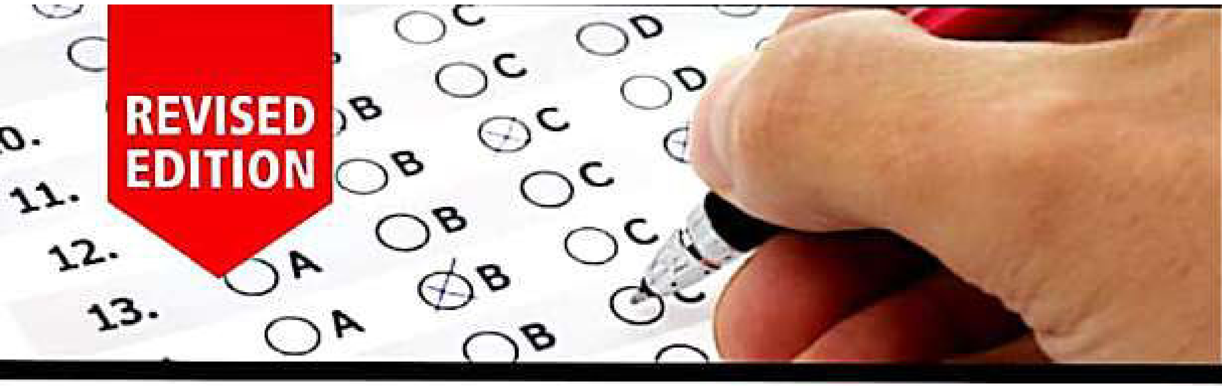

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With Latest Questions and Solutions

S. CHAND

Dr. R.S. AGGARWAL

Contents

1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I..Numeral : In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	**8**	**9**	**7**	**4**	**5**	**1**	**3**	**2**

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II Place Value or Local Value of a Digit in a Numeral :

In the above numeral :

Place value of 2 is $(2 \times 1) = 2$; Place value of 3 is $(3 \times 10) = 30$;

Place value of 1 is $(1 \times 100) = 100$ and so on.

Place value of 6 is $6 \times 10^8 = 600000000$

III.Face Value : The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV.TYPES OF NUMBERS

1.Natural Numbers : Counting numbers 1, 2, 3, 4, 5,..... are called ***natural*** *numbers.*

2.Whole Numbers : All counting numbers together with zero form the set of ***whole numbers***. Thus,

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whole number.

3.Integers : All natural numbers, 0 and negatives of counting numbers *i.e.*, {..., -3, -2, -1, 0, 1, 2, 3,.....} together form the set of integers.

(i) **Positive Integers** : {1, 2, 3, 4, ...} is the set of all positive integers.

(ii) **Negative Integers** : {- 1, - 2, - 3,.....} is the set of all negative integers.

(iii) **Non-Positive and Non-Negative Integers** : 0 is neither positive nor negative. So, {0, 1, 2, 3,....} represents the set of non-negative integers, while {0, -1, -2, -3, ...} represents the set of non-positive integers.

4. Even Numbers : A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8, 10, etc.

5. Odd Numbers : A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. Prime Numbers : A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :
Find a whole number nearly greater than the square root of p. Let k > *jp. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.
e.g,,We have to find whether 191 is a prime number or not. Now, 14 > V191.
Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.
191 is not divisible by any of them. So, 191 is a prime number.
7.Composite Numbers : Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.
Note : (i) 1 is neither prime nor composite.
(ii) 2 is the only even number which is prime.
(iii) There are 25 prime numbers between 1 and 100.
8. Co-primes : Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g., (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes,

V.TESTS OF DIVISIBILITY

1. Divisibility By 2 : A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.
Ex. 84932 is divisible by 2, while 65935 is not.
2. Divisibility By 3 : A number is divisible by 3, if the sum of its digits is divisible by 3.
Ex.592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.
But, 864329 is not divisible by 3, since sum of its digits =(8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.
3. Divisibility By 4 : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.
Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.
But, 749282 is not divisible by 4, since the number formed by the last tv/o digits is 82, which is not divisible by 4.
4. Divisibility By 5 : A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.
5. Divisibility By 6 : A number is divisible by 6, if it is divisible by both 2 and 3. Ex. The number 35256 is clearly divisible by 2.
Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3. Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.
6. Divisibility By 8 : A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.
Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.
But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.
7. Divisibility By 9 : A number is divisible by 9, if the sum of its digits is divisible by 9.
Ex. 60732 is divisible by 9, since sum of digits * (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.
But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is

not divisible by 9.

8. Divisibility By 10 : A number is divisible by 10, if it ends with 0.

Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. Divisibility By 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Ex. The number 4832718 is divisible by 11, since :

(sum of digits at odd places) - (sum of digits at even places)

$(8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

10. Divisibility By 12 ; A number is divisible by 12, if it is divisible by both 4 and 3.

Ex. Consider the number 34632.

(i) The number formed by last two digits is 32, which is divisible by 4,

(ii) Sum of digits = $(3 + 4 + 6 + 3 + 2) = 18$, which is divisible by 3. Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. Divisibility By 14 : A number is divisible by 14, if it is divisible by 2 as well as 7.

12. Divisibility By 15 : A number is divisible by 15, if it is divisible by both 3 and 5.

13. Divisibility By 16 : A number is divisible by 16, if the number formed by the last4 digits is divisible by 16.

Ex.7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. Divisibility By 24 : A given number is divisible by 24, if it is divisible by both3 and 8.

15. Divisibility By 40 : A given number is divisible by 40, if it is divisible by both 5 and 8.

16. Divisibility By 80 : A given number is divisible by 80, if it is divisible by both 5 and 16.

Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

If p arid q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4x6) = 24$, since 4 and 6 are not co-primes.

VI MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :

(i) $a \times (b + c) = a \times b + a \times c$ (ii) $ax(b-c) = a \times b-a \times c$.

Ex. (i) $567958 \times 99999 = 567958 \times (100000 - 1)$

$= 567958 \times 100000 - 567958 \times 1 = (56795800000 - 567958) = 56795232042$. (ii) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000$.

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n

Ex. 975436 x 625 = 975436 x 5^4 = $\frac{9754360000}{16}$ = 609647600

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a^2 - b^2) = (a + b)(a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor x Quotient) + Remainder

IX. (i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a progression.

1. Arithmetic Progression (A.P.) : If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference* of the A.P.

An A.P. with first term a and common difference d is given by a, (a + d), (a + 2d),(a + 3d),.....

The **nth term of this A.P. is given by T_n =a (n - 1) d.**

The sum of n terms of this A.P.

$S_n = n/2\ [2a + (n - 1)d] = n/2$ (first term + last term).

SOME IMPORTANT RESULTS :

(i) $(1 + 2 + 3 + \ldots + n) = n(n+1)/2$

(ii) $(1^2 + 2^2 + 3^2 + \ldots + n^2) = n(n+1)(2n+1)/6$

(iii) $(1^3 + 2^3 + 3^3 + \ldots + n^3) = n^2(n+1)^2$

2. Geometrical Progression (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a geometrical progression.

The constant ratio is called the common ratio of the G.P. A G.P. with first term a and common ratio r is :

$a, ar, ar^2,$

In this G.P. $T_n = ar^{n-1}$

sum of the n terms, Sn= $\frac{a(1-r^n)}{(1-r)}$

SOLVED EXAMPLES

Ex. 1. Simplify : (i) 8888 + 888 + 88 + 8
(ii) 11992 - 7823 - 456

Sol. i)
```
   8888
    888
     88
+     8
   9872
```
ii) 11992 - 7823 - 456 = 11992 - (7823 + 456)
= 11992 - 8279 = 3713-
```
  7823        11992
+  456      -  8279
  8279         3713
```

Ex. 2, What value will replace the question mark in each of the following equations ?
(i) ? - 1936248 = 1635773 (ii) 8597 - ? = 7429 - 4358

Sol. (i) Let x - 1936248=1635773.Then, x = 1635773 + 1936248=3572021.
(ii) Let 8597 - x = 7429 - 4358.
Then, x = (8597 + 4358) - 7429 = 12955 - 7429 = 5526.

Ex. 3. What could be the maximum value of Q in the following equation? 5P9 + 3R7 + 2Q8 = 1114

Sol. We may analyse the given equation as shown :
Clearly, 2 + P + R + Q = ll.
So, the maximum value of Q can be
(11 - 2) i.e., 9 (when P = 0, R = 0);
```
  1 2
  5 P 9
    3 R 7
  2 Q 8
 11 1 4
```

Ex. 4. Simplify : (i) 5793405 x 9999 (ii) 839478 x 625
Sol.
i)5793405x9999=5793405(10000-1)=57934050000-5793405=57928256595.b

ii) 839478 x 625 = 839478 x 5^4 = $\frac{8394780000}{16}$ = 524673750.

Ex. 5. Evaluate : (i) 986 x 237 + 986 x 863 (ii) 983 x 207 - 983 x 107
Sol.
(i) 986 x 137 + 986 x 863 = 986 x (137 + 863) = 986 x 1000 = 986000.
(ii) 983 x 207 - 983 x 107 = 983 x (207 - 107) = 983 x 100 = 98300.

Ex. 6. Simplify : (i) 1605 x 1605 ii) 1398 x 1398
Sol.
i) 1605 x 1605 = $(1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5$
= 2560000 + 25 + 16000 = 2576025.
(ii) 1398 x 1398 - $(1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2$
=1960000 + 4 - 5600 = 1954404.

Ex. 7. Evaluate : (313 x 313 + 287 x 287).

Sol.

$(a^2 + b^2) = 1/2\ [(a + b)^2 + (a - b)^2]$

$(313)^2 + (287)^2 = 1/2\ [(313 + 287)^2 + (313 - 287)^2] = ½[(600)^2 + (26)^2]$

$= 1/2\ (360000 + 676) = 180338.$

Ex. 8. Which of the following are prime numbers ?

(i) 241 (ii) 337 (Hi) 391 (iv) 571

Sol.

(i) Clearly, 16 > Ö241. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.
241 is not divisible by any one of them.
241 is a prime number.

(ii) Clearly, 19>Ö337. Prime numbers less than 19 are 2, 3, 5, 7, 11,13,17.
337 is not divisible by any one of them.
337 is a prime number.

(iii) Clearly, 20 > Ö391". Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
We find that 391 is divisible by 17.
391 is not prime.

(iv) Clearly, 24 > Ö57T. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
571 is not divisible by any one of them.
571 is a prime number.

Ex. 9. Find the unit's digit in the product (2467)163 x (341)72.

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.

Now, 74 gives unit digit 1.

7^{152} gives unit digit 1,

$\therefore$ 7^{153} gives unit digit (1 x 7) = 7. Also, 1^{72} gives unit digit 1.

Hence, unit's digit in the product = (7 x 1) = 7.

Ex. 10. Find the unit's digit in $(264)^{102} + (264)^{103}$

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$

Now, 4^2 gives unit digit 6.

$\therefore (4)^{102}$ gives unjt digit 6.

$\therefore$(4)103 gives unit digit of the product (6 x 4) i.e., 4.

Hence, unit's digit in (264)m + (264)103 = unit's digit in (6 + 4) = 0.

Ex. 11. Find the total number of prime factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$.

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{11} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$

Total number of prime factors = (22 + 5 + 2) = 29.

Ex.12. Simplify : (i) 896 x 896 - 204 x 204

(ii) 387 x 387 + 114 x 114 + 2 x 387 x 114
(iii) 81 X 81 + 68 X 68-2 x 81 X 68.

Sol.

(i) Given exp = $(896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(ii) Given exp = $(387)^2 + (114)^2 + (2 \times 387 \times 114)$
$= a^2 + b^2 + 2ab$, where $a = 387, b=114$
$= (a+b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(iii) Given exp = $(81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, Where $a = 81, b=68$
$= (a-b)^2 = (81 - 68)^2 = (13)^{2=169}$

Ex.13. Which of the following numbers is divisible by 3 ?
(i) 541326 (ii) 5967013

Sol.

(i) Sum of digits in 541326 = (5 + 4 + 1 + 3 + 2 + 6) = 21, which is divisible by 3.
Hence, 541326 is divisible by 3.

(ii) Sum of digits in 5967013 =(5+9 + 6 + 7 + 0+1 +3) = 31, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Ex.14.What least value must be assigned to * so that the number 197*5462 is r 9 ?

Sol.

Let the missing digit be x.
Sum of digits = (1 + 9 + 7 + x + 5 + 4 + 6 +»2) = (34 + x).
For (34 + x) to be divisible by 9, x must be replaced by 2 .
Hence, the digit in place of * must be 2.

Ex. 15. Which of the following numbers is divisible by 4 ?
(i) 67920594 (ii) 618703572

Sol.

(i) The number formed by the last two digits in the given number is 94, which is not divisible by 4.
Hence, 67920594 is not divisible by 4.

(ii) The number formed by the last two digits in the given number is 72, which is divisible by 4.
Hence, 618703572 is divisible by 4.

Ex. 16. Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5 ?

Sol.
Since the given number is divisible by 5, so 0 or 5 must come in place of \$. But, a number ending with 5 is never divisible by 8. So, 0 will replace \$.
Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.
Hence, digits in place of * and \$ are 4 and 0 respectively.

Ex. 17. Show that 4832718 is divisible by 11.
Sol. (Sum of digits at odd places) - (Sum of digits at even places)
$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.
Hence, 4832718 is divisible by 11.

Ex. 18. Is 52563744 divisible by 24 ?
Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.
The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.
The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.
Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3×8, i.e., 24.

Ex. 19. What least number must be added to 3000 to obtain a number exactly divisible by 19 ?
Sol. On dividing 3000 by 19, we get 17 as remainder.
$\therefore$ Number to be added $= (19 - 17) = 2$.

Ex. 20. What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?
Sol. On dividing 2000 by 17, we get 11 as remainder.
$\therefore$ Required number to be subtracted = 11.

Ex. 21. Find the number which is nearest to 3105 and is exactly divisible by 21.
Sol. On dividing 3105 by 21, we get 18 as remainder.
$\therefore$ Number to be added to 3105 = (21 - 18) - 3.
Hence, required number $= 3105 + 3 = 3108$.

Ex. 22. Find the smallest number of 6 digits which is exactly divisible by 111.
Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

∴ Number to be added = (111 - 100) - 11.

Hence, required number = 100011.-

Ex. 23. On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.

Sol. $$\text{Divisor} = \frac{\text{Dividend - Remainder}}{\text{.Quotient}} = \frac{15968-37}{89} = 179.$$

Ex. 24. A number when divided by 342 gives a remainder 47. When the same number ift divided by 19, what would be the remainder ?

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number – 342k + 47 = (19 x 18k + 19 x 2 + 9) = 19 (18k + 2) + 9.

∴ The given number when divided by 19, gives (18k + 2) as quotient and 9 as remainder.

Ex. 25. A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed,

Sol.

3	X	
5	y	- 1
8	z	- 4
	1	- 7

∴ z = (8 x 1 + 7) = 15; y = {5z + 4) = (5 x 15 + 4) = 79; x = (3y + 1) = (3 x 79 + 1) = 238.

Now,

8	238	
5	29	- 6
3	5	- 4
	1	- 9,

∴ Respective remainders are 6, 4, 2.

Ex. 26. Find the remainder when 2^{31} is divided by 5.

Sol. 2^{10} = 1024. Unit digit of 2^{10} x 2^{10} x 2^{10} is 4 [as 4 x 4 x 4 gives unit digit 4].

∴ Unit digit of 231 is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 231 when divided by 5, gives 3 as remainder.

Ex. 27. How many numbers between 11 and 90 are divisible by 7 ?

Sol. The required numbers are 14, 21, 28, 35, ... 77, 84.

This is an A.P. with $a = 14$ and $d = (21 - 14) = 7$.

Let it contain n terms.

Then, $T_n = 84 \Rightarrow a + (n - 1)\,d = 84$

$\Rightarrow 14 + (n - 1) \times 7 = 84$ or $n = 11$.

$\therefore$ Required number of terms = 11.

Ex. 28. Find the sum of all odd numbers upto 100.

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with $a = 1$ and $d = 2$.

Let it contain n terms. Then,

$1 + (n - 1) \times 2 = 99$ or $n = 50$.

$\therefore$ Required sum $= \frac{n}{2}$ (first term + last term)

$= \frac{50}{2}(1 + 99) = 2500.$

Ex. 29. Find the sum of all 2 digit numbers divisible by 3.

Sol. All 2 digit numbers divisible by 3 are :

12, 51, 18, 21, ..., 99.

This is an A.P. with $a = 12$ and $d = 3$.

Let it contain n terms. Then,

$12 + (n - 1) \times 3 = 99$ or $n = 30$.

$\therefore$ Required sum $= \frac{30}{2} \times (12+99) = 1665.$

Ex.30.How many terms are there in 2,4,8,16......1024?

Sol. Clearly 2,4,8,16........1024 form a GP. With a=2 and $r = 4/2 = 2$.

Let the number of terms be n . Then

$2 \times 2^{n-1} = 1024$ or $2^{n-1} = 512 = 2^9$.

$\therefore n-1=9$ or $n=10$.

Ex. 31. $2 + 2^2 + 2^3 + ... + 2^8 = ?$

Sol. Given series is a G.P. with $a = 2$, $r = 2$ and $n = 8$.

$\therefore$ sum $= \frac{a(r^n-1)}{(r-1)} = \frac{2 \times (2^8-1)}{(2-1)} = (2 \times 255) = 510$

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. Factors and Multiples : If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a.

II. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.): The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are *two methods* of finding the H.C.F. of a given set of numbers :

1. Factorization Method : Express each one of the given numbers as the product of prime factors.The product of least powers of common prime factors gives H.C.F.

2. Division Method: Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. Least Common Multiple (L.C.M.) : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. Factorization Method of Finding L.C.M.: Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors,

2. Common Division Method {Short-cut Method) of Finding L.C.M.: Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers,

IV. Product of two numbers =Product of their H.C.F. and L.C.M.

V. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

VI. H.C.F. and L.C.M. of Fractions:

1. $\text{H C F} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2. $\text{L C M} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

VII. H.C.F. and L.C.M. of Decimal Fractions: In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

VIII. Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

SOLVED EXAMPLES

Ex. 1. Find the H.C.F. of $2^3 \text{ X } 3^2 \text{ X } 5 \text{ X } 7^4$, $2^2 \text{ X } 3^5 \text{ X } 5^2 \text{ X } 7^3$, $2^3 \text{ X } 5^3 \text{ X } 7^2$

Sol. The prime numbers common to given numbers are 2,5 and 7.

H.C.F. = $2^2 \times 5 \times 7^2 = 980$.

Ex. 2. Find the H.C.F. of 108, 288 and 360.

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$.

H.C.F. = $2^2 \times 3^2 = 36$.

Ex. 3. Find the H.C.F. of 513, 1134 and 1215.

Sol.

```
1134 ) 1215 ( 1
        1134
       ------
          81 ) 1134 ( 14
                 81
                324
                324
                 x
```

∴ H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

```
81 ) 513 ( 6
     486
     27) 81 ( 3
         81
          0
```

H.C.F. of given numbers = 27.

Ex. 4. Reduce $\frac{391}{667}$ to lowest terms.

to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}$

Ex.5.Find the L.C.M. of $2^2 \times 3^3 \times 5 \times 7^2 \cdot 2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^{3 \times 7 \times 11}$.

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = $2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$

Ex.6. Find the L.C.M. of 72, 108 and 2100.

Sol. $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$.

L.C.M. = $2^3 \times 3^3 \times 5^2 \times 7 = 37800$.

Ex.7.Find the L.C.M. of 16, 24, 36 and 54.

Sol.

2	16	24	36	54
2	8	12	18	27
2	4	6	9	27
3	2	3	9	27
3	2	1	3	9
	2	1	1	3

$\therefore$ L.C.M. = 2 x 2 x 2 x 3 x 3 x 2 x 3 = 432.

Ex. 8. Find the H.C.F. and L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Sol. H.C.F. of given fractions = $\frac{\text{H.C.F. of } 2,8,16,10}{\text{L.C.M. of } 3,9,81,27} = \frac{2}{81}$

L.C.M of given fractions = $\frac{\text{L.C.M. of } 2,8,16,10}{\text{H.C.F. of } 3,9,81,27} = \frac{80}{3}$

Ex. 9. Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, find the numbers.

Sol. Let the required numbers be 15.x and llx.

Then, their H.C.F. is x. So, x = 13.

The numbers are (15 x 13 and 11 x 13) i.e., 195 and 143.

Ex. 11. TheH.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77,find the other.

Sol. Other number = $\frac{11 \times 693}{77} = 99$

Ex. 12. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

$\therefore$ H.C.F. = 32 x 5 = 45.

Hence, required length = 45 cm.

Ex. 13. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Sol. Required number = H.C.F. of (1657 - 6) and (2037 - 5) = H.C.F. of 1651 and 2032

```
1651 ) 2032 ( 1 1651
       1651
        381 ) 1651 ( 4
              1524
               127 )  381 ( 3
                      381
                        0
```

Required number = 127.

Ex. 14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Sol. Required number = H.C.F. of (132 - 62), (237 - 132) and (237 - 62)

= H.C.F. of 70, 105 and 175 = 35.

Ex.15.Find the least number exactly divisible by 12,15,20,27.

Sol.

3	12	15	20	27
4	4	5	20	9
5	1	5	5	9
	1	1	1	9

Ex.16.Find the least number which when divided by 6,7,8,9, and 12 leave the same remainder 1 each case

Sol. Required number = (L.C.M OF 6,7,8,9,12) + 1

3	6 - 7 - 8 - 9 - 12
4	2 - 7 - 8 - 3 - 4
5	1 - 7 - 4 - 3 - 2
	1 - 7 - 2 - 3 - 1

∴L.C.M = 3 X 2 X 2 X 7 X 2 X 3 = 504.
Hence required number = (504 +1) = 505.

Ex.17. Find the largest number of four digits exactly divisible by 12,15,18 and 27.
Sol. The Largest number of four digits is 9999.
Required number must be divisible by L.C.M. of 12,15,18,27 i.e. 540.
On dividing 9999 by 540,we get 279 as remainder .
∴Required number = (9999-279) = 9720.

Ex.18.Find the smallest number of five digits exactly divisible by 16,24,36 and 54.
Sol. Smallest number of five digits is 10000.
Required number must be divisible by L.C.M. of 16,24,36,54 i.e 432,
On dividing 10000 by 432,we get 64 as remainder.
∴Required number = 10000 +(432 – 64) = 10368.

Ex.19.Find the least number which when divided by 20,25,35 and 40 leaves remainders 14,19,29 and 34 respectively.
Sol. Here,(20-14) = 6,(25 – 19)=6,(35-29)=6 and (40-34)=6.
∴Required number = (L.C.M. of 20,25,35,40) – 6 =1394.

Ex.20.Find the least number which when divided by 5,6,7, and 8 leaves a remainder 3, but when divided by 9 leaves no remainder .
Sol. L.C.M. of 5,6,7,8 = 840.
∴ Required number is of the form 840k + 3
Least value of k for which (840k + 3) is divisible by 9 is k = 2.
∴Required number = (840 X 2 + 3)=1683

Ex.21.The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec.respectively .If they all change simultaneously at 8:20:00 hours,then at what time they again change simultaneously .
Sol. Interval of change = (L.C.M of 48,72,108)sec.=432sec.
So, the lights will agin change simultaneously after every 432 seconds i.e,7 min.12sec
Hence , next simultaneous change will take place at 8:27:12 hrs.

Ex.22.Arrange the fractions $\frac{17}{18}$, $\frac{31}{36}$, $\frac{43}{45}$, $\frac{59}{60}$ in the ascending order.
Sol.L.C.M. of 18,36,45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}$;

Since, 155<170<172<177, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$

3. DECIMAL FRACTIONS

IMPORTANT FACTS AND FORMULAE

I. Decimal Fractions : Fractions in which denominators are powers of 10 are known as **decimal fractions.**

Thus ,1/10=1 tenth=.1;1/100=1 hundredth =.01;

99/100=99 hundreths=.99;7/1000=7 thousandths=.007,etc

II. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

Thus, 0.25=25/100=1/4;2.008=2008/1000=251/125.

III. 1. Annexing zeros to the extreme right of a decimal fraction does not change its value
Thus, 0.8 = 0.80 = 0.800, etc.

2. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.
Thus, 1.84/2.99 = 184/299 = 8/13; 0.365/0.584 = 365/584=5

IV. Operations on Decimal Fractions :

1. Addition and Subtraction of Decimal Fractions : The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. Multiplication of a Decimal Fraction By a Power of 10 : Shift the decimal point to the right by as many places as is the power of 10.
Thus, 5.9632 x 100 = 596,32; 0.073 x 10000 = 0.0730 x 10000 = 730.

3.**Multiplication of Decimal Fractions** : Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.
Suppose we have to find the product (.2 x .02 x .002). Now, 2x2x2 = 8. Sum of decimal places = (1 + 2 + 3) = 6. .2 x .02 x .002 = .000008.

4.**Dividing a Decimal Fraction By a Counting Number :** Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.
Suppose we have to find the quotient (0.0204 + 17). Now, 204 ^ 17 = 12. Dividend contains 4 places of decimal. So, 0.0204 + 17 = 0.0012.

5. **Dividing a Decimal Fraction By a Decimal Fraction :** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, 0.00066/0.11 = (0.00066*100)/(0.11*100) = (0.066/11) = 0.006V

V. Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions 3/5, 6/7 and 7/9 in descending order.

now, 3/5=0.6,6/7 = 0.857,7/9 = 0.777....

since 0.857>0.777...>0.6, so 6/7>7/9>3/5

VI. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a *recurring decimal.*

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set

Thus 1/3 = 0.3333….= 0.3; 22 /7 = 3.142857142857.....= $3.\overline{142857}$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

thus ,0.5 = 5/9; 0.53 = 53/59 ;0.067 = 67/999;etc...

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., 0.17333.= 0.173.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated, In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus 0.16 = (16-1) / 90 = 15/19 = 1/6;

$0.22\overline{73}$ = (2273 – 22)/9900 = 2251/9900