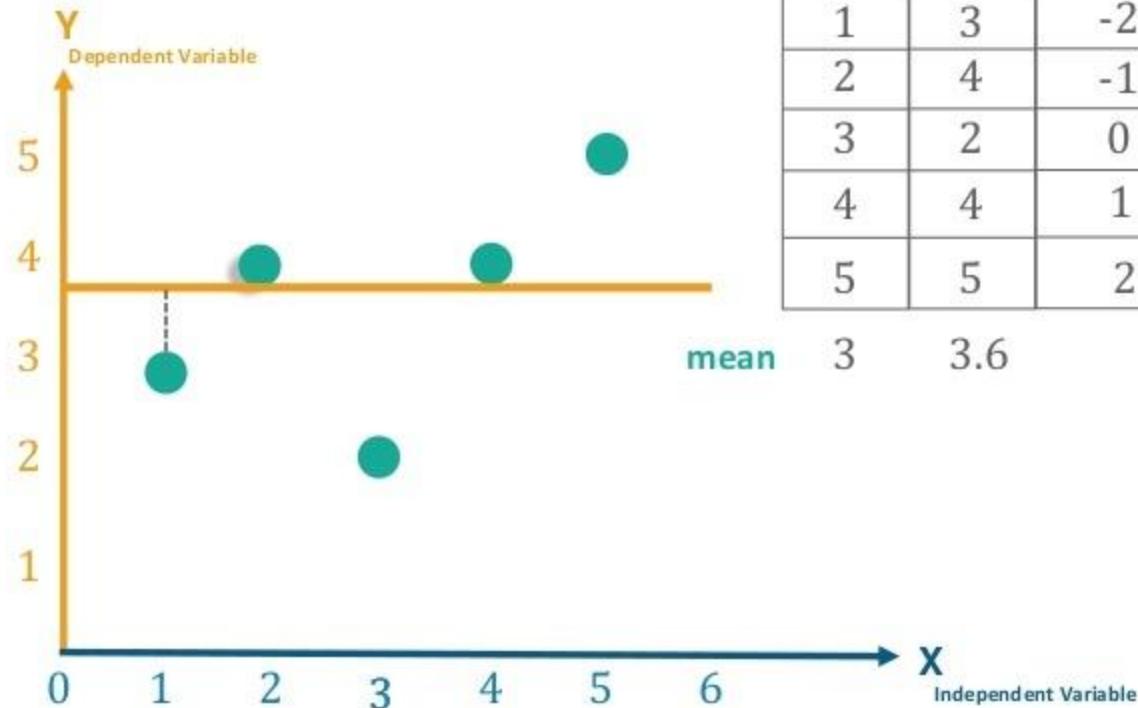


$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	3 - 3.6
2	4	-1	
3	2	0	
4	4	1	
5	5	2	

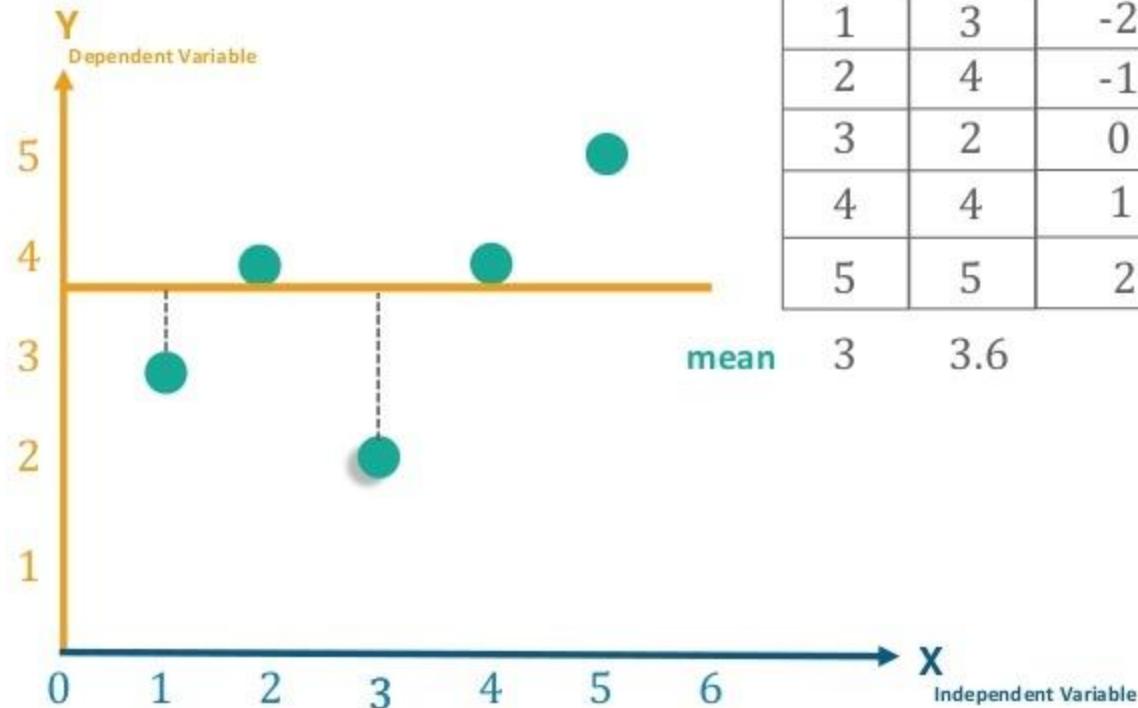
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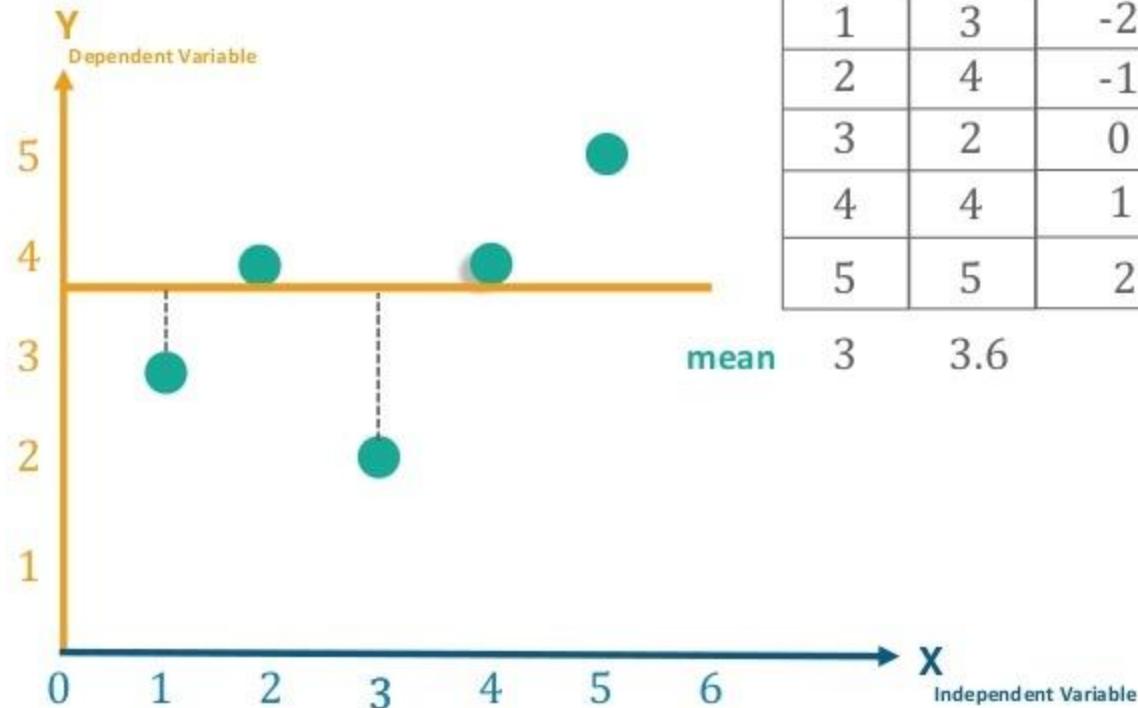


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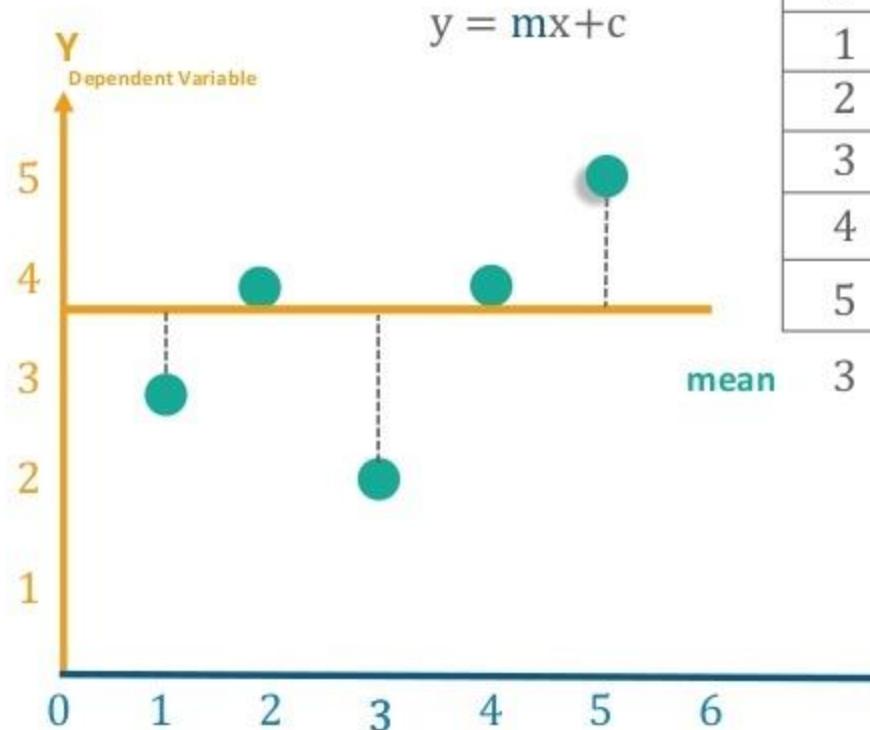
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	-0.6
2	4	-1	0.4
3	2	0	-1.6
4	4	1	
5	5	2	

# Understanding Linear Regression Algorithm



$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	-0.6
2	4	-1	0.4
3	2	0	-1.6
4	4	1	0.4
5	5	2	

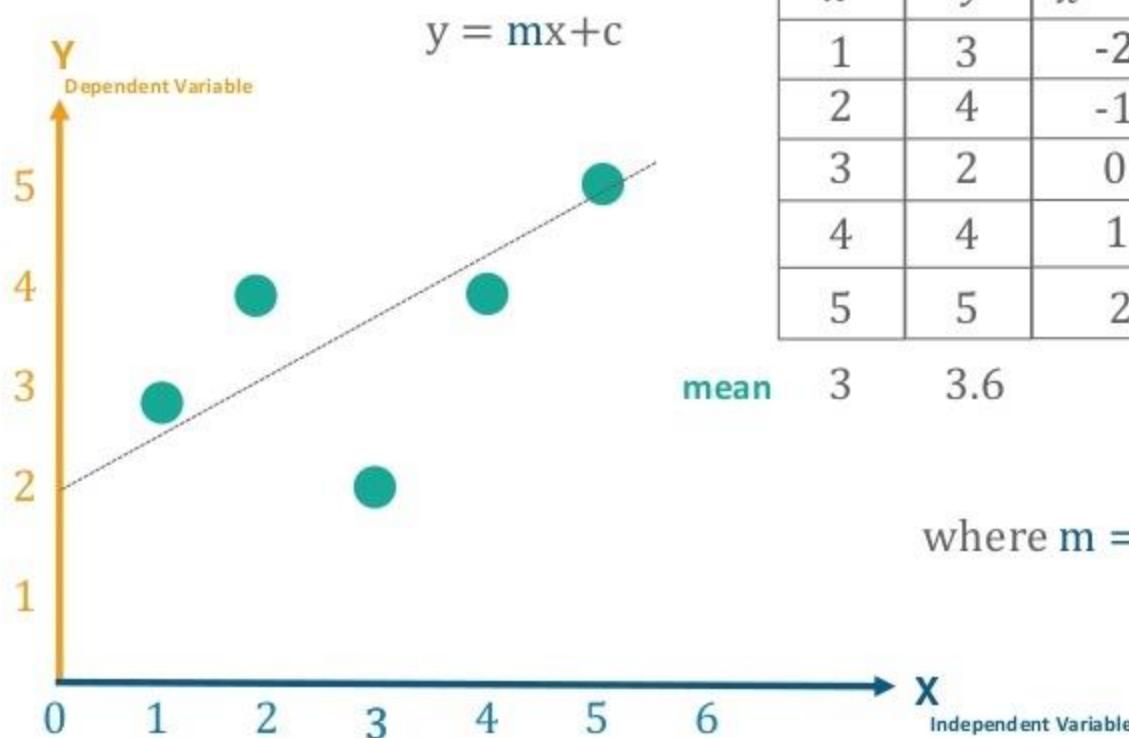
# Understanding Linear Regression Algorithm



$x$	$y$	$x - \bar{x}$	$y - \bar{y}$
1	3	-2	-0.6
2	4	-1	0.4
3	2	0	-1.6
4	4	1	0.4
5	5	2	1.4

$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

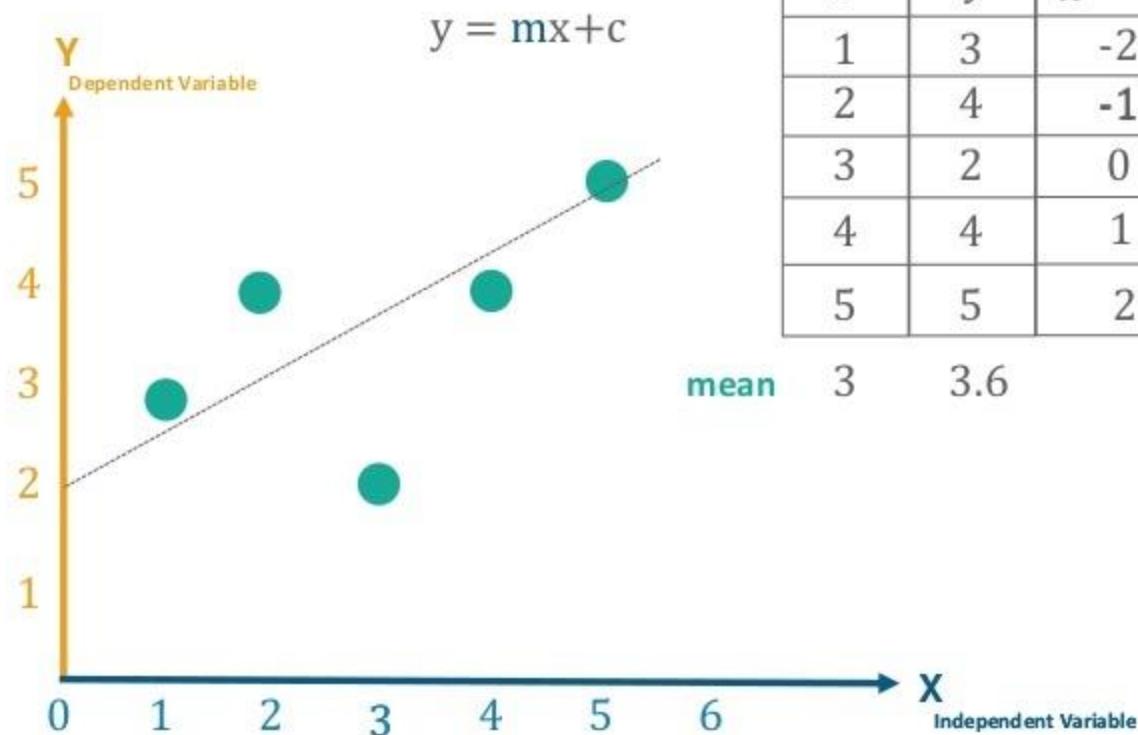
# Understanding Linear Regression Algorithm



$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	
2	4	-1	0.4	
3	2	0	-1.6	
4	4	1	0.4	
5	5	2	1.4	

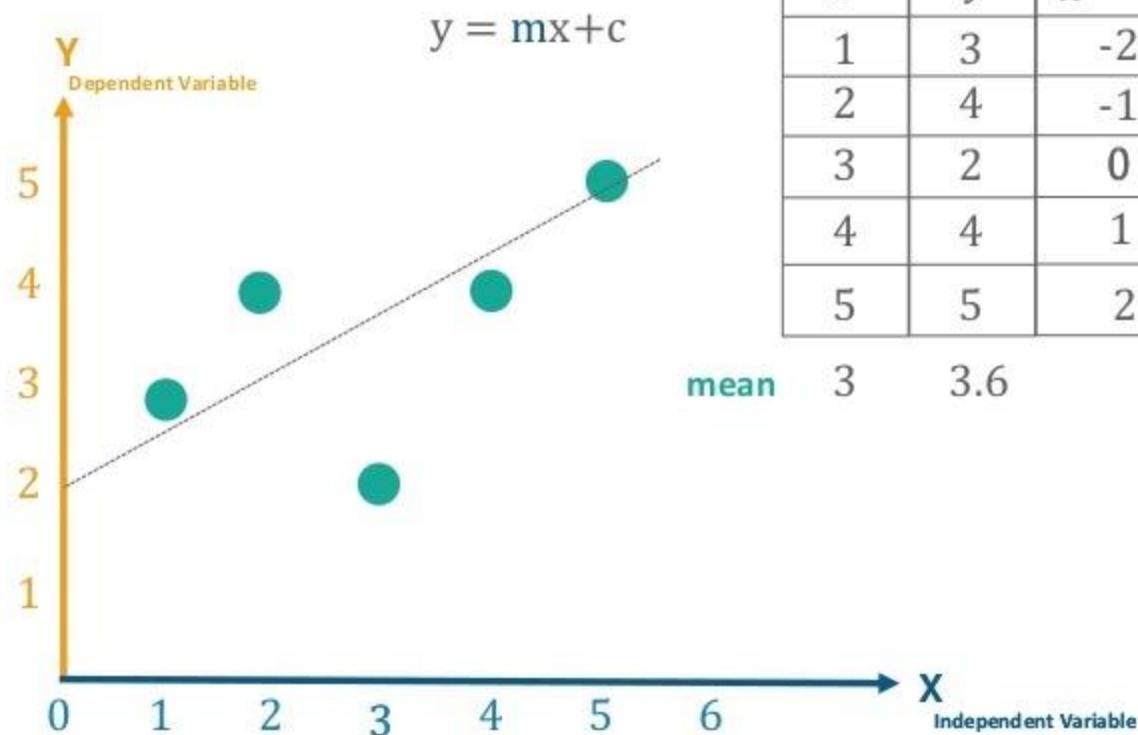
$$\text{where } m = \sum \frac{(x - \bar{x})(y - \bar{y})}{(x - \bar{x})^2}$$

# Understanding Linear Regression Algorithm



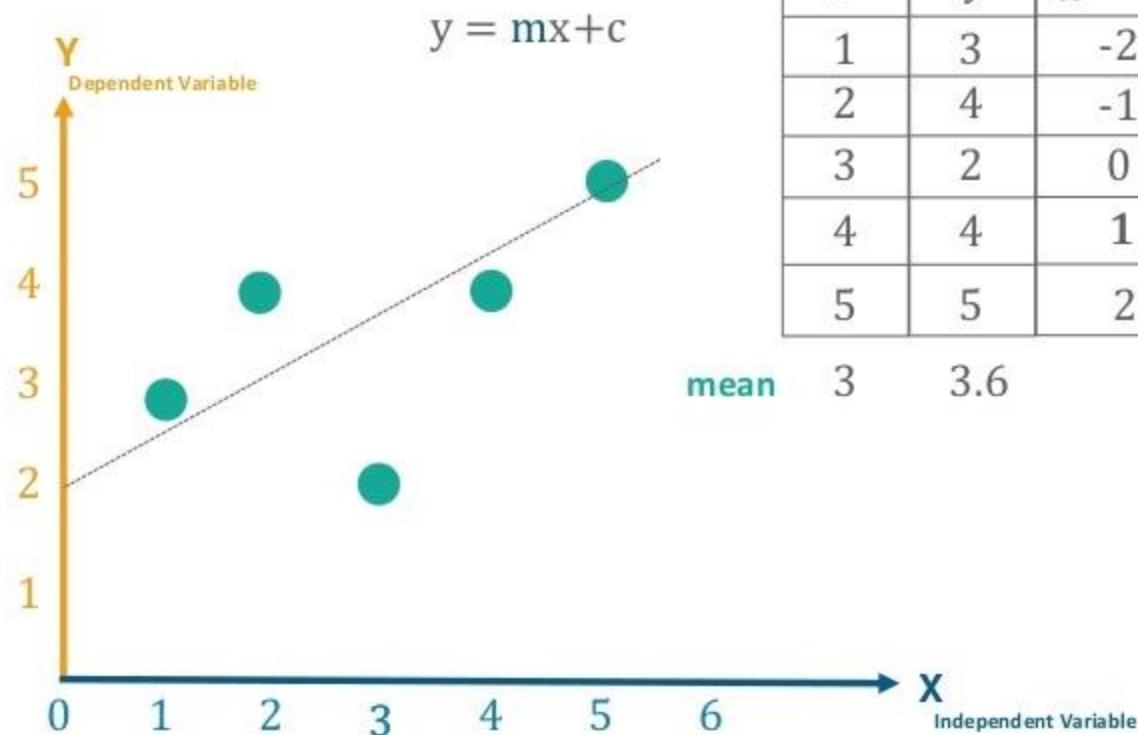
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	4
2	4	-1	0.4	
3	2	0	-1.6	
4	4	1	0.4	
5	5	2	1.4	

# Understanding Linear Regression Algorithm



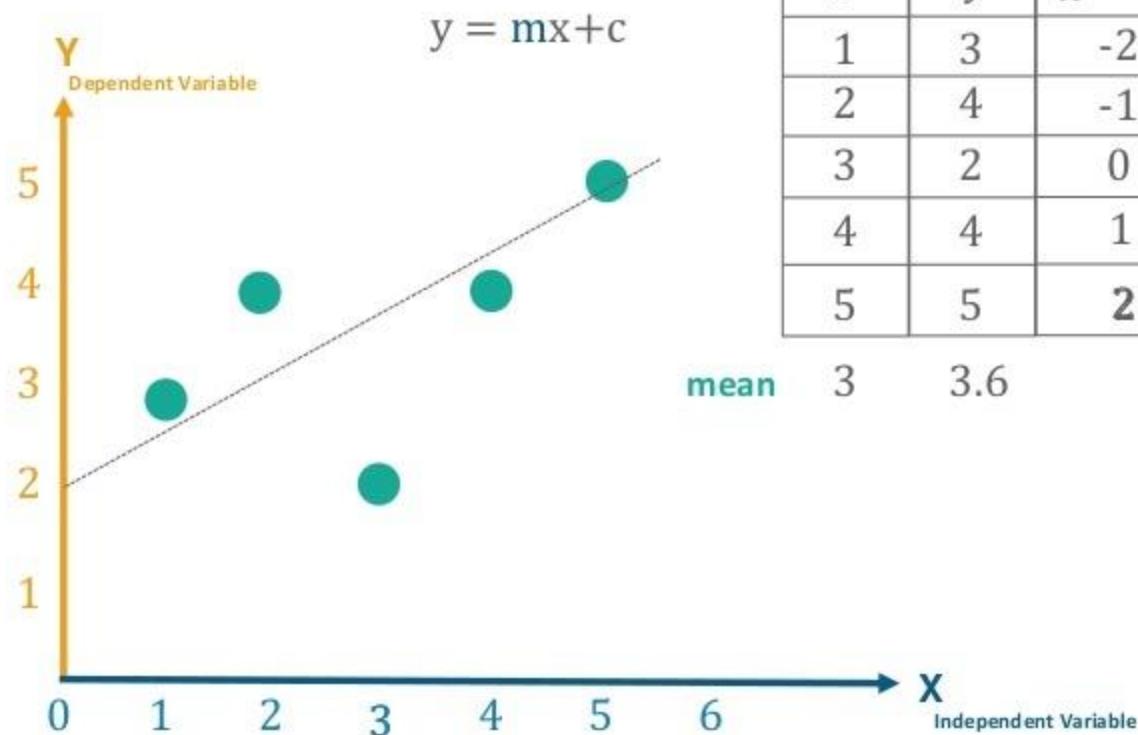
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	4
2	4	-1	0.4	1
3	2	0	-1.6	
4	4	1	0.4	
5	5	2	1.4	

# Understanding Linear Regression Algorithm



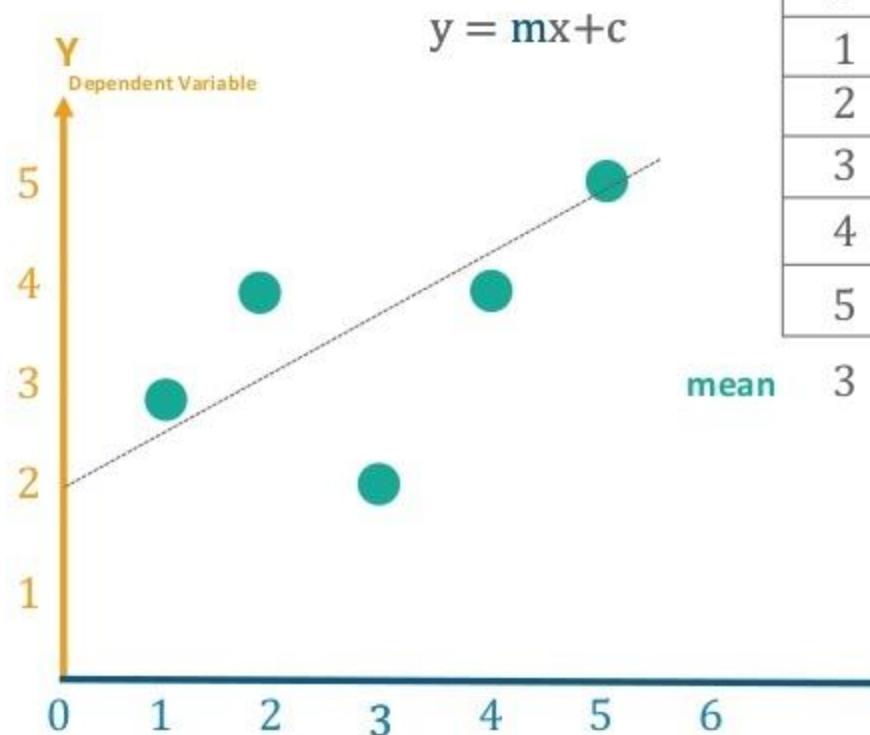
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	4
2	4	-1	0.4	1
3	2	0	-1.6	0
4	4	1	0.4	
5	5	2	1.4	

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$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	4
2	4	-1	0.4	1
3	2	0	-1.6	0
4	4	1	0.4	1
5	5	2	1.4	

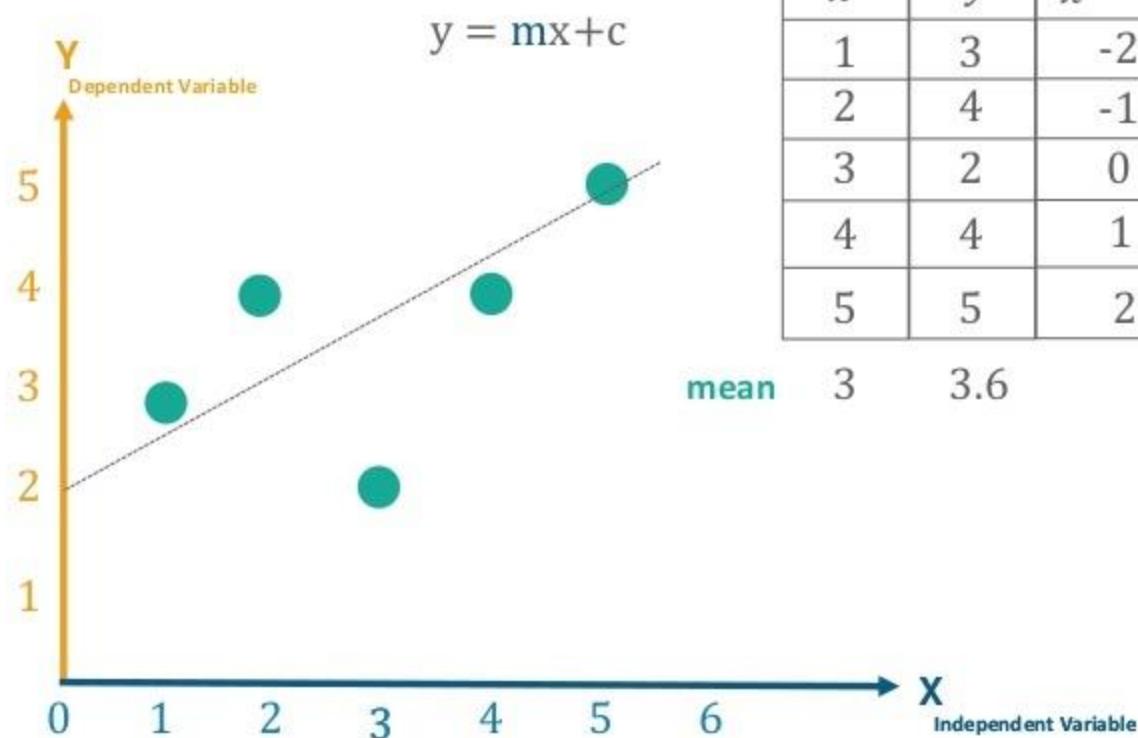
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$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	4
2	4	-1	0.4	1
3	2	0	-1.6	0
4	4	1	0.4	1
5	5	2	1.4	4

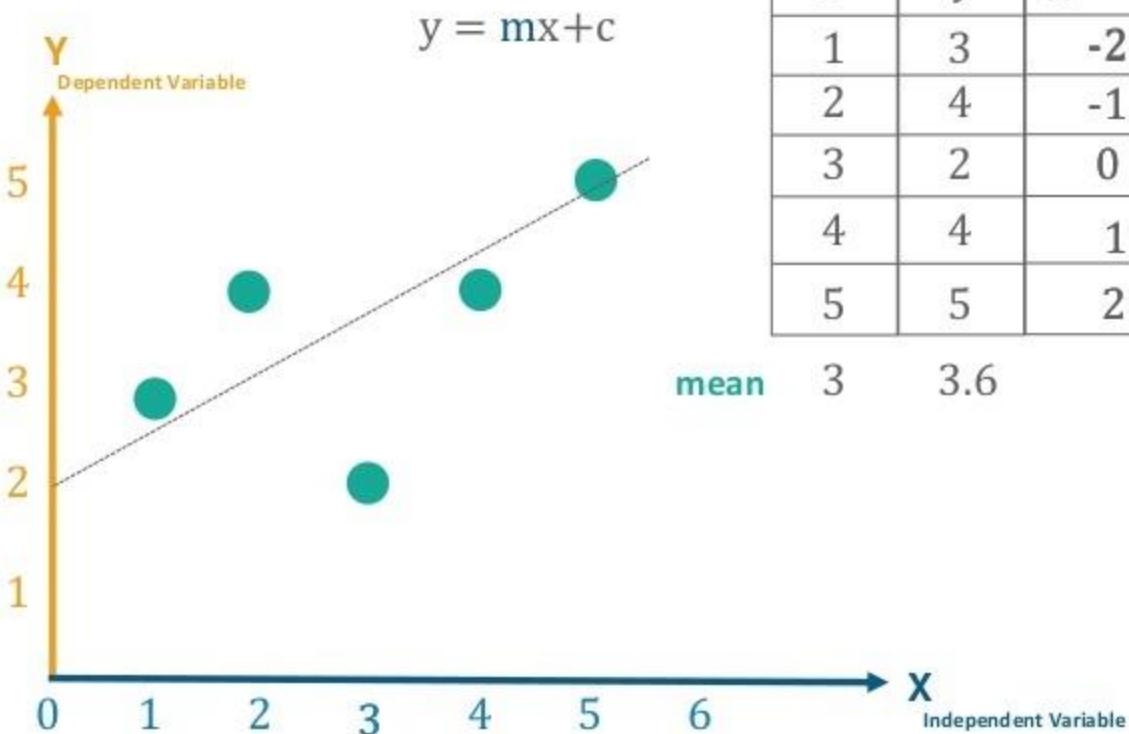
$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

# Understanding Linear Regression Algorithm



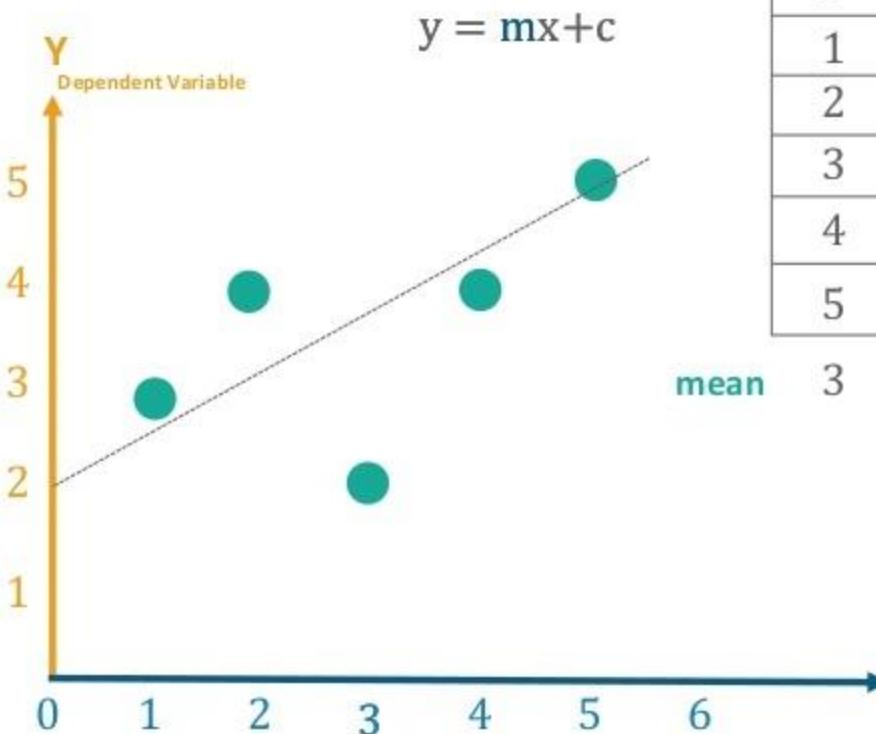
$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$
1	3	-2	-0.6	4
2	4	-1	0.4	1
3	2	0	-1.6	0
4	4	1	0.4	1
5	5	2	1.4	4

# Understanding Linear Regression Algorithm



$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	-2	-0.6	4	
2	4	-1	0.4	1	
3	2	0	-1.6	0	
4	4	1	0.4	1	
5	5	2	1.4	4	

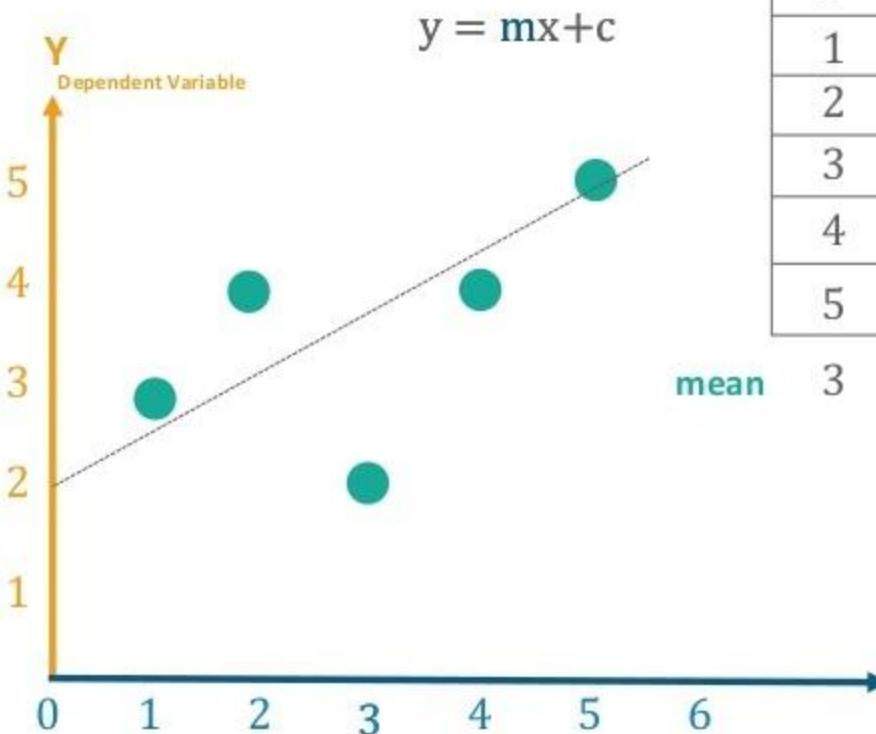
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x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	-2	-0.6	4	1.2
2	4	-1	0.4	1	-0.4
3	2	0	-1.6	0	0
4	4	1	0.4	1	0.4
5	5	2	1.4	4	2.8

$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

# Understanding Linear Regression Algorithm

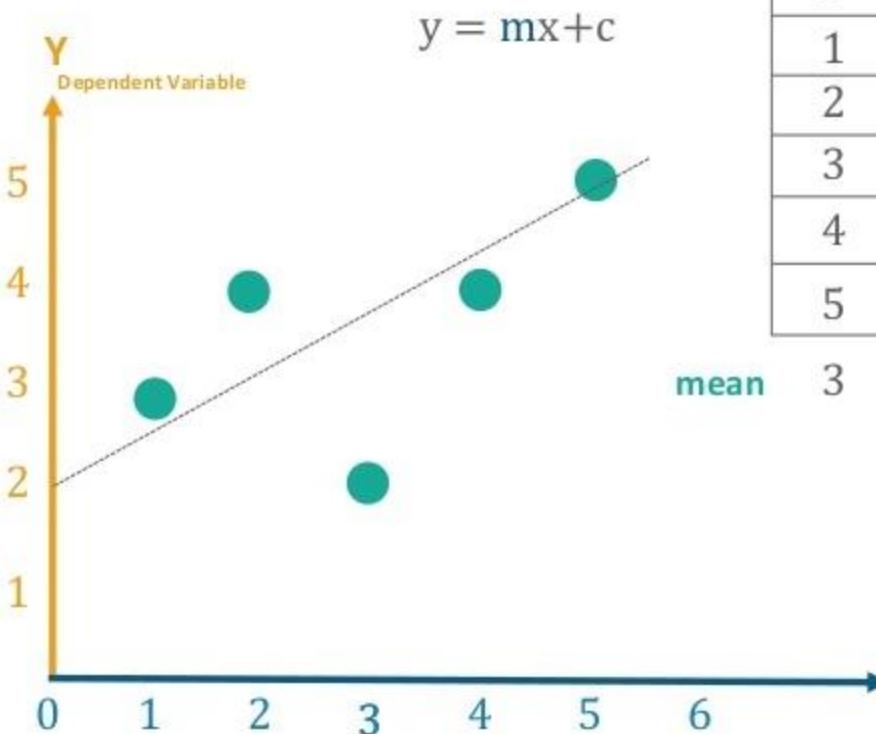


x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	-2	-0.6	4	1.2
2	4	-1	0.4	1	-0.4
3	2	0	-1.6	0	0
4	4	1	0.4	1	0.4
5	5	2	1.4	4	2.8

mean    3    3.6     $\Sigma = 10$      $\Sigma = 4$

$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

# Understanding Linear Regression Algorithm

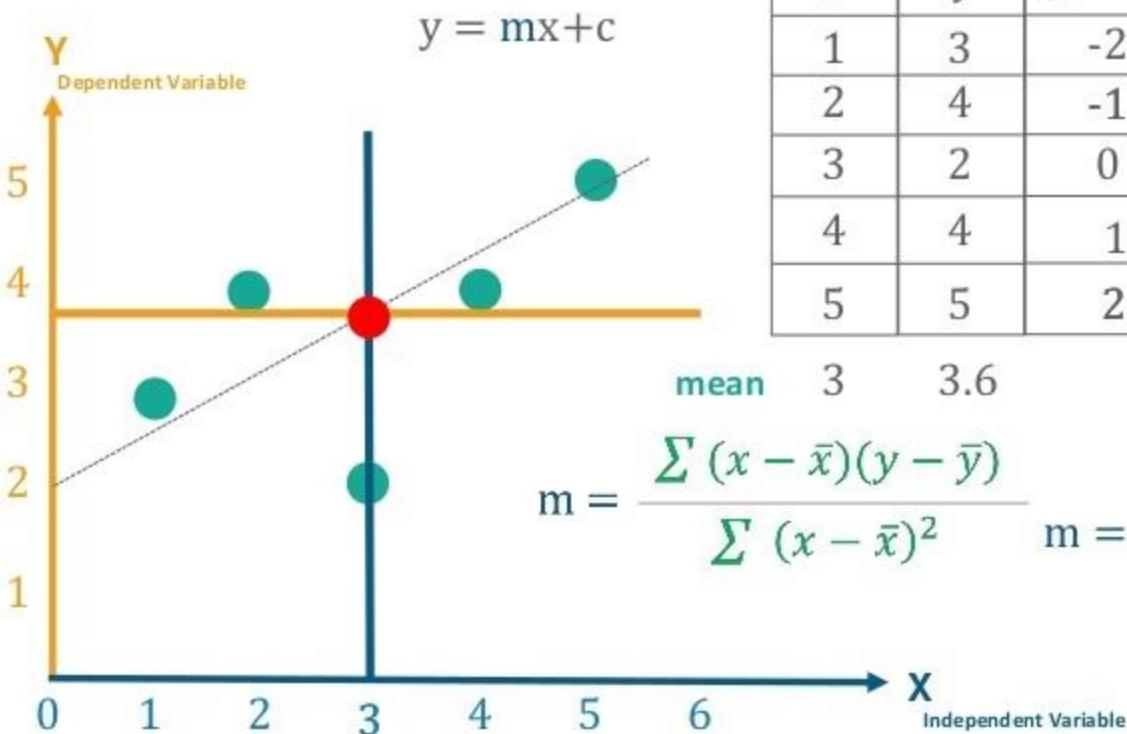


x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	-2	-0.6	4	1.2
2	4	-1	0.4	1	-0.4
3	2	0	-1.6	0	0
4	4	1	0.4	1	0.4
5	5	2	1.4	4	2.8

mean    3    3.6     $\Sigma = 10$      $\Sigma = 4$

$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{4}{10}$$

# Understanding Linear Regression Algorithm

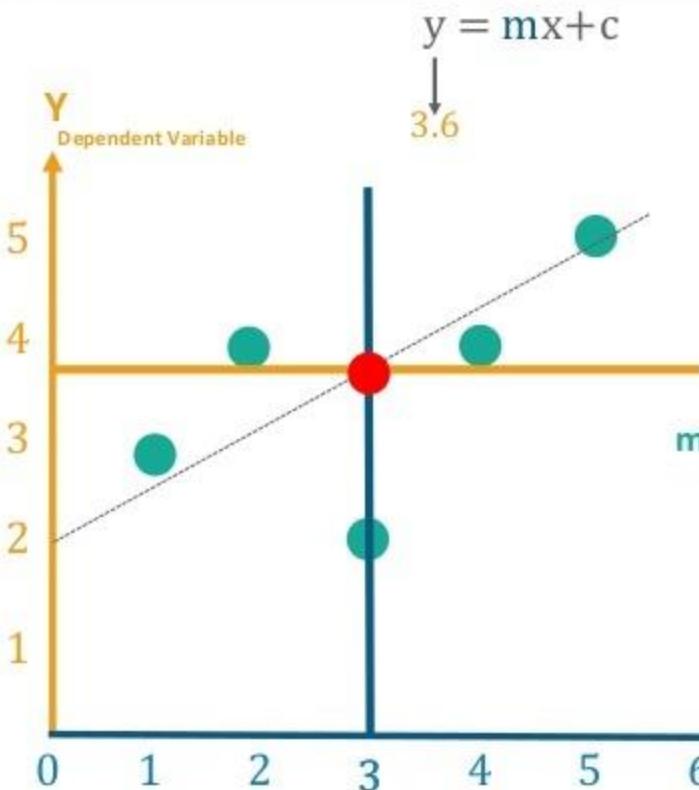


x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
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# Understanding Linear Regression Algorithm

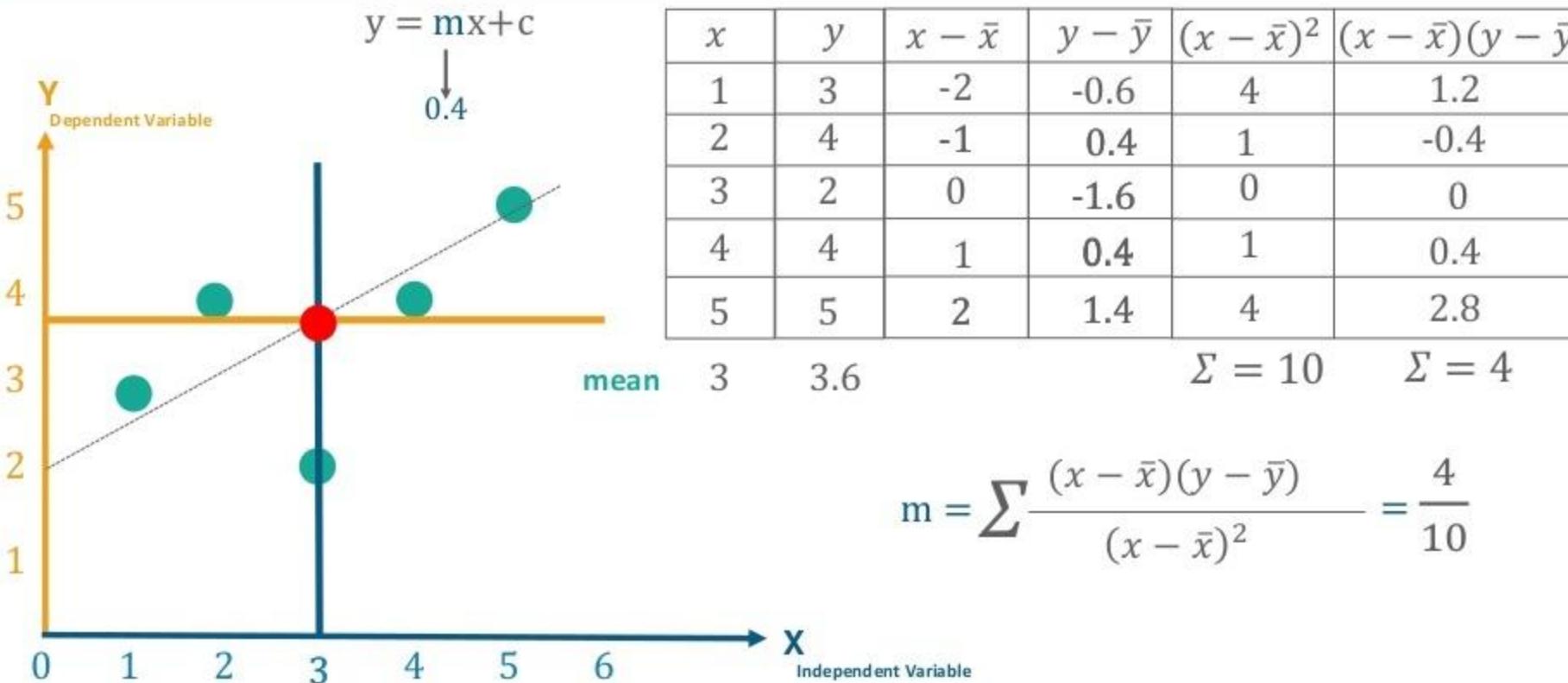


x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
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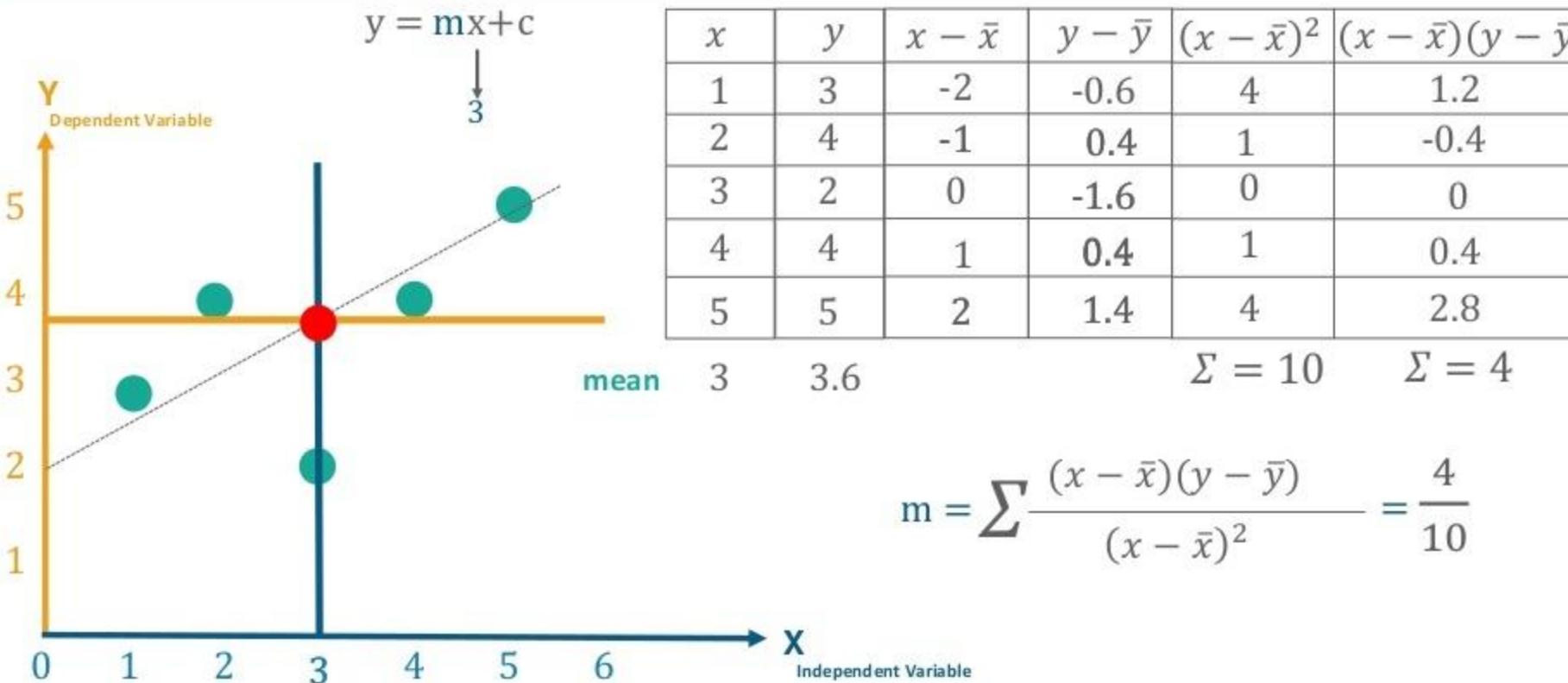
$\Sigma = 10$        $\Sigma = 4$

$$m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{4}{10}$$

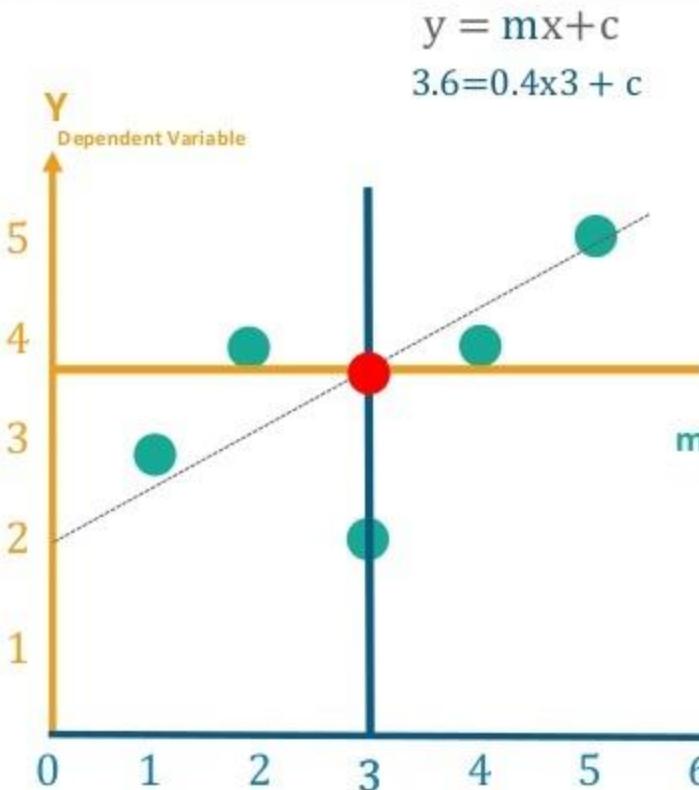
# Understanding Linear Regression Algorithm



# Understanding Linear Regression Algorithm



# Understanding Linear Regression Algorithm

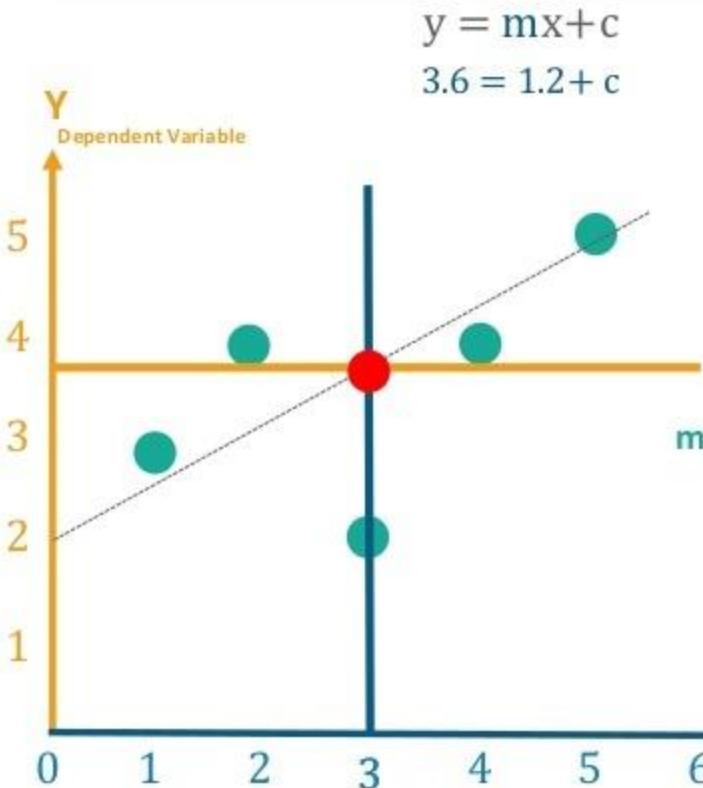


$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	-2	-0.6	4	1.2
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3	2	0	-1.6	0	0
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5	5	2	1.4	4	2.8

$\Sigma = 10$        $\Sigma = 4$

$$m = \sum \frac{(x - \bar{x})(y - \bar{y})}{(x - \bar{x})^2} = \frac{4}{10}$$

# Understanding Linear Regression Algorithm

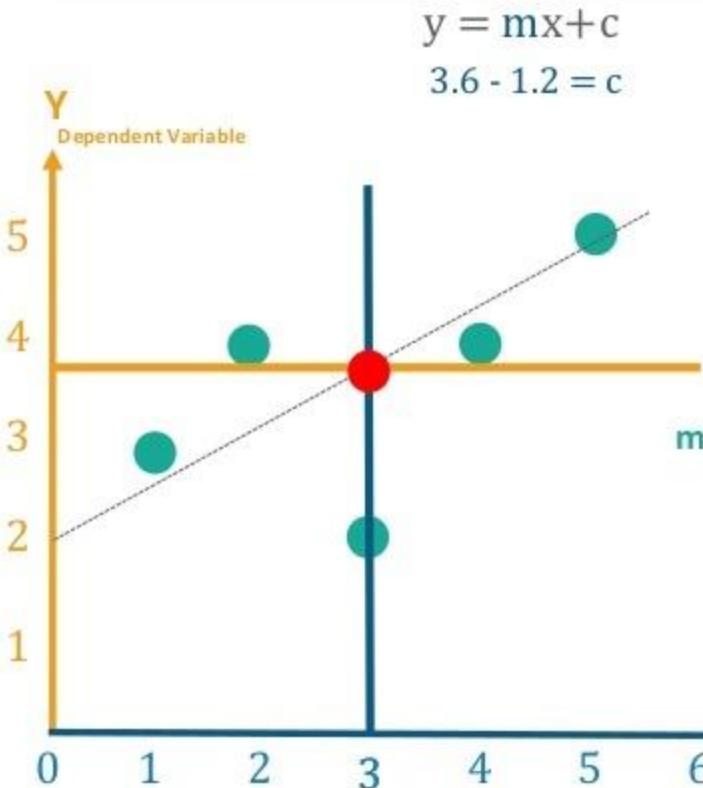


$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
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# Understanding Linear Regression Algorithm

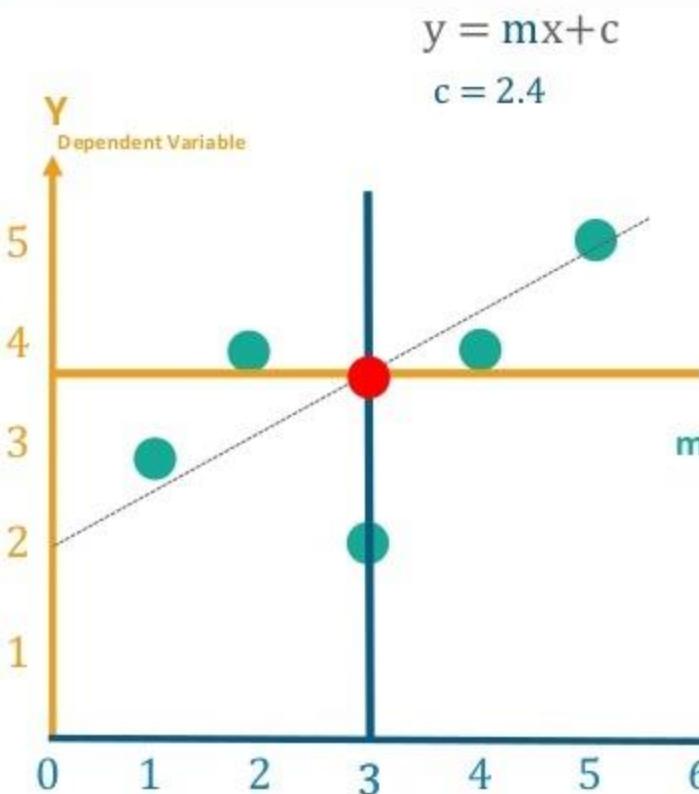


$x$	$y$	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	-2	-0.6	4	1.2
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# Understanding Linear Regression Algorithm

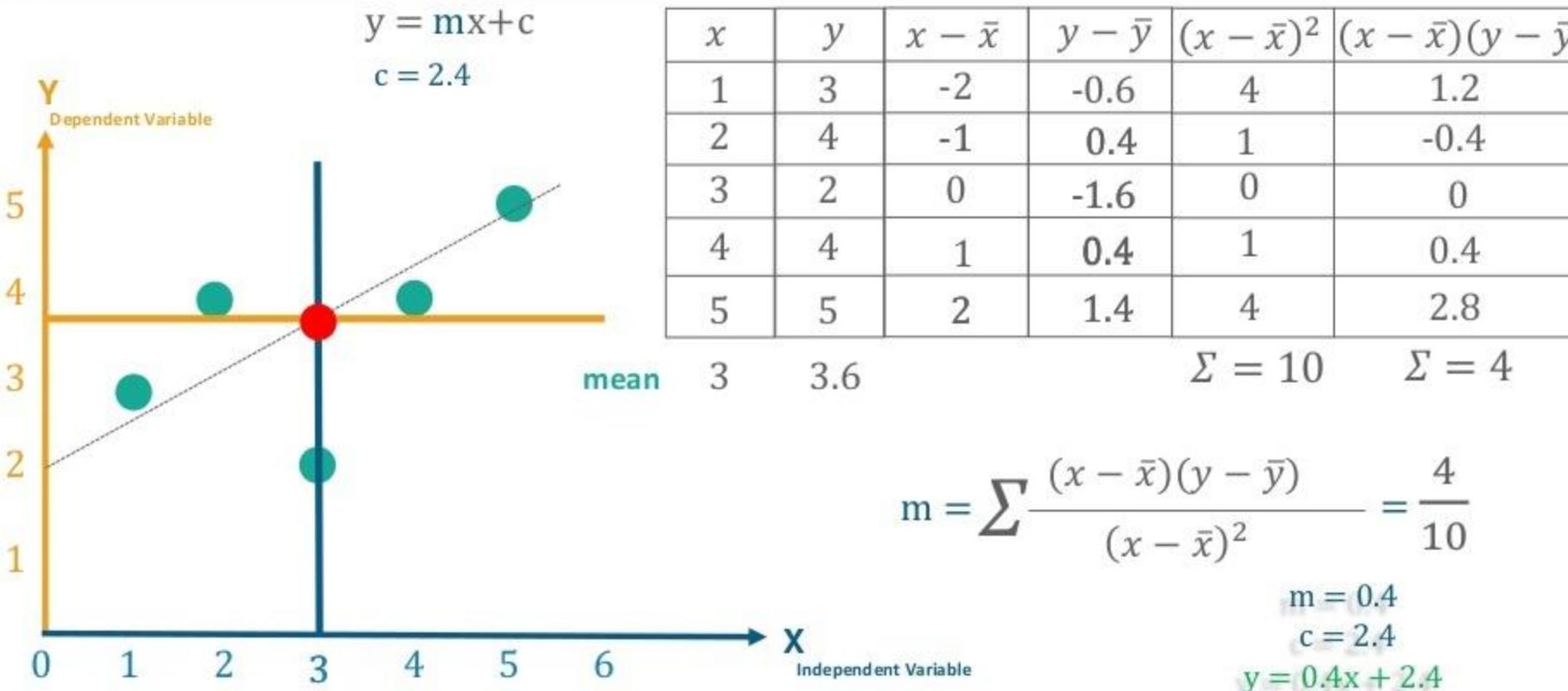


x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
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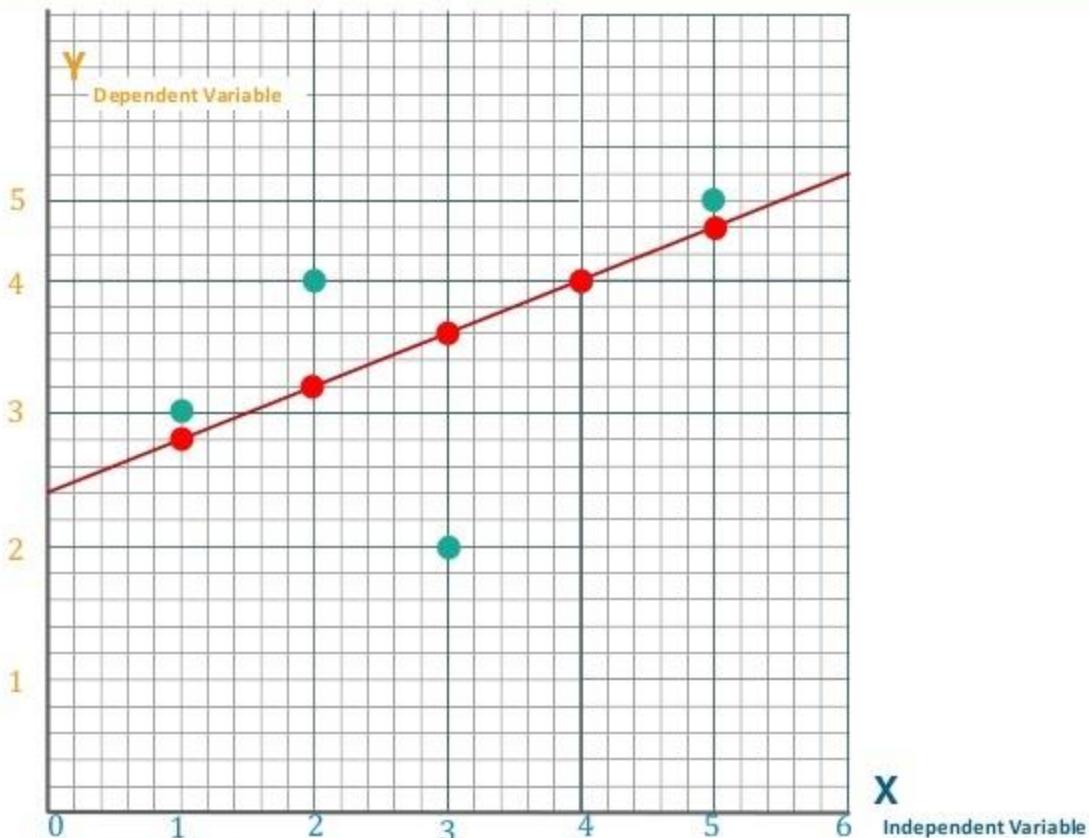
$\Sigma = 10$        $\Sigma = 4$

$$m = \sum \frac{(x - \bar{x})(y - \bar{y})}{(x - \bar{x})^2} = \frac{4}{10}$$

# Understanding Linear Regression Algorithm



# Mean Square Error



$$m = 0.4$$

$$c = 2.4$$

$$y = 0.4x + 2.4$$

For given  $m = 0.4$  &  $c = 2.4$ , lets predict values for  $y$  for  $x = \{1,2,3,4,5\}$

$$y = 0.4 \times 1 + 2.4 = 2.8$$

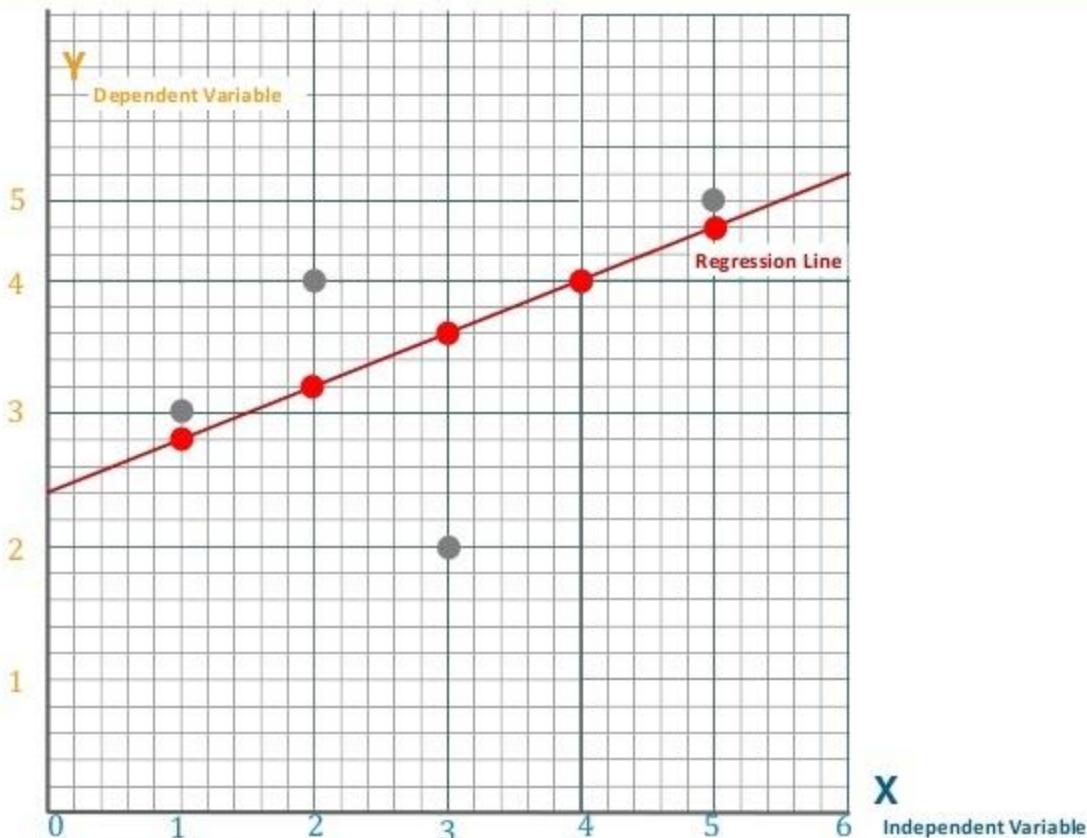
$$y = 0.4 \times 2 + 2.4 = 3.2$$

$$y = 0.4 \times 3 + 2.4 = 3.6$$

$$y = 0.4 \times 4 + 2.4 = 4.0$$

$$y = 0.4 \times 5 + 2.4 = 4.4$$

# Mean Square Error



$$m = 0.4$$

$$c = 2.4$$

$$y = 0.4x + 2.4$$

For given  $m = 0.4$  &  $c = 2.4$ , lets predict values for  $y$  for  $x = \{1, 2, 3, 4, 5\}$

$$y = 0.4 \times 1 + 2.4 = 2.8$$

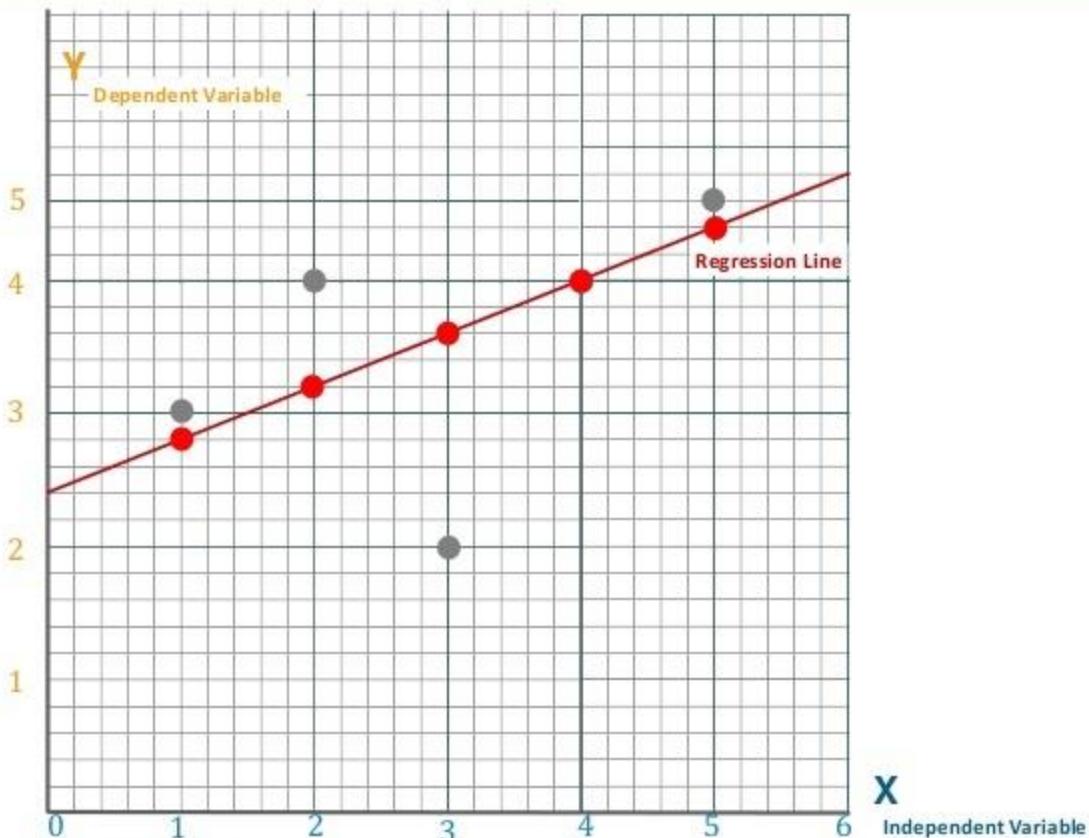
$$y = 0.4 \times 2 + 2.4 = 3.2$$

$$y = 0.4 \times 3 + 2.4 = 3.6$$

$$y = 0.4 \times 4 + 2.4 = 4.0$$

$$y = 0.4 \times 5 + 2.4 = 4.4$$

# Mean Square Error



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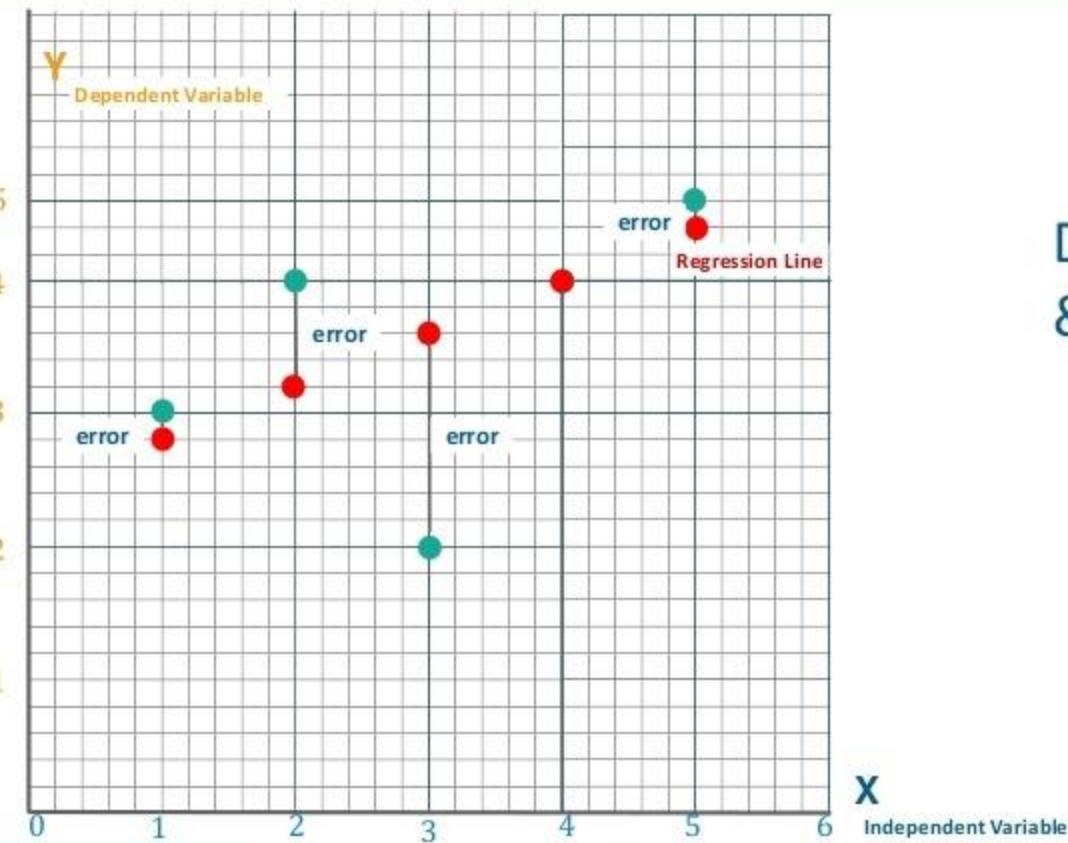
$$y = 0.4 \times 2 + 2.4 = 3.2$$

$$y = 0.4 \times 3 + 2.4 = 3.6$$

$$y = 0.4 \times 4 + 2.4 = 4.0$$

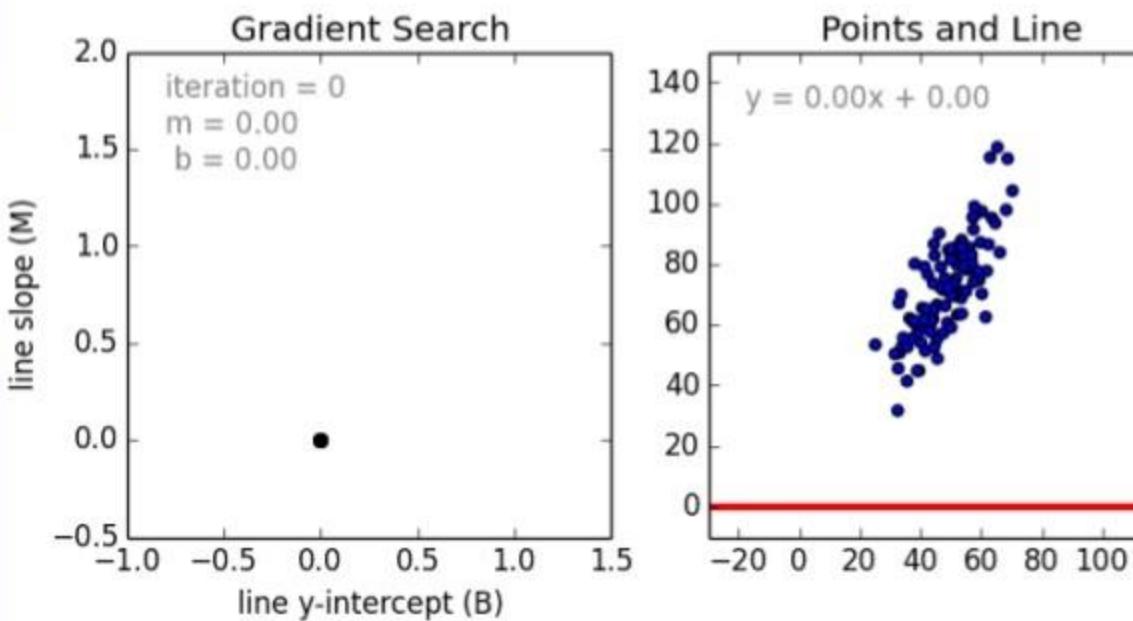
$$y = 0.4 \times 5 + 2.4 = 4.4$$

# Mean Square Error



Distance between actual  
& predicted value

# Finding the best fit line



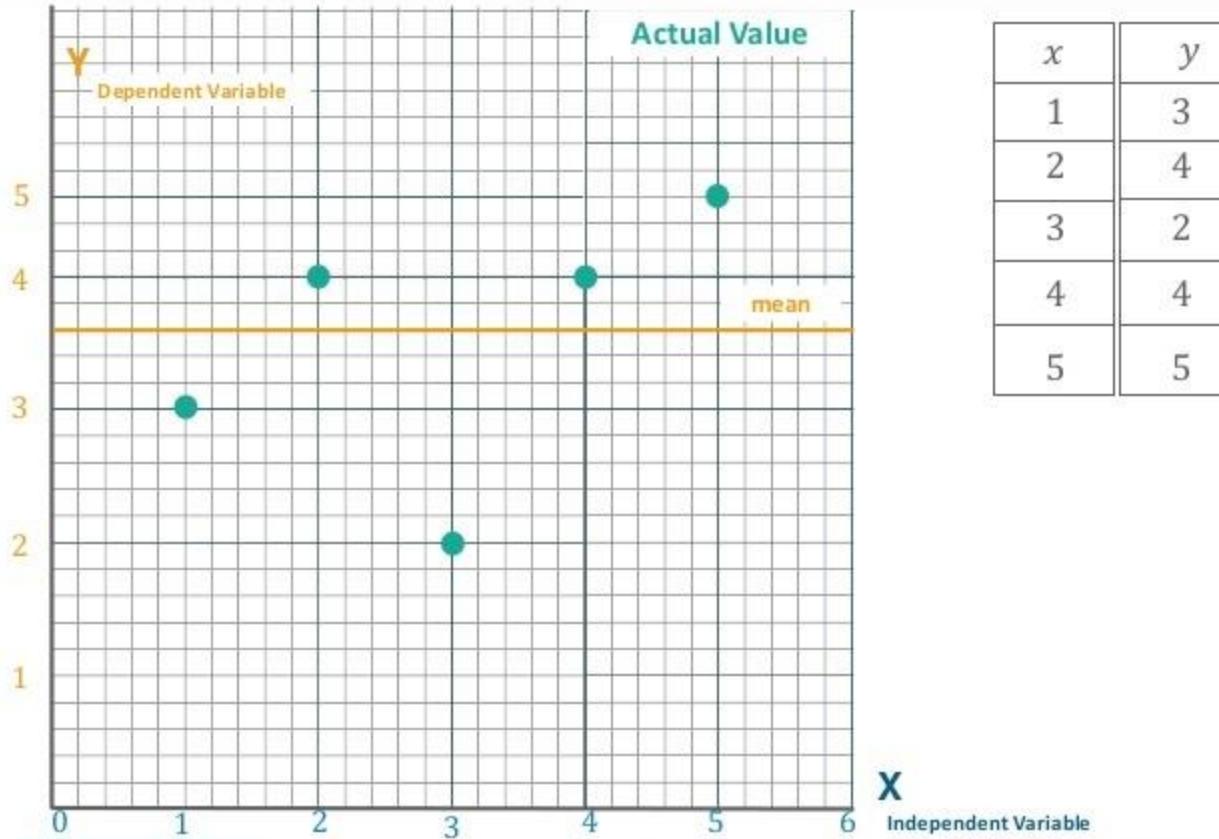


**Let's check the Goodness of fit**

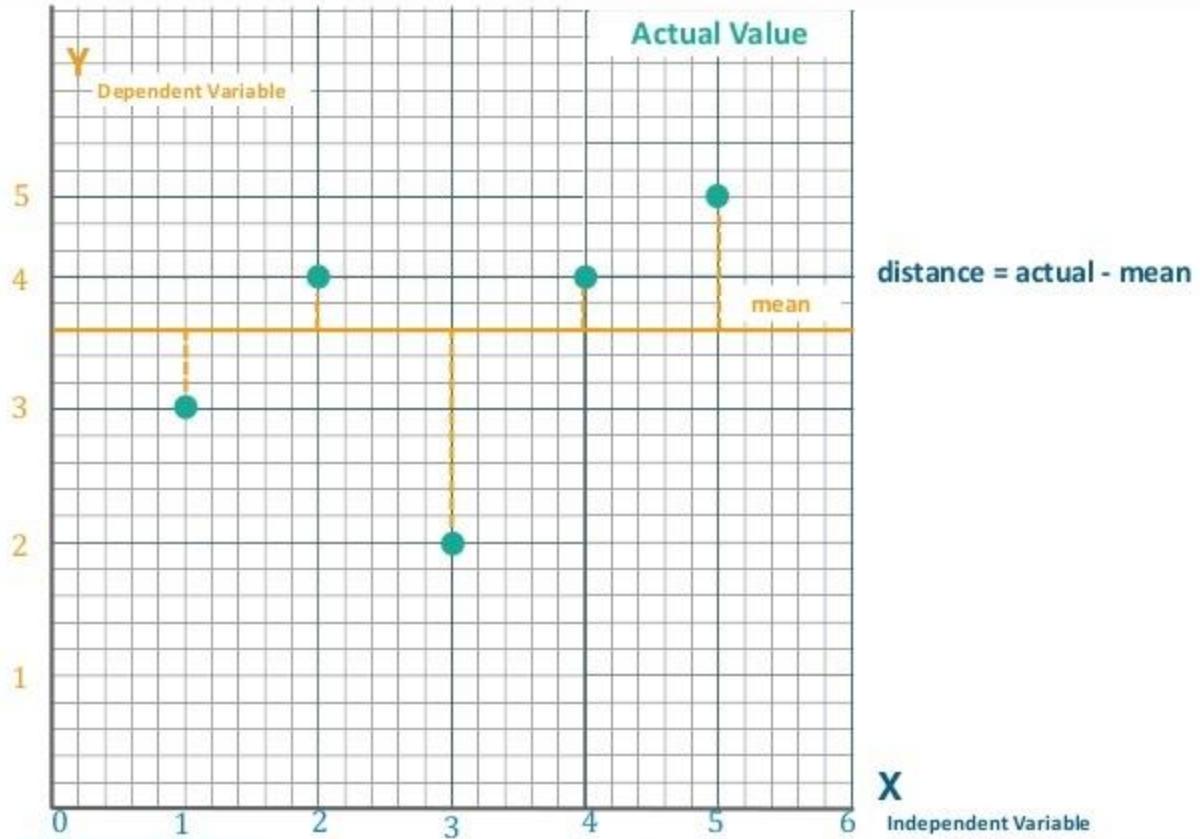
# What is R-Square?

- **R-squared** value is a statistical measure of how close the data are to the fitted regression line
- It is also known as **coefficient of determination**, or the **coefficient of multiple determination**

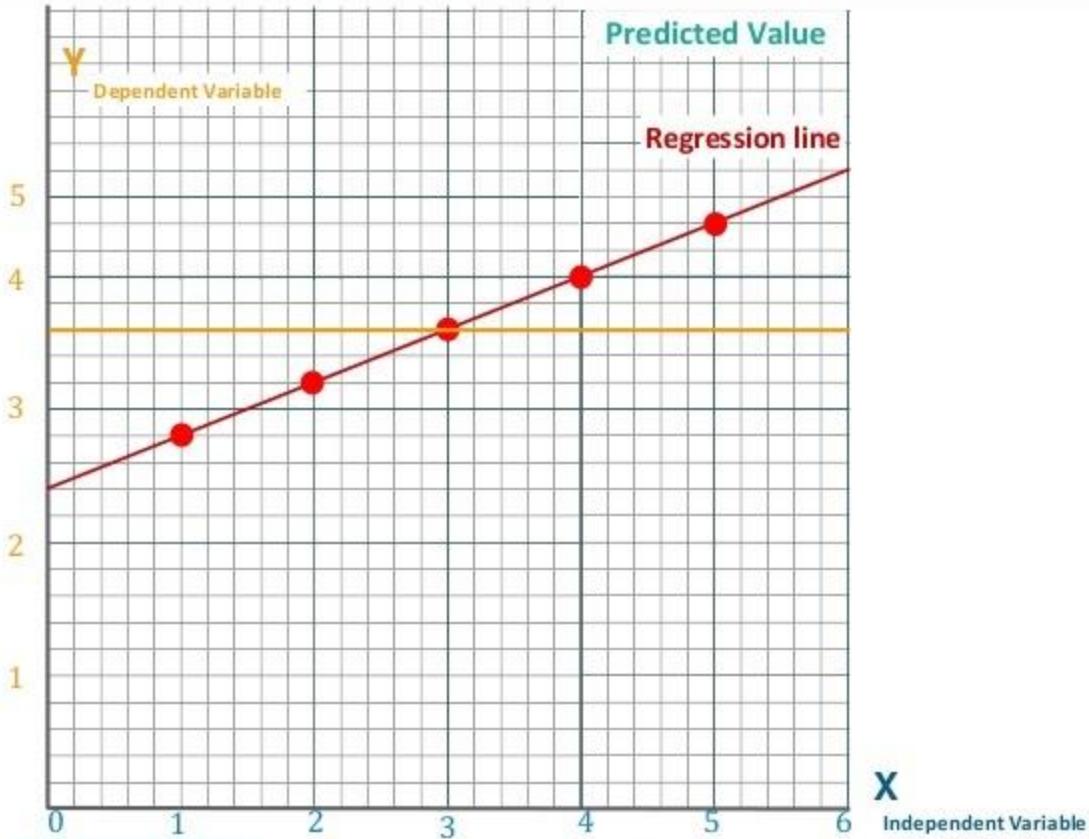
# Calculation of $R^2$



# Calculation of $R^2$

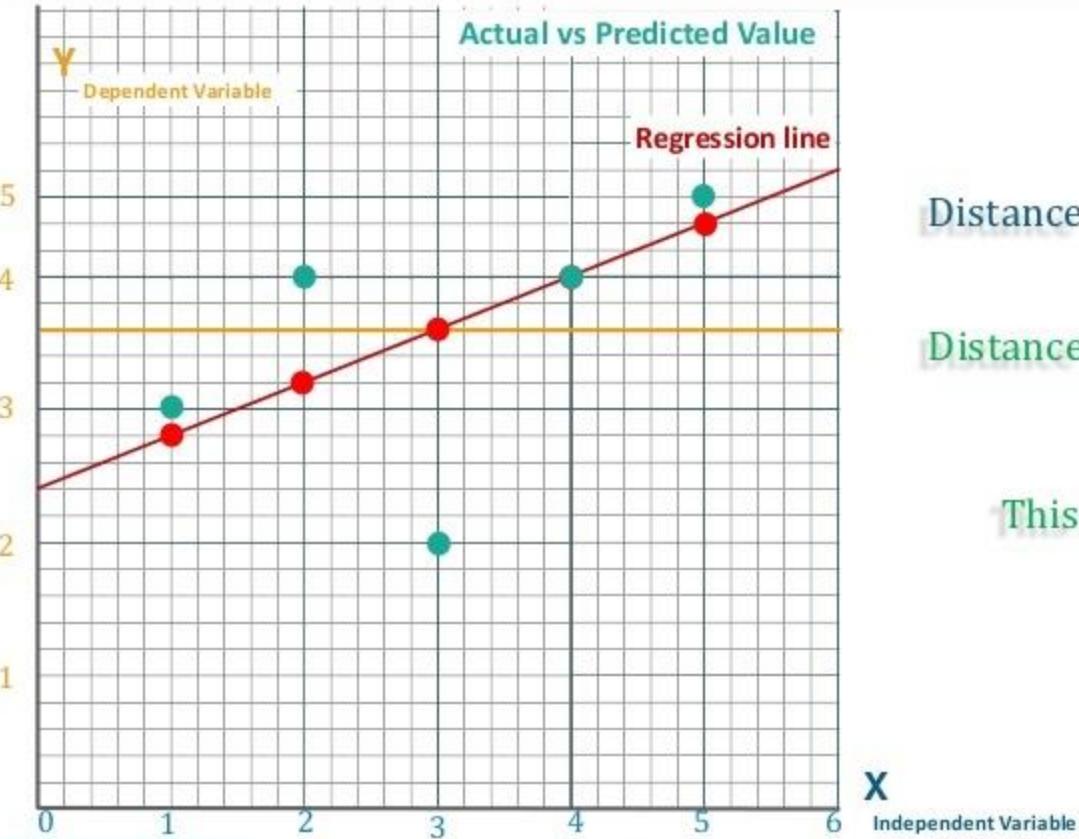


# Calculation of $R^2$



$x$	$y_p$
1	2.8
2	3.2
3	3.6
4	4.0
5	4.4

# Calculation of $R^2$



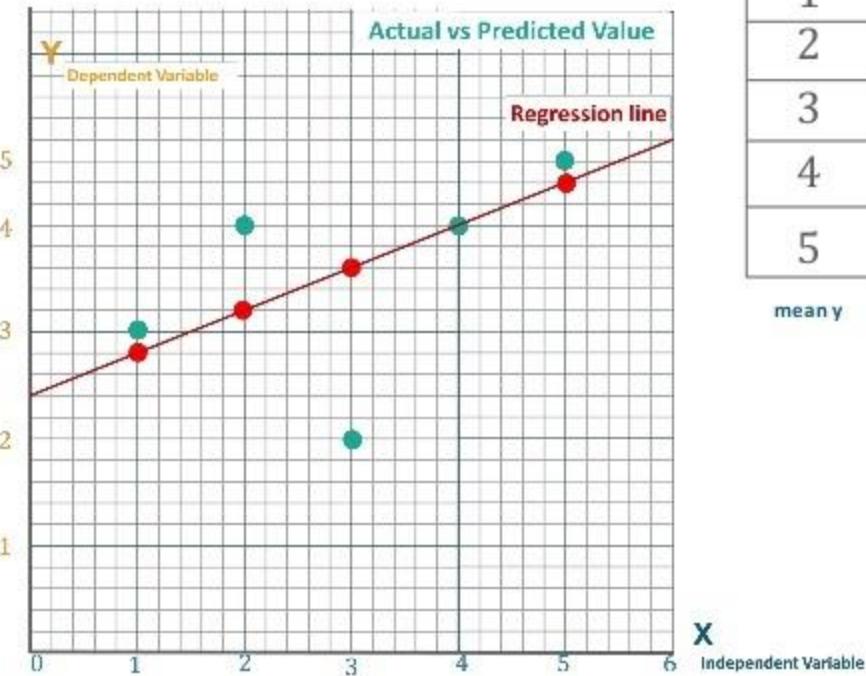
Distance actual - mean

vs

Distance predicted - mean

This is nothing but  $R^2 = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$

# Calculation of $R^2$

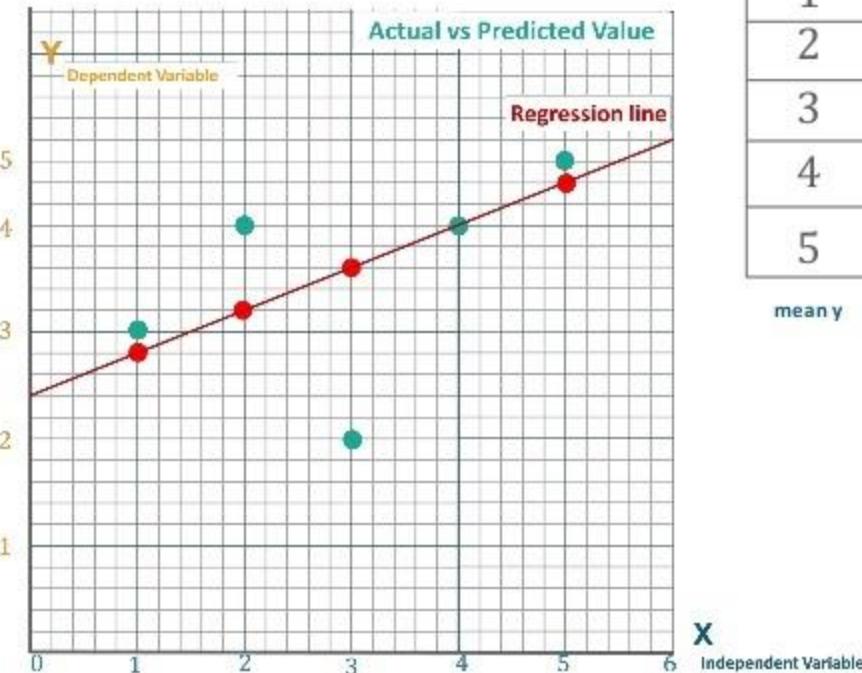


$x$	$y$	$y - \bar{y}$
1	3	- 0.6
2	4	0.4
3	2	-1.6
4	4	0.4
5	5	1.4

mean  $y$  3.6

$$R^2 = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$$

# Calculation of $R^2$

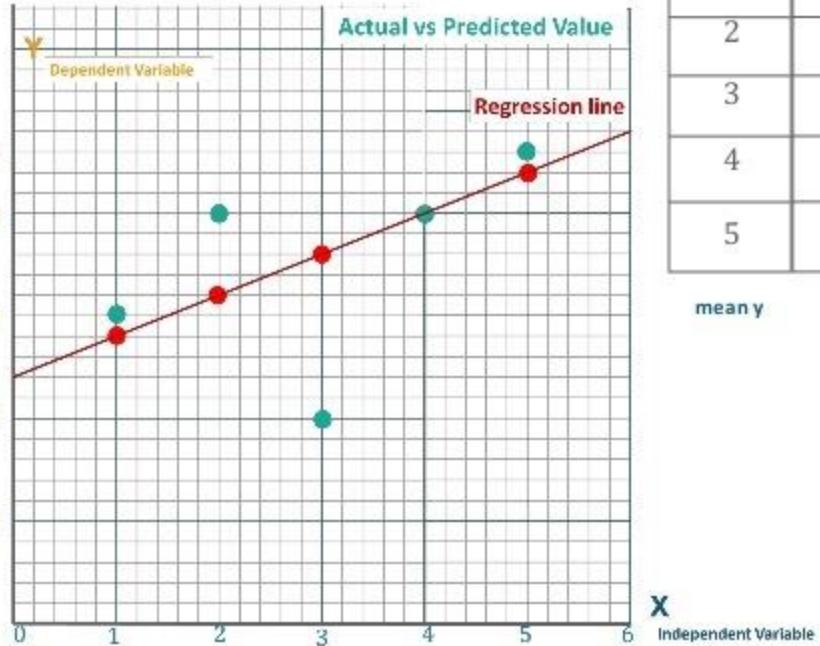


x	y	$y - \bar{y}$	$(y - \bar{y})^2$	$y_p$	$(y_p - \bar{y})$
1	3	-0.6	3.6	2.8	-0.8
2	4	0.4	1.6	3.2	-0.4
3	2	-1.6	2.56	3.6	0
4	4	0.4	1.6	4.0	0.4
5	5	1.4	1.96	4.4	0.8

mean y 3.6

$$R^2 = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$$

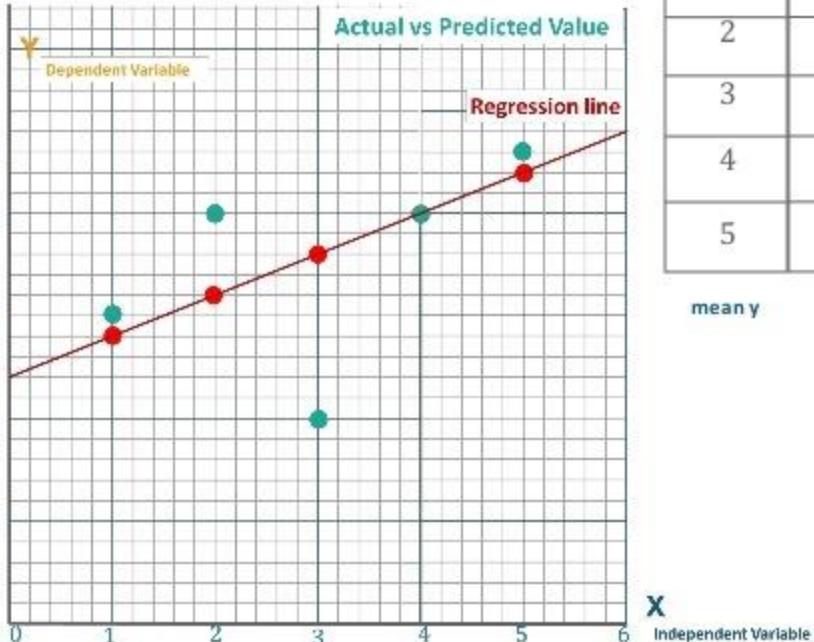
# Calculation of $R^2$



x	y	$y - \bar{y}$	$(y - \bar{y})^2$	$y_p$	$(y_p - \bar{y})$	$(y_p - \bar{y})^2$
1	3	-0.6	3.6	2.8	-0.8	6.4
2	4	0.4	1.6	3.2	-0.4	1.6
3	2	-1.6	2.56	3.6	0	0
4	4	0.4	1.6	4.0	0.4	1.6
5	5	1.4	1.96	4.4	0.8	6.4

$$R^2 = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$$

# Calculation of $R^2$

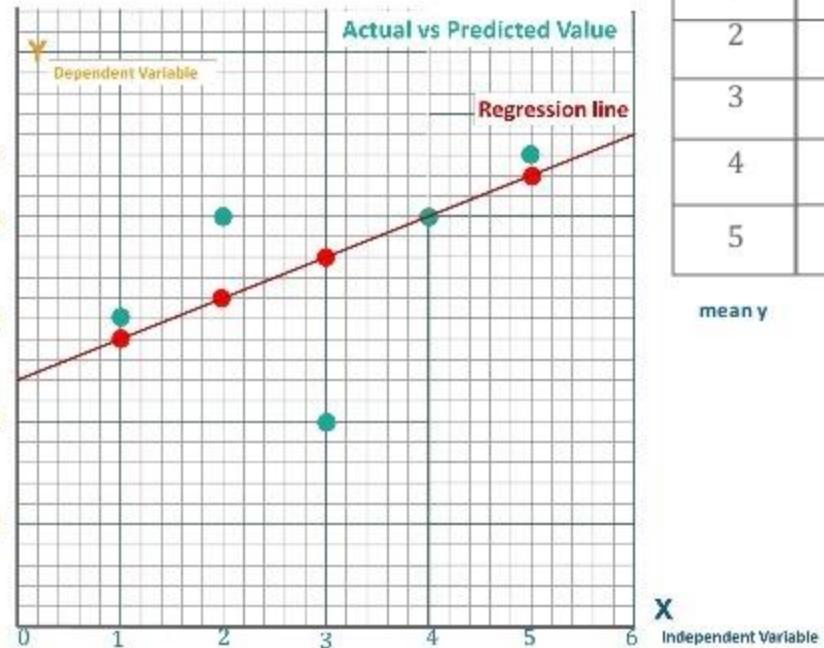


x	y	$y - \bar{y}$	$(y - \bar{y})^2$	$y_p$	$(y_p - \bar{y})$	$(y_p - \bar{y})^2$
1	3	-0.6	0.36	2.8	-0.8	0.64
2	4	0.4	0.16	3.2	-0.4	0.16
3	2	-1.6	2.56	3.6	0	0
4	4	0.4	0.16	4.0	0.4	0.16
5	5	1.4	1.96	4.4	0.8	0.64

mean y      3.6       $\sum$  5.2       $\sum$  1.6

$$R^2 = \frac{1.6}{5.2} = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$$

# Calculation of $R^2$

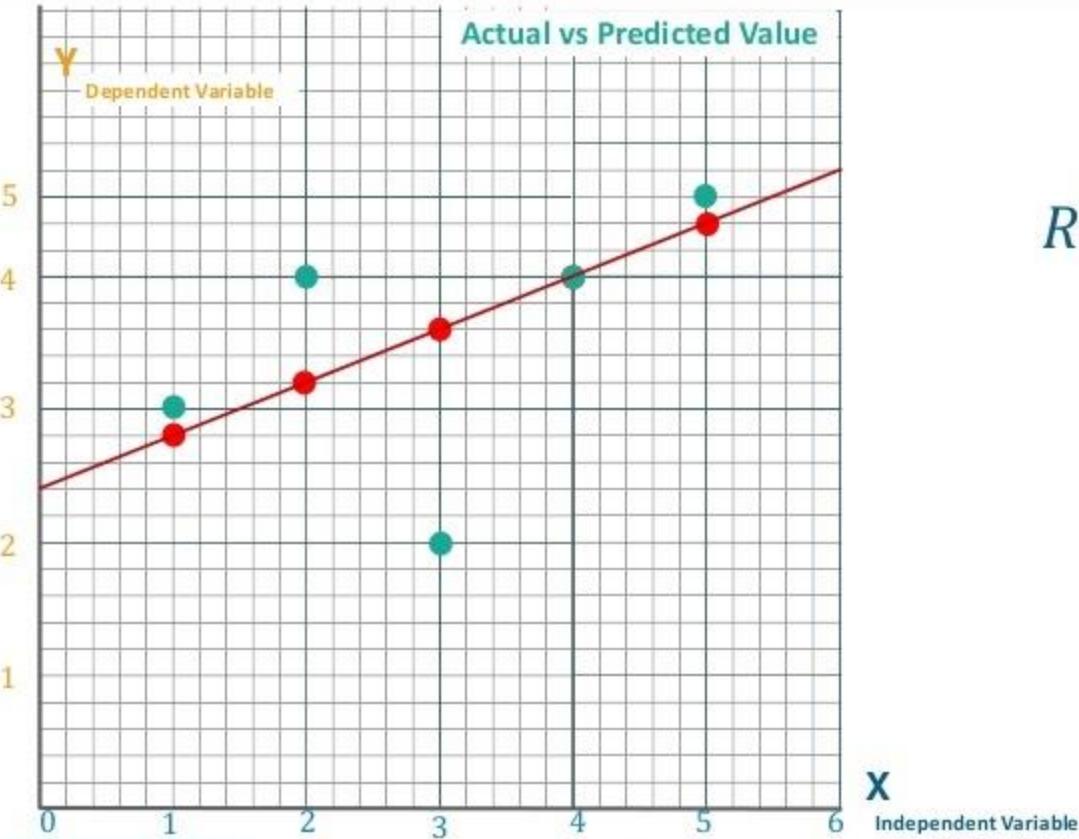


x	y	$y - \bar{y}$	$(y - \bar{y})^2$	$y_p$	$(y_p - \bar{y})$	$(y_p - \bar{y})^2$
1	3	-0.6	3.6	2.8	-0.8	6.4
2	4	0.4	1.6	3.2	-0.4	1.6
3	2	-1.6	2.56	3.6	0	0
4	4	0.4	1.6	4.0	0.4	1.6
5	5	1.4	1.96	4.4	0.8	6.4

mean y = 3.6       $\sum (y_p - \bar{y})^2 = 11.32$

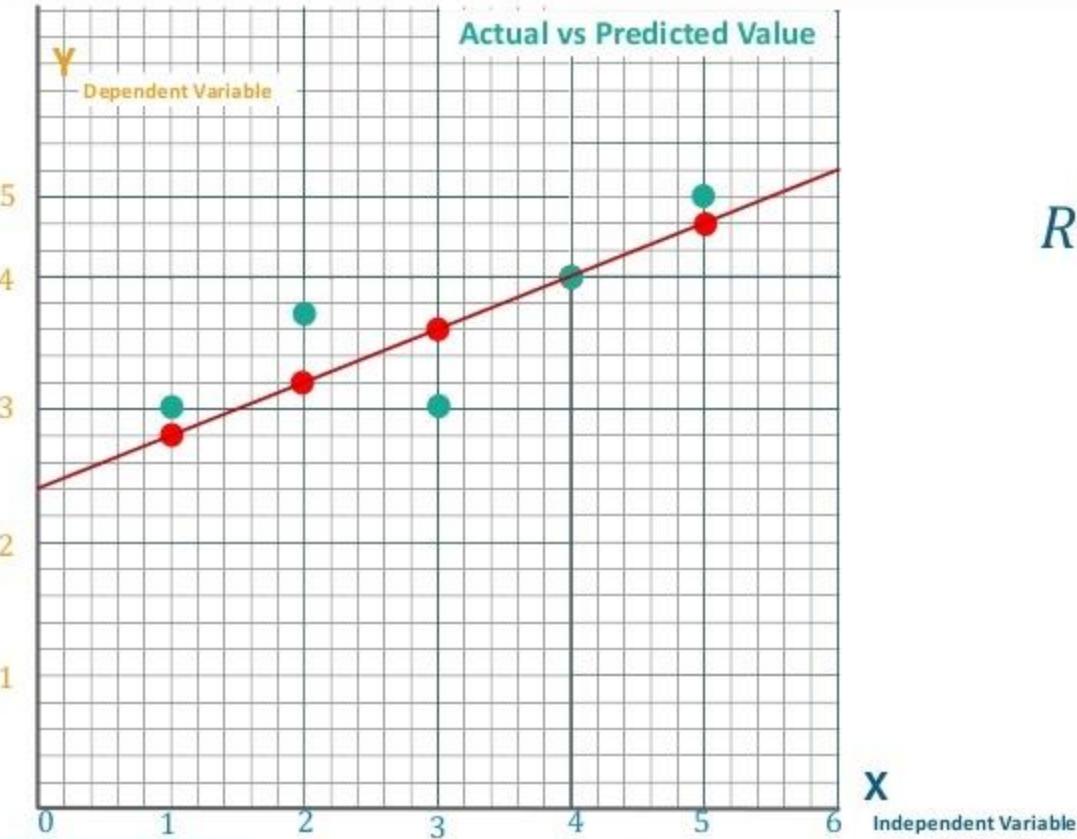
$$R^2 \approx 0.3$$

# Calculation of $R^2$



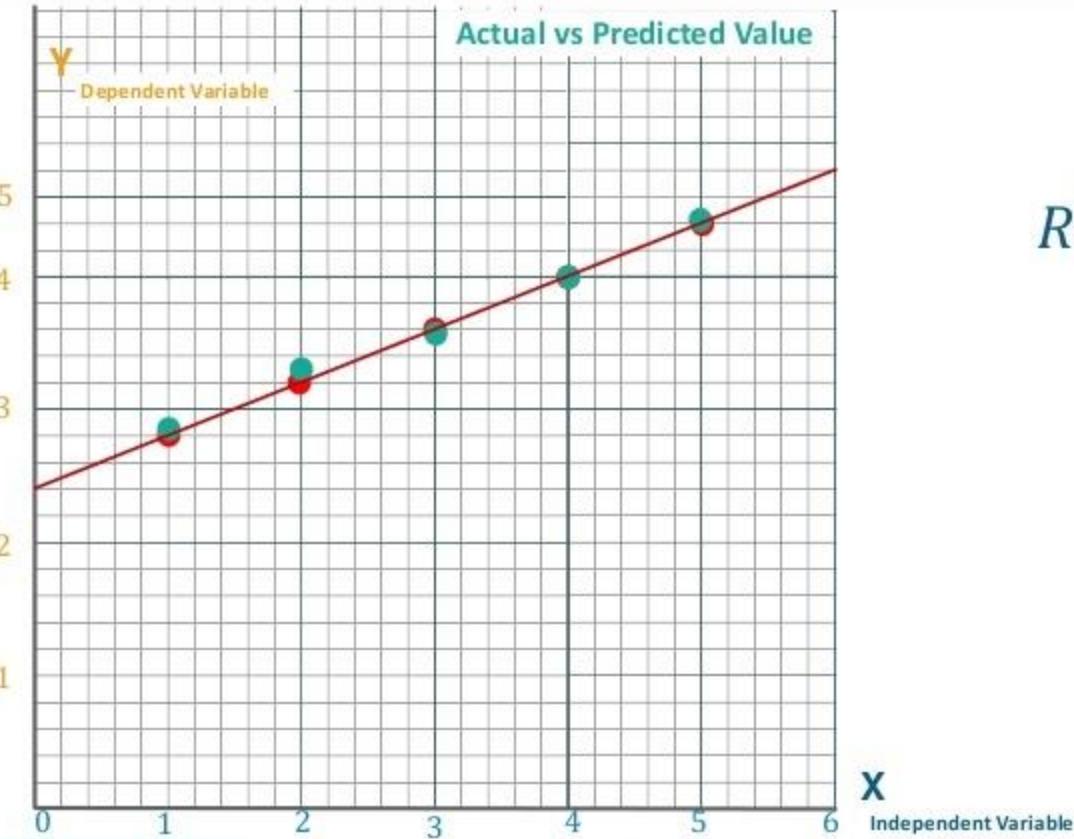
$$R^2 \approx 0.3$$

# Calculation of $R^2$



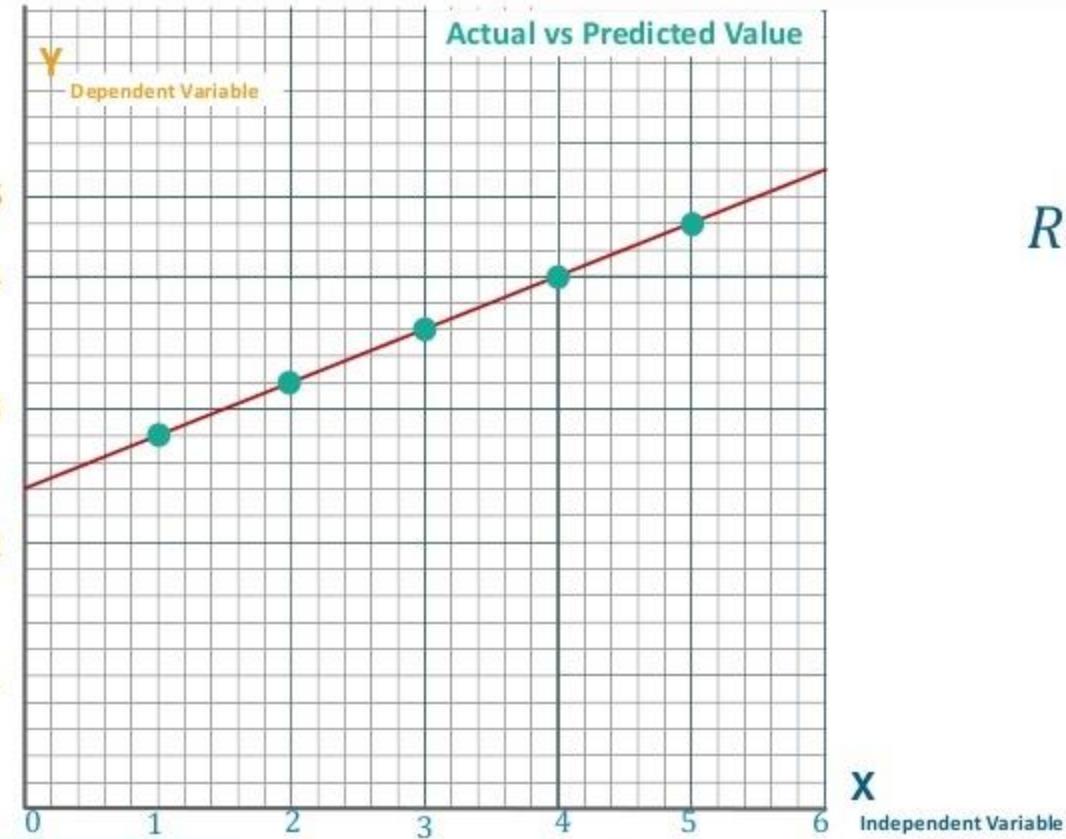
$$R^2 \approx 0.7$$

# Calculation of $R^2$



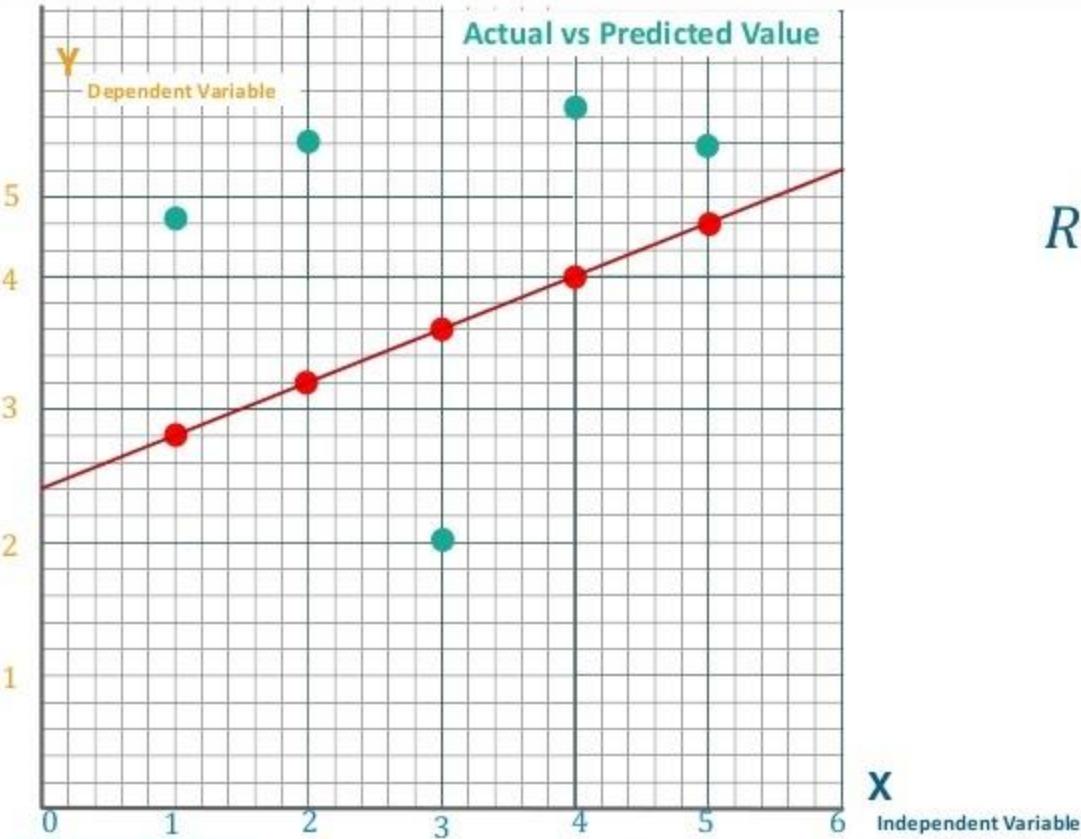
$$R^2 \approx 0.9$$

# Calculation of $R^2$



$$R^2 \approx 1$$

# Calculation of $R^2$



$$R^2 \approx 0.02$$

Are  
Low R-squared  
values always  
bad?



Are  
High R-squared  
values always  
good?



A stylized illustration of two hands, rendered in a teal color matching the background, are shown from the side and slightly from above, as if wearing gloves. They are positioned over a white computer keyboard, which is also outlined in black. The hands appear to be in the middle of typing or coding.

**Let's learn to  
code**

600,000+  
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