

### **Question:-**

**Methanol vapor at saturation temperature and 1 atm pressure is to be condensed and sub-cooled to 40°C at a rate of 100 g/s. Water at 20°C and 3 kg/s is available as a coolant flowing through 2 mm thick AISI 302 stainless steel tubes of 20 mm outer diameter. Design a shell and tube heat exchanger for this purpose.**

### **Solution:-**

#### **Assumptions**

- 1) Steady operating conditions exist.
  - 2) The heat exchanger is well insulated so that heat loss to the surroundings is negligible.
  - 3) Changes in the kinetic and potential energies of fluid streams are negligible.
  - 4) Fluids' properties are constant.
- & along with the solution

## Group - 1

Since methanol vapor is condensing and also sub-cooled  
So we divide the heat exchanger in to two parts

: firstly we solve/calculate the length of tube that involves condensation of methanol vapor

: Secondly, we calculate the length of tube that involving the sub cooling of methanol to  $40^{\circ}\text{C}$ .

### Part - A : Condensation

At saturation temperature and 1 atm pressure ;  $T_{\text{sat}} = 65^{\circ}\text{C}$

Water is at  $20^{\circ}\text{C}$

$\therefore$  methanol is undergoing phase change.

So, rate of heat transfer :  $\dot{Q} = \dot{m} h_{fg}$

where,  $\dot{m}$  = rate of condensation of fluid

$h_{fg}$  = enthalpy of vaporization of fluid at specified temperature.

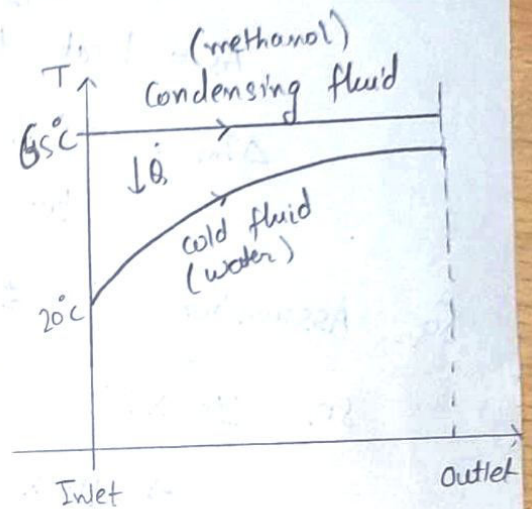
Now, in the problem :  $\dot{m} = 100 \text{ g/s} = 0.1 \text{ kg/s}$

and  $h_{fg} (\text{at } 65^{\circ}\text{C}) = 35.21 \text{ kJ/mol}$

Now, molar mass of methanol ( $\text{CH}_3\text{OH}$ ) =  $32 \text{ g/mol}$   
 $= 32 \times 10^{-3} \text{ kg/mol}$

$$\text{Hence } h_{fg} = \frac{35.21}{32 \times 10^{-3}} \text{ kJ/kg}$$

$$\Rightarrow h_{fg} = \frac{1.1 \text{ kJ/kg}}{\times 10^3} \approx 1100 \text{ kJ/kg}$$





So,  $\dot{Q} = 0.1 \times 1100 \text{ kJ/s}$   
 $\Rightarrow \dot{Q} = 110 \text{ kJ/s}$

from Newton's law of cooling, rate of heat transfer in a heat exchanger,

$$\dot{Q} = UA_s \Delta T_m$$

where  $U$  = overall heat transfer coefficient  
~~(800-1700)~~ for ~~water-water~~ Alcohol condensor  
 250-700

$A_s$  = heat transfer surface area

$\Delta T_m$  = mean temperature difference between the two fluids.

So, Assuming  $U = \cancel{1250} 475$

$$\text{So, } 110 \times 10^3 = \cancel{1250} \times (\pi \times D_o \times L) \times (65 - 20)$$

$$\Rightarrow 110 \times 10^3 = 1250 \times \pi \times 20 \times 10^{-3} \times L \times 45$$

$$\Rightarrow L = 31 \text{ m}$$

$$\Rightarrow 110 \times 10^3 = 475 \times \pi \times 20 \times 10^{-3} \times L \times 45 \Rightarrow \boxed{L = 81.57 \text{ m}}$$

Part-B : Subcooling of methanol

Since the outlet temperature of coolant (water) is not known, so we will take the limiting case.

for methanol :  $T_{avg} = \frac{65 + 40}{2} = 52.5^\circ \text{C}$

at  $T_{avg}$ ,  $C_p = 2738 \text{ J/kg}\cdot\text{K}$  (using interpolation between  $50^\circ\text{C}$  &  $60^\circ\text{C}$ )

$$\text{So, } C_h = \dot{m}_h C_{ph} = 0.1 \frac{\text{kg}}{\text{s}} \times 2738 \frac{\text{J}}{\text{kg}\cdot\text{K}} = 273.8 \frac{\text{J}}{\text{s}\cdot\text{K}}$$



for water: Assuming, ~~rate~~  $C_{pw} = 4182 \text{ J/kg}\cdot\text{K}$

$$\text{then, } C_w = \dot{m}_w C_{pw} = 3 \frac{\text{kg}}{\text{s}} \times 4182 \frac{\text{J}}{\text{kg}\cdot\text{K}} = 12546 \frac{\text{J}}{\text{s}\cdot\text{K}}$$

$$\text{Therefore } C_{\min} = C_h = 273.8 \text{ J/s}\cdot\text{K}$$

$\therefore$  maximum heat transfer rate

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min} (T_{h,\text{in}} - T_{w,\text{in}}) \\ &= 273.8 (65 - 20) \\ &= 273.8 \times 45 = 12321 \text{ J/s} \approx 12.321 \text{ kJ/s}\end{aligned}$$

maximum temperature difference in this heat exchanger is

$$\Delta T_{\max} = T_{h,\text{in}} - T_{w,\text{in}} = 45^\circ\text{C}$$

The outlet temperature of the cold stream (water)

In this limiting case:

$$\begin{aligned}\dot{Q}_{\max} &= C_w (T_{w,\text{out}} - T_{w,\text{in}}) \Rightarrow T_{w,\text{out}} = T_{w,\text{in}} + \frac{\dot{Q}_{\max}}{C_w} \\ \Rightarrow T_{w,\text{out}} &= 20 + \frac{12321}{12546} \approx 20.98 \approx 21^\circ\text{C}\end{aligned}$$

$$\text{So, } Q = UA \Delta T_m \quad (\text{without fouling})$$

where  $U$  = overall heat transfer coefficient;  $A$  = surface area

$\Delta T_m = \Delta T_{\text{em}} =$  logarithmic mean difference

$$\text{So, } \Delta T_{\text{em}} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \quad \text{with} \quad \begin{aligned}\Delta T_1 &= T_{h,\text{in}} - T_{w,\text{out}} \\ \Delta T_2 &= T_{h,\text{out}} - T_{w,\text{in}}\end{aligned}$$

$$\text{So, } \Delta T_1 = 65 - 21 = 44^\circ\text{C} \quad ; \quad \Delta T_2 = 40 - 20 = 20^\circ\text{C}$$

$$\text{So, } \Delta T_{\text{em}} = \frac{44 - 20}{\ln\left(\frac{44}{20}\right)} = \frac{24}{0.788} = 30.45^\circ\text{C}$$

$$\text{Also, } Q = C_h \Delta T = 273.8 \times (65 - 40) \approx 6845 \text{ J/s}$$



for Calculating U

cold fluid  $\rightarrow$  water ; Hot fluid  $\rightarrow$  methanol

$$\text{So, } U = 50 - 150 \text{ Btu/hr-ft}^2\text{-F}$$

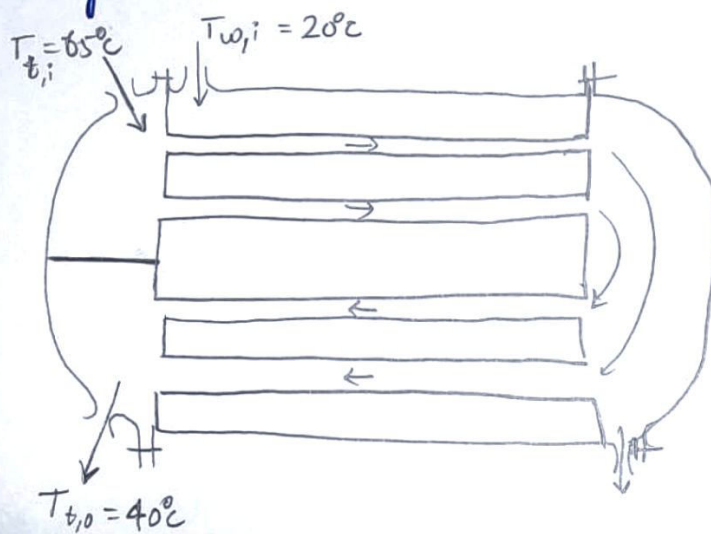
$$\text{So, } U_{\text{avg}} = 100 \text{ Btu/hr-ft}^2\text{-F} = 567.826 \text{ J/s-m}^2\text{-K}$$

$$\text{So, } Q = U A_s \Delta T_{\text{em}}$$

$$\Rightarrow 6845 = 567.826 \times \pi (D_o - 2 \times 2 \times 10^{-3}) L \times 30.45$$

$$\Rightarrow \boxed{L \geq 7.87 \text{ m}}$$

Hence the total length of Shell & tube heat Exchanger would be  $81.57 + 7.87 = \underline{89.44 \text{ m}}$



$$\begin{aligned} \text{Outer surface area} &= \pi D_o (89.44) \\ &= \underline{5.6168 \text{ m}^2} \end{aligned}$$