CIT-24-01-0110

Vectors _ Madule.

01) A vector is a quantity that has both magnitude (size) and direction. Vectors are commonly used in physics and mathematics to represent things like force, velocity, acceleration and displacement.

A scaler has only magnitude, no direction (eg: mass, temperature)

02) The dot product shows how much one vector goes in the direction of another. If they point in the same direction, the value is big, if they are at 90°, its zero. If they are opposite Its negative.

03) If two vectors A and B are perpendicular their dot product is 0.

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 90° = 0$$

04) The cross product is equal to the product of their magnitudes.

$$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin(90°) = |\vec{A}| \cdot |\vec{B}|$$

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05) False

The magnitude of the cross product of two vectors is zero when they are parallel, not maximum.

$$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin\theta$$

If vectors are parallel, the $\theta = 0°$ or $180°$, so.

$$\sin 0° = \sin(180°) = 0$$

06) A unit vector is a vector that has a magnitude (length) of exactly

It only shows direction, not size

$$i = \langle 1, 0, 0 \rangle$$

$$\sqrt{1^2 + 0^2 + 0^2} = 1$$

other examples

$j = \langle 0, 1, 0 \rangle \rightarrow$ along y-axis

$k = \langle 0, 0, 1 \rangle \rightarrow$ along z-axis.

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7) The scaler triple product of three vectors ~~represent~~ $\vec{A} \cdot (\vec{B} \times \vec{C})$

represents the volume of a parallelepiped formed by the three vectors.

8) Dot product

1) Commutative Property

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

2) Distributive over Addition

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Cross Product. — Two properties

1. Anti-commutative Property

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

2). Distributive Over Addition

$$A \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

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9) $\vec{A} = 3i + 2j - k$, $\vec{B} = i - 4j + 2k$

$\vec{A} + \vec{B}$

add i components $\rightarrow$ $3 + 1 = 4$
add j components $\rightarrow$ $2 + (-4) = -2$
add $\hat{k}$ components $\rightarrow$ $-1 + 2 = 1$

$\vec{A} + \vec{B} = 4i - 2j + k$

10) $\vec{V} = \cancel{4i} \; 4i - 3j + 12k$

$|V| = \sqrt{(4)^2 + (-3)^2 + (12)^2}$

$= \sqrt{16 + 9 + 144}$

$= \sqrt{169}$

$|\vec{V}| = 13$

11) $\vec{A} \cdot \vec{B} = (A_x \cdot B_x) + (A_y \cdot B_y)$

$A_x = 2, \quad B_x = -1 \Rightarrow 2 \cdot (-1) = -2$
$A_y = 3, \quad B_y = 4 \Rightarrow 3 \cdot 4 = 12$

$\vec{A} \cdot \vec{B} = -2 + 12 = \cancel{12} \; 10$

12) $\vec{P} = 2i + j$, $\vec{Q} = i - j$

I] $\vec{P}.\vec{Q} = (2)(1) + (1)(-1) = 2 - 1 = 1$

$$\boxed{\vec{P}\cdot\vec{Q} = |\vec{P}|\cdot|\vec{Q}|\cdot\cos\theta \Rightarrow \cos\theta = \frac{\vec{P}\cdot\vec{Q}}{|\vec{P}||\vec{Q}|}}$$

II] Find magnitude

$|\vec{P}| = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

$|\vec{Q}| = \sqrt{1^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$

III] $\cos\theta = \frac{1}{\sqrt{5}.\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right) \approx \cos^{-1}(0.316) \approx 71.57^\circ$

$\theta' \approx 71.57$

13) $\vec{W} = \cancel{6i+}2\ 6i - 2j + 3k$

$|\vec{W}| = \sqrt{6^2+(-2)^2+3^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

$= 7$

Final answer $= \frac{6}{7}i - \frac{2}{7}j + \frac{3}{7}k$

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14) $\vec{A} = i + 2j$, $B = 2i + j$

$$\text{Proj}_{\vec{B}} \vec{A} = \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B}$$

Dot Product $\vec{A} \cdot \vec{B}$

$\vec{A} \cdot \vec{B} = (1)(2) + (2)(-1) = 2 - 2 = 0$

Magnitude ~~squ~~ Squared of $\vec{B}$

$|\vec{B}|^2 = 2^2 + (-1)^2 = 4 + 1 = 5$

$\text{Proj}_B \vec{A} = \left(\frac{0}{5}\right) \vec{B} = 0 \cdot \vec{B} = \vec{0}$

Projection $= \vec{0}$ (A is perpendicular to B)

15) $\vec{U} = i + 2j + 3k$, $\vec{V} = 2i - j + 4k$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} - j \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

~~R~~

i : $(2)(4) - (3)(-1) = 8 + 3 = 11$

j : $(1)(4) - (3)(2) = 4 - 6 = -6$

k : $(1)(-1) - (2)(2) = -1 - 4 = -5$

$\vec{U} \times \vec{V} = 11i + 2j - 5k$

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16) Force = $10i + 5j$

Displacement = $4i$ ← only the i component matter for work.

Work = $10 \times 4 = 40$

17) $\vec{A} = i + 2j + 3k$, $B = 2i + 4j + 6k$

$B = 2.\vec{A}$

Therefore A and B are collinear

18) Area $|\vec{A} \times \vec{B}|$

$A = i + j = (1,1,0)$, $B = j + k = (0,1,1)$

Magnitude

$|\vec{A} \times \vec{B}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\sqrt{3}$ //

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19) $\vec{A} = 2i - 3j + k$

$\vec{B} = -i + 4j - 2k$

$\vec{C} = 3i - j - k$

$B \times C = 2i - 5j - 11k$

Do dot product with $\vec{A}$

$A \cdot (B \times C) = (2)(2) + (-3)(-5) + (1)(-11) = 4 + 15 - 11 = 8$

Answer 8

20) $A = (1, 0, 0)$

$B = (0, 1, 0)$

$C = (0, 0, 1)$

$AB = B - A = (-1, 1, 0)$, $AC = C - A = (-1, 0, 1)$.

Cross product $\Rightarrow AB \times AC = i + j + k$

$|AB \times AC| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Area $= \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2}$