Project Report

PH 556 : Astrophysics

Sandeepan Naskar

Team : UzUMa-KI

Overview

Study an RR Lyrae variable and calculate luminosity, temperature, radius, distance and spectral class.

Aim

Evaluate the following metrics for the star, considering the information and data previously available.

Variability period - Using images across varying brightness.

Total Luminosity from multiple period-luminosity relations / period-total-luminosity relation.

Distance by comparing luminosity and apparent brightness from the images.

Radius using a radius-period relation [1].

Temperature using total luminosity and radius.

Spectral Class using temperature co-relations.

Assumptions and Givens

The key assumption is the fact that the variable star follows the known period-luminosity relations and radius-period relations, **taking generalized empirical relations with taking metallicity as a representative value**.

We also required the average magnitude to fine tune our period estimations, which we did not consider during the project proposal. Hence, this can be considered as an additional given that we required.

Verification

Compared obtained values to the ones from existing literature: [1] [2] [3] [4]

Target Star

We have chosen the target star UZ UMa (Simbad link), which is an RRab variable[2].

Magnitude and Coordinates

[RA] = 08°18' 53".9127454200 **[DEC]** = +73°05' 47".926302000

Apparent Magnitudes : [3] **[g]** 14.7030 **[r]** 14.1269 **[i]** 14.2903 **[z]** 14.3070

Methods

Photometry

Source Selection

From the Panstarrs-1 catalogue, we queried Vizier for the known magnitudes of the sources, and used SExtractor to select good sources from our image. We limited the maximum magnitude to 18 and selected point-like sources by limiting the FWHM to 2 arcseconds.

PSF Photometry

Using the package PSFex, we fit various point-spread functions to the good sources selected earlier and finally went with the standard Moffat function.

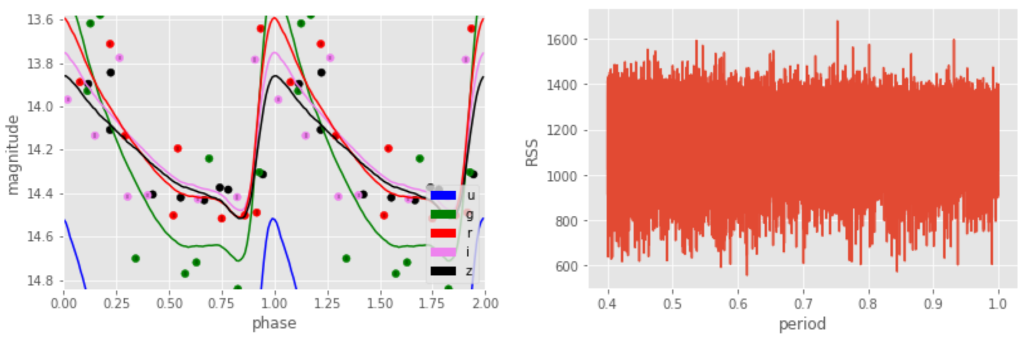
We then use the best-fit PSF on our "good" sources to obtain instrumental magnitudes and error bars for each of them. Comparing with Panstarrs-1 magnitudes, we find the expected linear relation not passing through the origin which needs to be fixed in the zero point correction step.

Zero-point corrections

For aperture photometry, we manually selected 10 good stars for zero-point correction. For PSF photometry, our cross-matching of good sources from SExtractor and Panstarrs-1 greatly simplifies and improves our correction procedure. As with our target, we use the best-fit PSF to determine instrumental magnitudes of all of the good sources, subtract from the catalog magnitudes, and perform 3-σ clipping to obtain the mean and error of the zero-point (which we add in quadrature to instrumental error for total error).

Curve Fitting

We started out with template matching[5]. The code takes in our data points for the light curve, and by using previously sampled data, tries to construct a statistical fit to our points.



However, the errors (above) in the templates were too high and volatile.

After trying out standard curves like Triangular and Sawtooth signals, followed by Periodic Gaussian and standard Sinusoidal curves, we achieved satisfactory results by using sine harmonics, as a combination of two waves.

In the above-described methods, we have used the `scipy.optimize.curve_fit`[6] to fit these functions to our data. In order to keep our data from being over-fitted in our functions, we loop through the values of frequency w for a given range in order to find the time periods.

This can be done since we know most RR Lyrae periods range from a few hours to a few days. In this method, as we get a good estimate of the period we narrow the range and increase the degree of precision/ step values in the loop to get a more precise estimate.

We also fixed the values of A_0 (y-shifting constant/zero point of the wave) with the same motivation. However, this was done in different fashions for each fit function. The error minimization in the looping of frequency was done using the χ^2 minimization[7] on each of the fits.

Triangular and Sawtooth signals

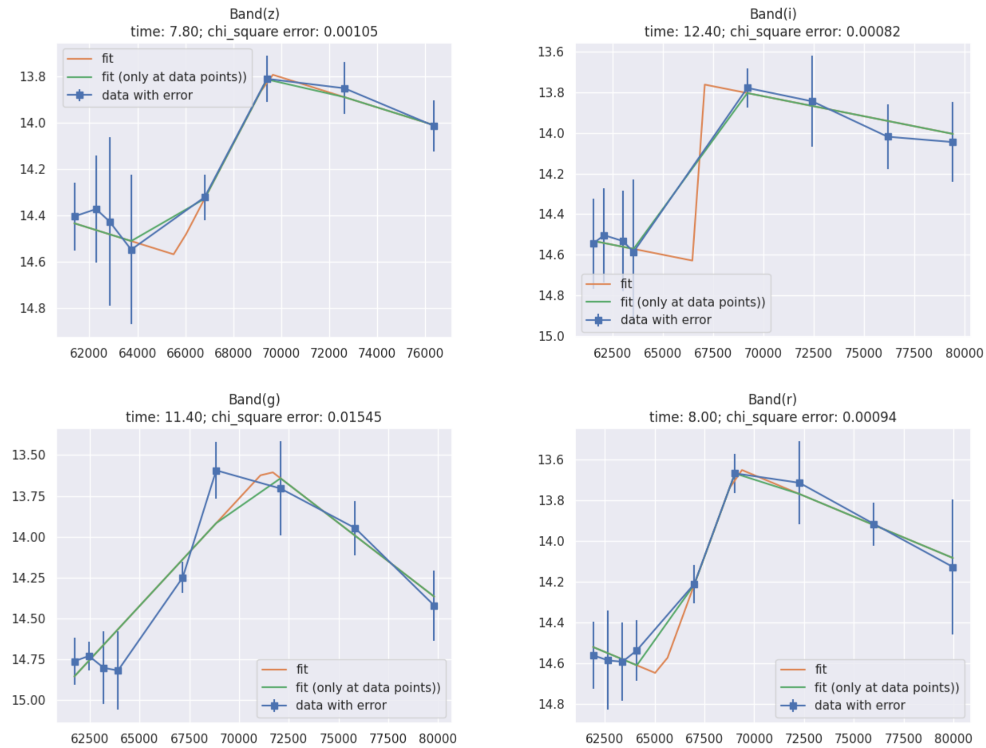
Here we used the `scipy.signal.sawtooth` function[8] for generating our waves. We parametrized it in the following way:

$$m = A_0 + A_1 sawtooth(\omega t + \phi, w) \tag{1}$$

where m is the apparent magnitude, A_i are the scaling/amplitude constants, ω, ϕ are the frequency, phase and w is the ratio of width of the sawtooth(distance of peak from a trough) and the wavelength.
Note: Taking $w = 0.5$ gives us a triangular wave.

First of all we fix the value of A_0 to be the approximate mean of the highest and lowest points in our data, since the final function fit would be symmetric about this value.

For triangular waves, we fix the value of $w = 0.5$ and then we fit for A_1, ϕ which naturally gave us slightly worse results as opposed to sawtooth where we keep the value w to be a fittable parameter bounded in the interval $[0, 1]$. Now fitting for A_1, ϕ, w we get the following plots:



Periodic Gaussian signals

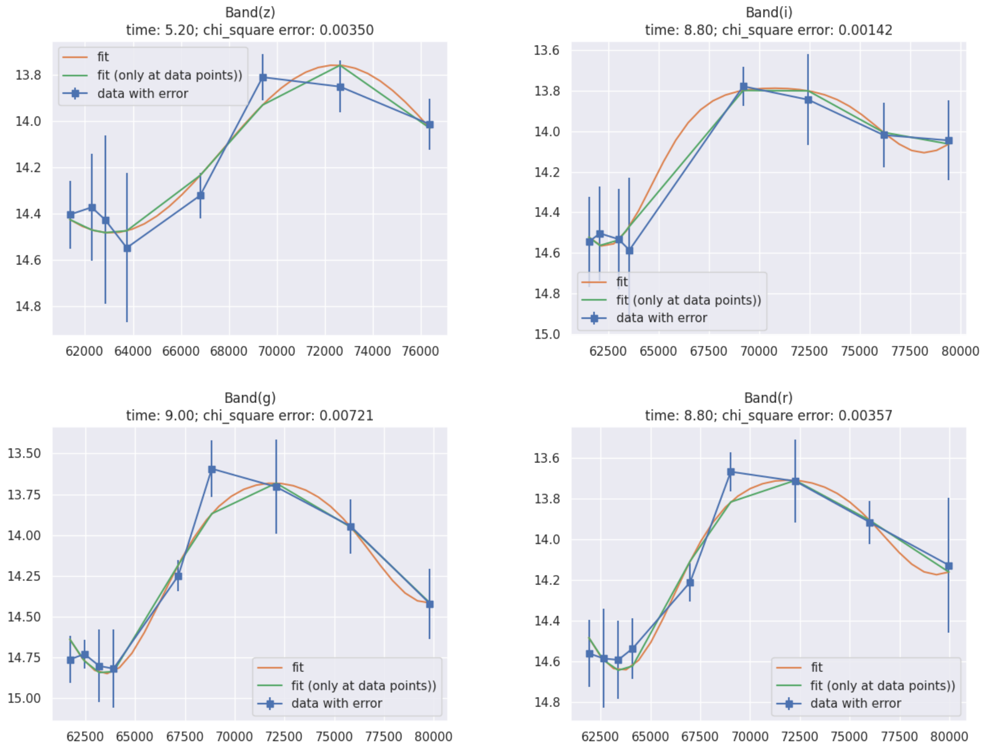
In this method, we use the below function in order to fit our curve

$$m = A_0 + \sum_{i=0}^{N-1} A_{i+1} \frac{e^{\kappa[2\pi(x - \frac{i}{N})]}}{2\pi I_0(\kappa)} \tag{2}$$

[9] where $I_0(k)$ is the modified Bessel function of order zero. In our case we truncate the above summation to the first two terms for the second harmonic and only the first term for the first harmonic. We also truncate the expansion of $I_0(\kappa)$ two the first 3-4 terms.

$$I_0(x) = \sum_{m=0}^{\inf} \frac{x^{2m}}{2^{2m}.(m!)^2} \tag{3}$$

And as expected we get a better fit for the second harmonic than the first with plots being as follows:



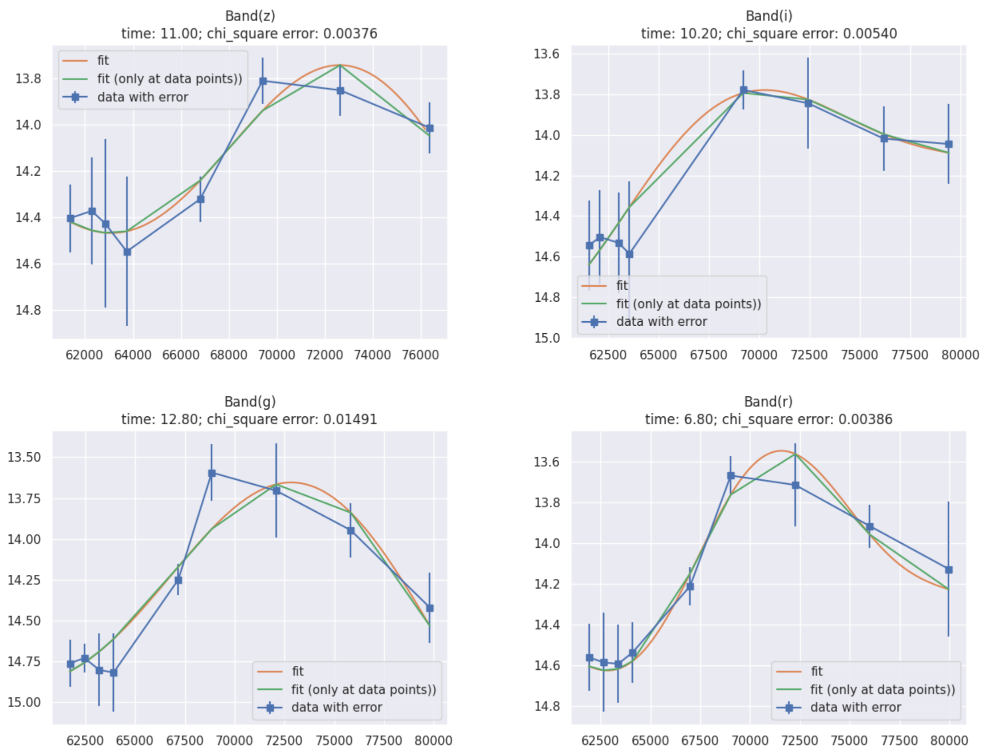
Sinusoidal signals

Finally, we come to the sinusoidal waves and try our hands at fitting with first and second harmonics. The formula that we use for second harmonic being:

$$m = A_0 + A_1 sin(\omega t + \phi_1) + A_0 sin(2\omega t + \phi_2) \tag{4}$$

In this case, we need to have a better mean value A_0 for our data in order to get a better period estimate without over-fitting. Finally after much discussion, we went with assigning the values of A_0 to be the mean apparent magnitudes taken from data catalogs.[3]

Working with this, we get the following plots for each band:



Following this we chose to go with the sinusoidal functions as it is a tradeoff between:

1. The value over-fitting in periodic Gaussians which gives us the time period to be around the value our data is spread out to ($\approx 5hrs$) for the z-band. As for the rest of the bands, the data-points more or less coincide with the fit function at those points which is again a clear sign of overfitting.

2. Bad time estimates across the bands due to underfitting as we are essentially using a piece-wise linear function in the Sawtooth methods giving a period of $\approx 8hrs$ in the z,r-band while giving us $\approx 12hrs$ for the i,g-band.

In the sinusoidal estimates itself, we can see that the r-band is an outlier at period=6.8hrs. Since our parameter calculation starting from luminosity is done on the i-band due to availability constraints of PL relations in literature. We go ahead with the period as **10.2 hours** or **0.425 days**.

Calculation of parameters

Once we had the period, calculating all other parameters in a cascading manner was fairly easy, considering we already had all the relations from known literature.

$$M_I = 0.4711 - 1.1318log_{10}(P) + 0.2053log_{10}(Z) \tag{5}$$

From here [10]. We use the I band since it has a significant overlap with the i band. The period is taken to be in days.

$$log_{10}(\frac{R}{R_\odot}) = 0.774 + 0.580log_{10}(P) - 0.035log_{10}(Z) \tag{6}$$

[1] Z, the metallicity, in both cases is taken as 0.002 as the representative value. For the distance, we have:

$$m - M = 5log(d) - 5 \tag{7}$$

And temperature can be calculated by:

$$L = 4\pi R^2 \sigma T^4 \tag{8}$$

Results

UZ UMa Parameters: i Band

Parameter	**Value**	**Error**
Period	0.425 days	8.93%
Mean Absolute Magnitude	0.3376	-
Distance	6201.29 pc	8.77%
Luminosity	$62.36L_\odot$	-
Radius	$4.498\ R_\odot$	-
Temperature	7657.9 K	7.86%
Spectral Class	A8	A5 (actual value)

Discussion

We faced a series of challenges along the course of the project. Some of them are listed below:

- Some images had a high interference from the moon. This polluted the image source and made background correction difficult. Some images had artifacts, which had to be taken care of and one image had to be discarded.
- The fact that the images were not evenly distributed along the period, and a lot of them were bunched in a rather "static" phase (almost 4/8 datapoints between 61,000 and 64,000 and rest being sparsed out from 64,000 to 80,000), makes it harder to fit the curves.
- The non-uniformity of a typical RR Lyrae curve makes it harder to fit with any standard periodic curve if we do not have a lot of data points. We also had to come up with measures to fix constants to avoid over-fitting.

- Various empirical relations for absolute magnitude and Radius exist. A lot of them take metallicity into account, and we had to use a representative value for metallicity since that cannot be obtained without spectroscopy.

The results, which had corresponding values in the existing literature, corroborated those values, with errors at roughly 8%. This can be deemed to be satisfactory, considering the sparse data we have been working with.

References

[1] In: (). URL: https://adsabs.harvard.edu/full/2004ASPC..310..502M.

[2] In: (). URL: https://arxiv.org/abs/astro-ph/0601432.

[3] In: (). URL: https://ui.adsabs.harvard.edu/abs/2016arXiv161205560C/abstract.

[4] In: (). URL: https://articles.adsabs.harvard.edu/pdf/1960ApJ...131..632H.

[5] In: (). URL: https://github.com/longjp/rr-templates.

[6] In: (). URL: https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html.

[7] In: (). URL: https://ned.ipac.caltech.edu/level5/Wall2/Wal3_4.html.

[8] In: (). URL: https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.sawtooth.html.

[9] In: (). URL: https://iopscience.iop.org/article/10.3847/1538-4357/aab4fd/pdf.

[10] In: (). URL: https://arxiv.org/abs/astro-ph/0406067.