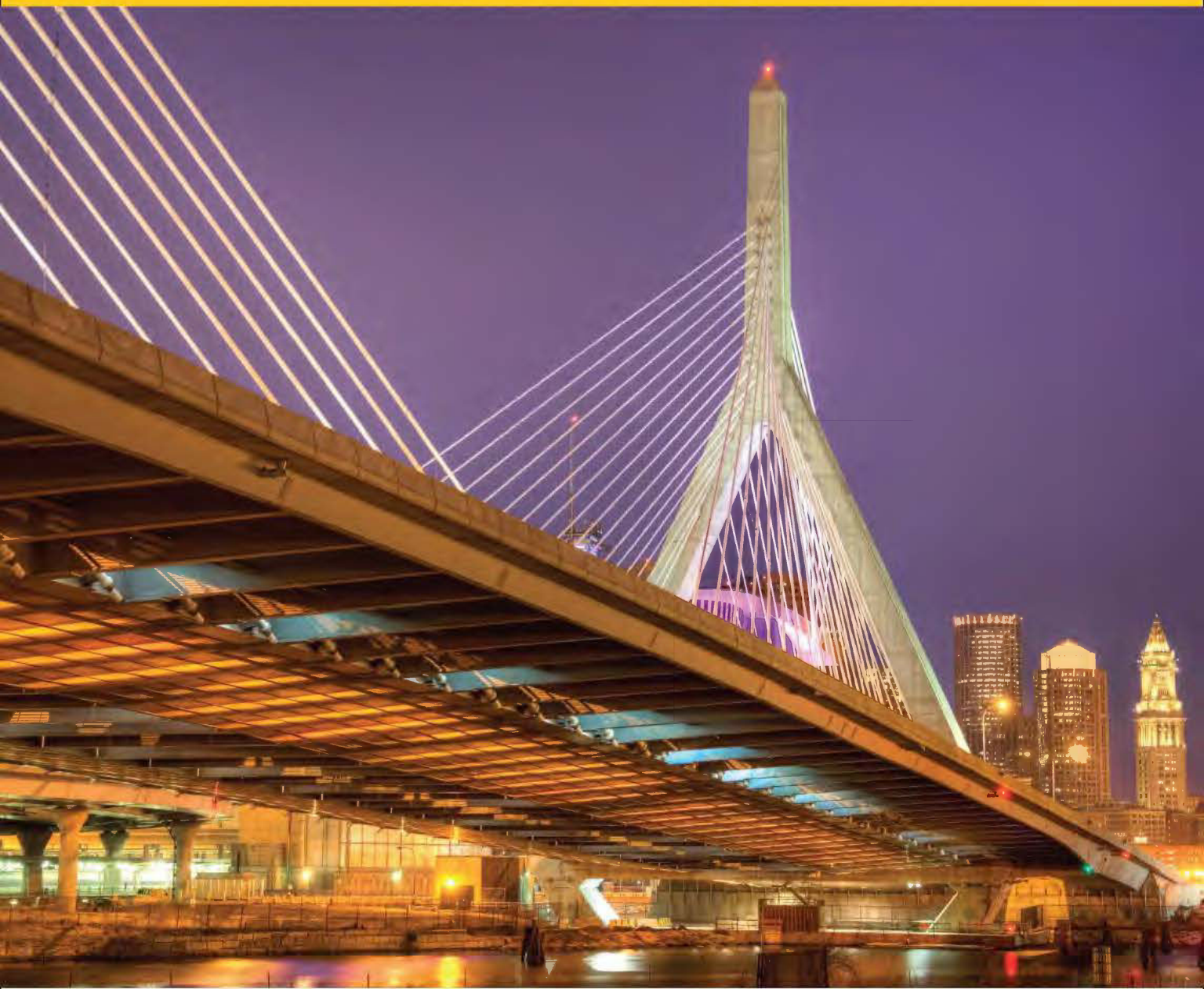


Tenth Edition



ERWIN KREYSZIG
ADVANCED ENGINEERING
MATHEMATICS

Systems of Units. Some Important Conversion Factors

The most important systems of units are shown in the table below. The mks system is also known as the *International System of Units* (abbreviated *SI*), and the abbreviations sec (instead of s), gm (instead of g), and nt (instead of N) are also used.

System of units	Length	Mass	Time	Force
cgs system	centimeter (cm)	gram (g)	second (s)	dyne
mks system	meter (m)	kilogram (kg)	second (s)	newton (nt)
Engineering system	foot (ft)	slug	second (s)	pound (lb)

$$1 \text{ inch (in.)} = 2.540000 \text{ cm}$$

$$1 \text{ foot (ft)} = 12 \text{ in.} = 30.480000 \text{ cm}$$

$$1 \text{ yard (yd)} = 3 \text{ ft} = 91.440000 \text{ cm}$$

$$1 \text{ statute mile (mi)} = 5280 \text{ ft} = 1.609344 \text{ km}$$

$$1 \text{ nautical mile} = 6080 \text{ ft} = 1.853184 \text{ km}$$

$$1 \text{ acre} = 4840 \text{ yd}^2 = 4046.8564 \text{ m}^2$$

$$1 \text{ mi}^2 = 640 \text{ acres} = 2.5899881 \text{ km}^2$$

$$1 \text{ fluid ounce} = 1.28 \text{ U.S. gallon} = 231 \text{ in.}^3 = 29.573730 \text{ cm}^3$$

$$1 \text{ U.S. gallon} = 4 \text{ quarts (liq)} = 8 \text{ pints (liq)} = 128 \text{ fl oz} = 3785.4118 \text{ cm}^3$$

$$1 \text{ British Imperial and Canadian gallon} = 1.200949 \text{ U.S. gallons} = 4546.087 \text{ cm}^3$$

$$1 \text{ slug} = 14.59390 \text{ kg}$$

$$1 \text{ pound (lb)} = 4.448444 \text{ nt}$$

$$1 \text{ newton (nt)} = 10^5 \text{ dynes}$$

$$1 \text{ British thermal unit (Btu)} = 1054.35 \text{ joules}$$

$$1 \text{ joule} = 10^7 \text{ ergs}$$

$$1 \text{ calorie (cal)} = 4.1840 \text{ joules}$$

$$1 \text{ kilowatt-hour (kWh)} = 3414.4 \text{ Btu} = 3.6 \cdot 10^6 \text{ joules}$$

$$1 \text{ horsepower (hp)} = 2542.48 \text{ Btu/h} = 178.298 \text{ cal/sec} = 0.74570 \text{ kW}$$

$$1 \text{ kilowatt (kW)} = 1000 \text{ watts} = 3414.43 \text{ Btu/h} = 238.662 \text{ cal/s}$$

$$^{\circ}\text{F} = ^{\circ}\text{C} \cdot 1.8 + 32$$

$$1^{\circ} = 60' = 3600'' = 0.017453293 \text{ radian}$$

For further details see, for example, D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics*. 9th ed., Hoboken, N. J.: Wiley, 2011. See also AN American National Standard, ASTM/IEEE Standard Metric Practice, Institute of Electrical and Electronics Engineers, Inc. (IEEE), 445 Hoes Lane, Piscataway, N. J. 08854, website at www.ieee.org.

Differentiation

$$(cu)' = cu' \quad (c \text{ constant})$$

$$(u + v)' = u' + v'$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} \quad (\text{Chain rule})$$

$$(x^n)' = nx^{n-1}$$

$$(e^x)' = e^x$$

$$(e^{ax})' = ae^{ax}$$

$$(a^x)' = a^x \ln a$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\cot x)' = -\csc^2 x$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{\log_a e}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

Integration

$$\int uv' dx = uv - \int u'v dx \quad (\text{by parts})$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln |\cos x| + c$$

$$\int \cot x dx = \ln |\sin x| + c$$

$$\int \sec x dx = \ln |\sec x + \tan x| + c$$

$$\int \csc x dx = \ln |\csc x - \cot x| + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \operatorname{arcsinh} \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \operatorname{arccosh} \frac{x}{a} + c$$

$$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

$$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

$$\int \tan^2 x dx = \tan x - x + c$$

$$\int \cot^2 x dx = -\cot x - x + c$$

$$\int \ln x dx = x \ln x - x + c$$

$$\begin{aligned} \int e^{ax} \sin bx dx \\ = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c \end{aligned}$$

$$\begin{aligned} \int e^{ax} \cos bx dx \\ = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c \end{aligned}$$



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ADVANCED ENGINEERING MATHEMATICS

10TH EDITION

ADVANCED ENGINEERING MATHEMATICS

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Cover photo shows the Zakim Bunker Hill Memorial Bridge in
Boston, MA.

This book was set in Times Roman. The book was composed by PreMedia Global, and printed and bound by RR Donnelley & Sons Company, Jefferson City, MO. The cover was printed by RR Donnelley & Sons Company, Jefferson City, MO.

This book is printed on acid free paper. ∞

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ISBN 978-0-470-45836-5

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

See also <http://www.wiley.com/college/kreyszig>

Purpose and Structure of the Book

This book provides a comprehensive, thorough, and up-to-date treatment of *engineering mathematics*. It is intended to introduce students of engineering, physics, mathematics, computer science, and related fields to those areas of *applied mathematics* that are most relevant for solving practical problems. A course in elementary calculus is the sole *prerequisite*. (However, a concise refresher of basic calculus for the student is included on the inside cover and in Appendix 3.)

The subject matter is arranged into seven parts as follows:

- A. Ordinary Differential Equations (ODEs) in Chapters 1–6
- B. Linear Algebra. Vector Calculus. See Chapters 7–10
- C. Fourier Analysis. Partial Differential Equations (PDEs). See Chapters 11 and 12
- D. Complex Analysis in Chapters 13–18
- E. Numeric Analysis in Chapters 19–21
- F. Optimization, Graphs in Chapters 22 and 23
- G. Probability, Statistics in Chapters 24 and 25.

These are followed by five appendices: **1.** References, **2.** Answers to Odd-Numbered Problems, **3.** Auxiliary Materials (see also inside covers of book), **4.** Additional Proofs, **5.** Table of Functions. This is shown in a block diagram on the next page.

The parts of the book are kept independent. In addition, individual chapters are kept as independent as possible. (If so needed, any prerequisites—to the level of individual sections of prior chapters—are clearly stated at the opening of each chapter.) We give the instructor **maximum flexibility in selecting the material** and tailoring it to his or her need. *The book has helped to pave the way for the present development of engineering mathematics.* This new edition will prepare the student for the current tasks and the future by a modern approach to the areas listed above. We provide the material and learning tools for the students to get a good foundation of engineering mathematics that will help them in their careers and in further studies.

General Features of the Book Include:

- **Simplicity of examples** to make the book teachable—why choose complicated examples when simple ones are as instructive or even better?
- **Independence of parts and blocks of chapters** to provide flexibility in tailoring courses to specific needs.
- **Self-contained presentation**, except for a few clearly marked places where a proof would exceed the level of the book and a reference is given instead.
- **Gradual increase in difficulty of material with no jumps or gaps** to ensure an enjoyable teaching and learning experience.
- **Modern standard notation** to help students with other courses, modern books, and journals in mathematics, engineering, statistics, physics, computer science, and others.

Furthermore, we designed the book to be a **single, self-contained, authoritative, and convenient source** for studying and teaching applied mathematics, eliminating the need for time-consuming searches on the Internet or time-consuming trips to the library to get a particular reference book.

PARTS AND CHAPTERS OF THE BOOK

PART A

Chaps. 1–6
Ordinary Differential Equations (ODEs)

Chaps. 1–4
Basic Material



Chap. 5
Series Solutions

Chap. 6
Laplace Transforms



PART B

Chaps. 7–10
Linear Algebra. Vector Calculus

Chap. 7
Matrices,
Linear Systems

Chap. 9
Vector Differential
Calculus



Chap. 8
Eigenvalue Problems



Chap. 10
Vector Integral Calculus

PART C

Chaps. 11–12
Fourier Analysis. Partial Differential
Equations (PDEs)

Chap. 11
Fourier Analysis



Chap. 12
Partial Differential Equations

PART D

Chaps. 13–18
Complex Analysis,
Potential Theory

Chaps. 13–17
Basic Material



Chap. 18
Potential Theory

PART E

Chaps. 19–21
Numeric Analysis

Chap. 19
Numerics in
General

Chap. 20
Numeric
Linear Algebra

Chap. 21
Numerics for
ODEs and PDEs

PART F

Chaps. 22–23
Optimization, Graphs

Chap. 22
Linear Programming

Chap. 23
Graphs, Optimization

PART G

Chaps. 24–25
Probability, Statistics

Chap. 24
Data Analysis. Probability Theory



Chap. 25
Mathematical Statistics

GUIDES AND MANUALS

Maple Computer Guide
Mathematica Computer Guide

Student Solutions Manual
and Study Guide

Instructor's Manual

Four Underlying Themes of the Book

The driving force in engineering mathematics is the rapid growth of technology and the sciences. New areas—often drawing from several disciplines—come into existence. Electric cars, solar energy, wind energy, green manufacturing, nanotechnology, risk management, biotechnology, biomedical engineering, computer vision, robotics, space travel, communication systems, green logistics, transportation systems, financial engineering, economics, and many other areas are advancing rapidly. What does this mean for engineering mathematics? The engineer has to take a problem from any diverse area and be able to model it. This leads to the first of four underlying themes of the book.

1. Modeling is the process in engineering, physics, computer science, biology, chemistry, environmental science, economics, and other fields whereby a physical situation or some other observation is translated into a mathematical model. This mathematical model could be a system of differential equations, such as in population control (Sec. 4.5), a probabilistic model (Chap. 24), such as in risk management, a linear programming problem (Secs. 22.2–22.4) in minimizing environmental damage due to pollutants, a financial problem of valuing a bond leading to an algebraic equation that has to be solved by Newton’s method (Sec. 19.2), and many others.

The next step is **solving the mathematical problem** obtained by one of the many techniques covered in *Advanced Engineering Mathematics*.

The third step is **interpreting the mathematical result** in physical or other terms to see what it means in practice and any implications.

Finally, we may have to **make a decision** that may be of an industrial nature or **recommend a public policy**. For example, the population control model may imply the policy to stop fishing for 3 years. Or the valuation of the bond may lead to a recommendation to buy. The variety is endless, but the underlying mathematics is surprisingly powerful and able to provide advice leading to the achievement of goals toward the betterment of society, for example, by recommending wise policies concerning global warming, better allocation of resources in a manufacturing process, or making statistical decisions (such as in Sec. 25.4 whether a drug is effective in treating a disease).

While we cannot predict what the future holds, we do know that the student has to practice modeling by being given problems from many different applications as is done in this book. We teach modeling from scratch, right in Sec. 1.1, and give many examples in Sec. 1.3, and continue to reinforce the modeling process throughout the book.

2. Judicious use of powerful software for numerics (listed in the beginning of Part E) and statistics (Part G) is of growing importance. Projects in engineering and industrial companies may involve large problems of modeling very complex systems with hundreds of thousands of equations or even more. They require the use of such software. However, our policy has always been to leave it up to the instructor to determine the degree of use of computers, from none or little use to extensive use. More on this below.

3. The beauty of engineering mathematics. *Engineering mathematics relies on relatively few basic concepts and involves powerful unifying principles.* We point them out whenever they are clearly visible, such as in Sec. 4.1 where we “grow” a mixing problem from one tank to two tanks and a circuit problem from one circuit to two circuits, thereby also increasing the number of ODEs from one ODE to two ODEs. This is an example of an attractive mathematical model because the “growth” in the problem is reflected by an “increase” in ODEs.

4. To clearly identify the conceptual structure of subject matters. For example, complex analysis (in Part D) is a field that is not monolithic in structure but was formed by three distinct schools of mathematics. Each gave a different approach, which we clearly mark. The first approach is solving complex integrals by Cauchy's integral formula (Chaps. 13 and 14), the second approach is to use the Laurent series and solve complex integrals by residue integration (Chaps. 15 and 16), and finally we use a geometric approach of conformal mapping to solve boundary value problems (Chaps. 17 and 18). Learning the conceptual structure and terminology of the different areas of engineering mathematics is very important for three reasons:

a. It allows the student to *identify a new problem and put it into the right group of problems*. The areas of engineering mathematics are growing but most often retain their conceptual structure.

b. The student can *absorb new information more rapidly* by being able to fit it into the conceptual structure.

c. Knowledge of the conceptual structure and terminology is also important when *using the Internet to search for mathematical information*. Since the search proceeds by putting in key words (i.e., terms) into the search engine, the student has to remember the important concepts (or be able to look them up in the book) that identify the application and area of engineering mathematics.

Big Changes in This Edition

1 Problem Sets Changed

The problem sets have been revised and rebalanced with some problem sets having more problems and some less, reflecting changes in engineering mathematics. There is a greater emphasis on modeling. Now there are also problems on the discrete Fourier transform (in Sec. 11.9).

2 Series Solutions of ODEs, Special Functions and Fourier Analysis Reorganized

Chap. 5, on series solutions of ODEs and special functions, has been shortened. Chap. 11 on Fourier Analysis now contains Sturm–Liouville problems, orthogonal functions, and orthogonal eigenfunction expansions (Secs. 11.5, 11.6), where they fit better conceptually (rather than in Chap. 5), being extensions of Fourier's idea of using orthogonal functions.

3 Openings of Parts and Chapters Rewritten As Well As Parts of Sections

In order to give the student a better idea of the structure of the material (see Underlying Theme 4 above), we have entirely rewritten the openings of parts and chapters. Furthermore, large parts or individual paragraphs of sections have been rewritten or new sentences inserted into the text. This should give the students a better intuitive understanding of the material (see Theme 3 above), let them draw conclusions on their own, and be able to tackle more advanced material. Overall, we feel that the book has become more detailed and leisurely written.

4 Student Solutions Manual and Study Guide Enlarged

Upon the explicit request of the users, the answers provided are more detailed and complete. More explanations are given on how to learn the material effectively by pointing out what is most important.

5 More Historical Footnotes, Some Enlarged

Historical footnotes are there to show the student that many people from different countries working in different professions, such as surveyors, researchers in industry, etc., contributed

to the field of engineering mathematics. It should encourage the students to be creative in their own interests and careers and perhaps also to make contributions to engineering mathematics.

Further Changes and New Features

- Parts of Chap. 1 on first-order ODEs are rewritten. More emphasis on modeling, also new block diagram explaining this concept in Sec. 1.1. Early introduction of Euler's method in Sec. 1.2 to familiarize student with basic numerics. More examples of separable ODEs in Sec. 1.3.
- For Chap. 2, on second-order ODEs, note the following changes: For ease of reading, the first part of Sec. 2.4, which deals with setting up the mass-spring system, has been rewritten; also some rewriting in Sec. 2.5 on the Euler–Cauchy equation.
- Substantially shortened Chap. 5, Series Solutions of ODEs. Special Functions: combined Secs. 5.1 and 5.2 into one section called “Power Series Method,” shortened material in Sec. 5.4 Bessel's Equation (of the first kind), removed Sec. 5.7 (Sturm–Liouville Problems) and Sec. 5.8 (Orthogonal Eigenfunction Expansions) and moved material into Chap. 11 (see “Major Changes” above).
- New equivalent definition of *basis* (Sec. 7.4).
- In Sec. 7.9, completely new part on **composition of linear transformations** with two new examples. Also, more detailed explanation of the role of axioms, in connection with the definition of vector space.
- New table of orientation (opening of Chap. 8 “Linear Algebra: Matrix Eigenvalue Problems”) where eigenvalue problems occur in the book. More intuitive explanation of what an eigenvalue is at the beginning of Sec. 8.1.
- Better definition of *cross product* (in vector differential calculus) by properly identifying the degenerate case (in Sec. 9.3).
- **Chap. 11 on Fourier Analysis extensively rearranged:** Secs. 11.2 and 11.3 combined into one section (Sec. 11.2), old Sec. 11.4 on complex Fourier Series removed and new Secs. 11.5 (Sturm–Liouville Problems) and 11.6 (Orthogonal Series) put in (see “Major Changes” above). New problems (new!) in problem set 11.9 on **discrete Fourier transform**.
- **New section 12.5** on modeling heat flow from a body in space by setting up the heat equation. Modeling PDEs is more difficult so we separated the modeling process from the solving process (in Sec. 12.6).
- **Introduction to Numerics** rewritten for greater clarity and better presentation; new Example 1 on how to round a number. Sec. 19.3 on interpolation shortened by removing the less important central difference formula and giving a reference instead.
- Large new footnote with historical details in Sec. 22.3, honoring George Dantzig, the inventor of the **simplex method**.
- **Traveling salesman problem** now described better as a “difficult” problem, typical of combinatorial optimization (in Sec. 23.2). More careful explanation on how to compute the capacity of a cut set in Sec. 23.6 (Flows on Networks).
- In Chap. 24, material on data representation and characterization restructured in terms of five examples and enlarged to include empirical rule on distribution of

data, outliers, and the z -score (Sec. 24.1). Furthermore, new example on encryption (Sec. 24.4).

- Lists of **software** for numerics (Part E) and statistics (Part G) updated.
- References in **Appendix 1** updated to include new editions and some references to websites.

Use of Computers

The presentation in this book is *adaptable to various degrees of use of software, Computer Algebra Systems (CAS's), or programmable graphic calculators*, ranging from no use, very little use, medium use, to intensive use of such technology. The choice of how much computer content the course should have is left up to the instructor, thereby exhibiting our philosophy of maximum flexibility and adaptability. And, no matter what the instructor decides, there will be no gaps or jumps in the text or problem set. Some problems are clearly designed as routine and drill exercises and should be solved by hand (paper and pencil, or typing on your computer). Other problems require more thinking and can also be solved without computers. Then there are problems where the computer can give the student a hand. And finally, the book has **CAS projects**, **CAS problems** and **CAS experiments**, which *do require* a computer, and show its power in solving problems that are difficult or impossible to access otherwise. Here our goal is to combine intelligent computer use with high-quality mathematics. The computer invites visualization, experimentation, and independent discovery work. In summary, the high degree of flexibility of computer use for the book is possible since there are plenty of problems to choose from and the CAS problems can be omitted if desired.

Note that *information on software* (what is available and where to order it) is at the beginning of Part E on Numeric Analysis and Part G on Probability and Statistics. Since *Maple* and *Mathematica* are popular Computer Algebra Systems, there are two computer guides available that are specifically tailored to *Advanced Engineering Mathematics*: E. Kreyszig and E.J. Norminton, *Maple Computer Guide, 10th Edition* and *Mathematica Computer Guide, 10th Edition*. Their use is completely optional as the text in the book is written without the guides in mind.

Suggestions for Courses: A Four-Semester Sequence

The material, when taken in sequence, is suitable for four consecutive semester courses, meeting 3 to 4 hours a week:

1st Semester	ODEs (Chaps. 1–5 or 1–6)
2nd Semester	Linear Algebra. Vector Analysis (Chaps. 7–10)
3rd Semester	Complex Analysis (Chaps. 13–18)
4th Semester	Numeric Methods (Chaps. 19–21)

Suggestions for Independent One-Semester Courses

The book is also suitable for various independent one-semester courses meeting 3 hours a week. For instance,

- Introduction to ODEs (Chaps. 1–2, 21.1)
- Laplace Transforms (Chap. 6)
- Matrices and Linear Systems (Chaps. 7–8)

Vector Algebra and Calculus (Chaps. 9–10)
Fourier Series and PDEs (Chaps. 11–12, Secs. 21.4–21.7)
Introduction to Complex Analysis (Chaps. 13–17)
Numeric Analysis (Chaps. 19, 21)
Numeric Linear Algebra (Chap. 20)
Optimization (Chaps. 22–23)
Graphs and Combinatorial Optimization (Chap. 23)
Probability and Statistics (Chaps. 24–25)

Acknowledgments

We are indebted to former teachers, colleagues, and students who helped us directly or indirectly in preparing this book, in particular this new edition. We profited greatly from discussions with engineers, physicists, mathematicians, computer scientists, and others, and from their written comments. We would like to mention in particular Professors Y. A. Antipov, R. Belinski, S. L. Campbell, R. Carr, P. L. Chambré, Isabel F. Cruz, Z. Davis, D. Dicker, L. D. Drager, D. Ellis, W. Fox, A. Goriely, R. B. Guenther, J. B. Handley, N. Harbertson, A. Hassen, V. W. Howe, H. Kuhn, K. Millet, J. D. Moore, W. D. Munroe, A. Nadim, B. S. Ng, J. N. Ong, P. J. Pritchard, W. O. Ray, L. F. Shampine, H. L. Smith, Roberto Tamassia, A. L. Villone, H. J. Weiss, A. Wilansky, Neil M. Wigley, and L. Ying; Maria E. and Jorge A. Miranda, JD, all from the United States; Professors Wayne H. Enright, Francis L. Lemire, James J. Little, David G. Lowe, Gerry McPhail, Theodore S. Norvell, and R. Vaillancourt; Jeff Seiler and David Stanley, all from Canada; and Professor Eugen Eichhorn, Gisela Heckler, Dr. Gunnar Schroeder, and Wiltrud Stiefenhofer from Europe. Furthermore, we would like to thank Professors John B. Donaldson, Bruce C. N. Greenwald, Jonathan L. Gross, Morris B. Holbrook, John R. Kender, and Bernd Schmitt; and Nicholaiv Villalobos, all from Columbia University, New York; as well as Dr. Pearl Chang, Chris Gee, Mike Hale, Joshua Jayasingh, MD, David Kahr, Mike Lee, R. Richard Royce, Elaine Schattner, MD, Raheel Siddiqui, Robert Sullivan, MD, Nancy Veit, and Ana M. Kreyszig, JD, all from New York City. We would also like to gratefully acknowledge the use of facilities at Carleton University, Ottawa, and Columbia University, New York.

Furthermore we wish to thank John Wiley and Sons, in particular Publisher Laurie Rosatone, Editor Shannon Corliss, Production Editor Barbara Russiello, Media Editor Melissa Edwards, Text and Cover Designer Madelyn Lesure, and Photo Editor Sheena Goldstein for their great care and dedication in preparing this edition. In the same vein, we would also like to thank Beatrice Ruberto, copy editor and proofreader, WordCo, for the Index, and Joyce Franzen of PreMedia and those of PreMedia Global who typeset this edition.

Suggestions of many readers worldwide were evaluated in preparing this edition. Further comments and suggestions for improving the book will be gratefully received.

KREYSZIG

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