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Graph Search Algorithms

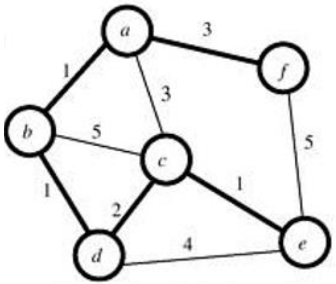
Lecture 8

Contents

- Brute-Force Algorithm
- Breadth-First Search (BFS) Algorithms
- Depth-First Search (DFS) Algorithms (Backtracking Algorithm)
- Parallel BFS and DFS

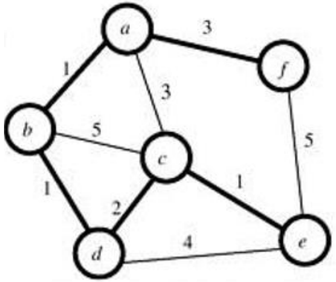
Brute-Force Algorithm

Minimum Spanning Tree

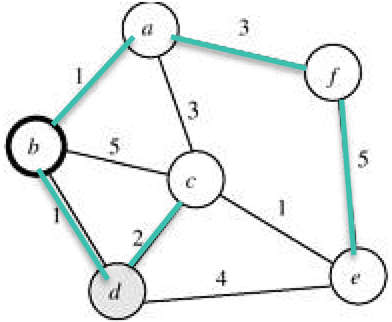


Prim’s algorithm

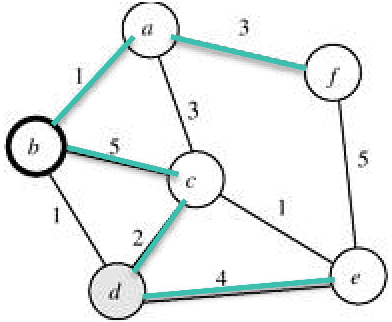
Minimum Spanning Tree



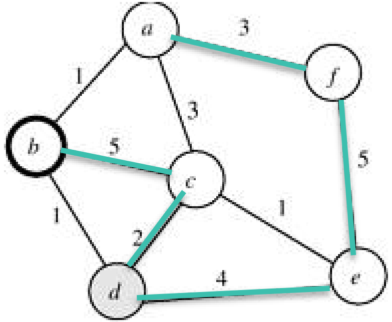
Prim's algorithm
Cost = 8



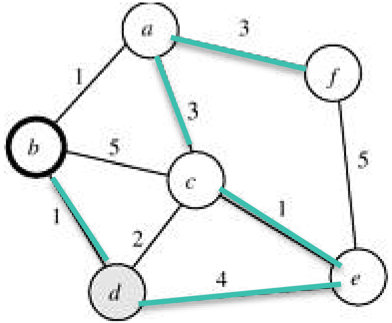
Cost = 13



Cost = 20



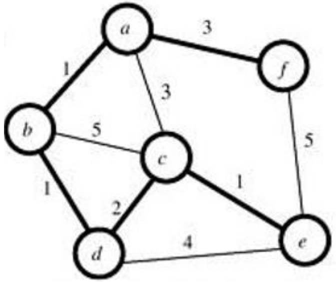
Cost = 19



Cost = 14

...

Minimum Spanning Tree



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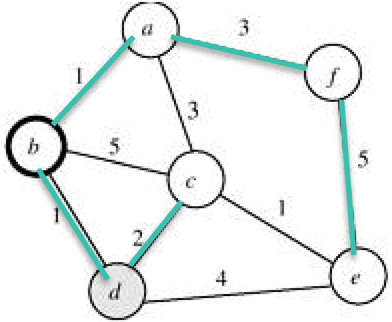
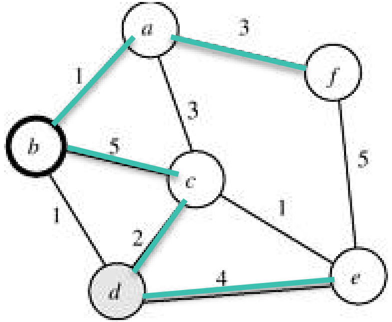
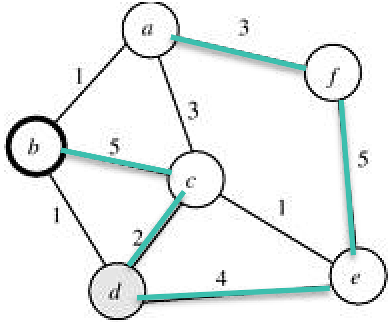


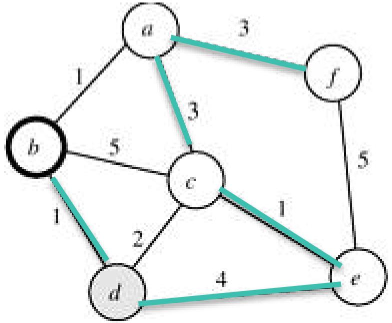
Cost = 12



Cost = 15



Cost = 19

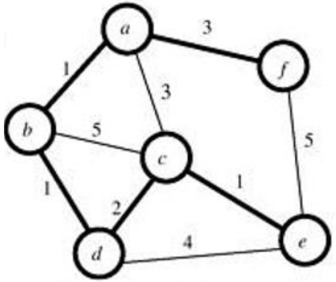


Cost = 14

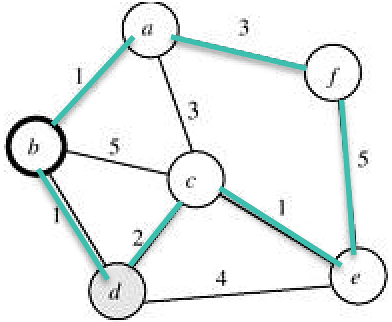
...

- Find all possible path which goes through all vertices in the graph
- Select the one with minimum cost

Minimum Spanning Tree



Prim's algorithm
Cost = 8



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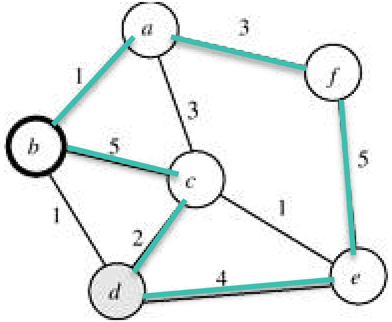

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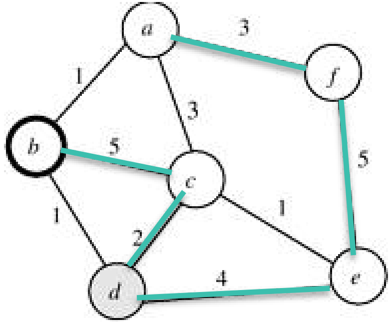

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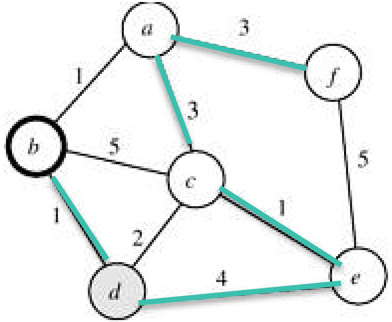
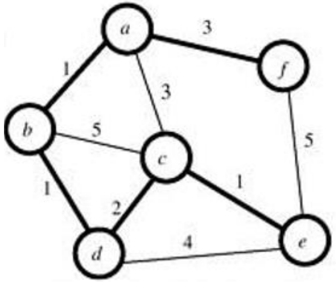


Cost = 14

...

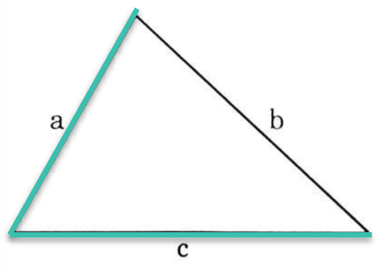
So many cases!!!

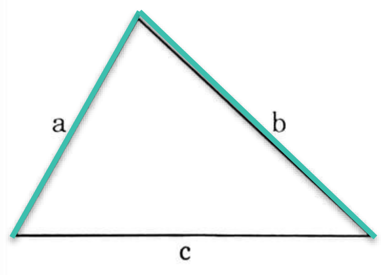
Minimum Spanning Tree

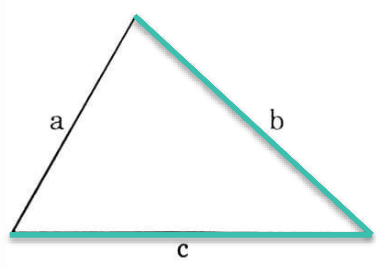


Prim's algorithm

But if the number of vertices is limited, then listing all cases is possible







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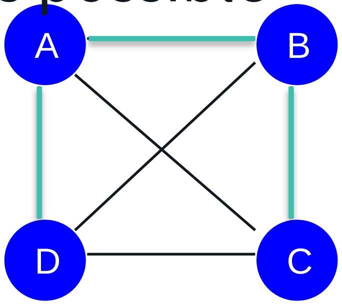

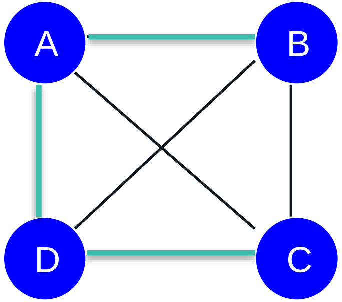

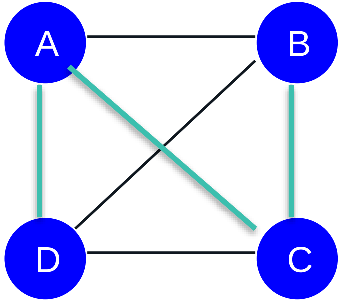

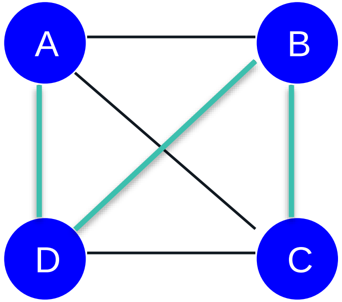

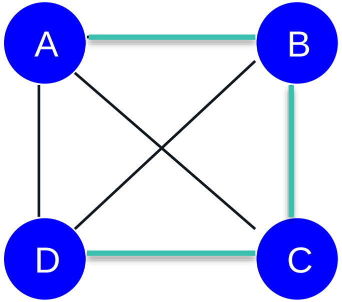

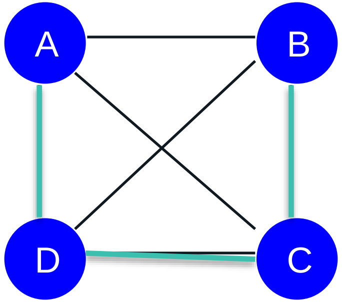

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Another example of Brute-Force solution

- There is a lock of 4-digit PIN. Each digit is chosen from 0-9
- You forgot the PIN and have to find the PIN
- The Brute-Force will try all possible combinations one by one: 0000, 0001, 0002,0003….
- The number of test cases: **10000**

4-digit PIN lock

6-digit PIN lock

Brute-Force Algorithm

- Exhaustive Search in entire searching space
- Search for an element with a special property until either a match is met, or the list is exhausted without finding a match
- Method
  - Generate a list of all protentional solutions to a problem
  - Evaluate each solution
  - When search ends, announce the solution

Brute-Force Algorithm - Pros

- Straightforward, usually directly based on the problem statement
- Simple
- Widely applicable (searching, sorting,…)

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Brute-Force Algorithm - Cons

- Exhaustive-search algorithms run in a reasonable amount of time only on very small instances, sometimes are unacceptable slow
- Usually not efficient
- Not as constructive as some other techniques
  - MST: Prim's algorithm
  - Dijkstra algorithm…
- But, in many cases, exhaustive search or its variation is the only known way to get exact solution

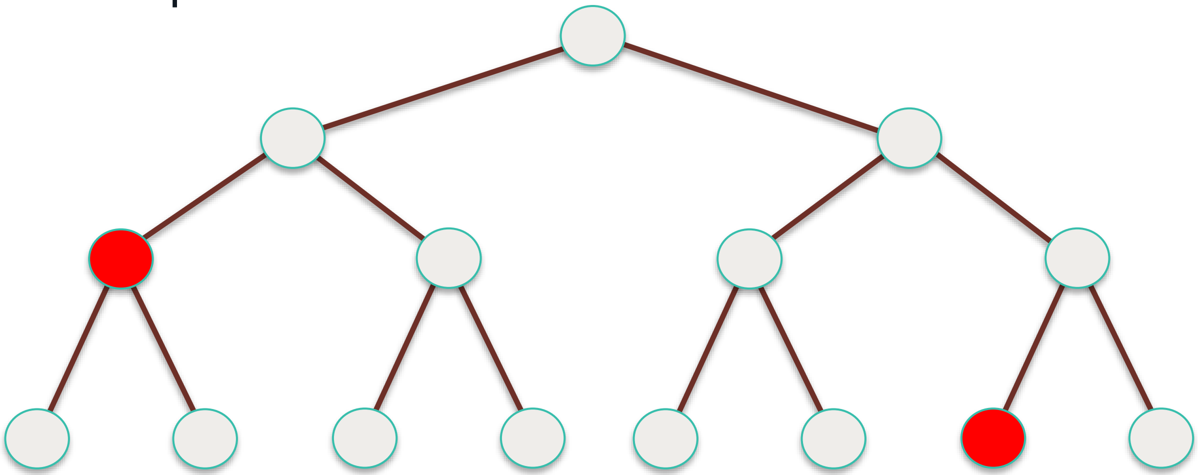
Breadth-First Search (BSF) Algorithms

Searching Algorithm

- Search algorithms can be used to solve discrete optimization problems (DOPs)
  - A class of computationally expensive problems with significant theoretical and practical interest
- Search algorithms systematically search the space of possible solutions subject to constraints

Searching Algorithm

- Look for a given node
  - Stop when node found, even if not all nodes were visited
  - Not guarantee optimization



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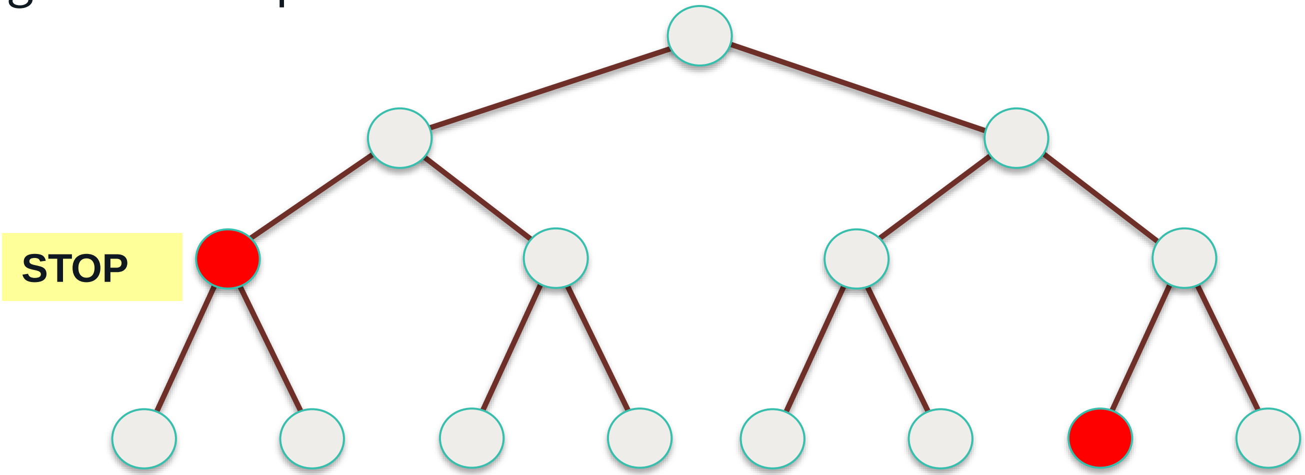
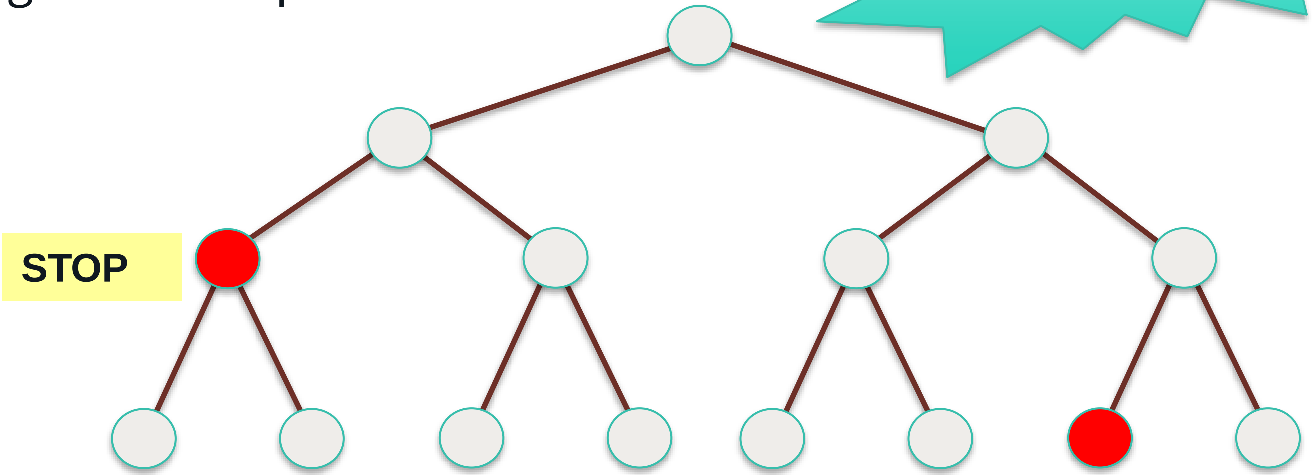


Searching Algorithm

- Look for a given node
  - Stop when node found, even if not all nodes
  - Not guarantee optimization

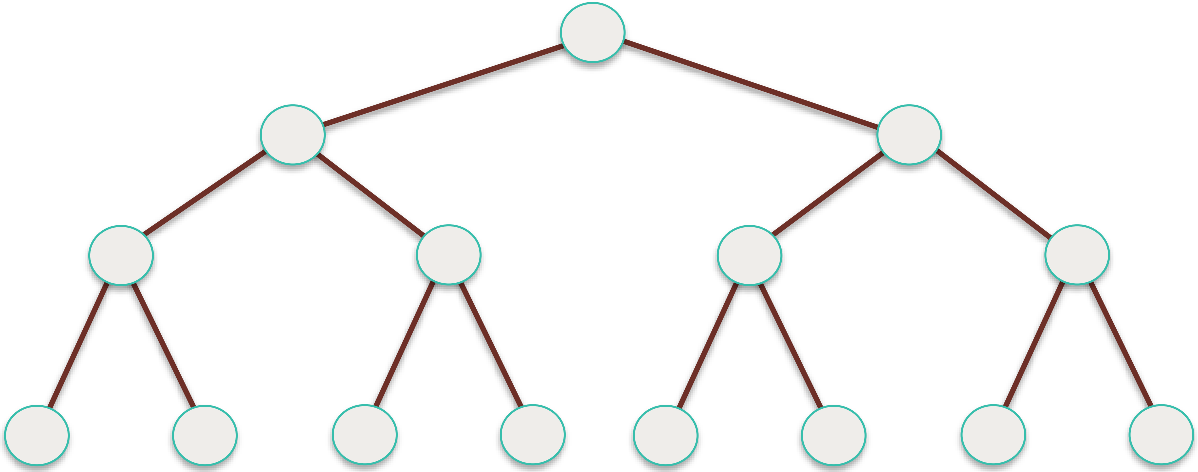
If solutions found while searching, STOP!



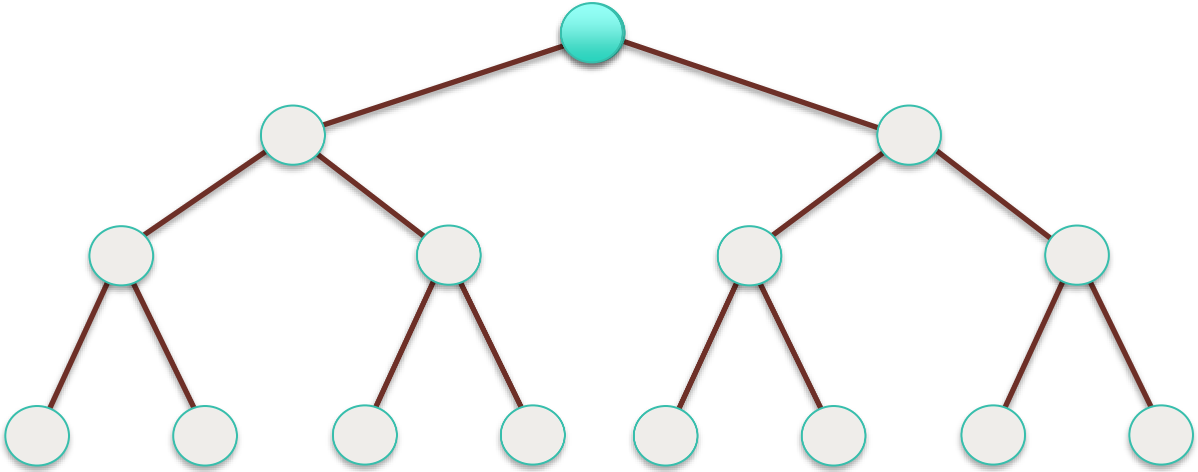
What it is and why

- Look for a given node
  - Stop when node found, even if not all nodes were visited
  - Not guarantee optimization

Breadth-First Search (BFS): Tree



Breadth-First Search (BFS): Tree



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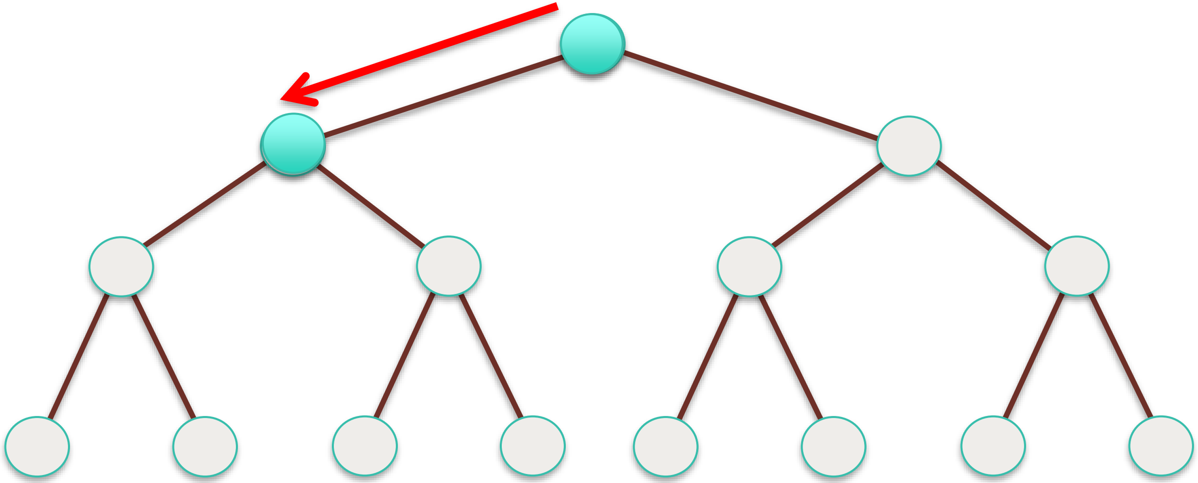

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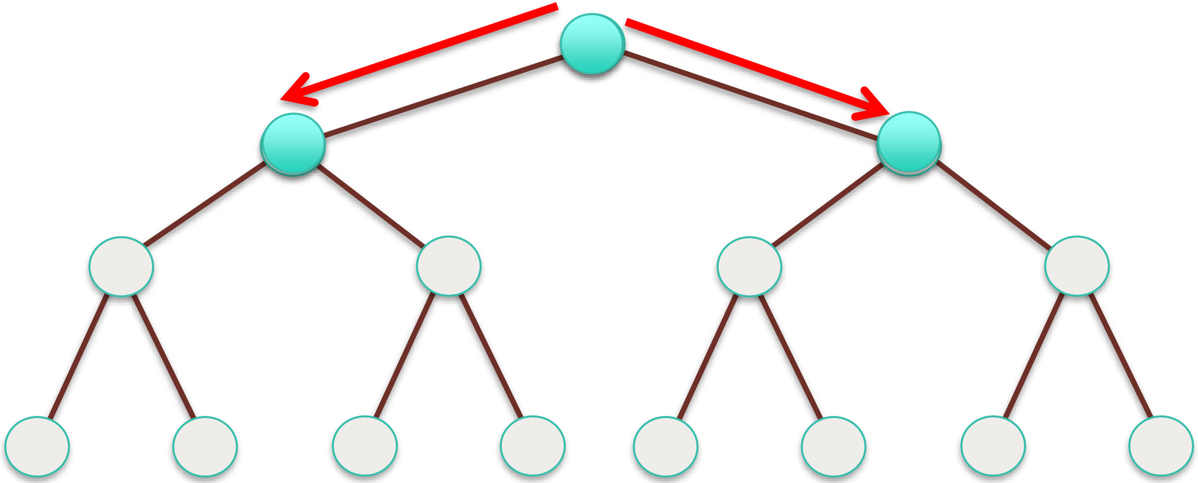

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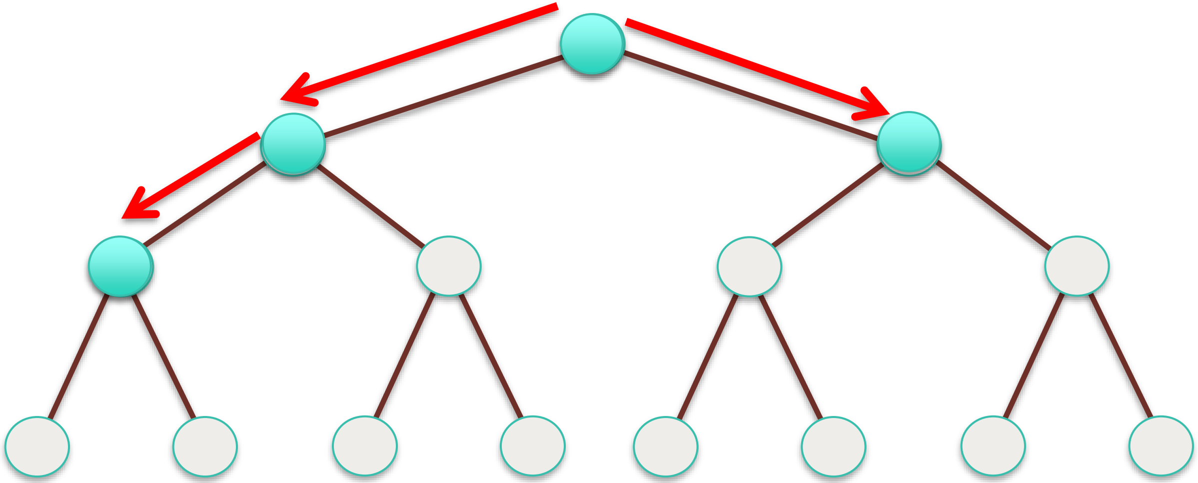

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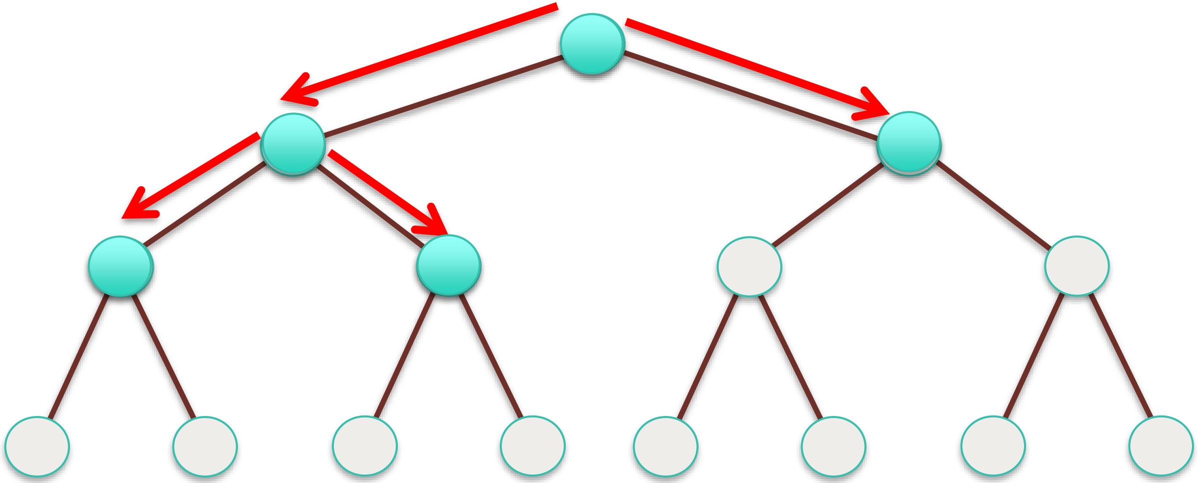

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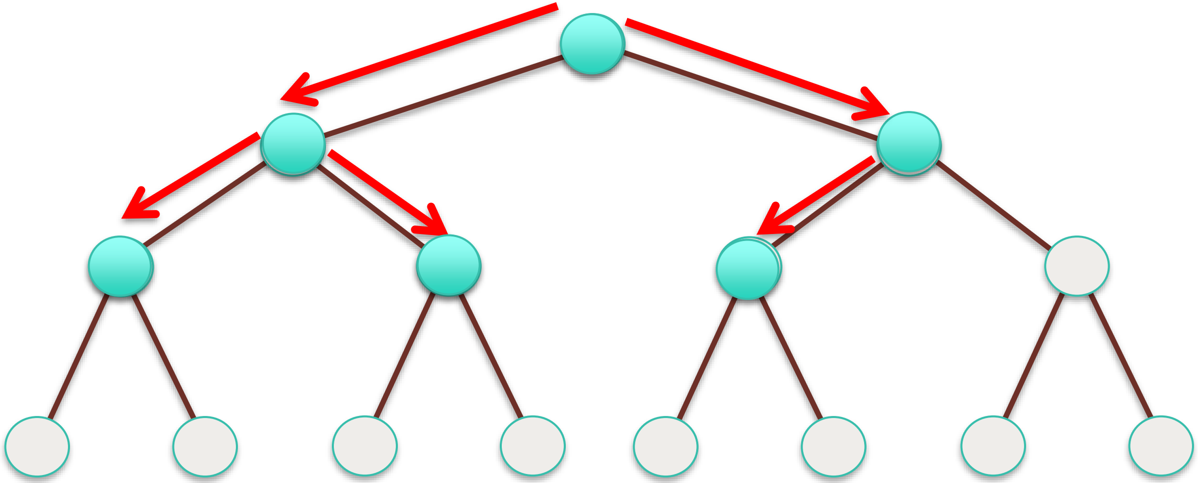

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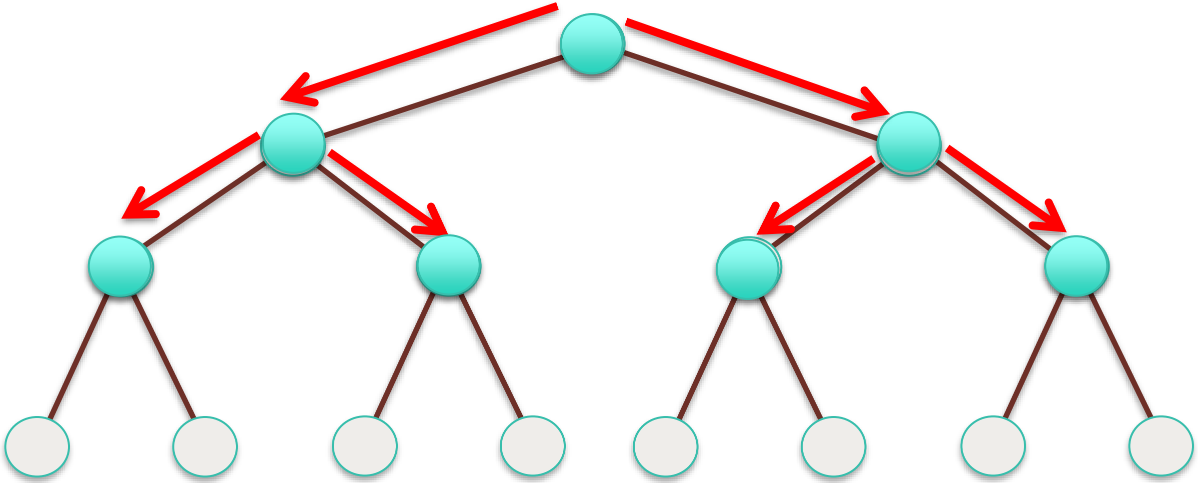

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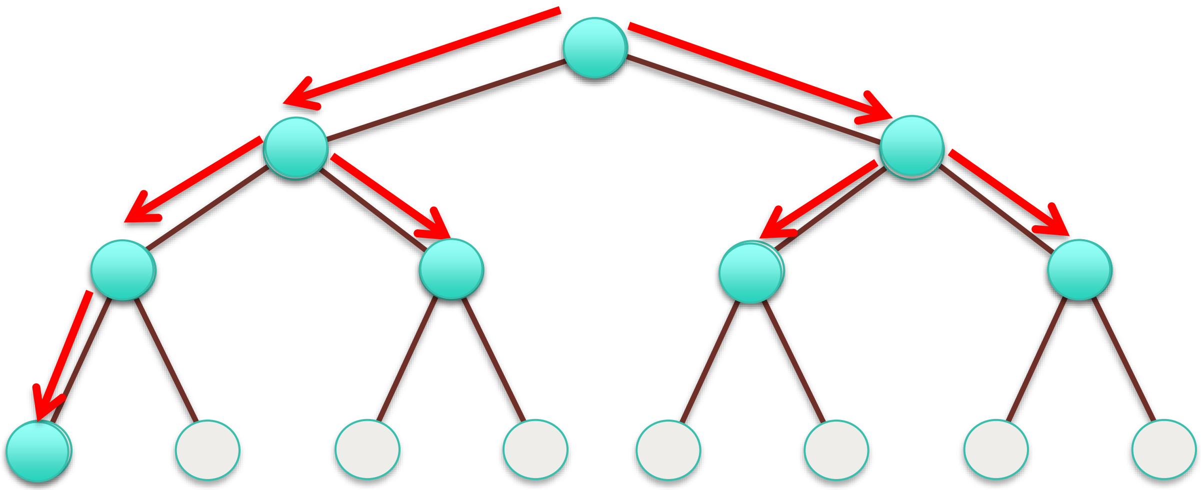

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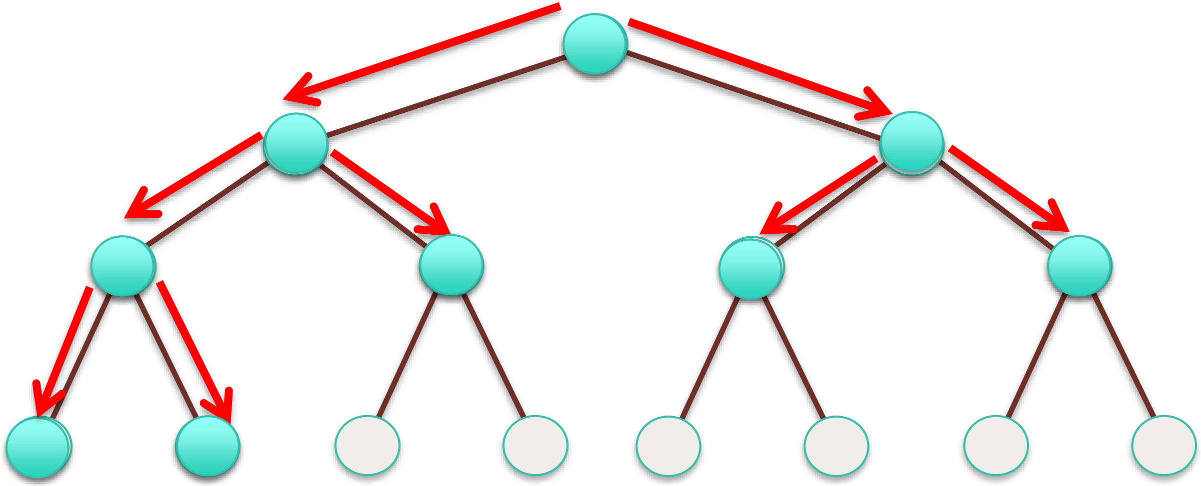

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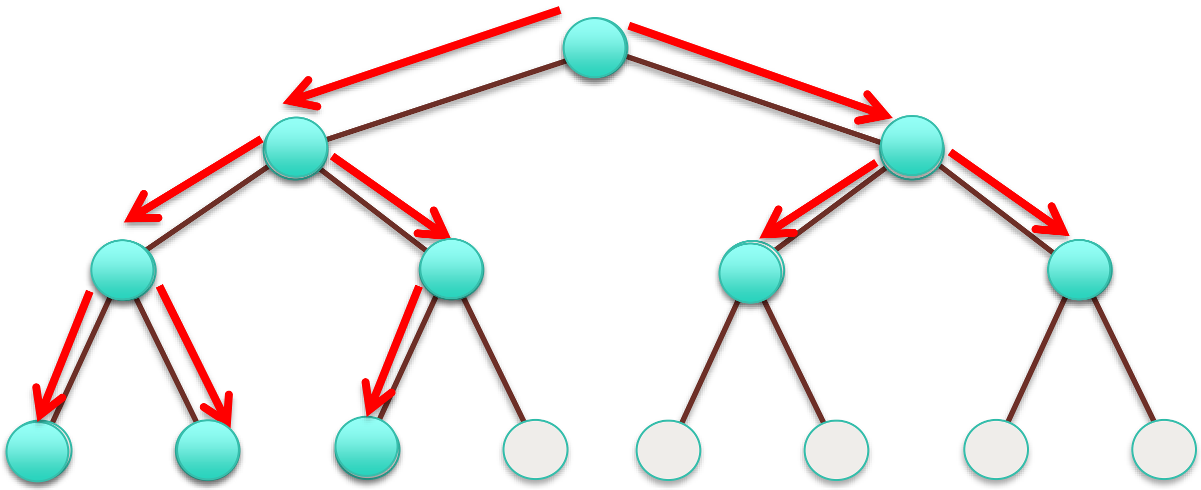

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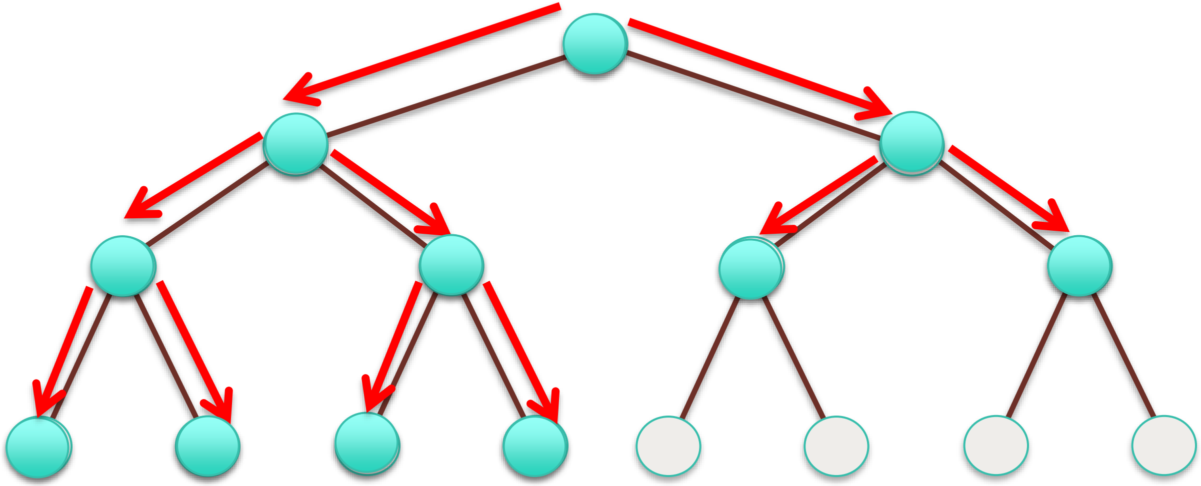

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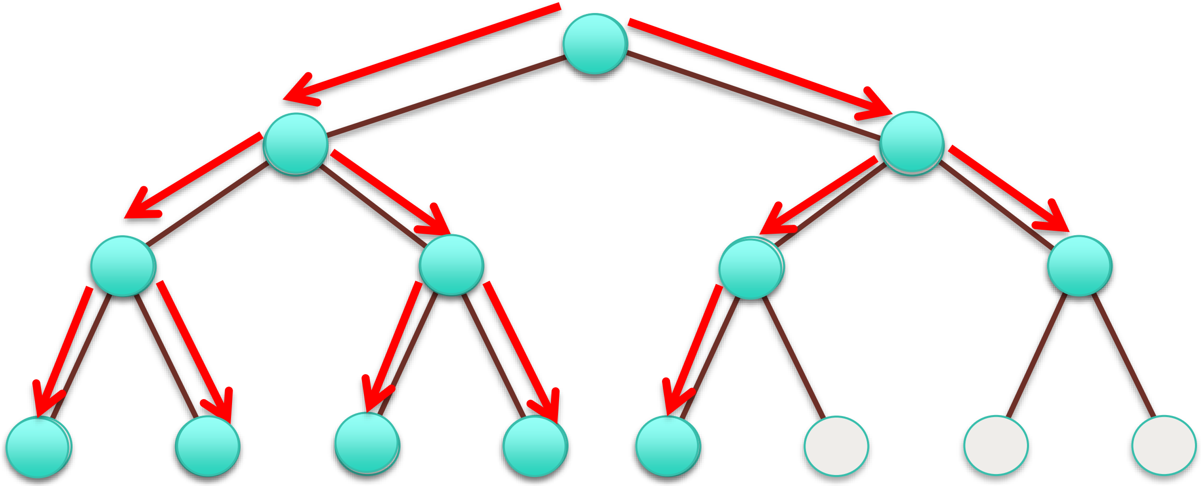

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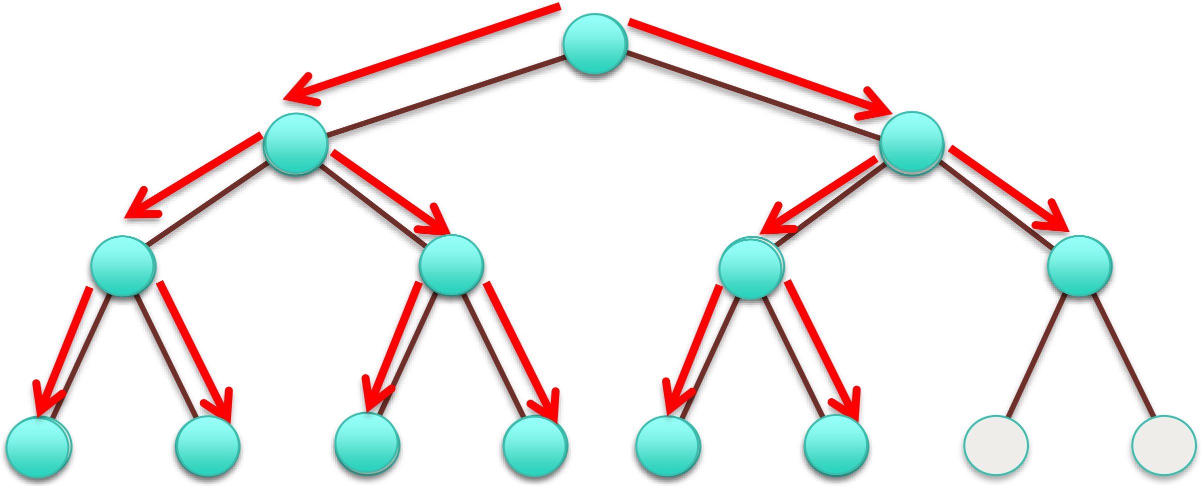

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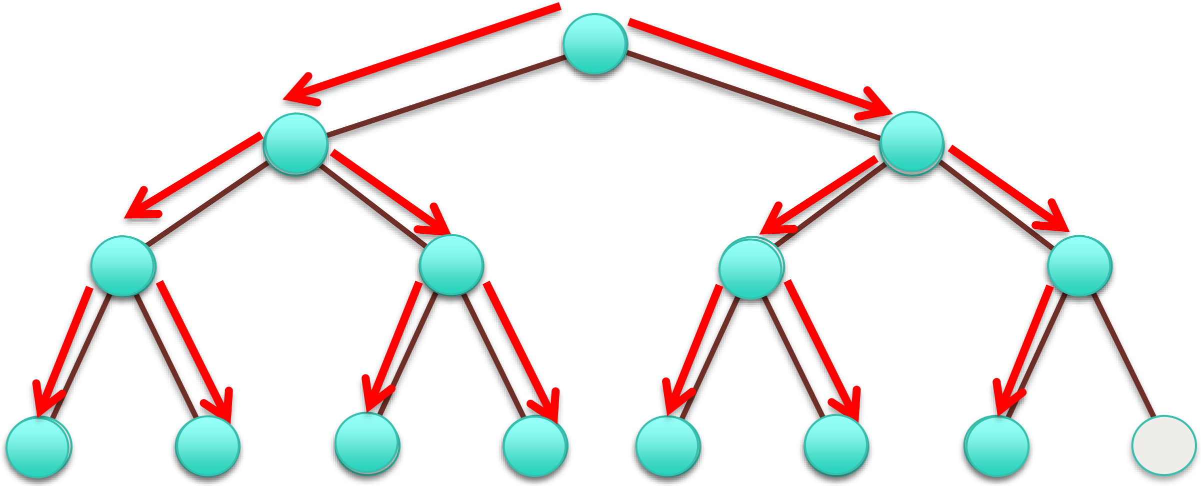

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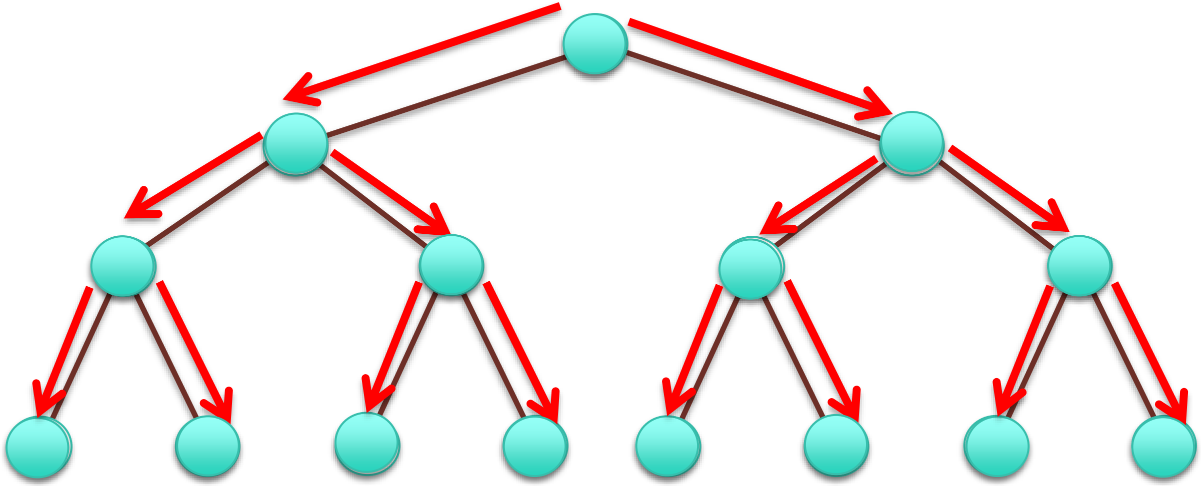

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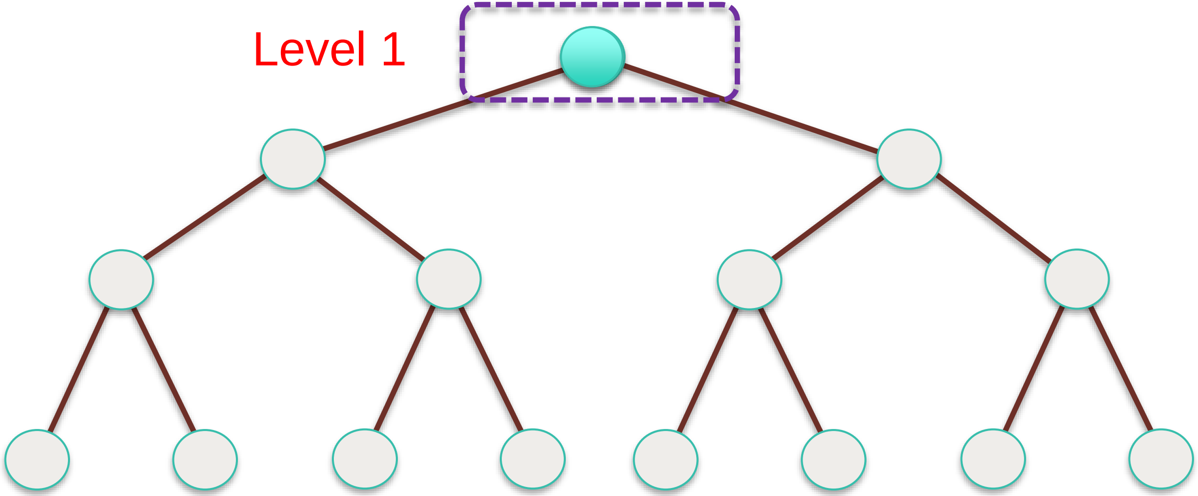

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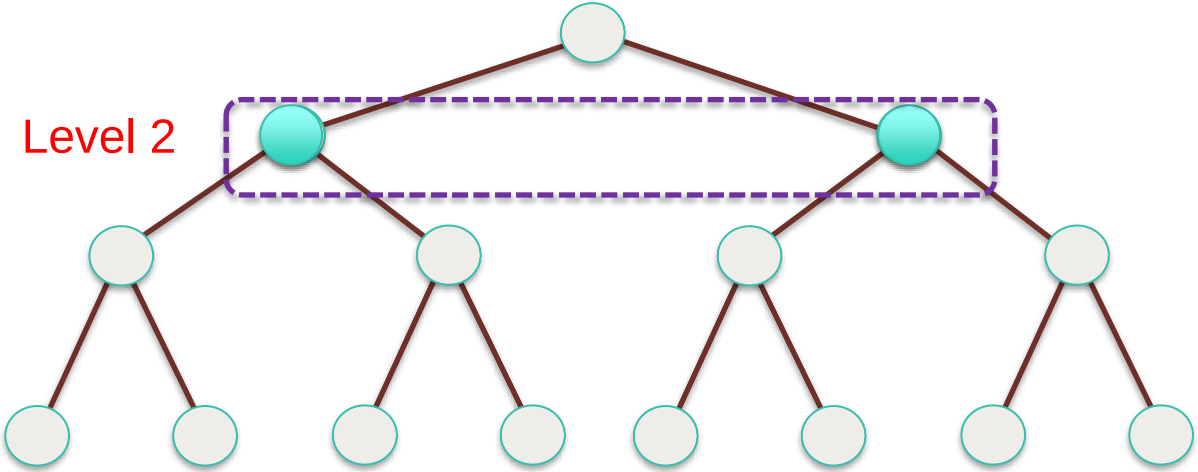
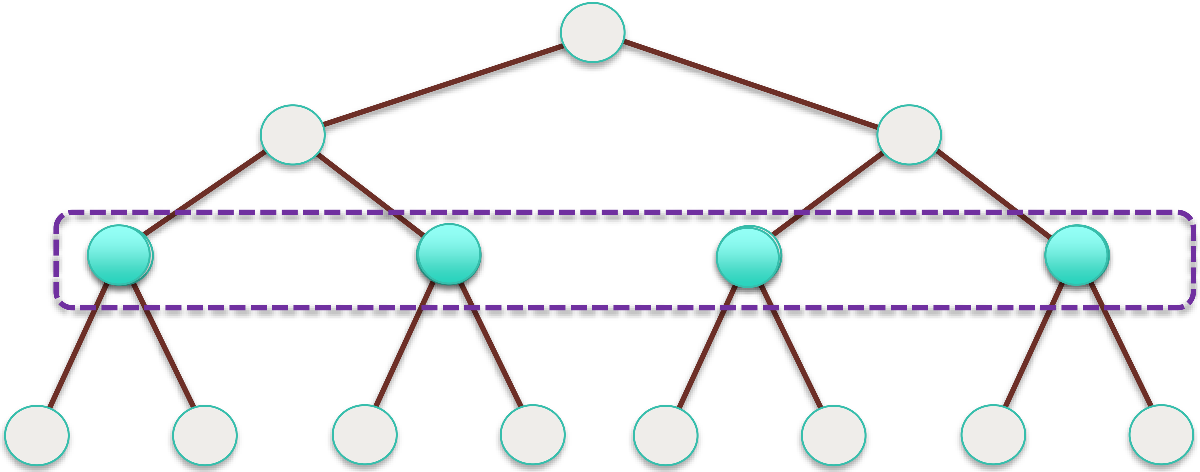


Breadth-First Search (BFS): Tree

Level 3



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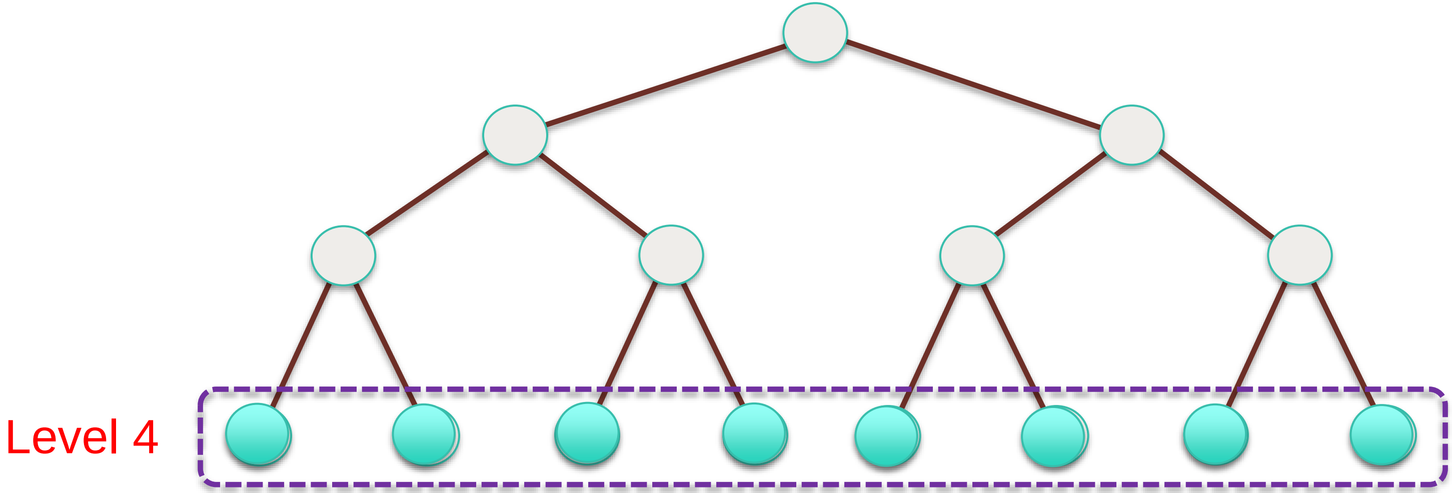
Breadth-First Search (BFS): Tree



Breadth- First Search (BFS)

- Used to search in undirected graph
- Start with a node and then visit all its adjacent nodes in the same level
- Move to the adjacent successor node in the next level
- Level-by-level search

Breadth- First Search (BFS) Steps

- Step 1: Start with the root node (any node in the graph), mark it visited
- Step 2: If that node has no node with same level, go to the next level
- Step 3: Visit all adjacent nodes and marked them visited
- Step 4: Go to the next level and visit all unvisited adjacent nodes
- Step 5: Continue until all nodes visited or solution found

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Notes

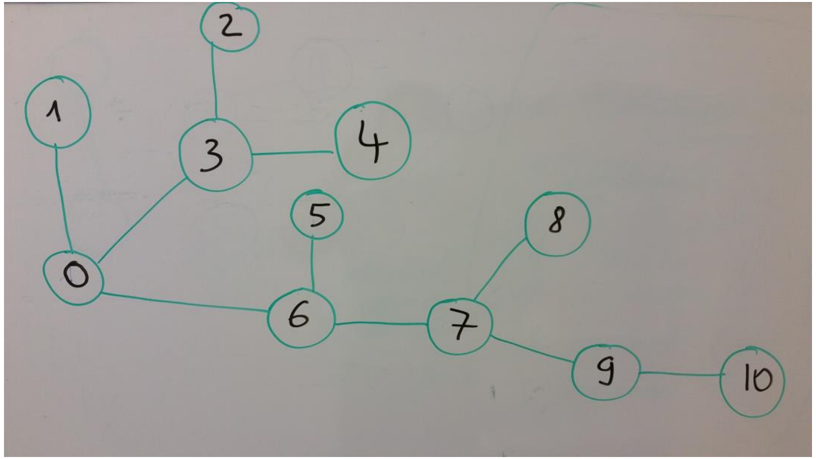
- Breadth- First Search (BFS) is different from Best-First Search (BFS) algorithm in the textbook
- Best-First Search is an algorithm that traverses a graph to reach a target in the shortest possible path.
- Best-First Search follows an evaluation function to determine which node is the most appropriate to traverse next
- But we don't look at Best-First Search (BFS)

BSF Pseudo-code (serial)

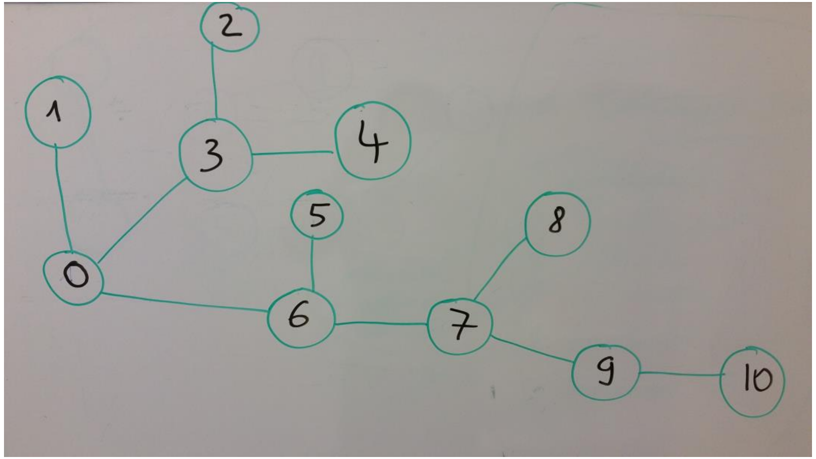
```
Input: G(V,E,w)
Output: List of visited nodes in G in ordered

Queue Q = { };
for a node u,
        set visited[u] = false;
        insert Q, u;
while ((Q ⊄∅) || ( u is not the target node)) // Q is not empty or u is not what we are searching for
        delete u from Q
        if (visited[u] = false)
              for each adjacent v of u then
                     insert Q, v;
       endif
endwhile
```

Sample ‘Implementation’



Sample 'Implementation'



Data structure first: adjacent matrix or adjacent list ?
Start vertex: 0 (but can be any other vertex)
Data Structure: Queue (First-in-First-Out)
Q = ?
Step 1: Q = {0}
Step 2: Q = {1,3,6}
Step 3: Q = {3,6}
Step 4: Q = {6,2,4}
Step 5: Q = {6,2,4}
Step 6: Q = {2,4,5,7}
Step 7: Q= {4,5,7}
Step 9: Q= {5,7}
Step 10: Q= {7}
Step 11: Q= {8,9}
Step 12: Q= {9}
Step 13: Q= {10}
Step 14: Q= {∅} **STOP**

Depth-First Search (DSF) Algorithms

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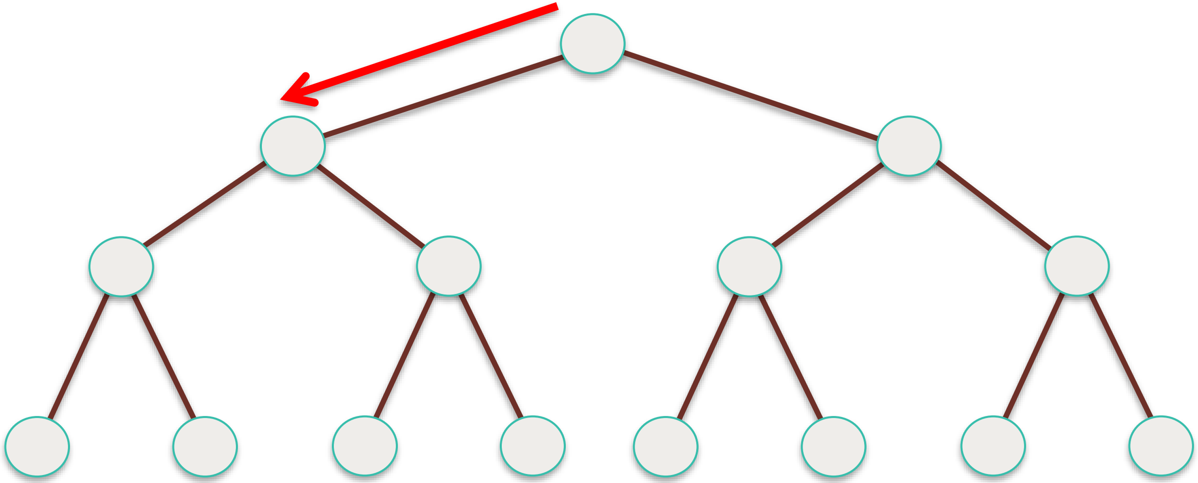
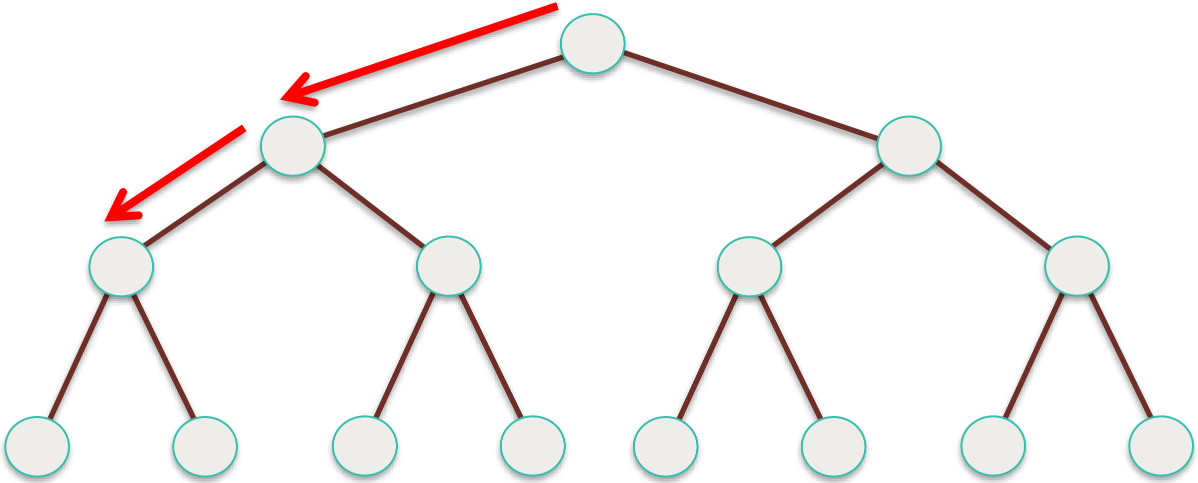
Depth-First Search (DFS): Tree



Depth-First Search (DFS): Tree



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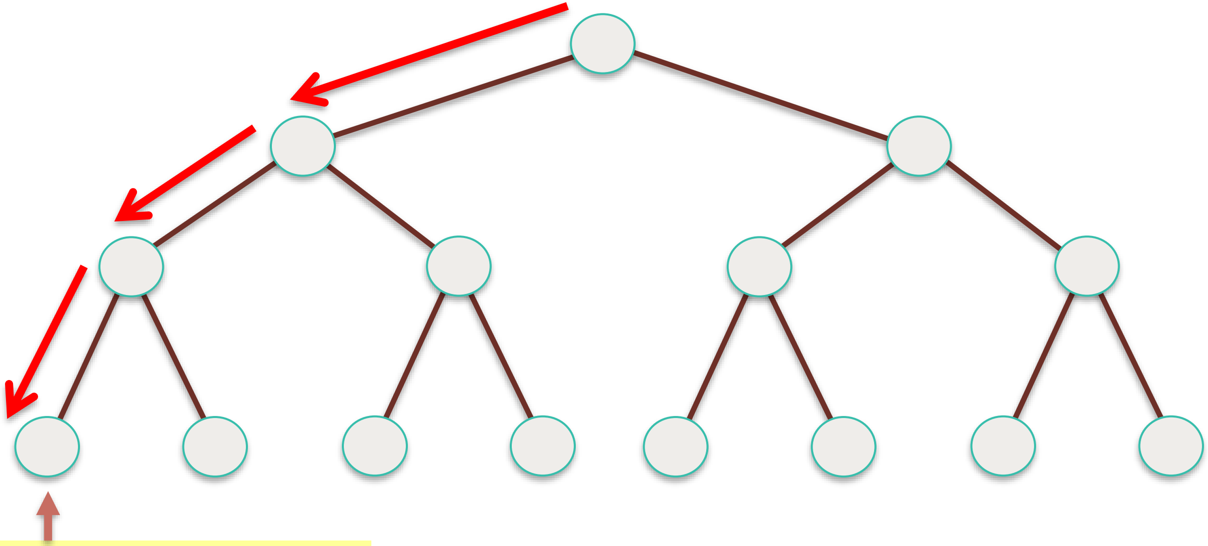
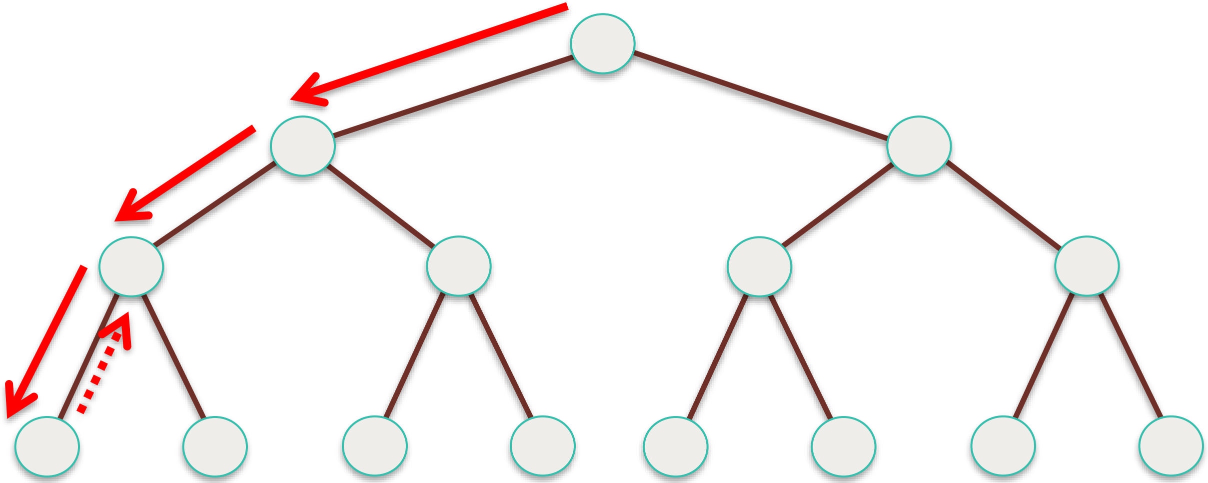


No successor, backtracking!

Depth-First Search (DFS): Tree



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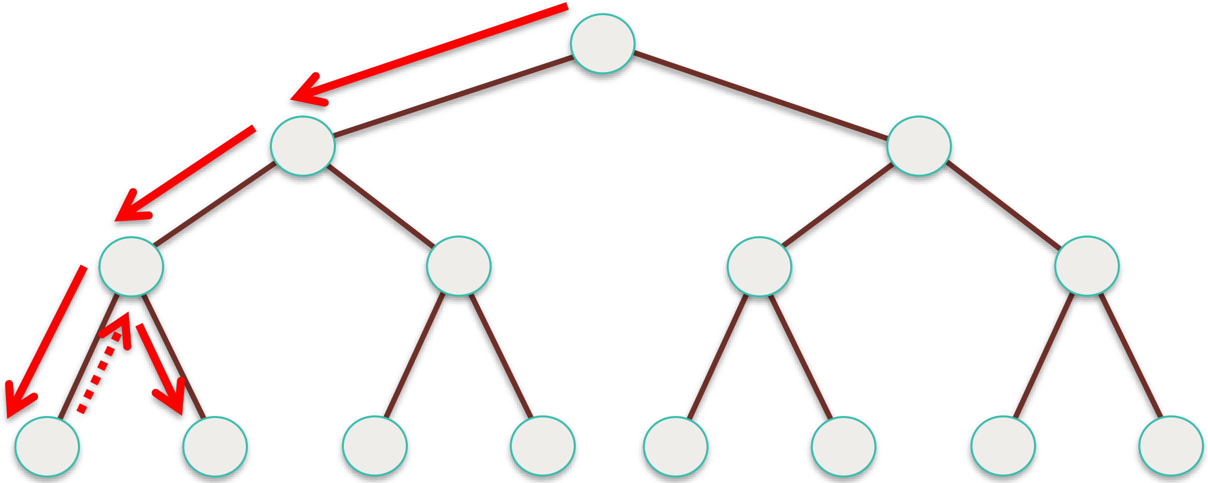

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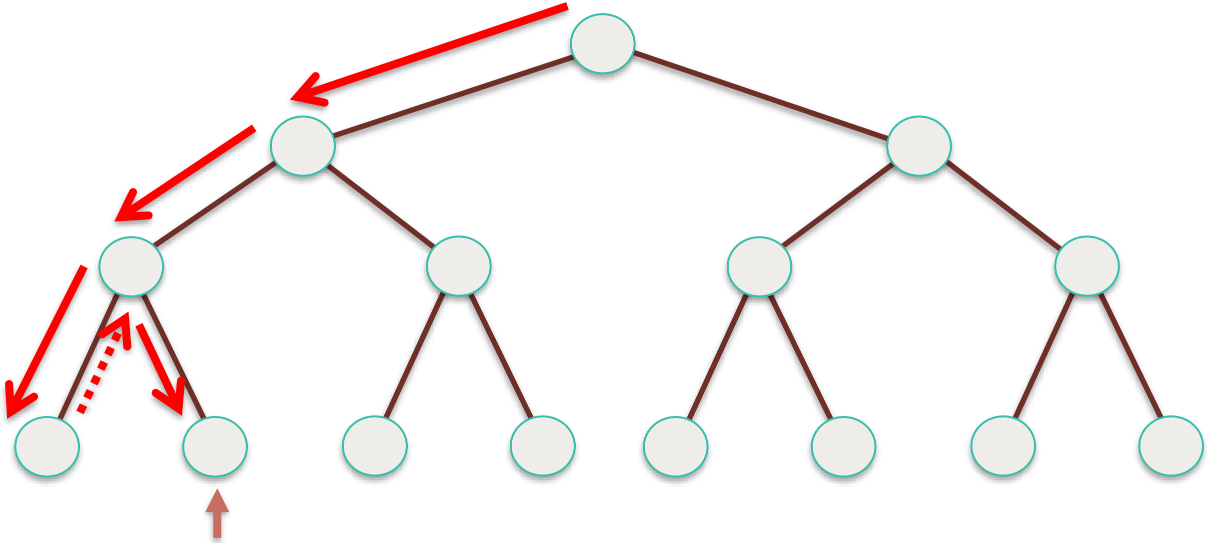
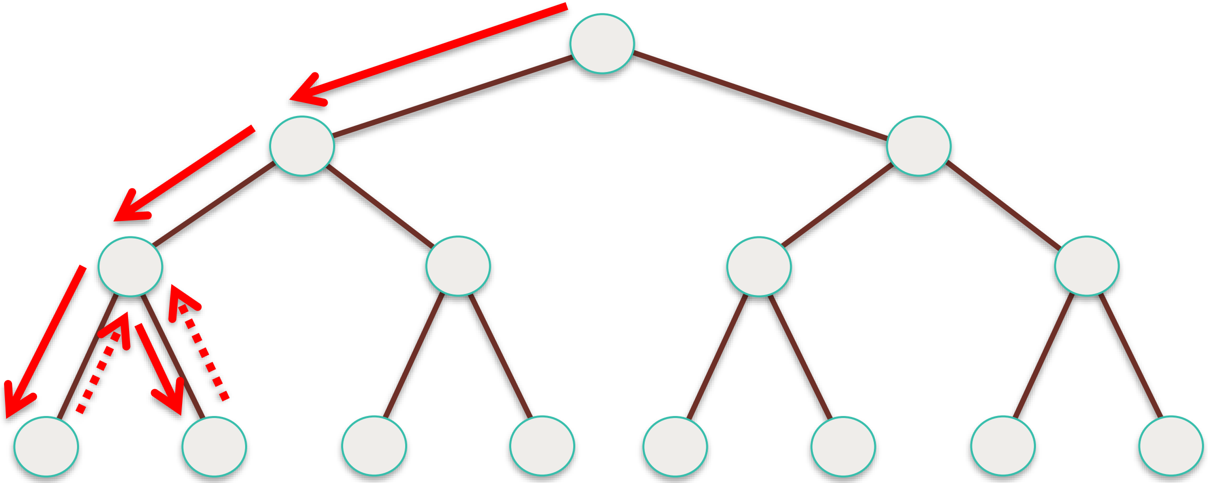


No successor, backtracking!

Depth-First Search (DFS): Tree



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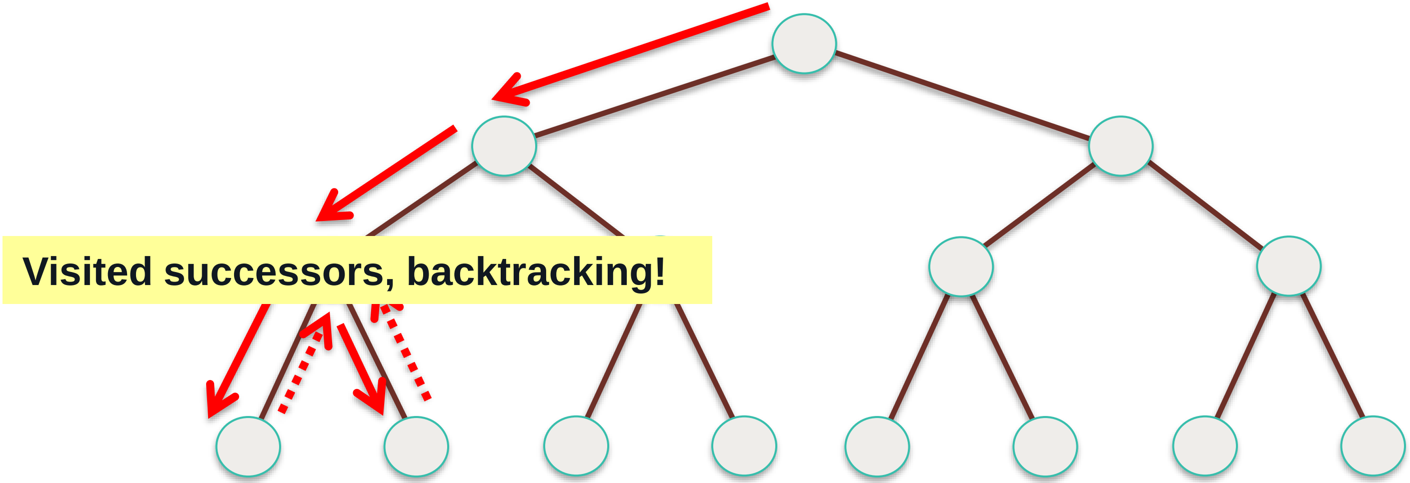
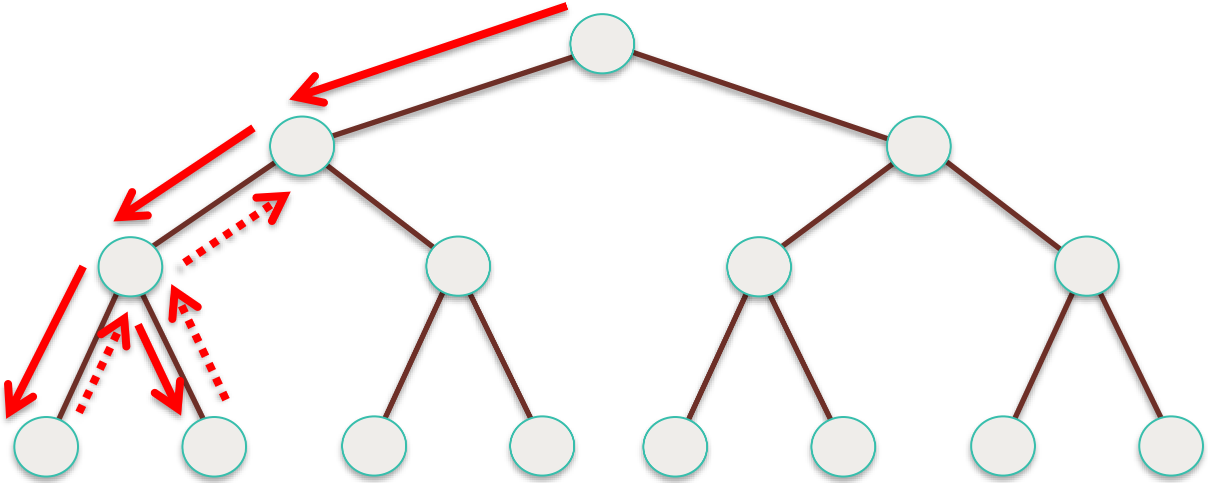


Depth-First Search (DFS): Tree



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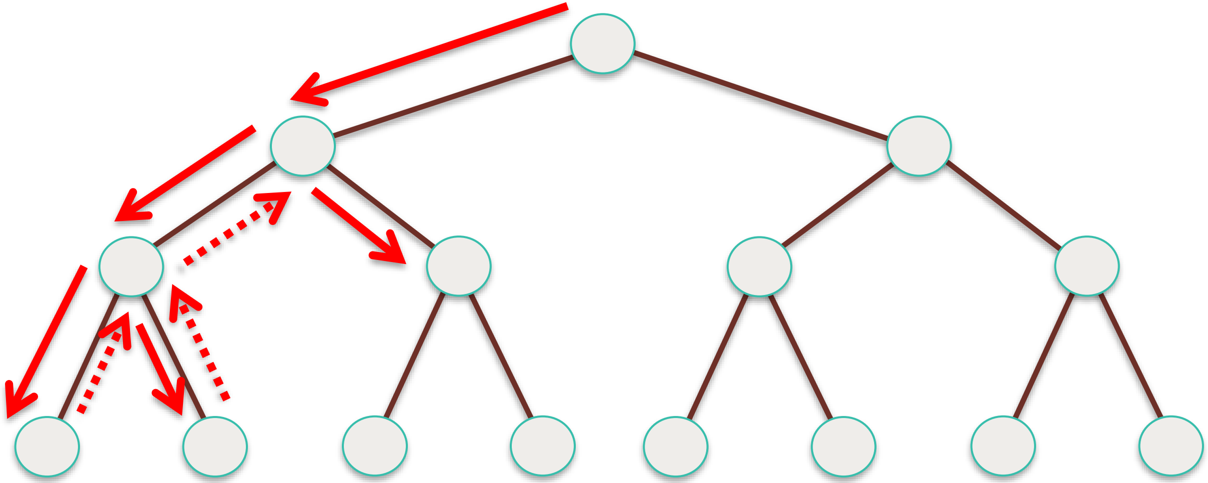

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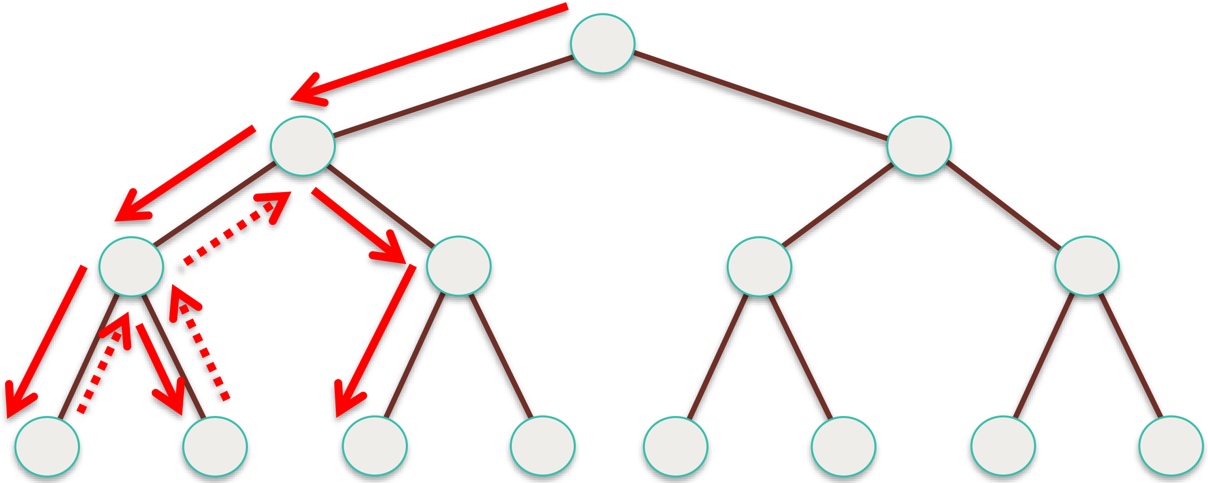

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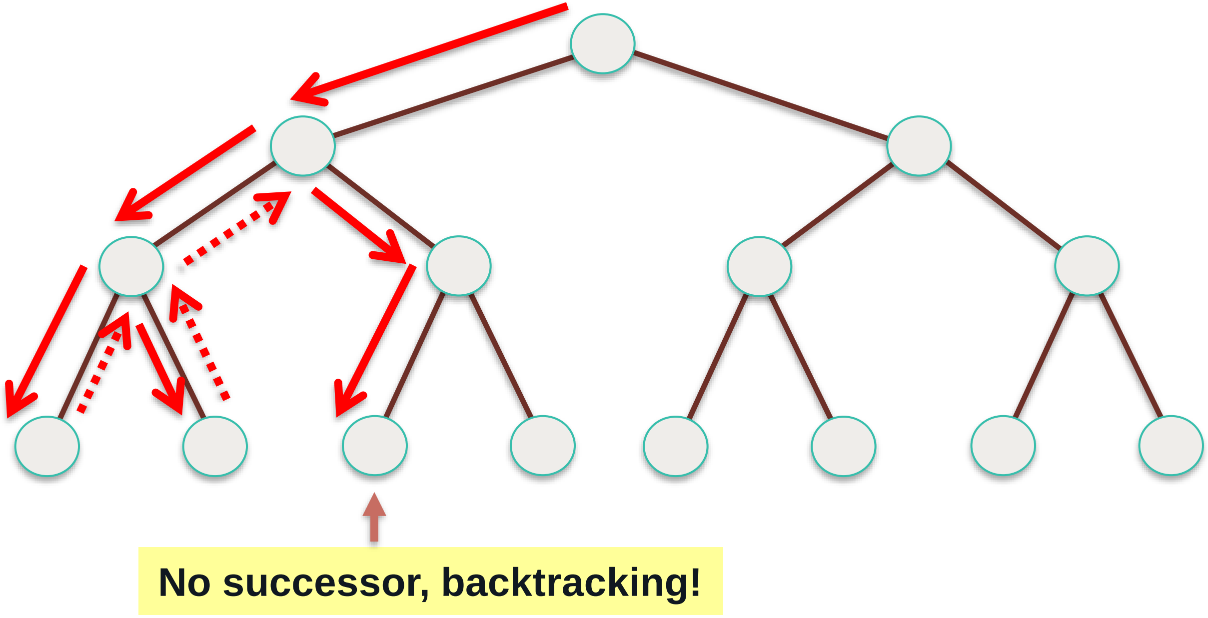

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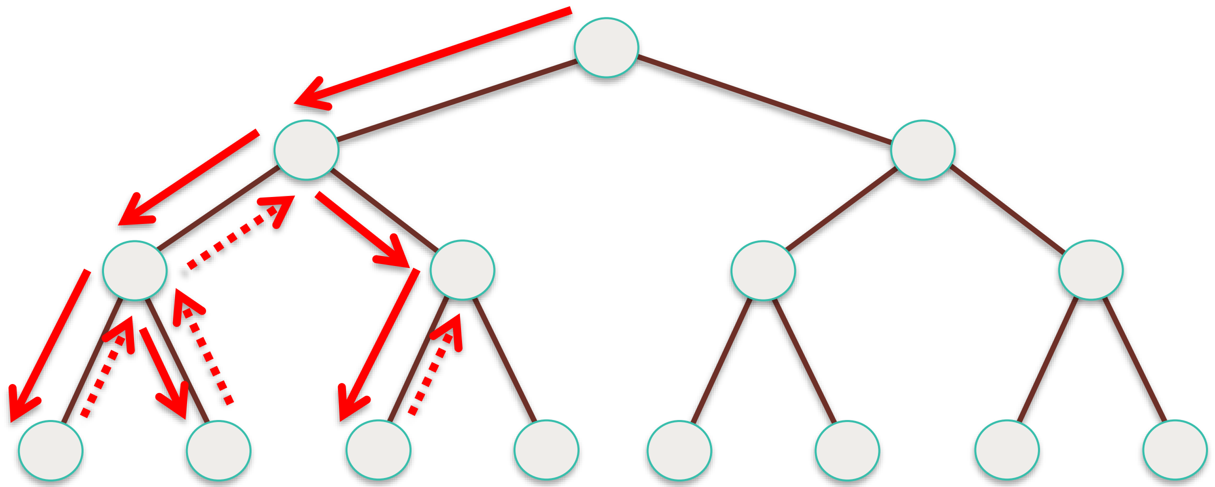

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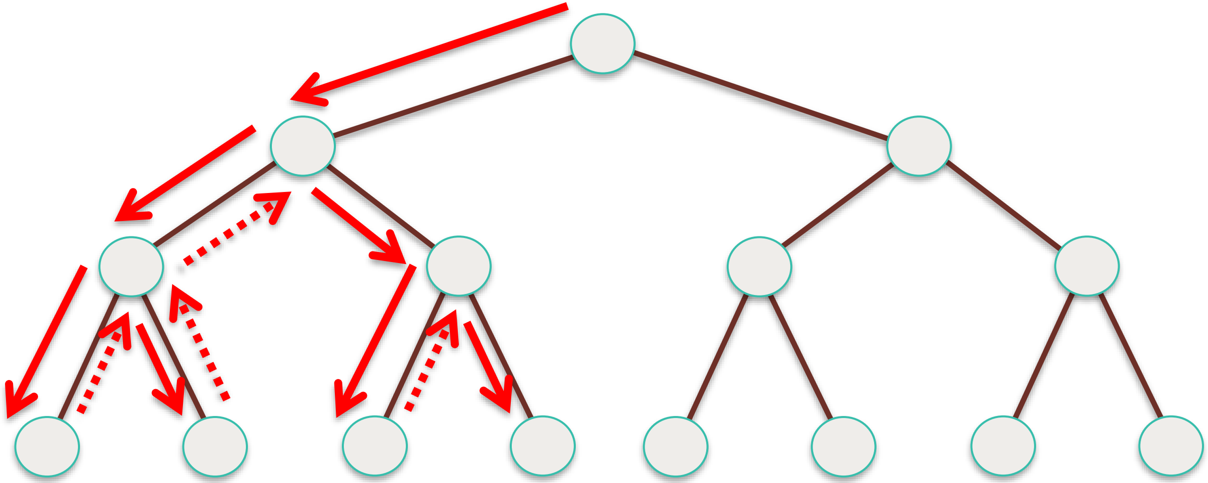

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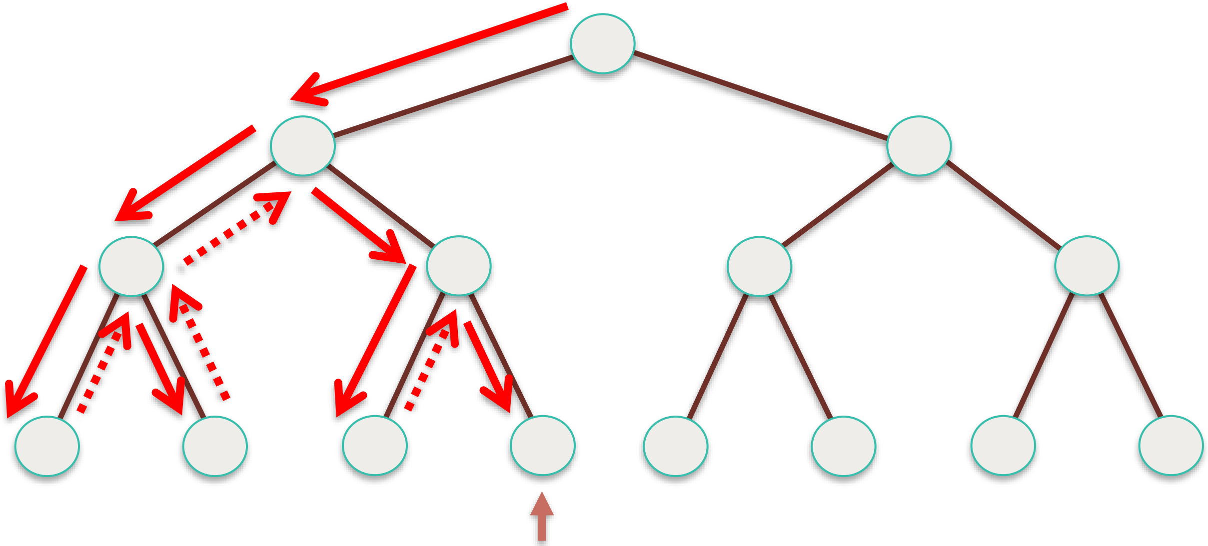
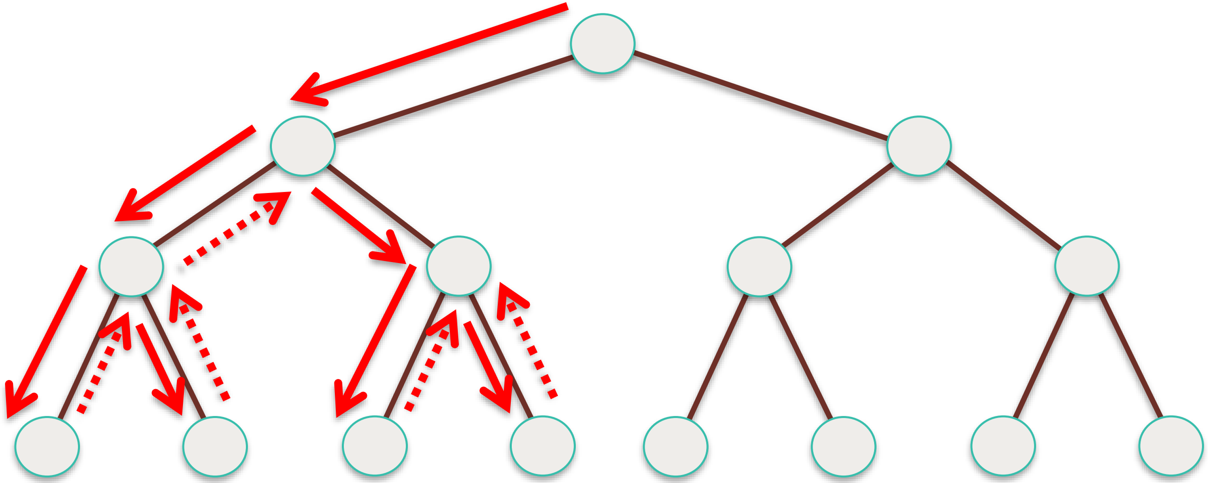


No successor, backtracking!

Depth-First Search (DFS): Tree



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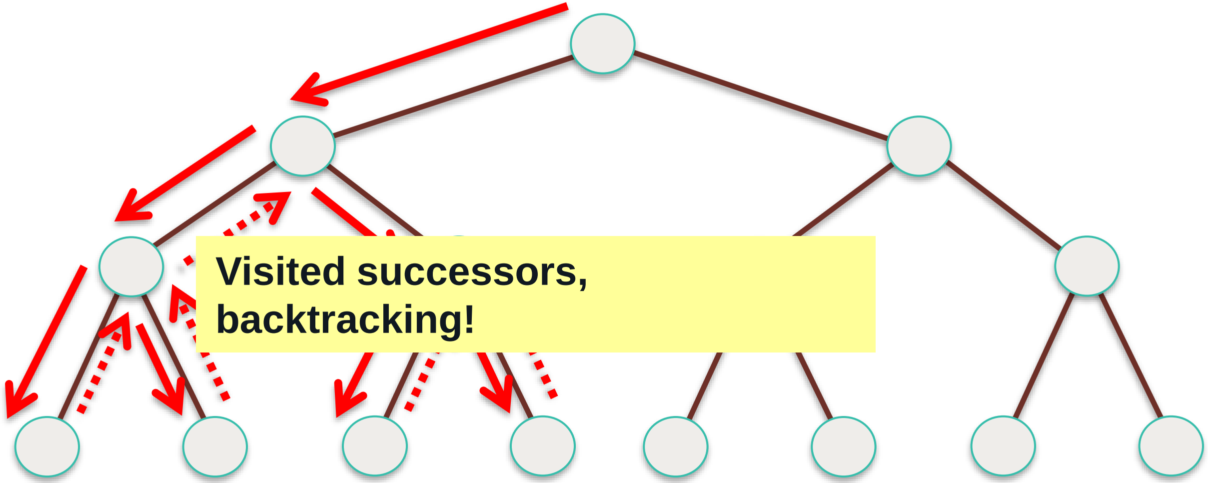

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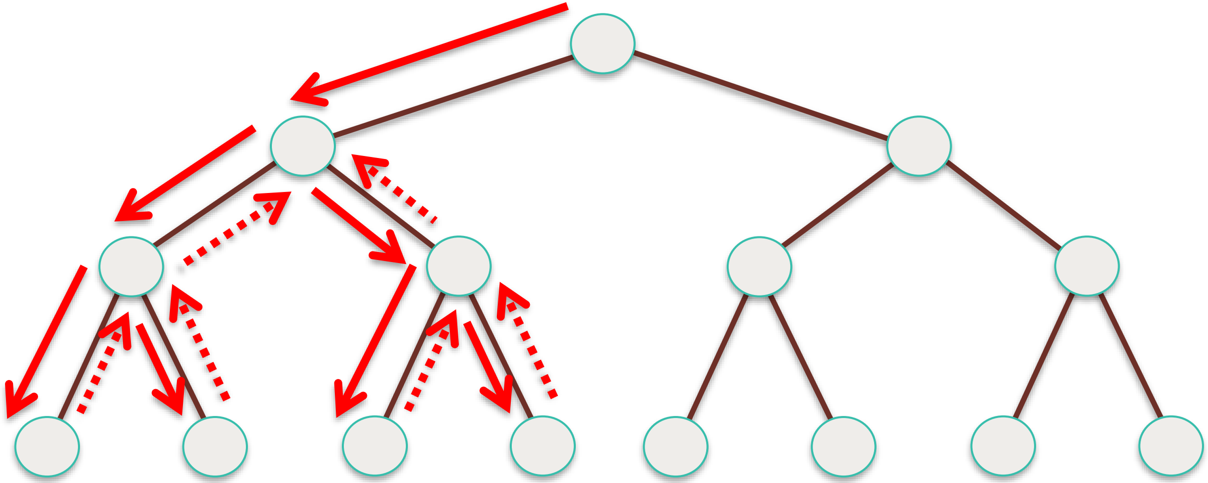

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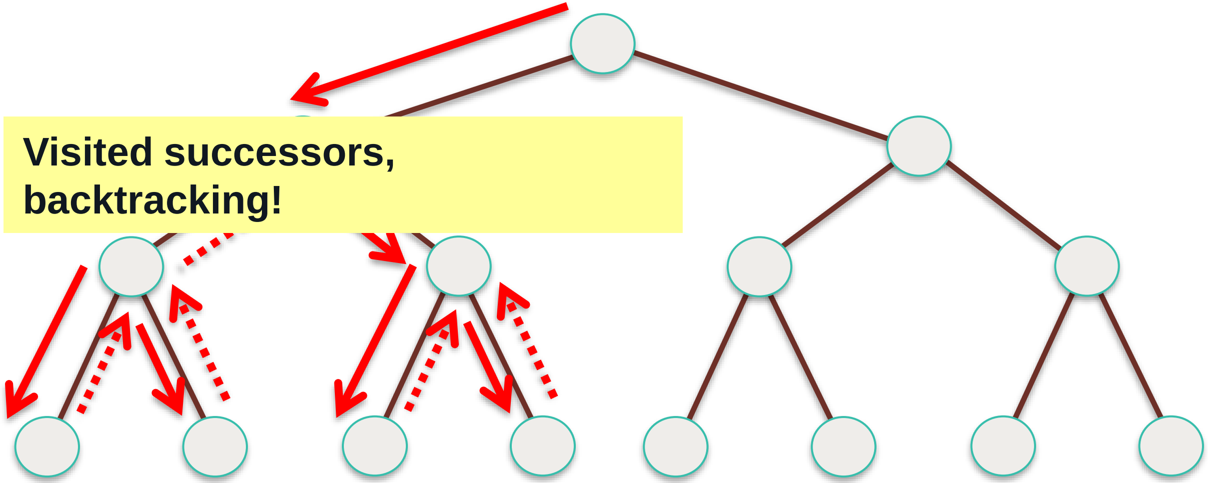

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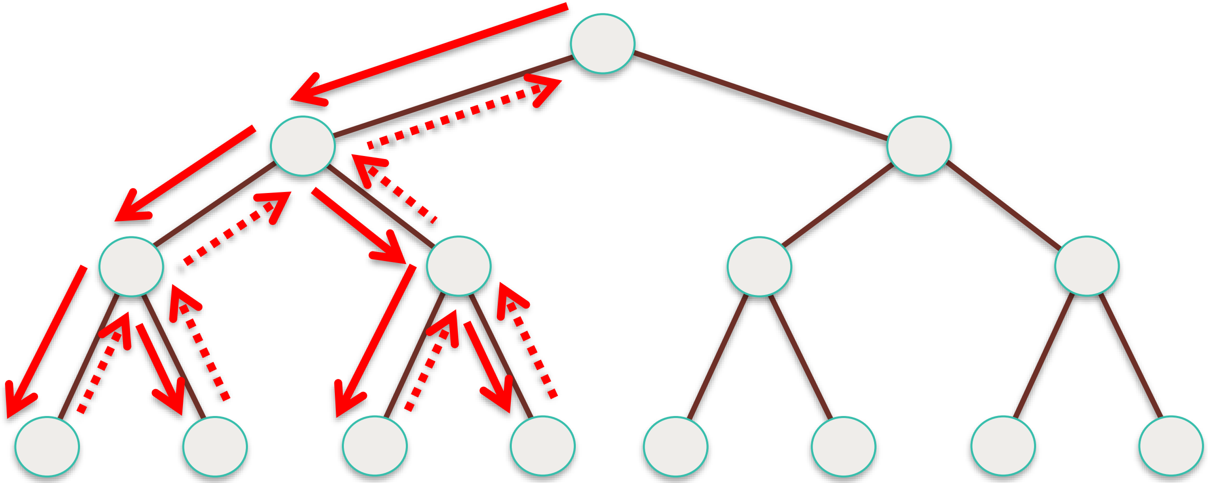

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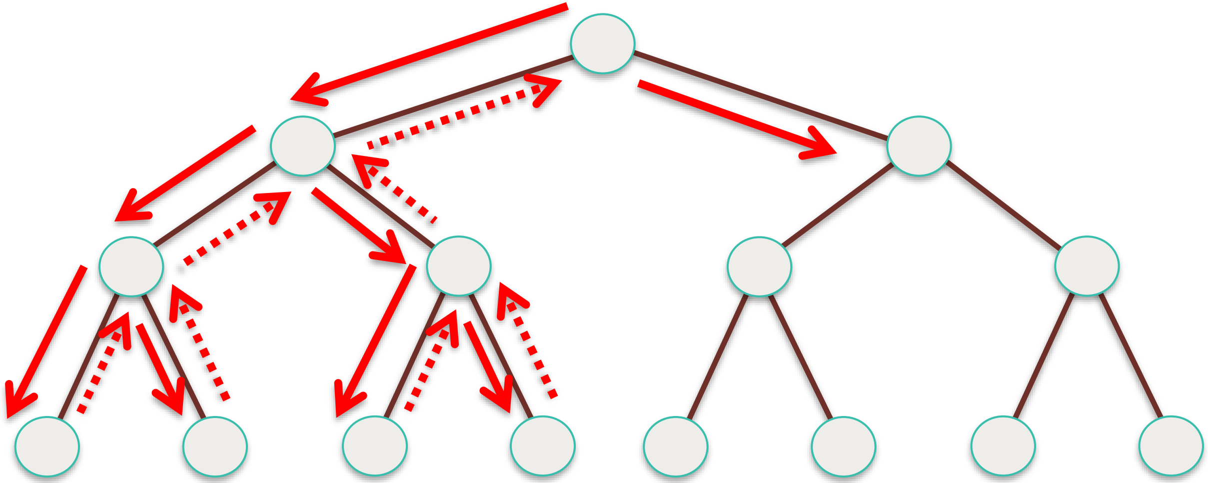

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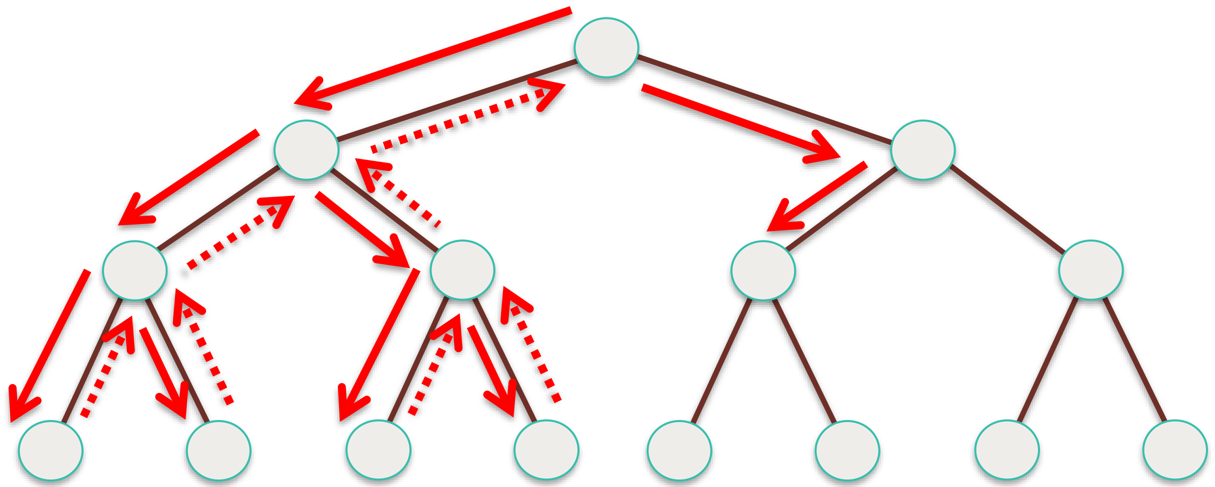

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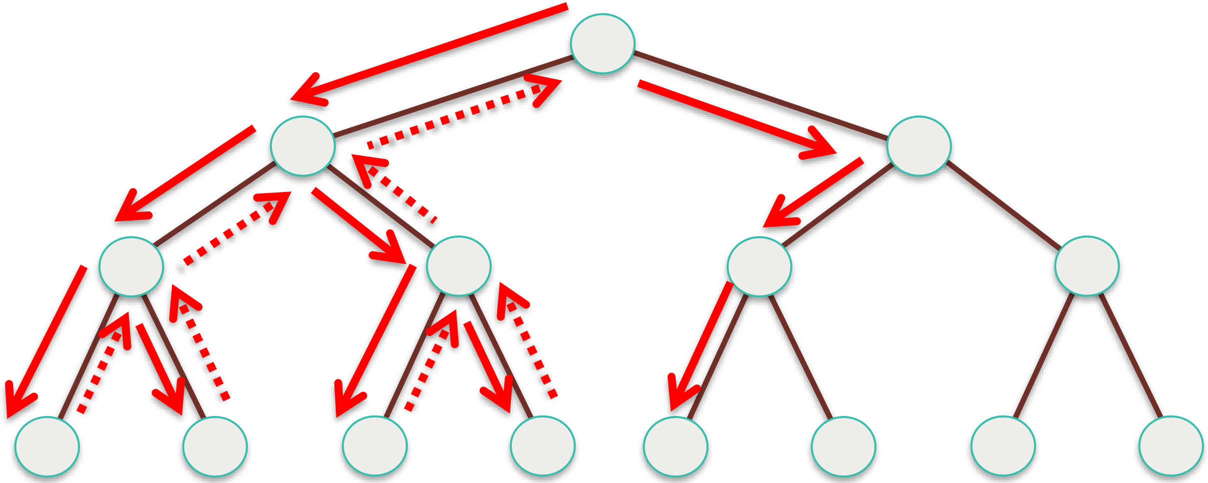

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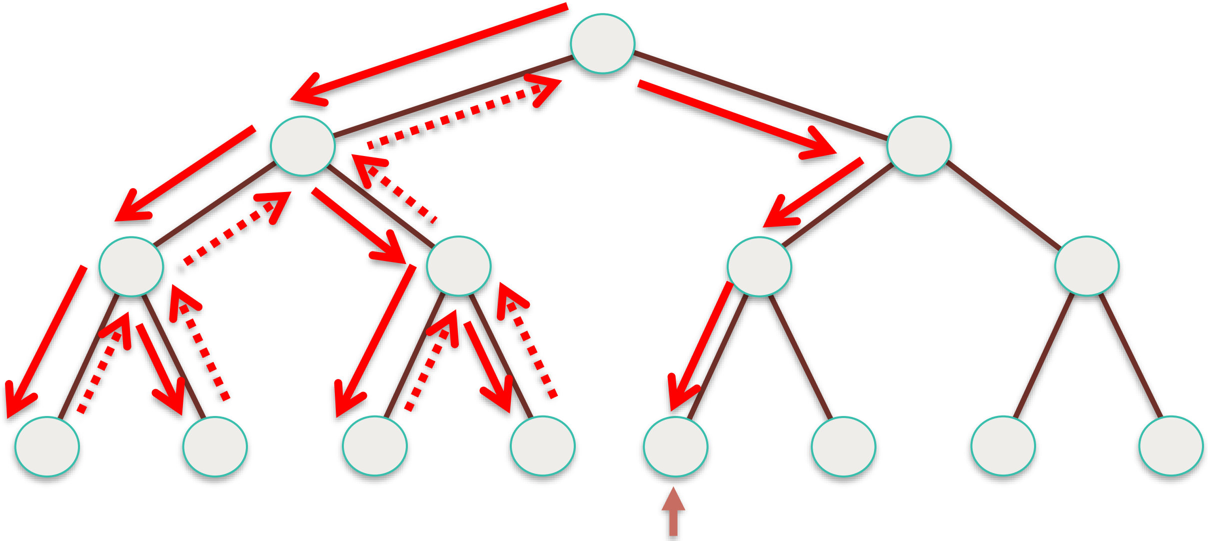

No successor, backtracking!

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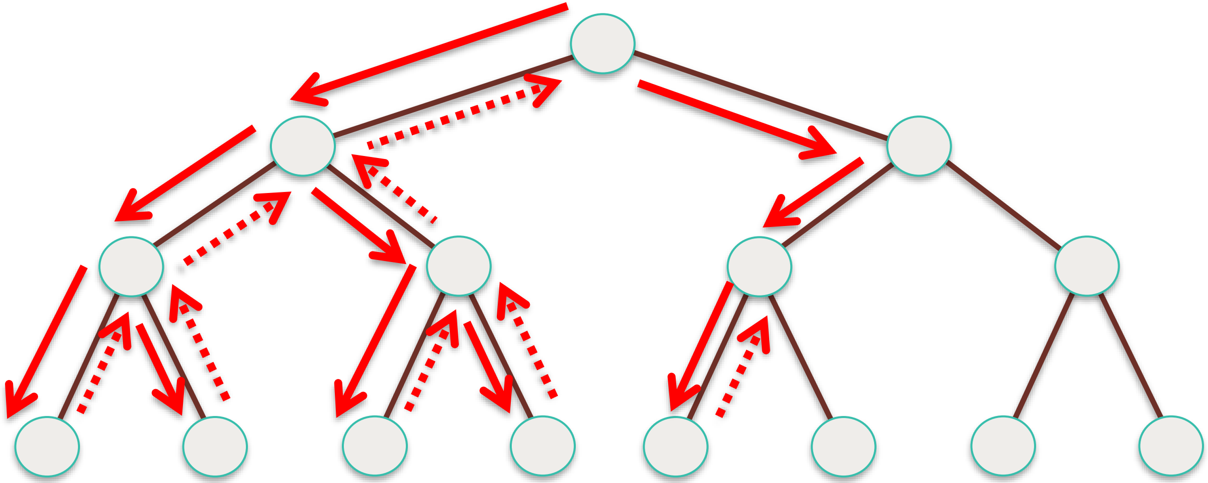

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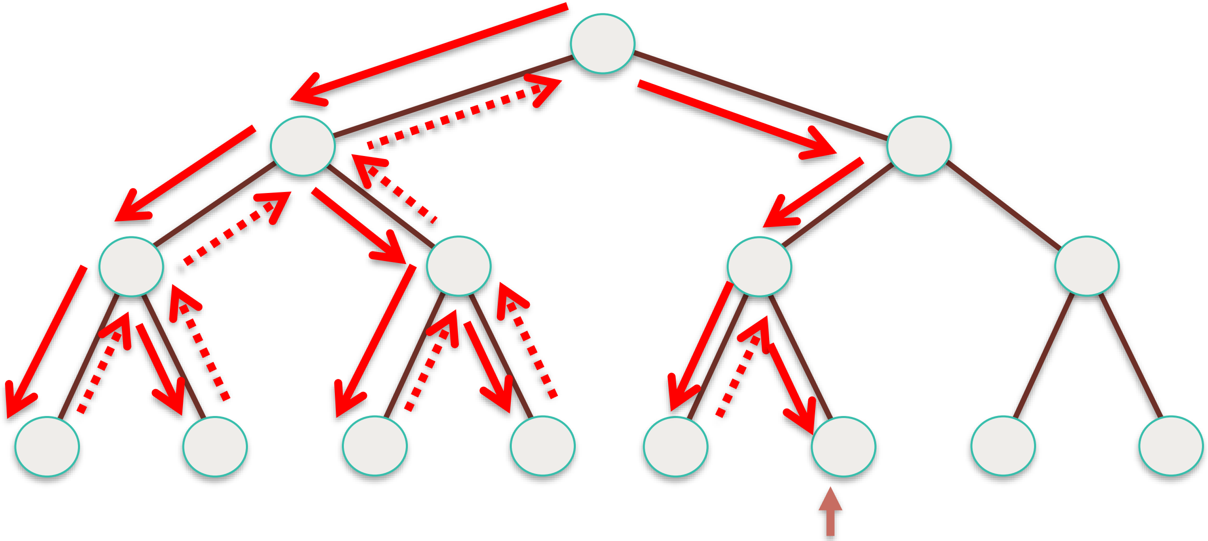

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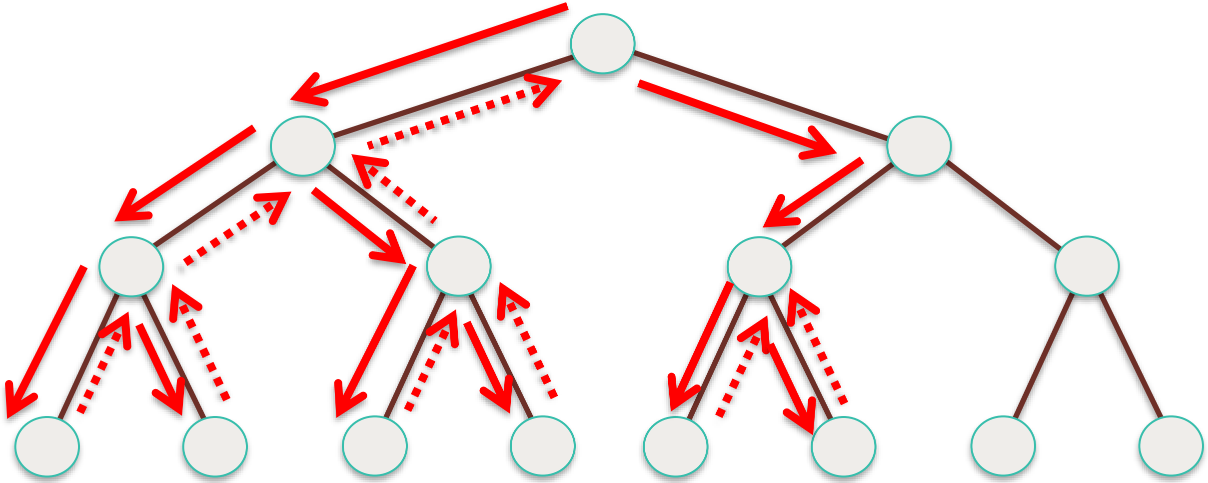

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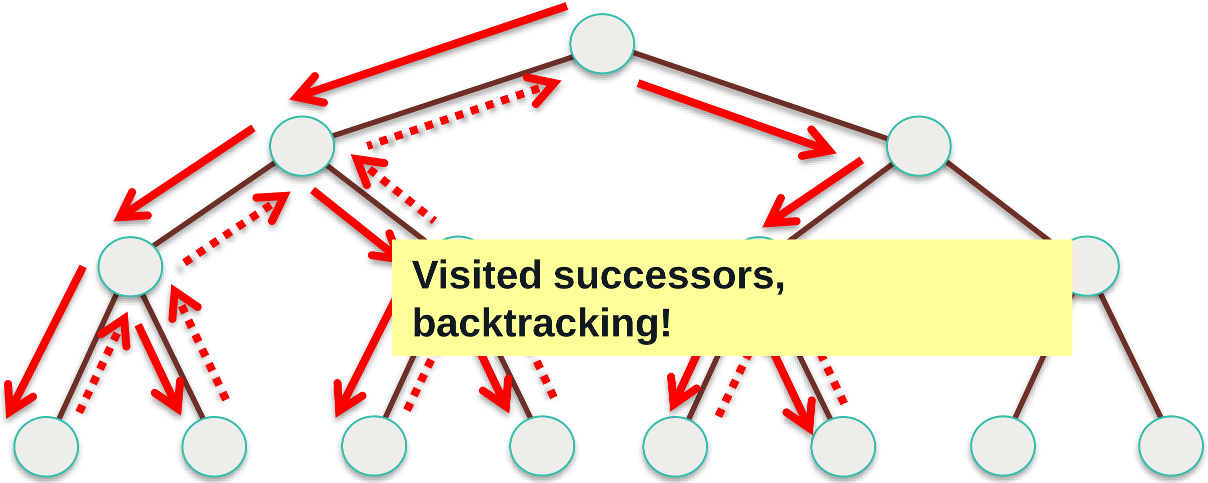

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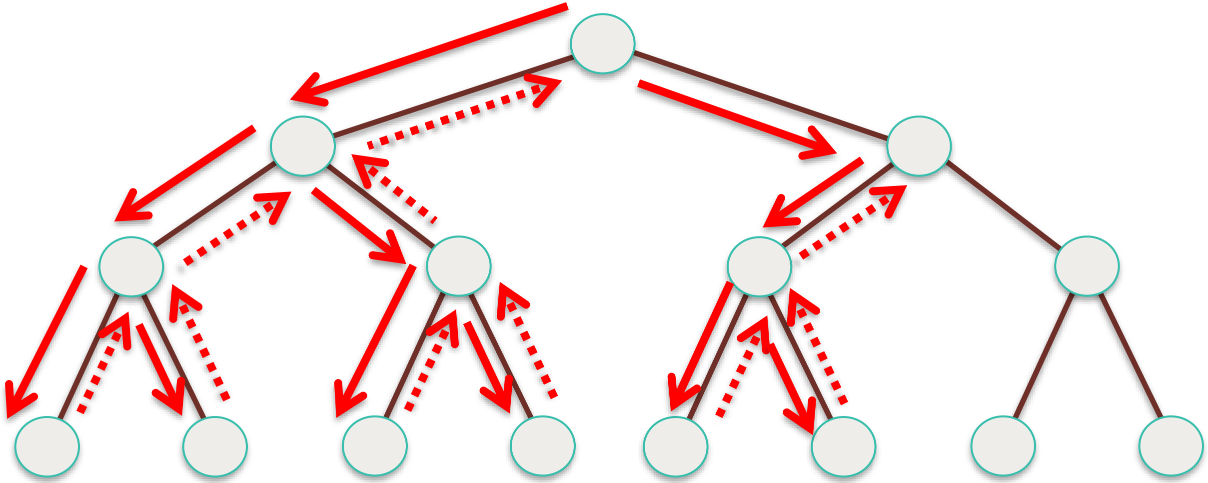

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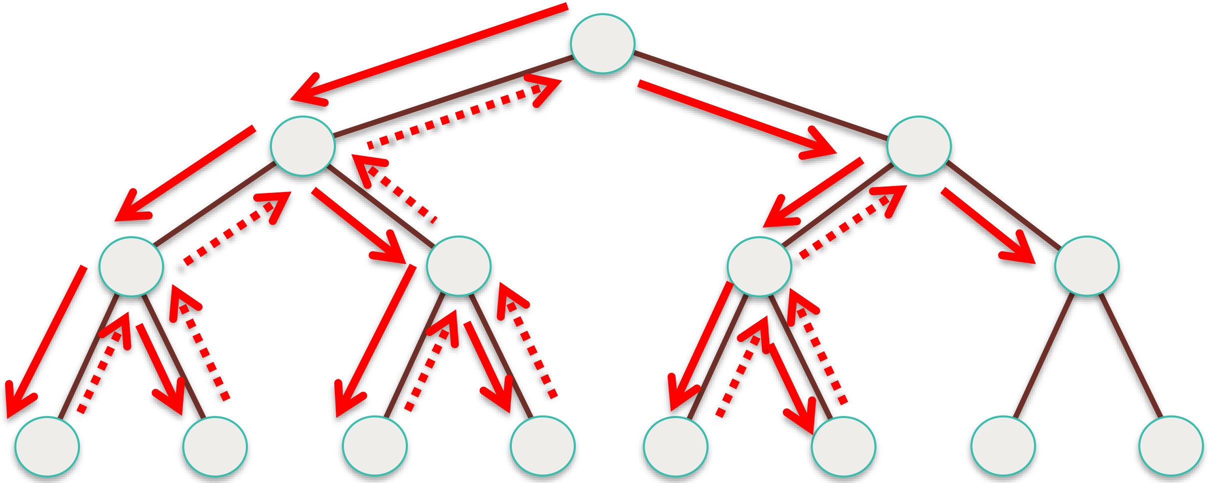

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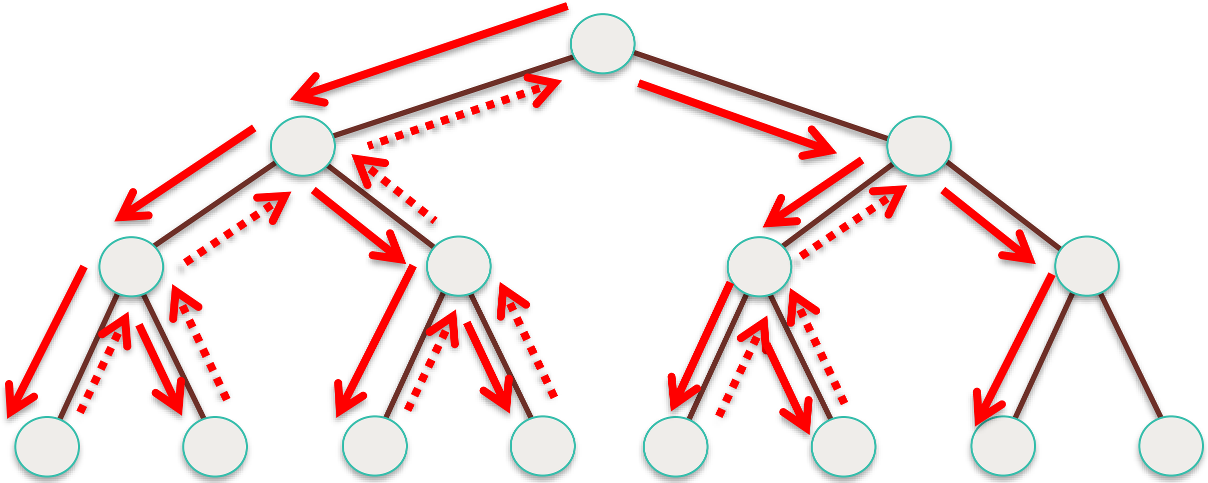

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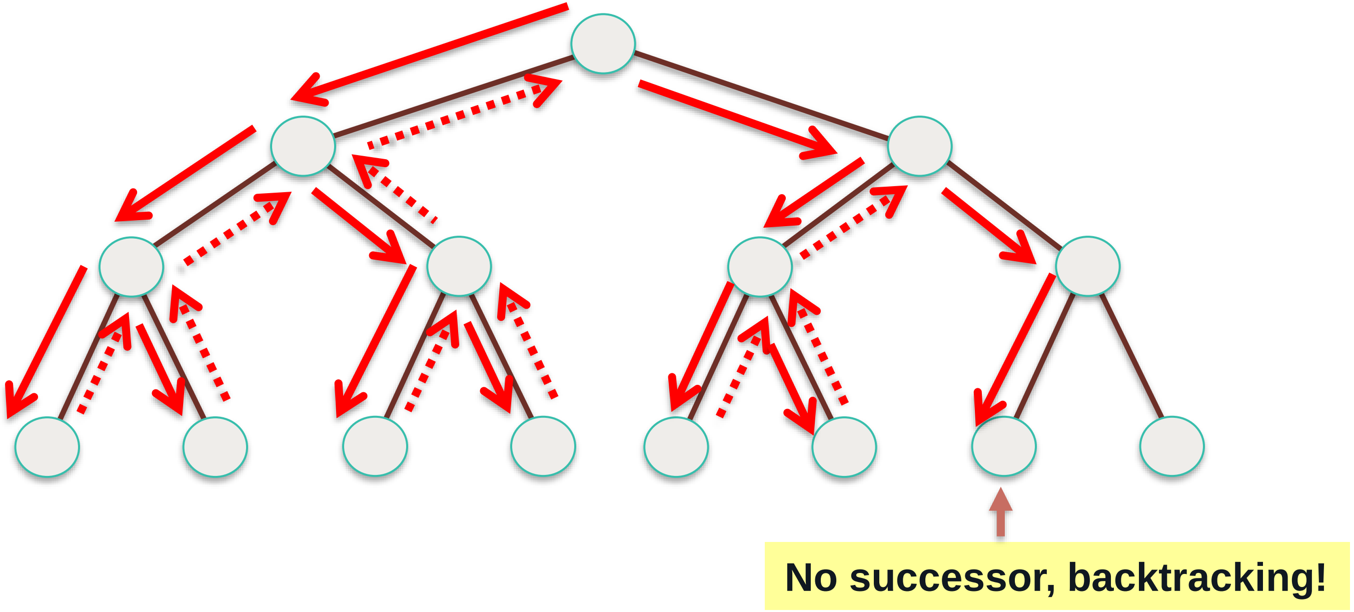

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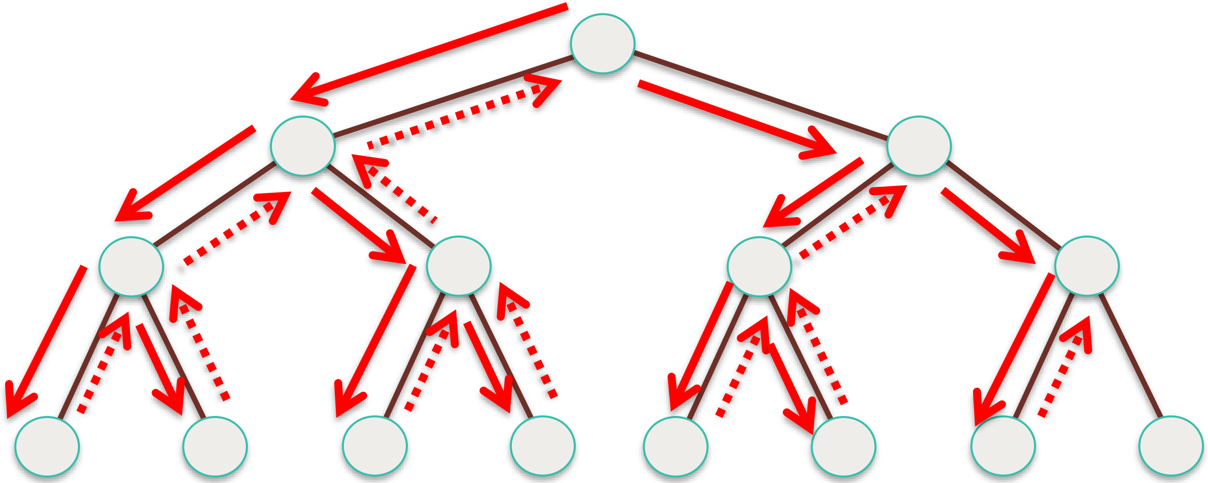

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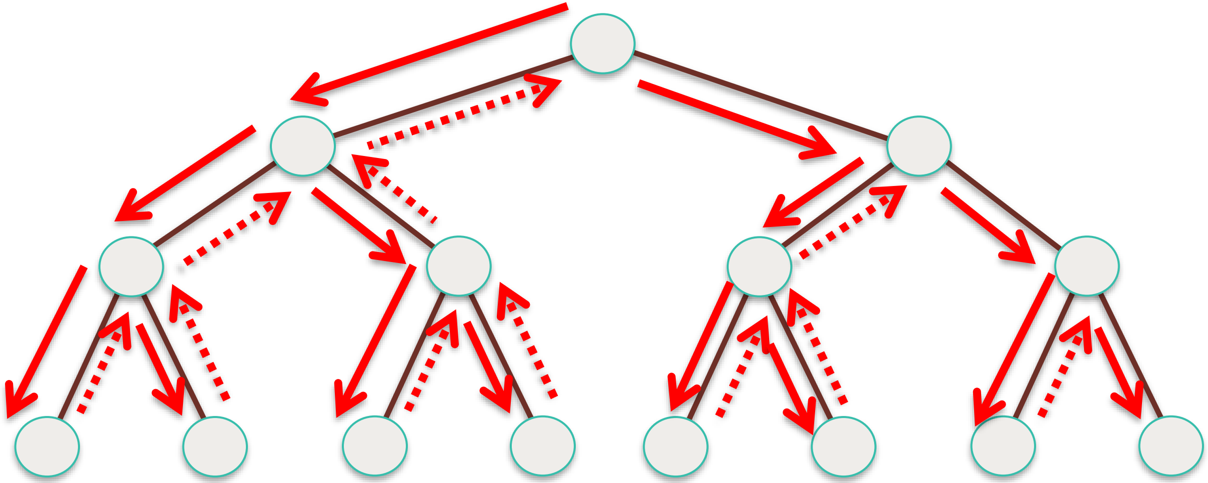

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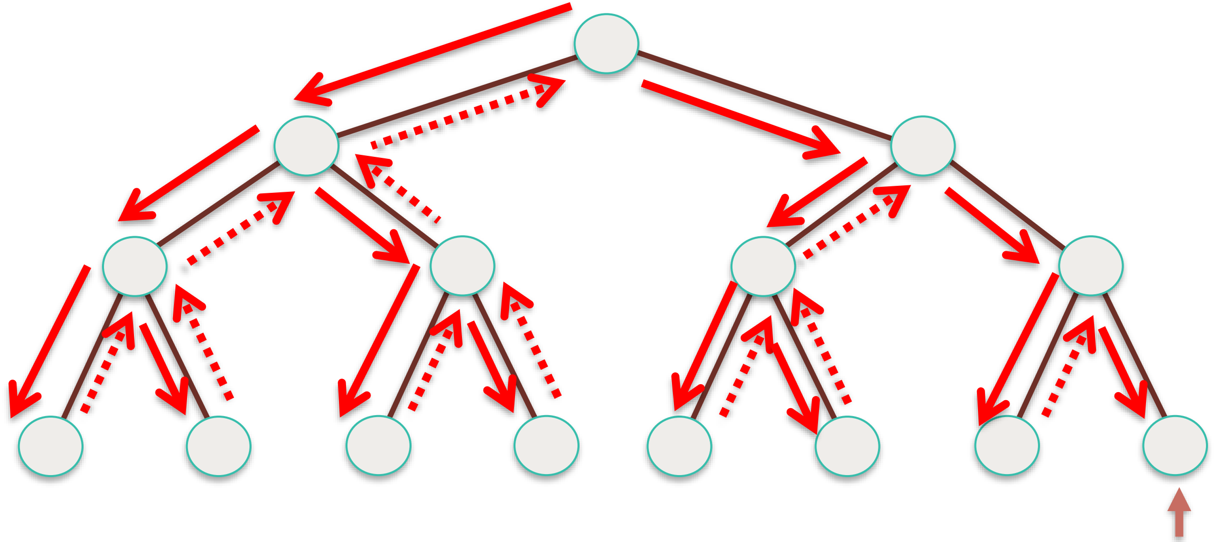
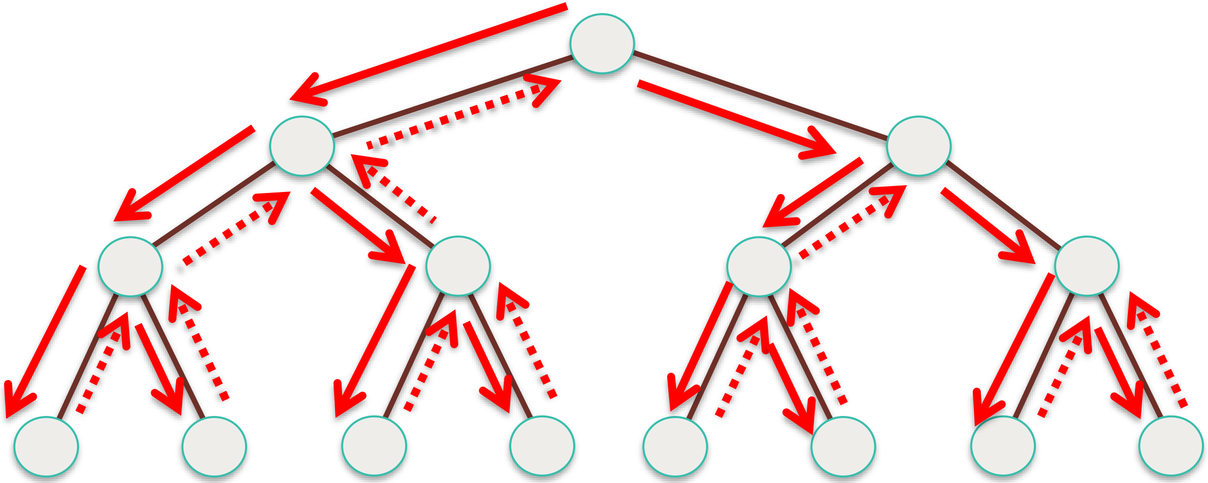


No successor, backtracking

Depth-First Search (DFS): Tree



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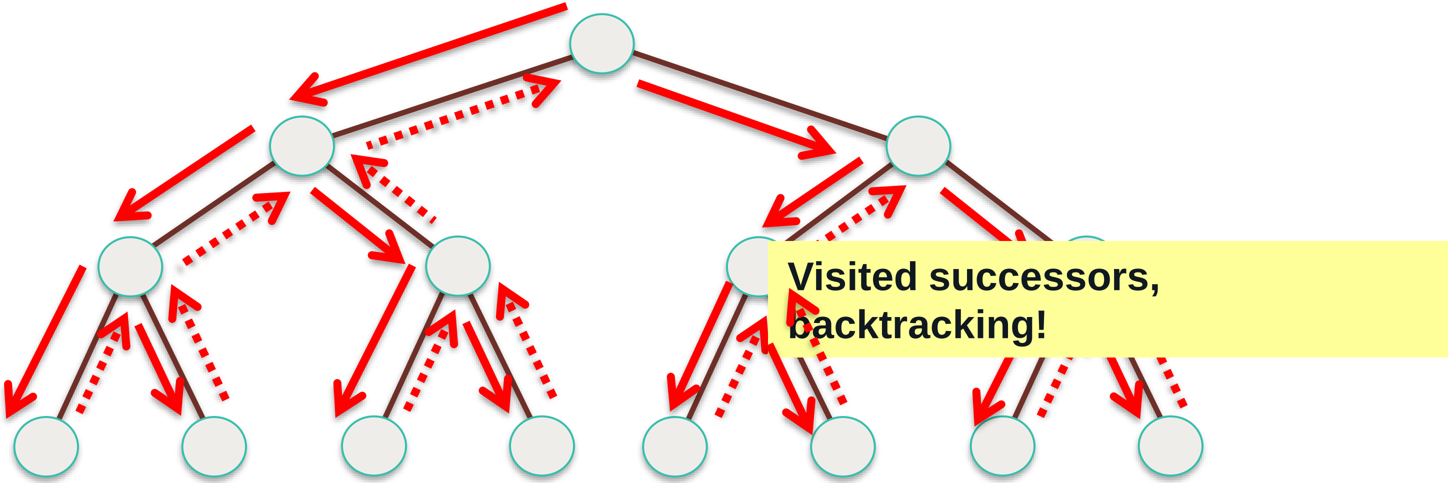

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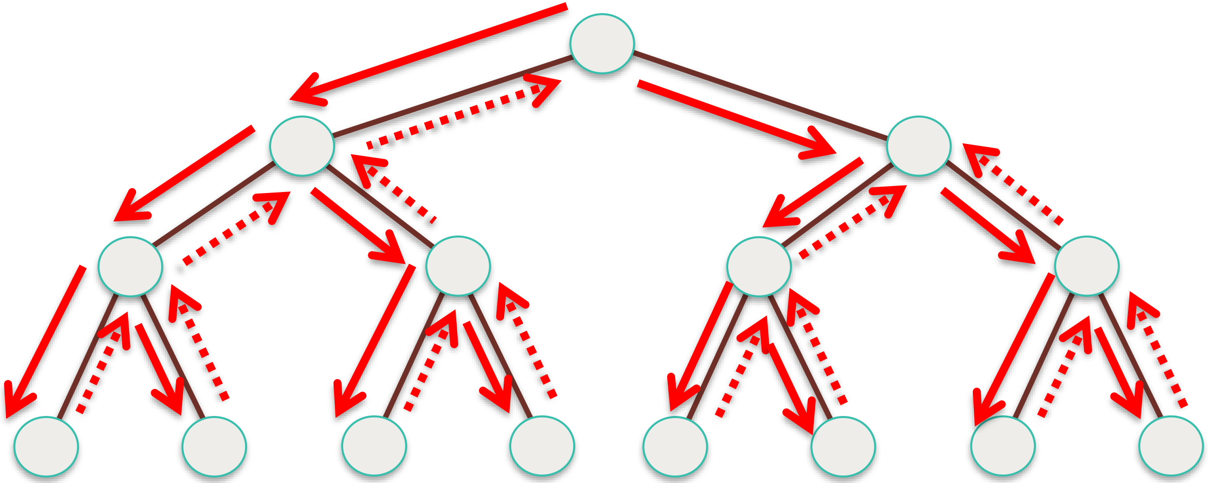

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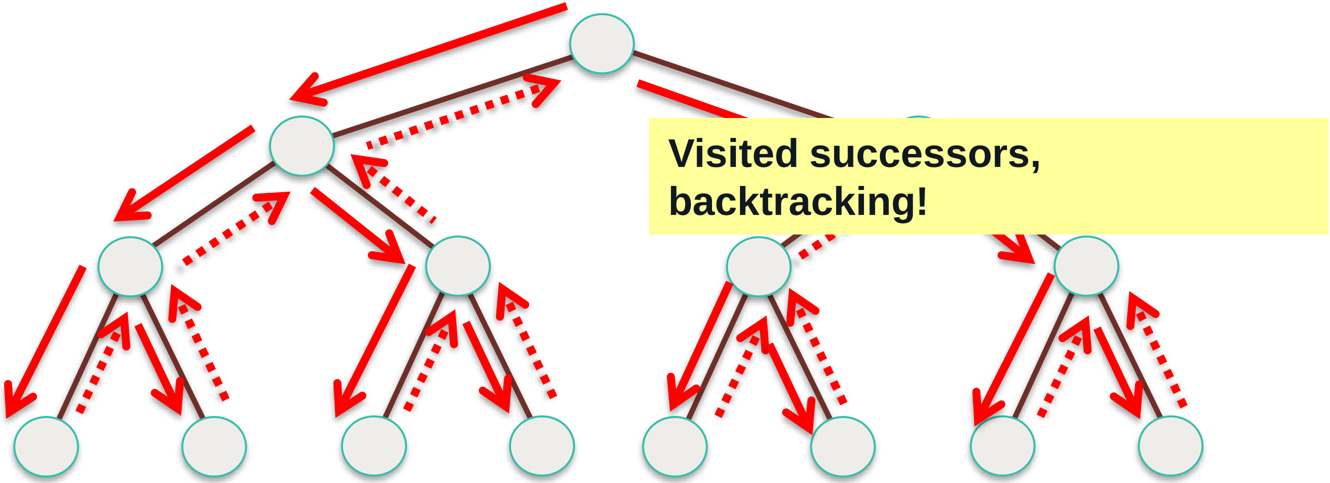

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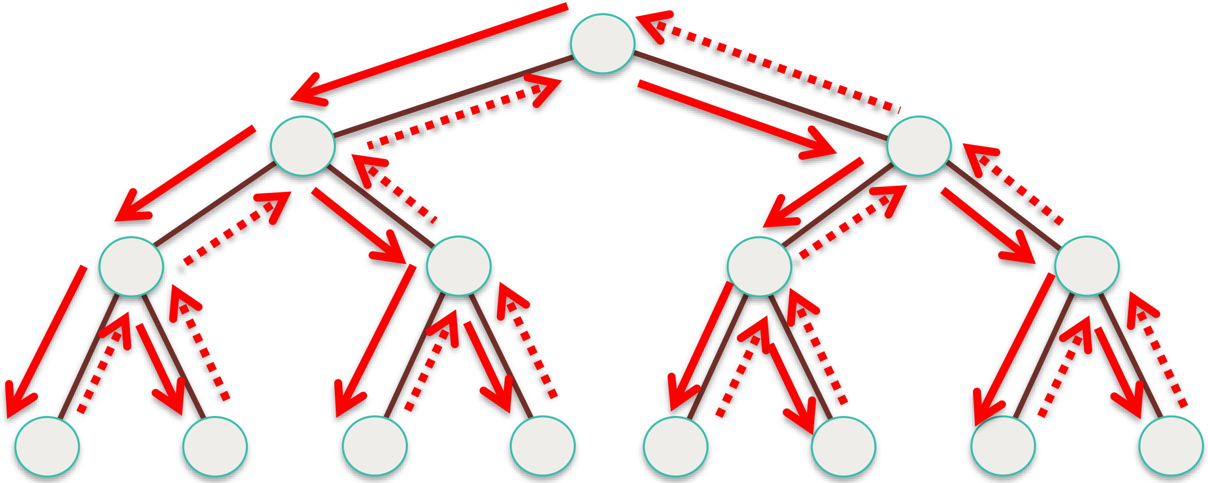

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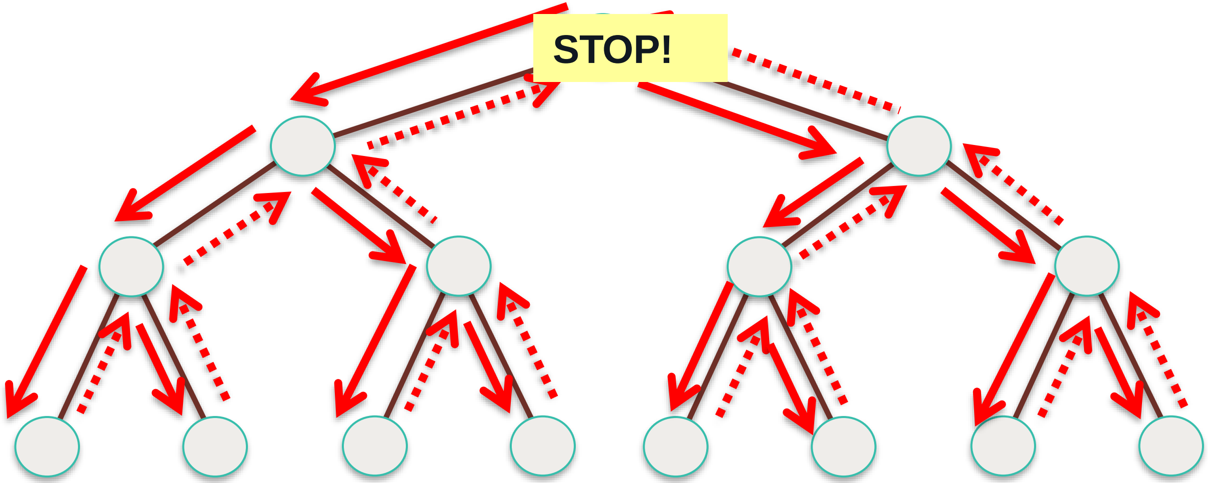

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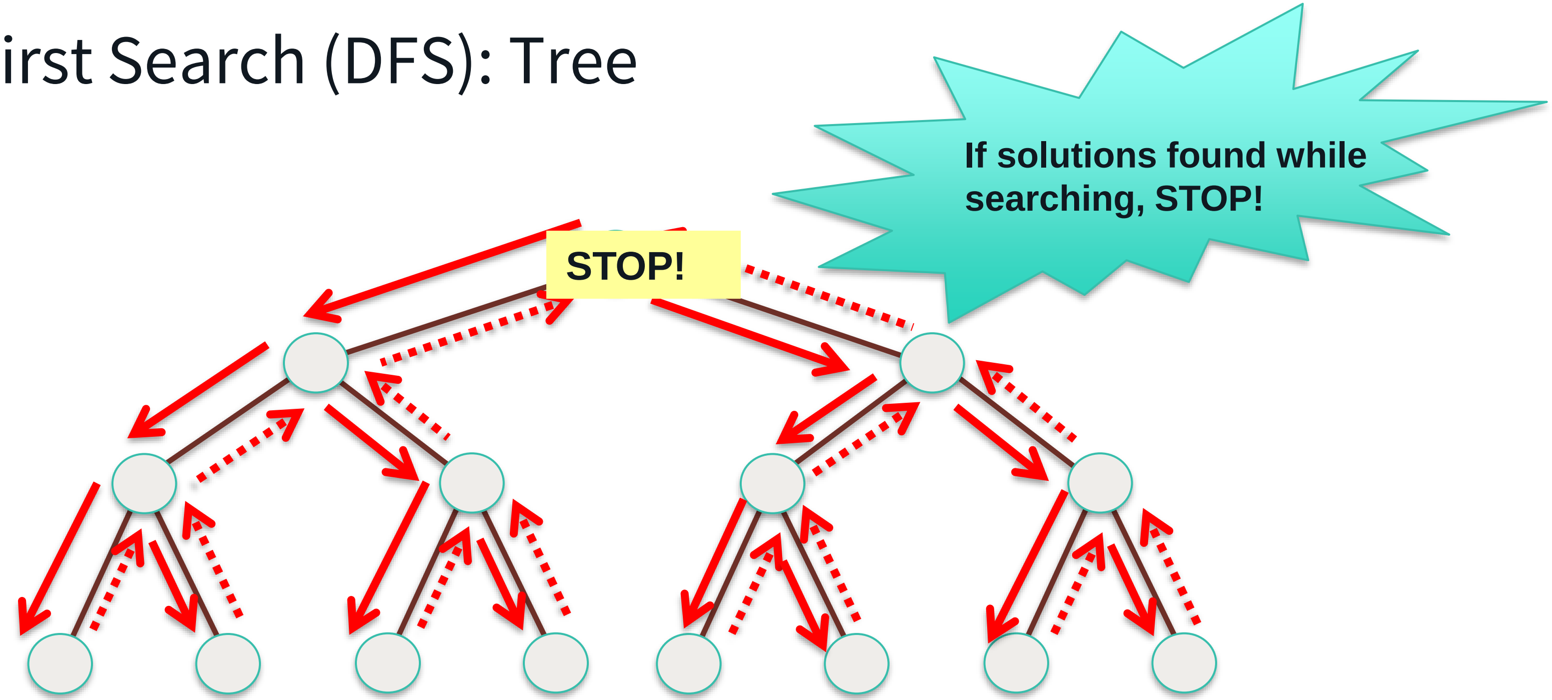


Depth-First Search (DFS)

- Expand the initial node & generate its successors
- In each subsequent step, DFS expands one of the most recently generated nodes.
  - If this node has no successors (or cannot lead to any solutions),
    - Backtracks
    - Expands an alternate child
- Successors of a node are often ordered based on their likelihood of reaching a solution

Depth-First Search (DFS) – Simple Backtracking

- Simple backtracking performs DFS until it finds the first feasible solution and terminates
- No optimal solution guaranteed
- Simple backtracking: uses no heuristic information to order the successors of an expanded node
- Ordered backtracking: uses heuristics to order the successors of an expanded node

Applications of DSF

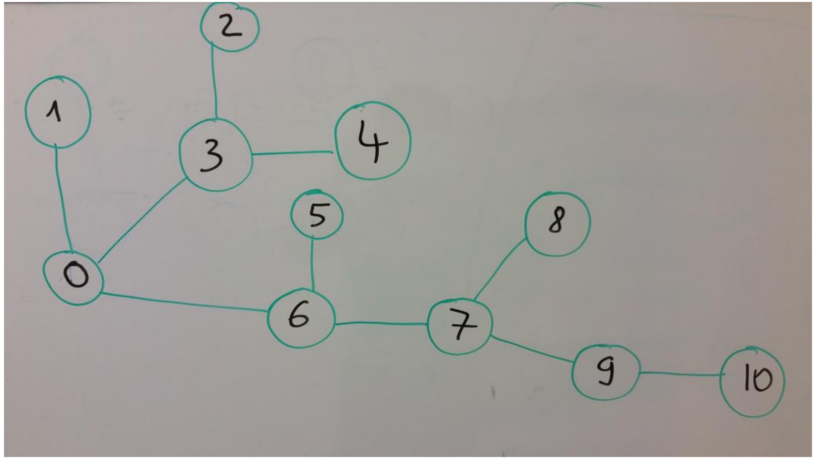
- Detecting cycle in a graph
- Path Finding: Find a path between two given vertices u and v.
  - Call DFS(G, u) with u as the start vertex
  - Keep track of the path between the start vertex and the current vertex
  - As soon as destination vertex z is encountered, return the path
- Finding Strongly Connected Components of a graph
  - A directed graph is called strongly connected if there is a path from each vertex in the graph to every other vertex

https://www.geeksforgeeks.org/applications-of-depth-first-search/

Depth-First Search (DFS): Pseudocode

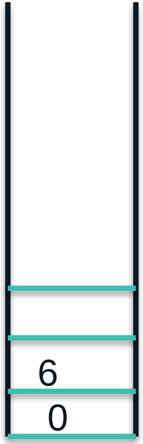
```
DFS-recursive(G, s):
     mark s as visited
     for all neighbours w of s in Graph G:
          if w is not visited:
               DFS-recursive(G, w)
```

Sample 'Implementation'



Data structure first: adjacent matrix or adjacent list ?
Start vertex: 0 (but can be any other vertex)
Use Stack (Last-in-First-Out)

DFS: 0, 6, 7, 9, 10, 8, 5, 3, 4, 2,1



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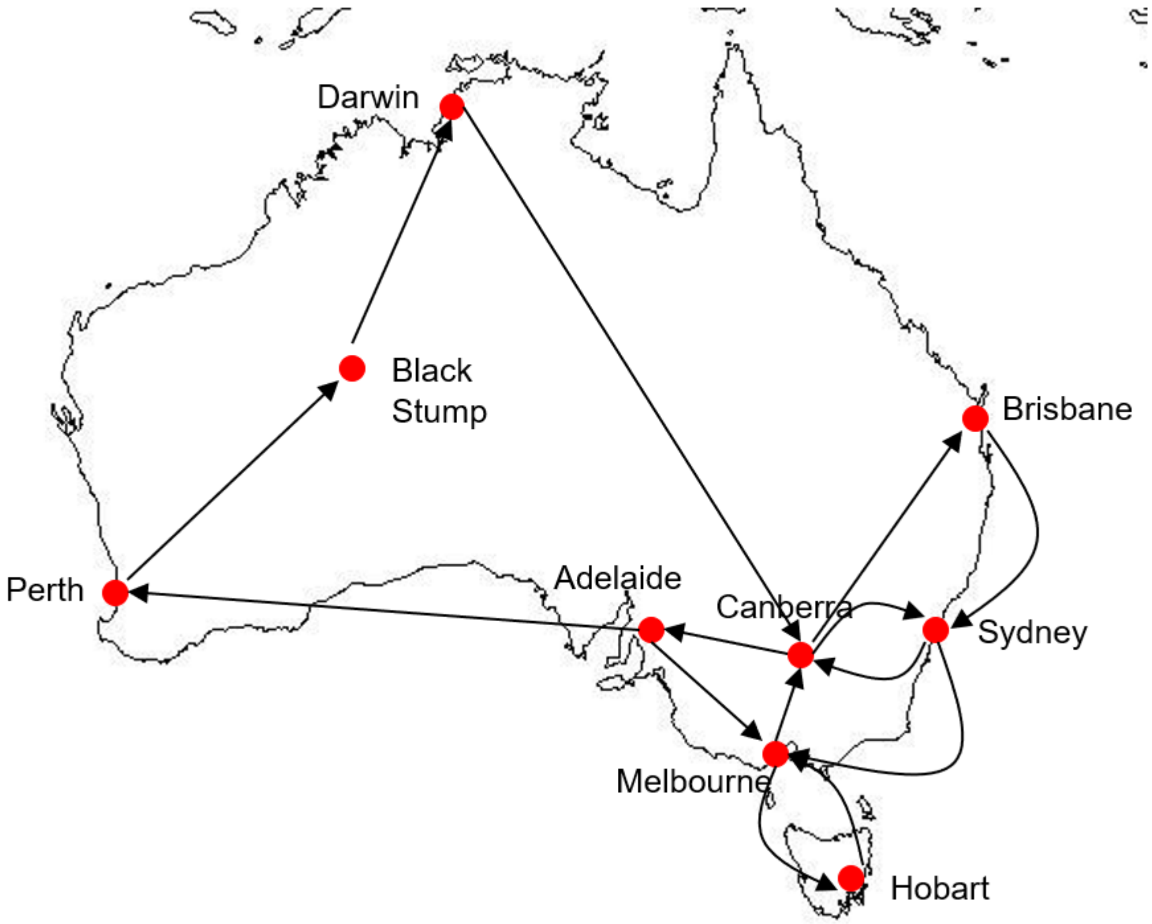

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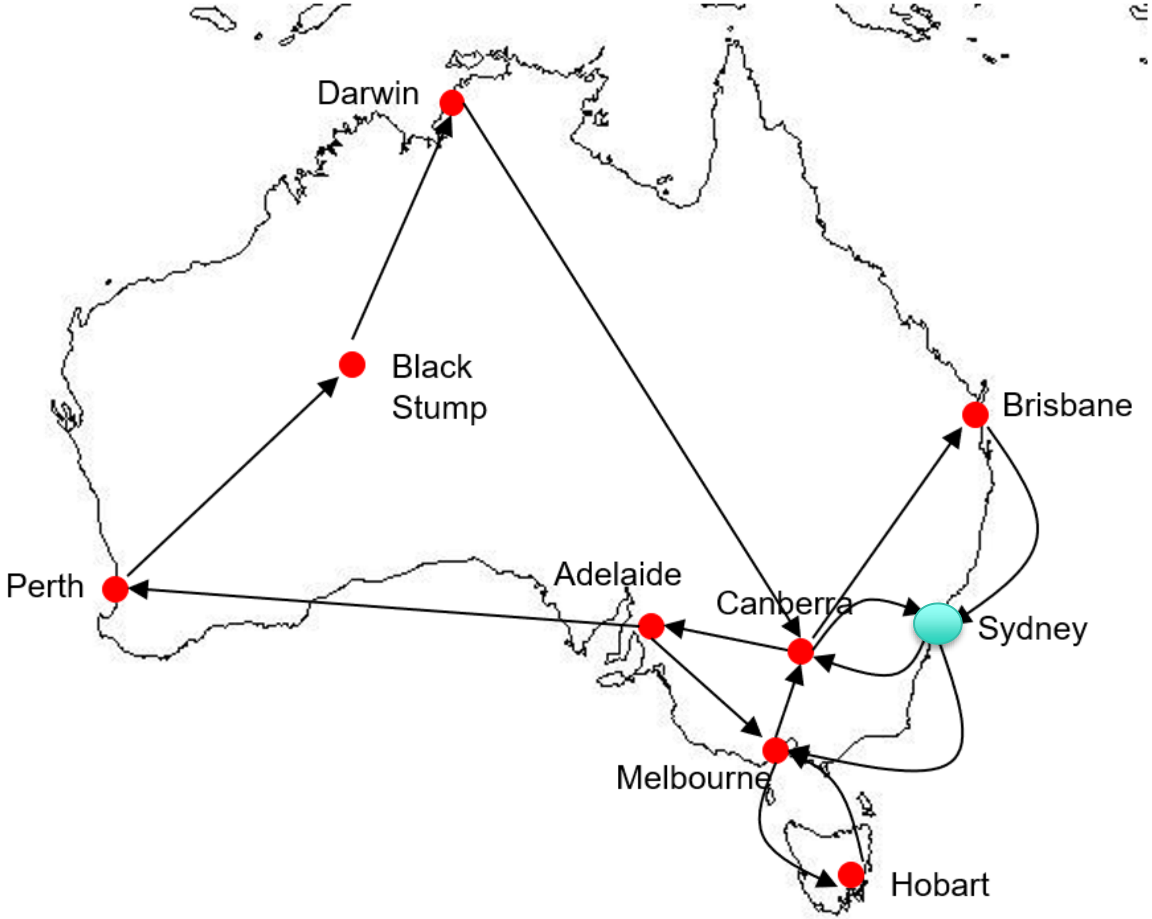
Hobart

- BSF

Sydney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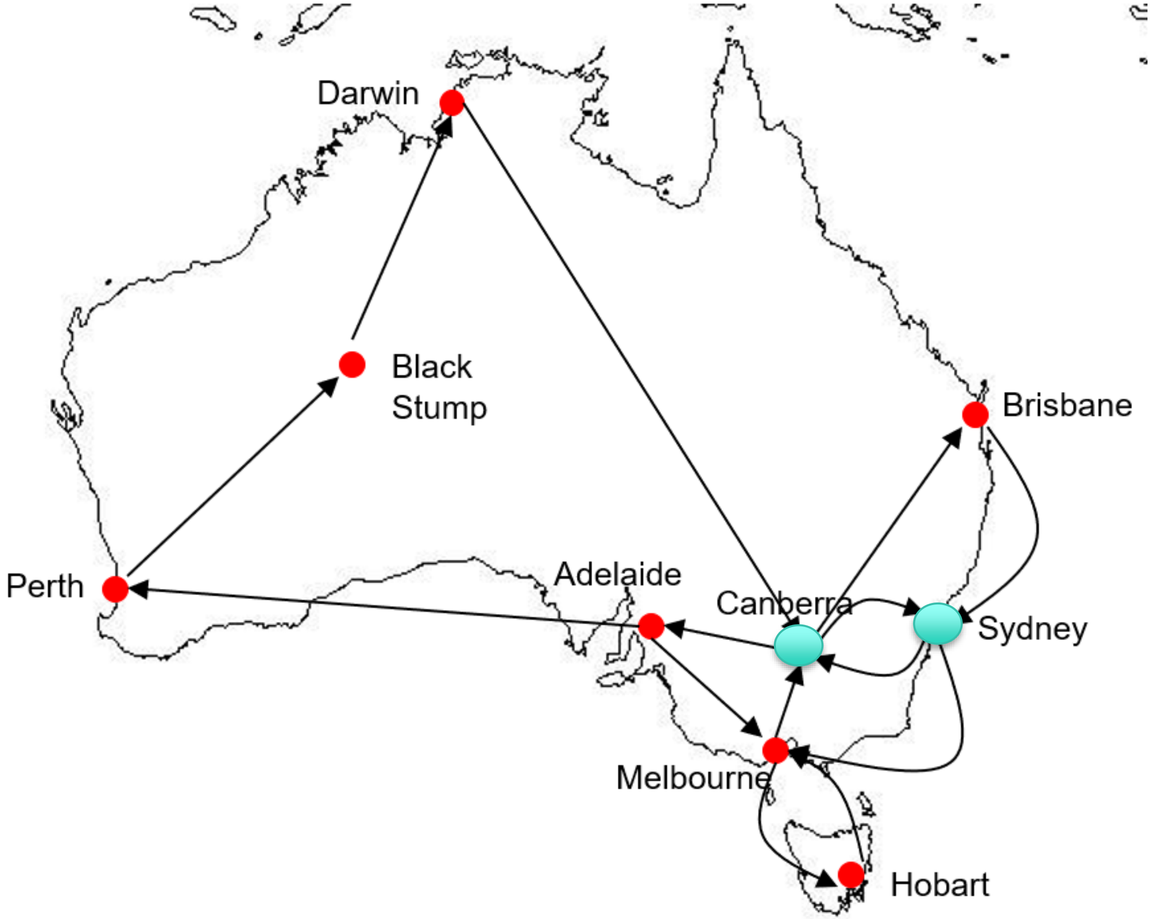

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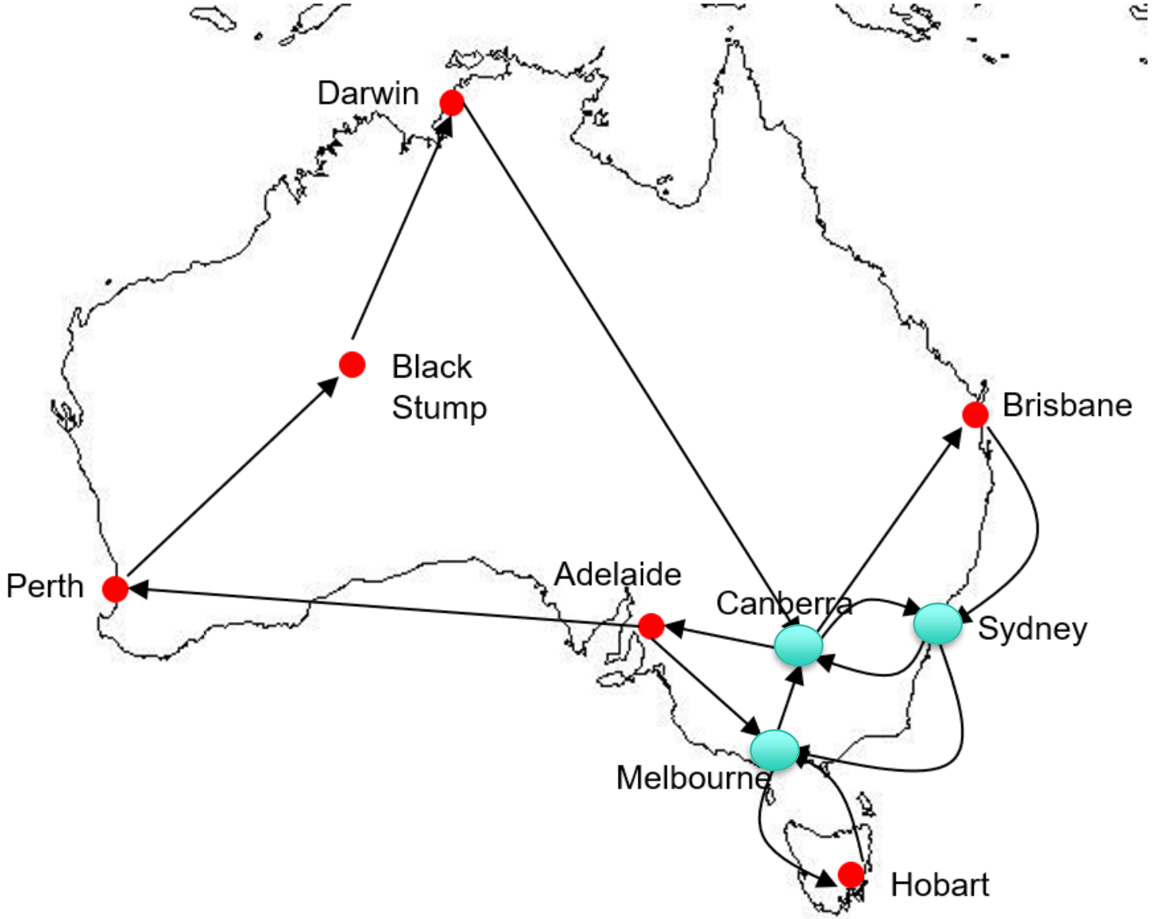

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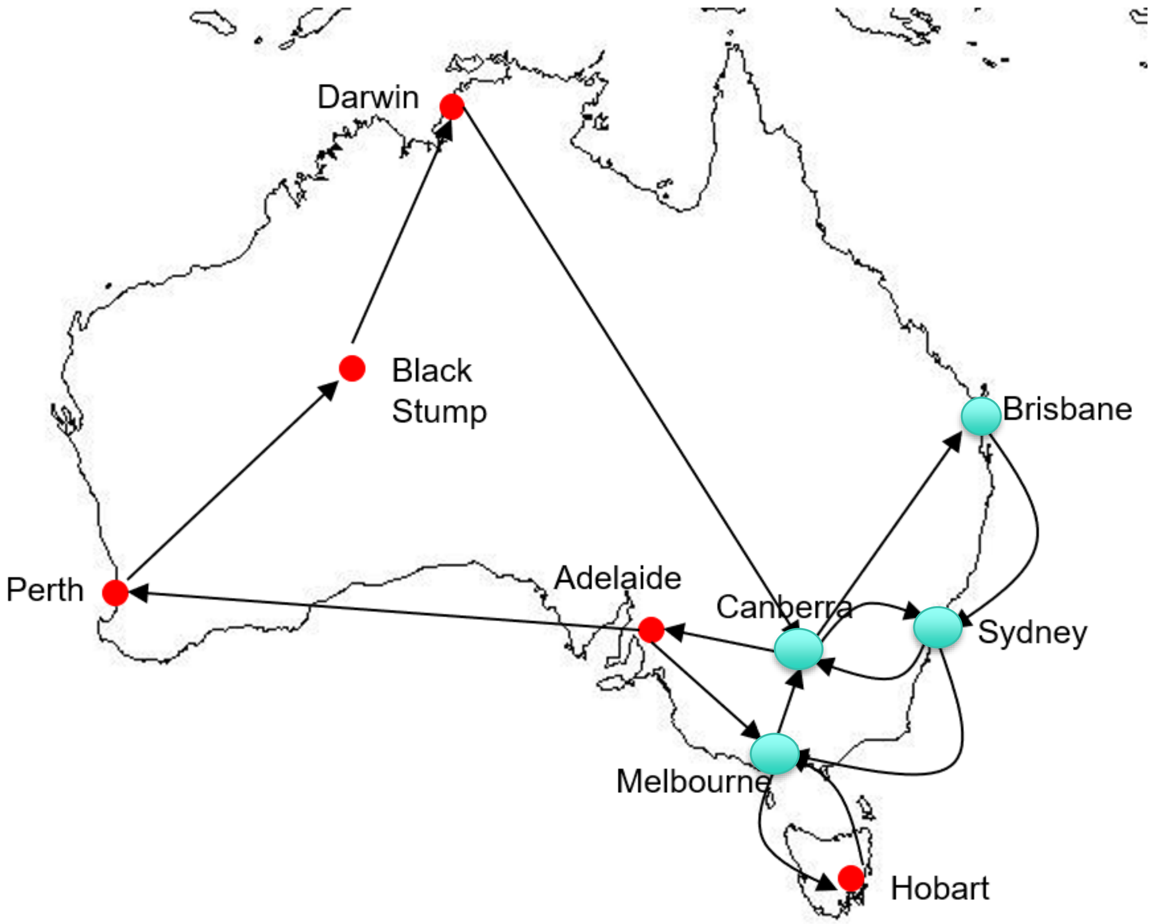

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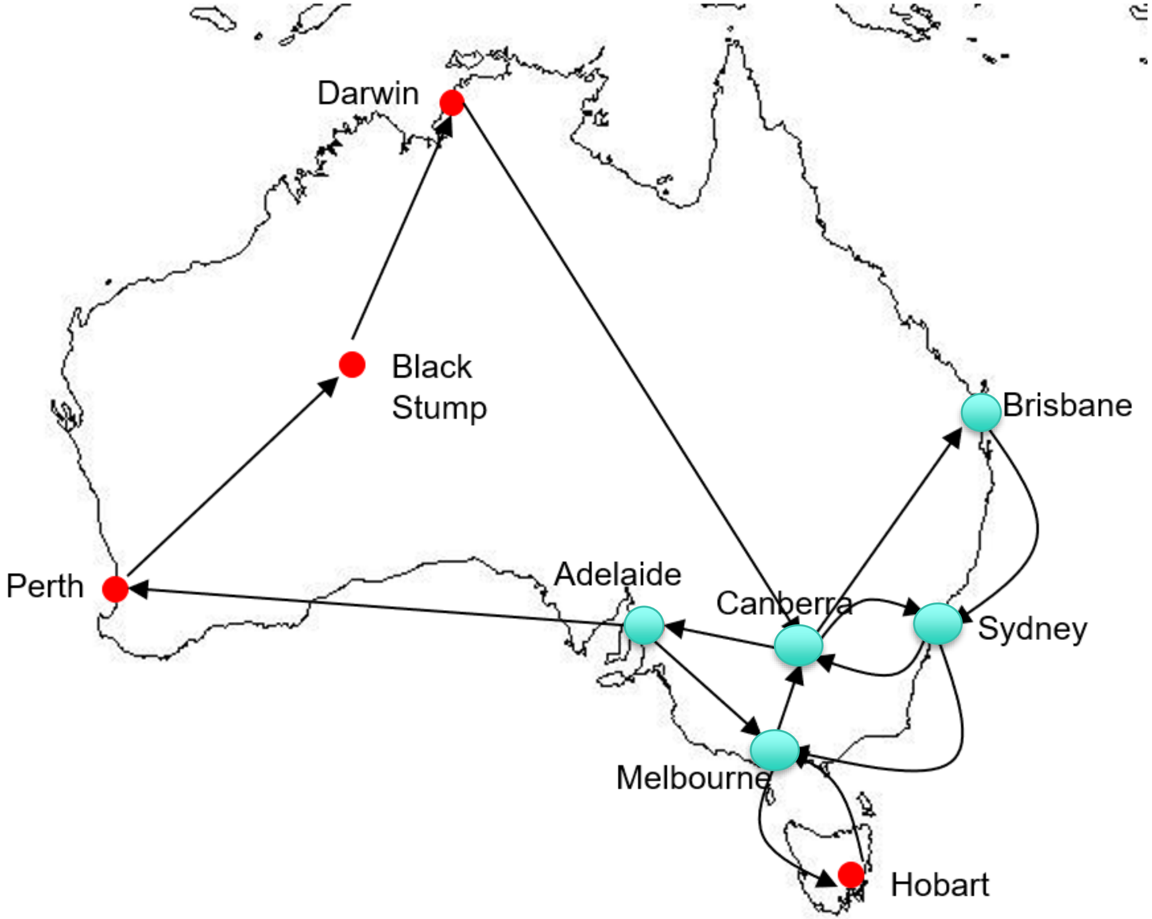

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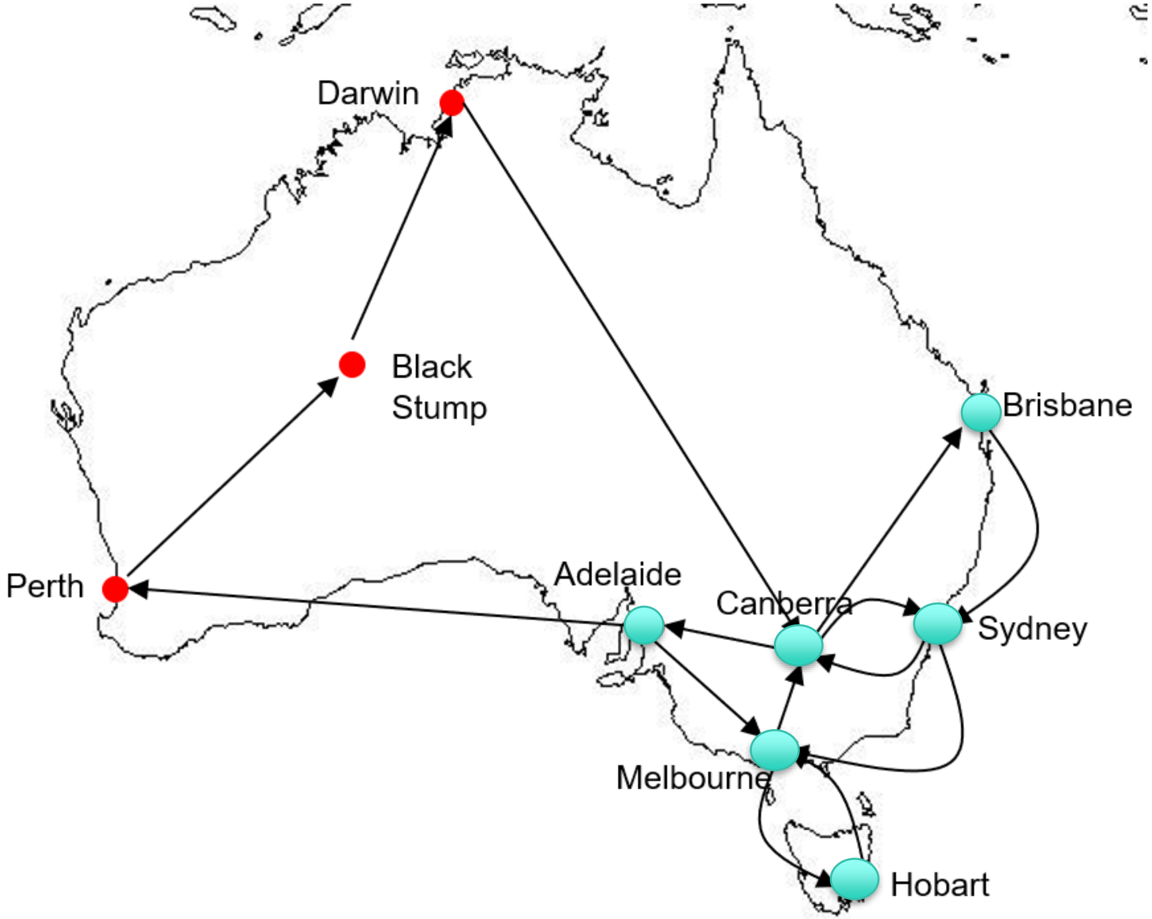

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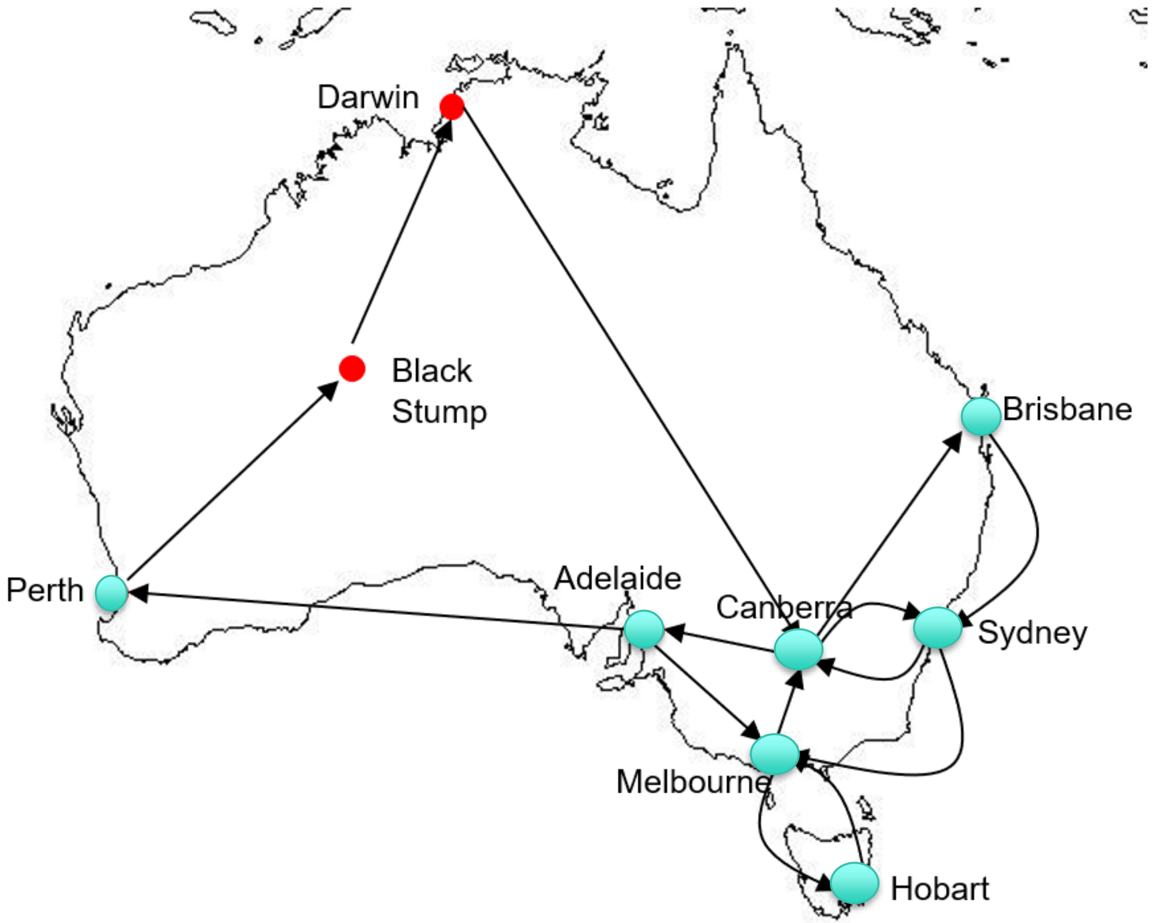

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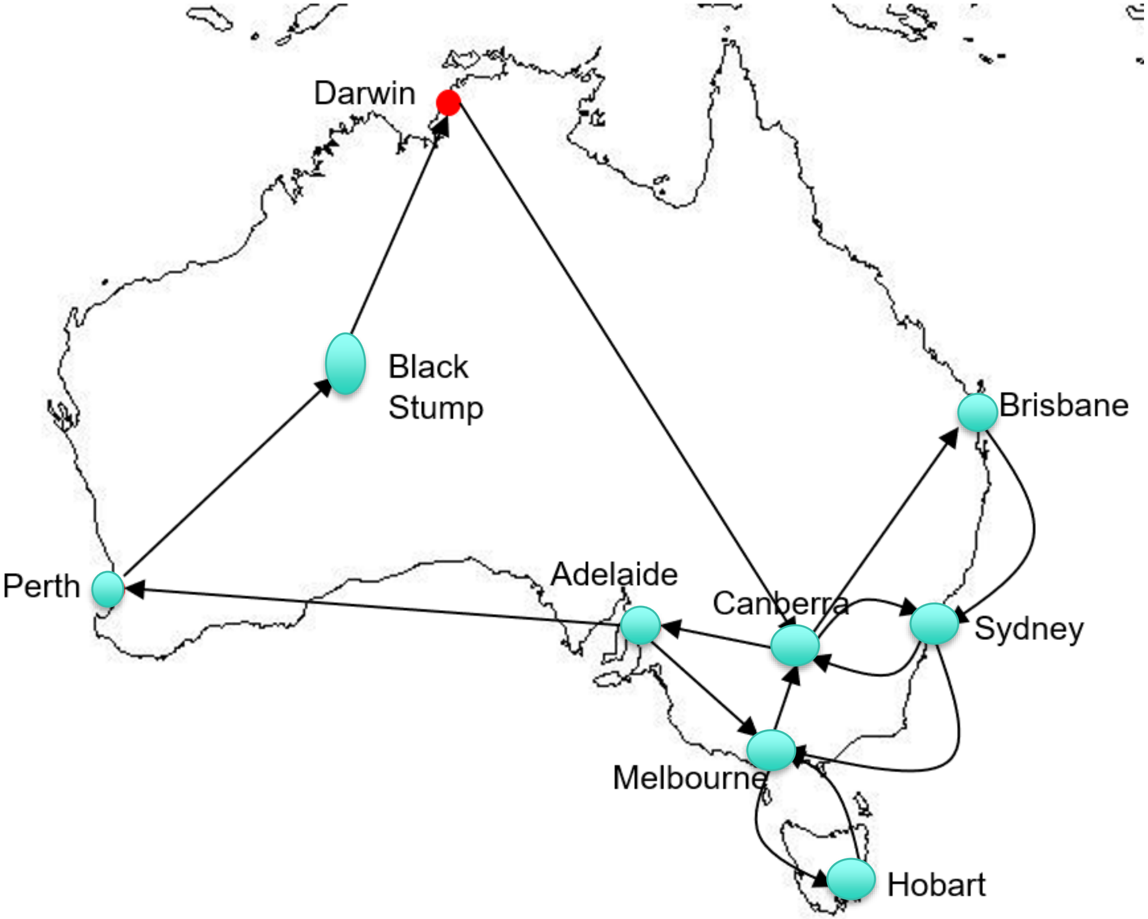

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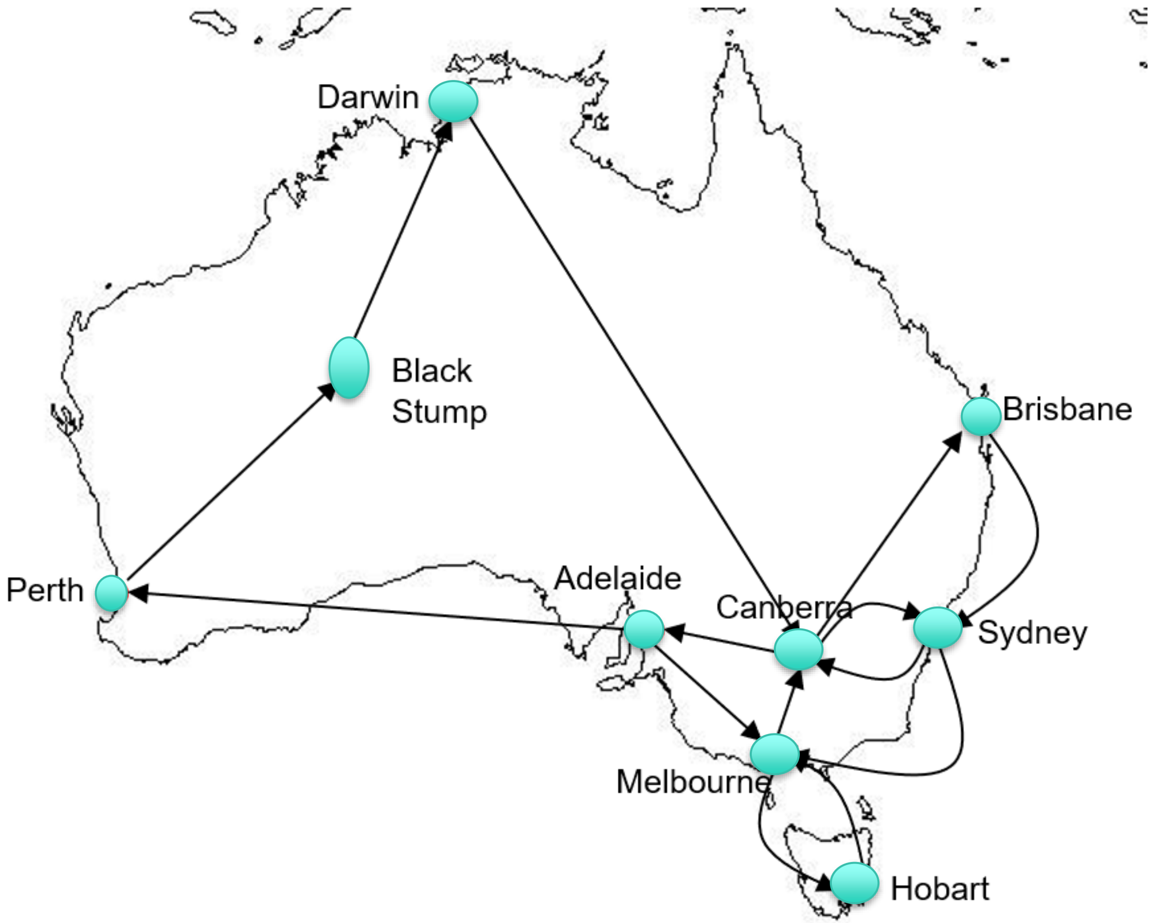

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- DSF
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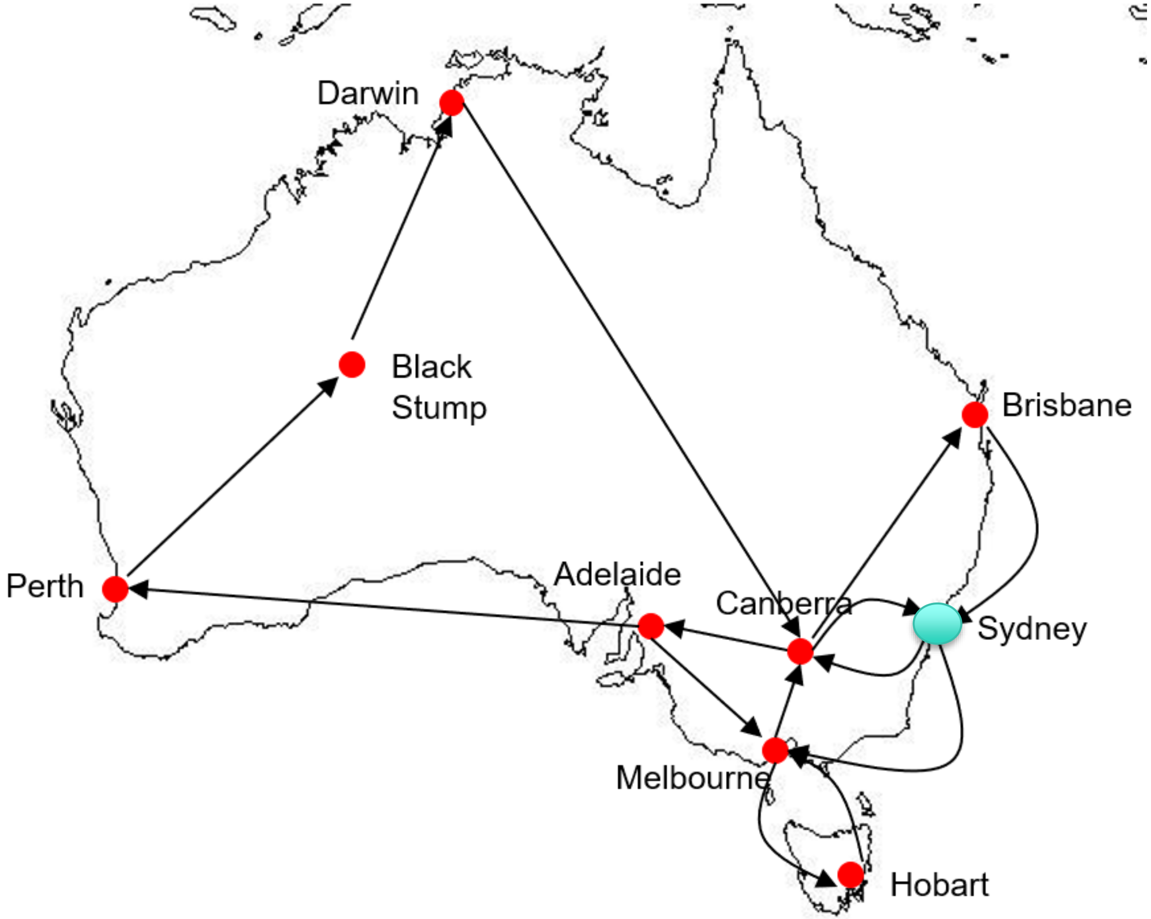
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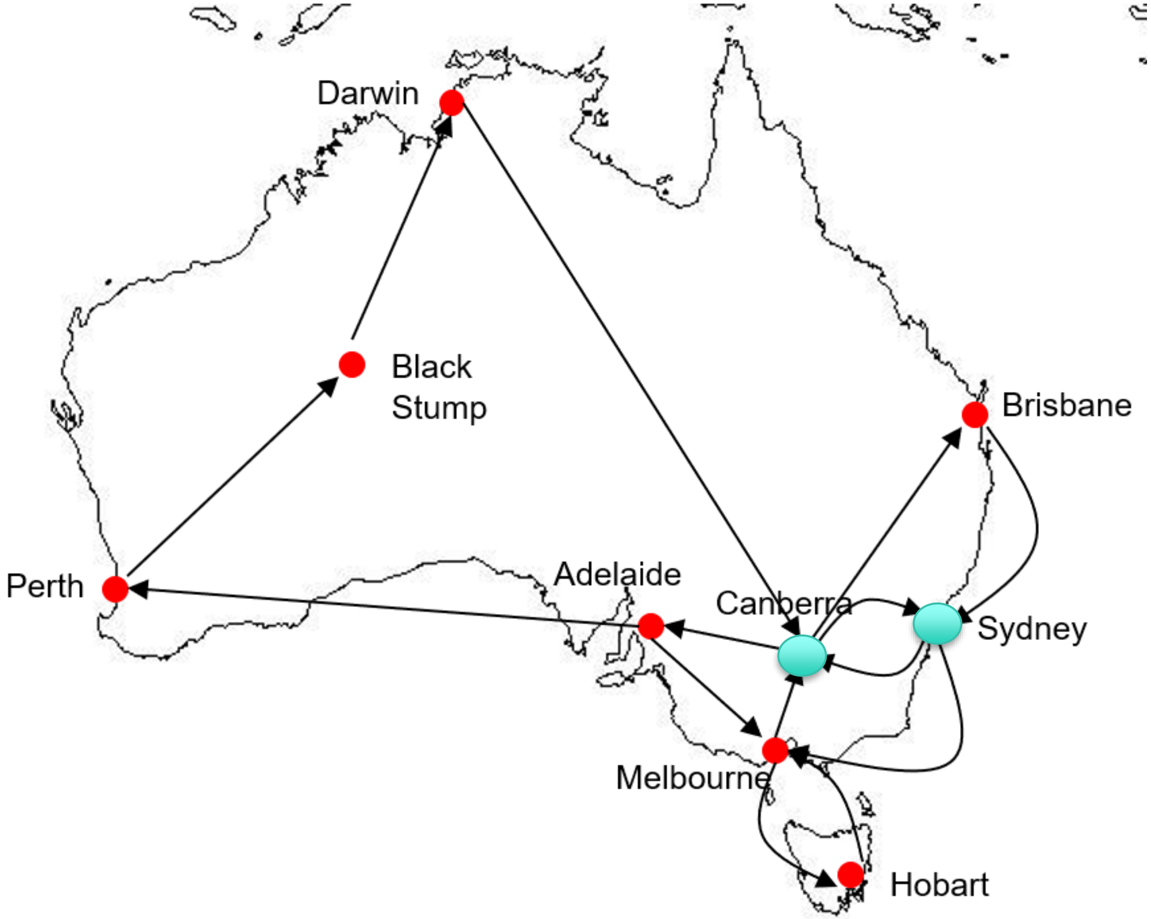

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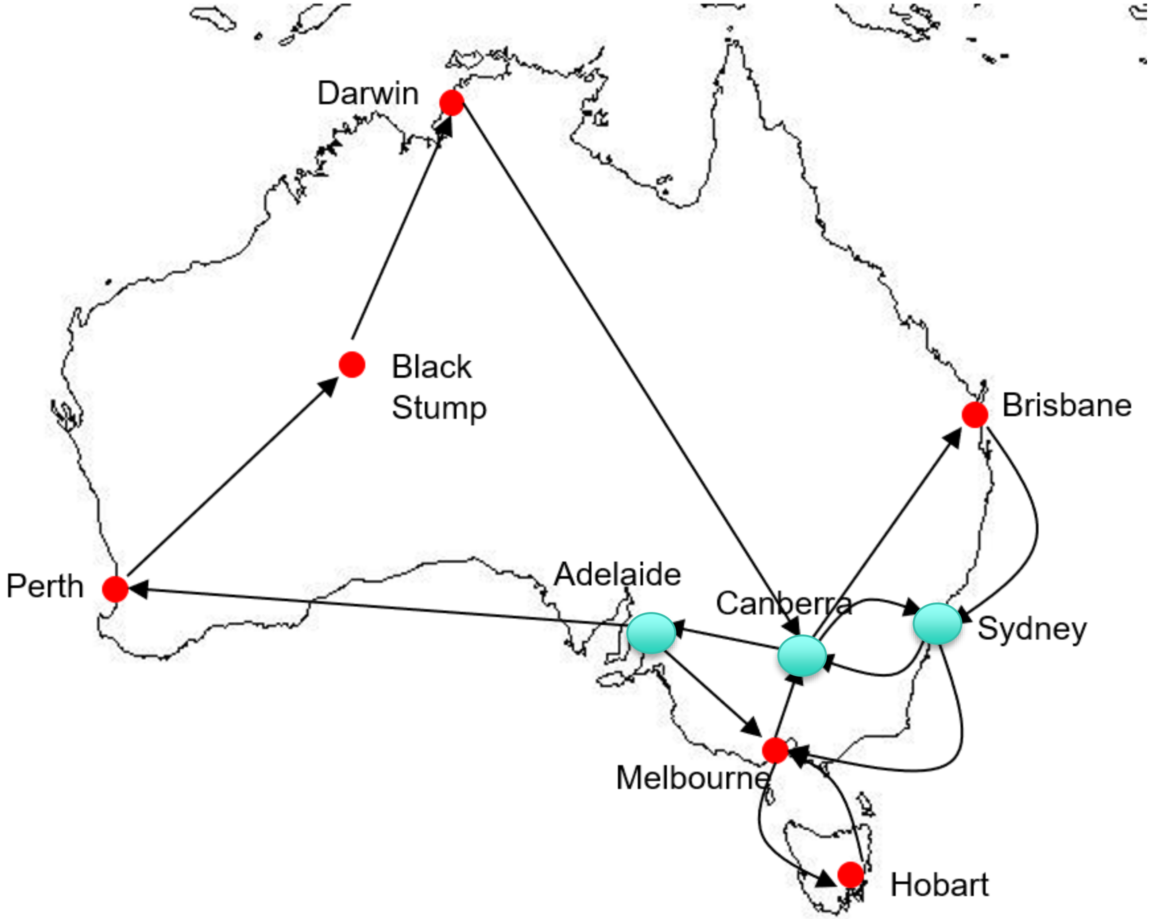

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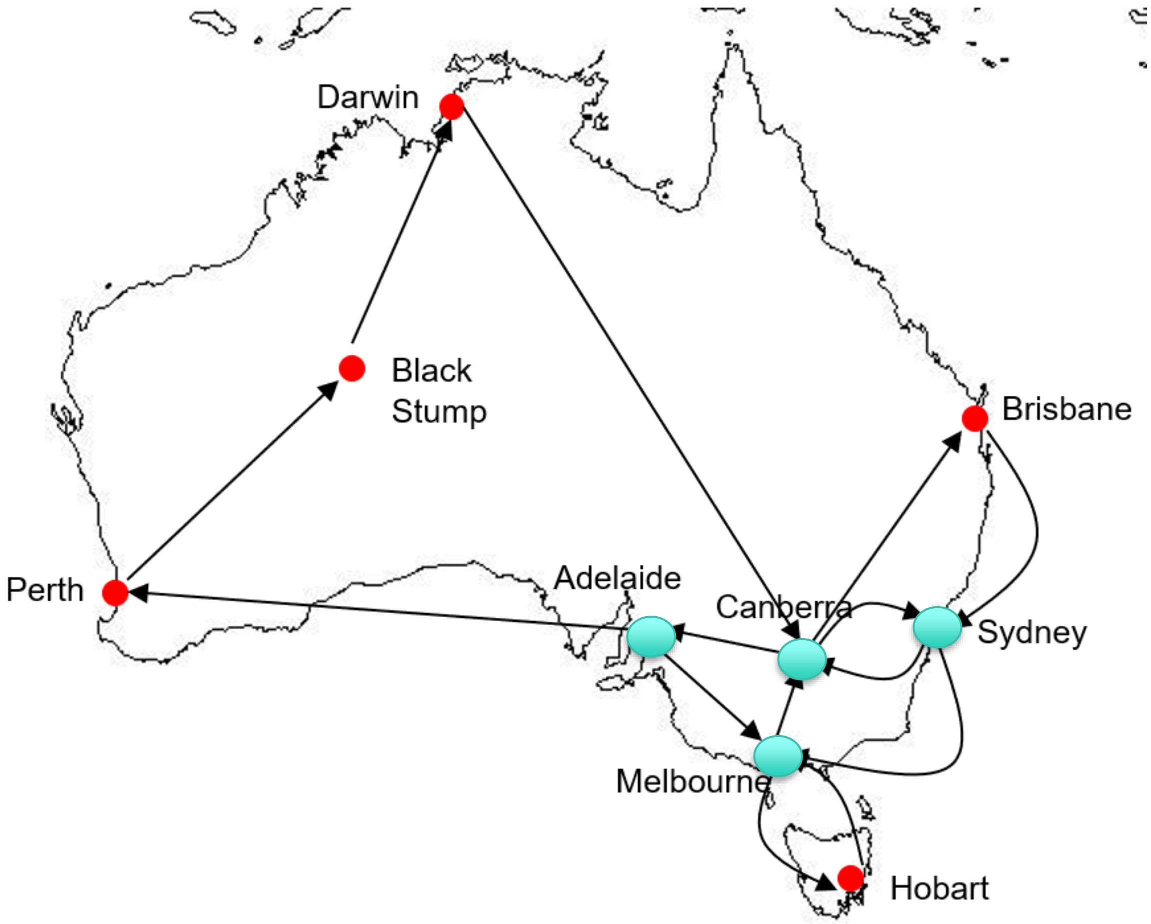

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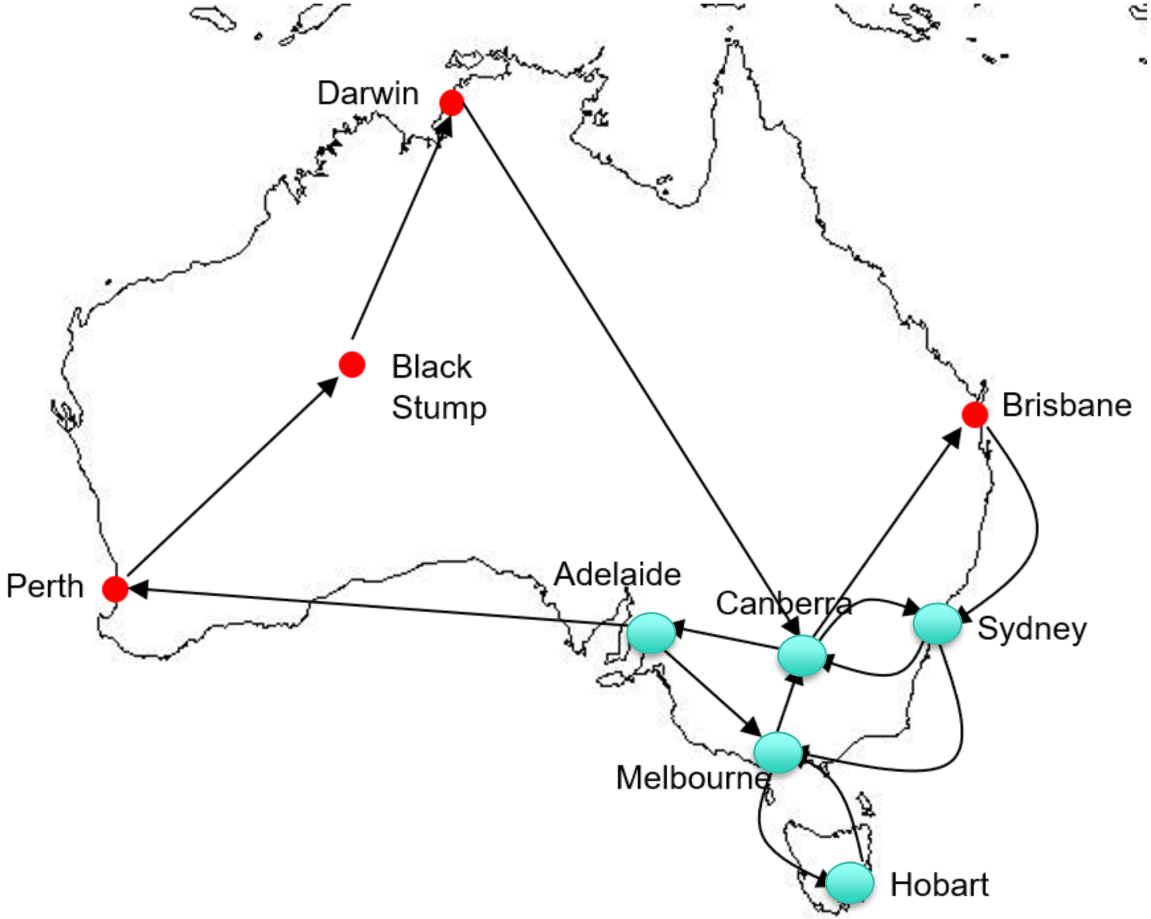

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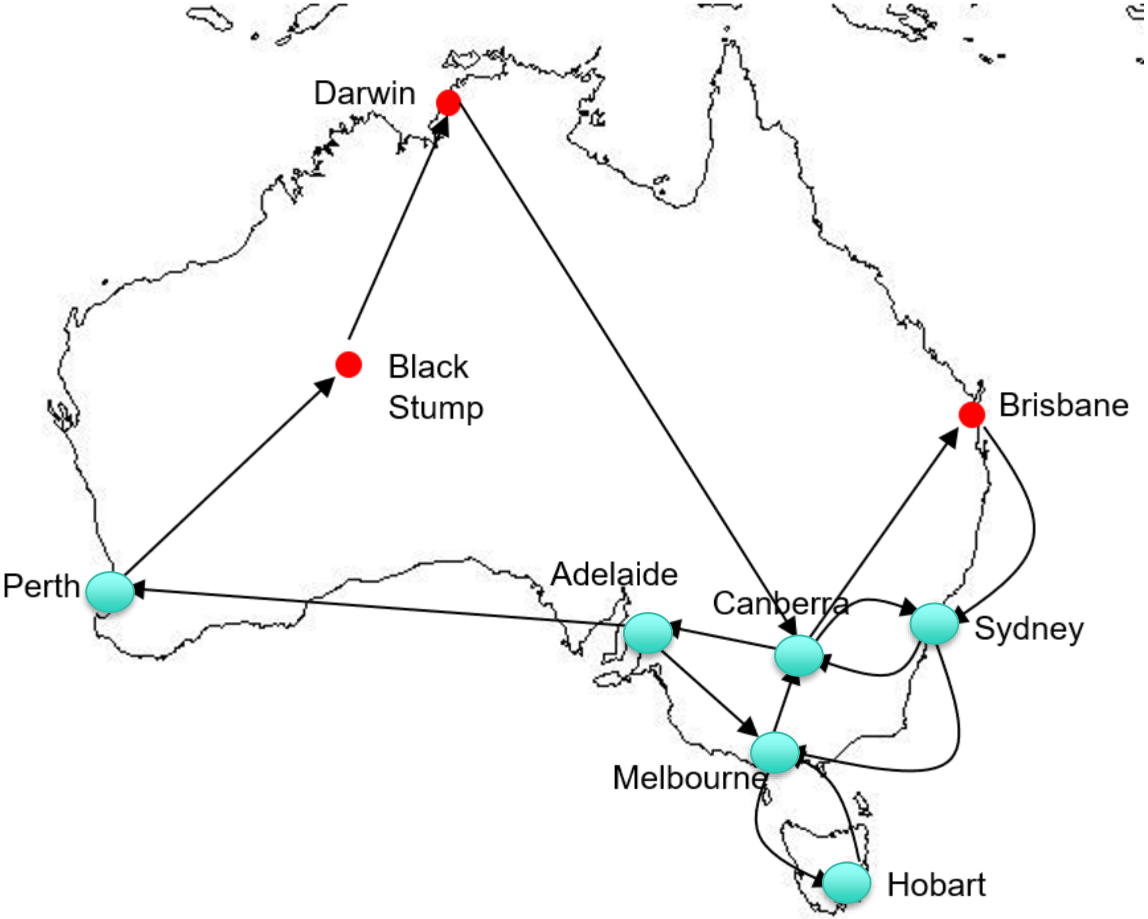

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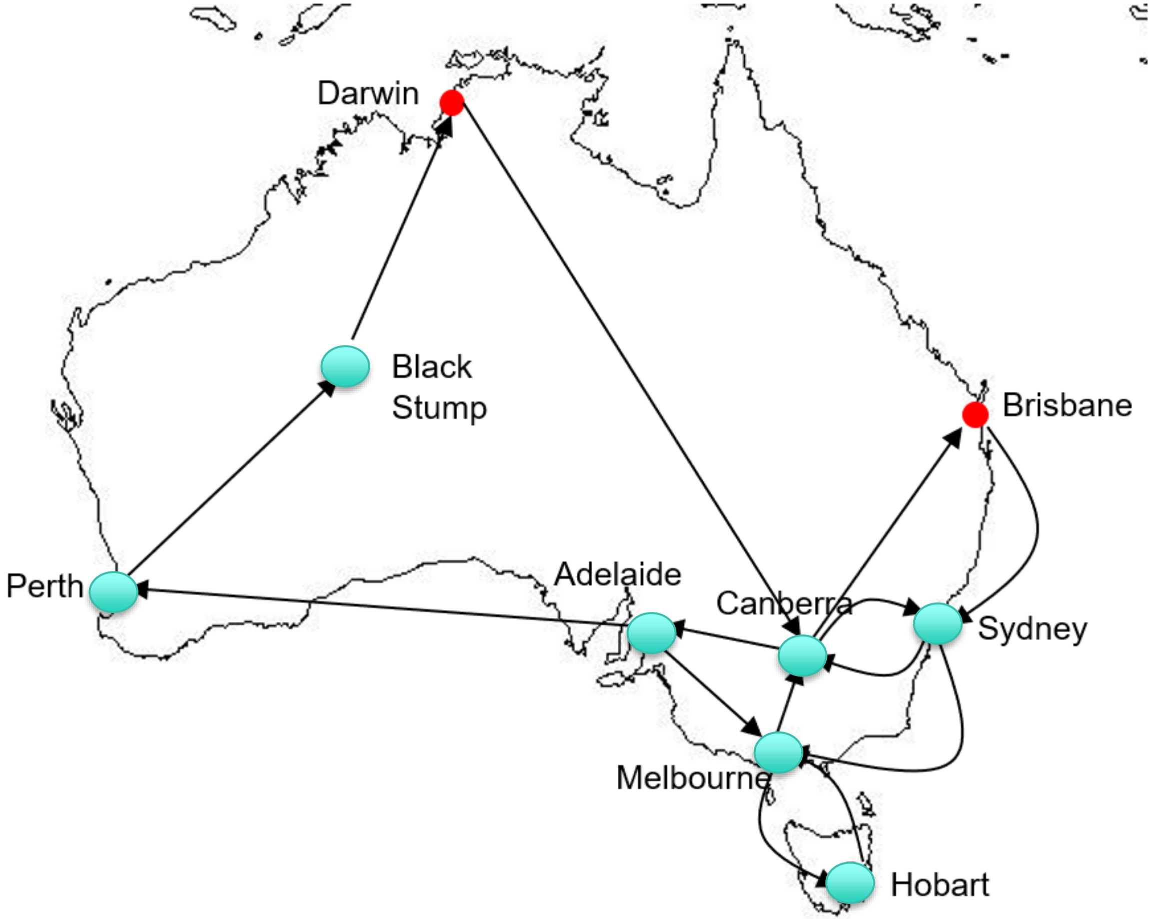

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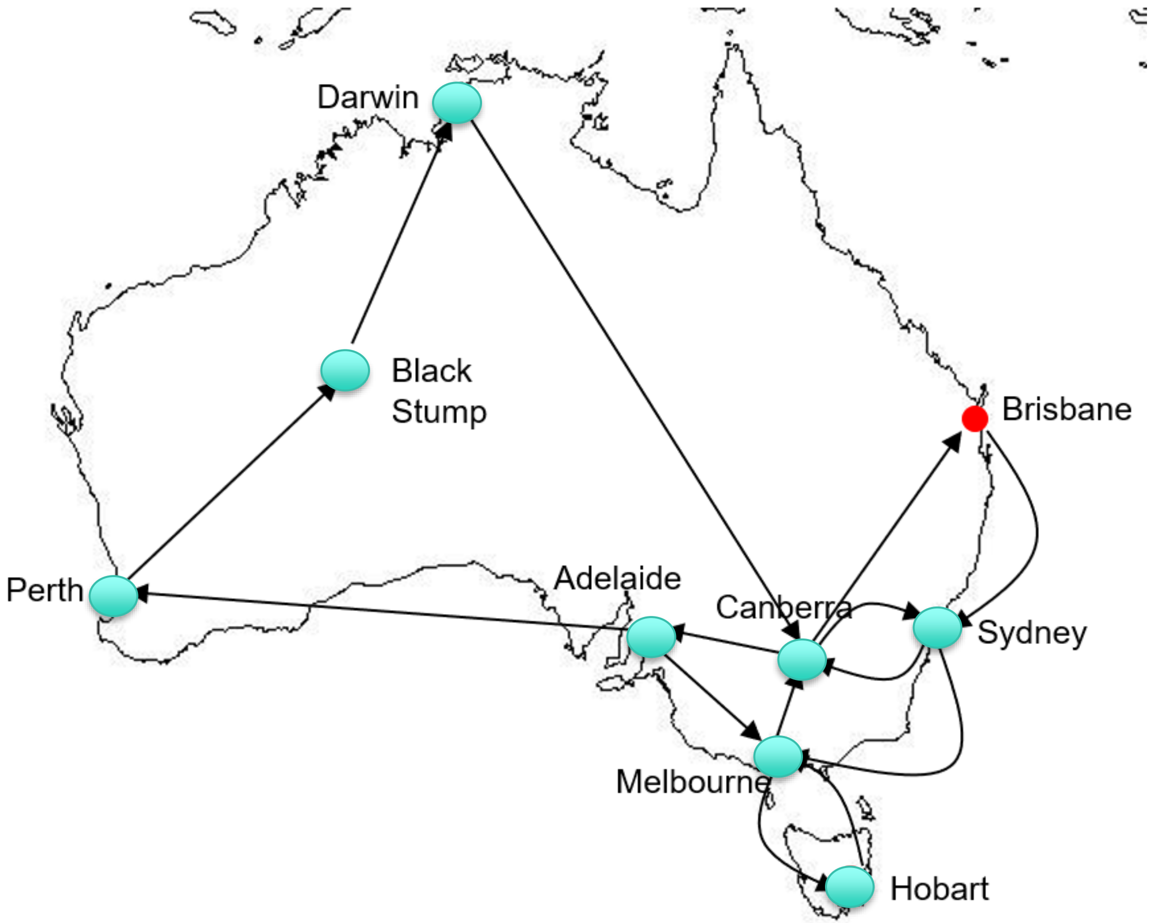

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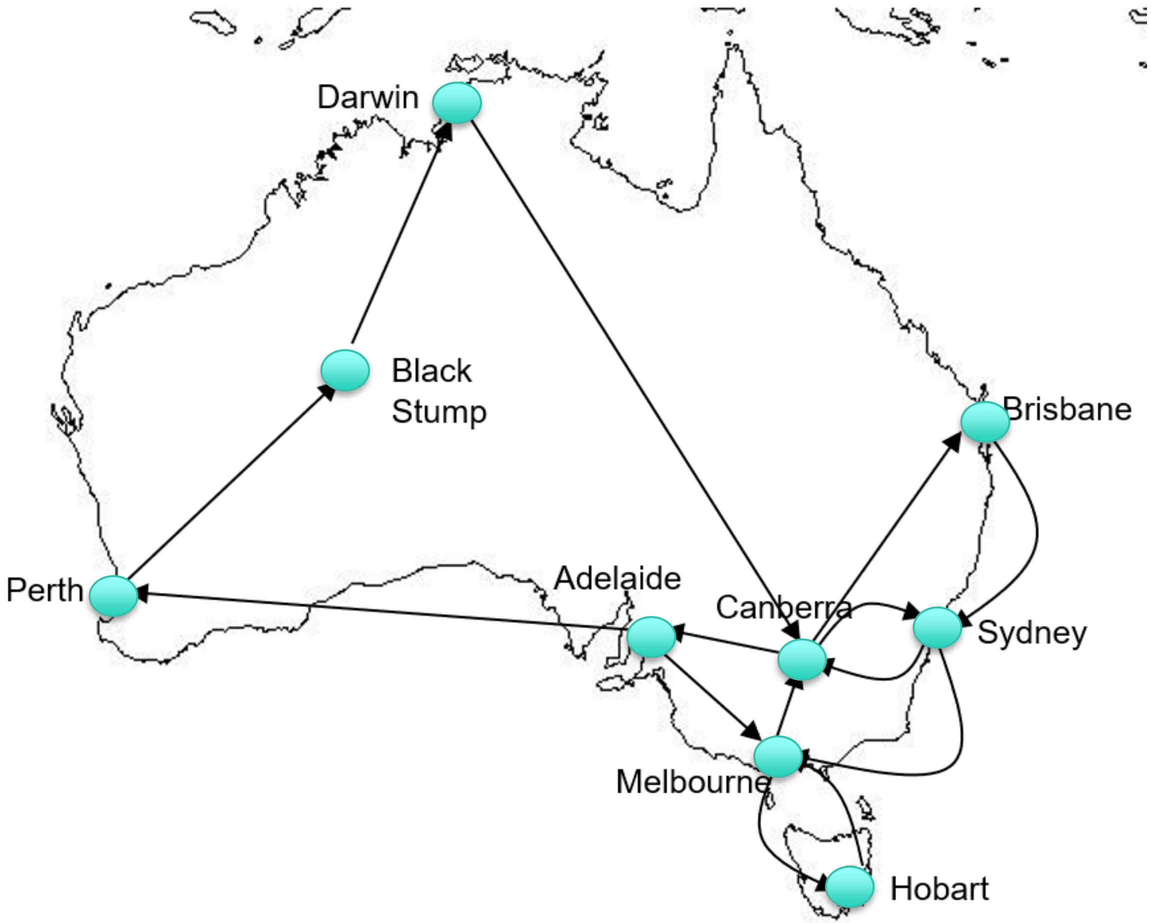

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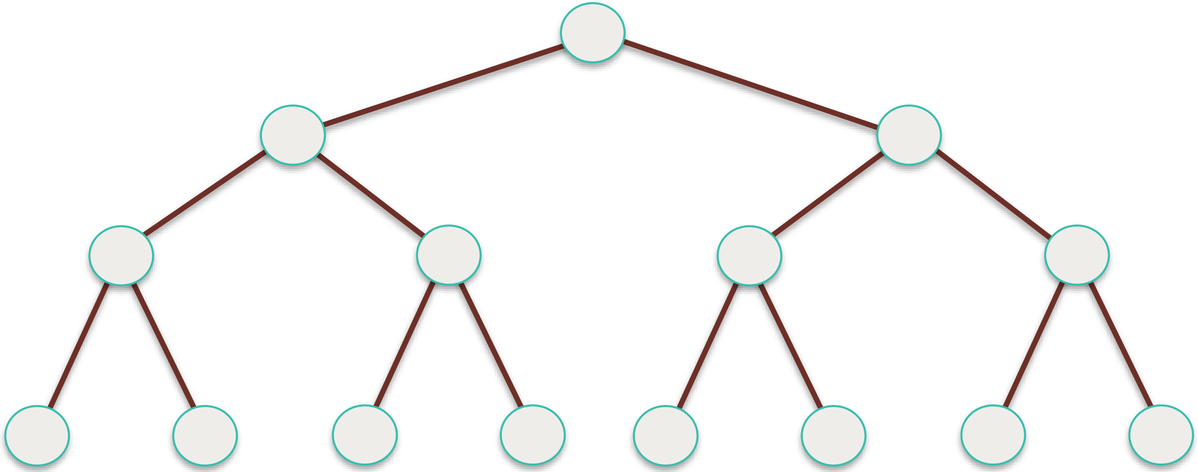
BFS and DFS Parallelism

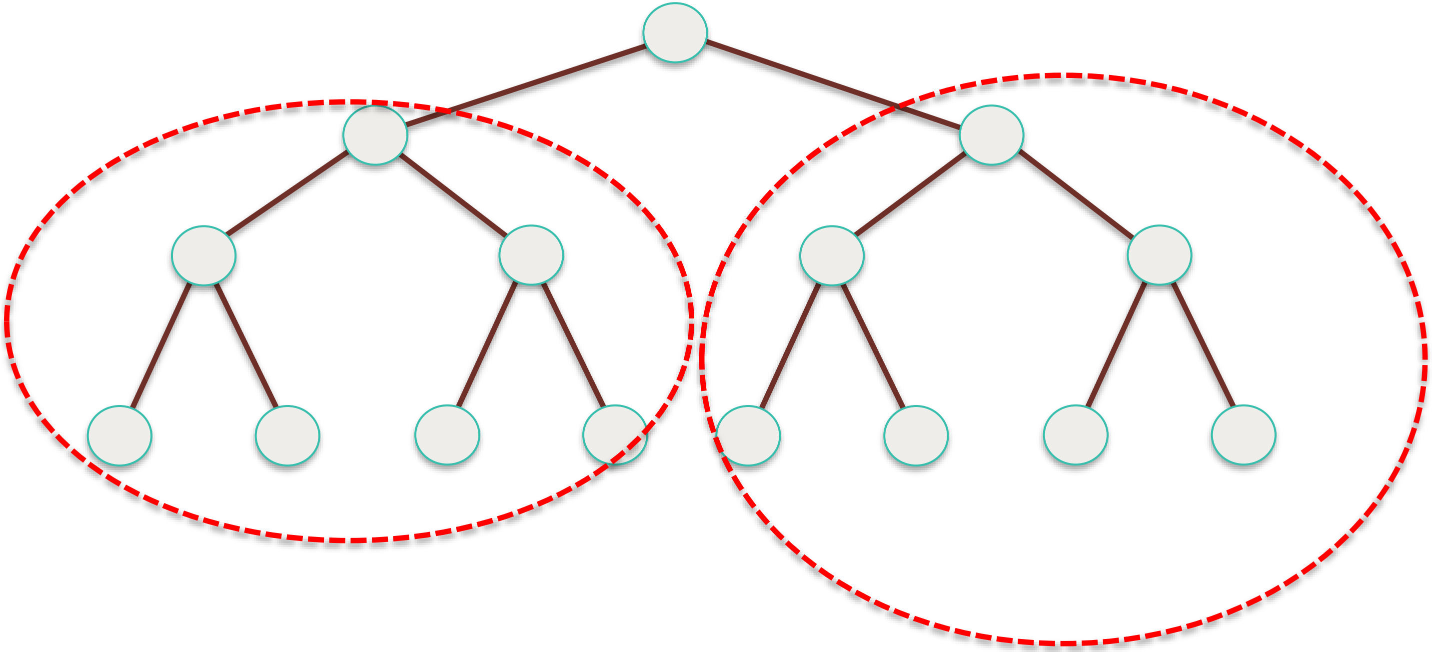
BSF and DFS Parallelism

- How the search space is splitted among processors?
- Different subtrees can be searched concurrently
- However, subtrees can be very different in size
- It is difficult to estimate the size of a subtree rooted at a node
- Dynamic load balancing is required

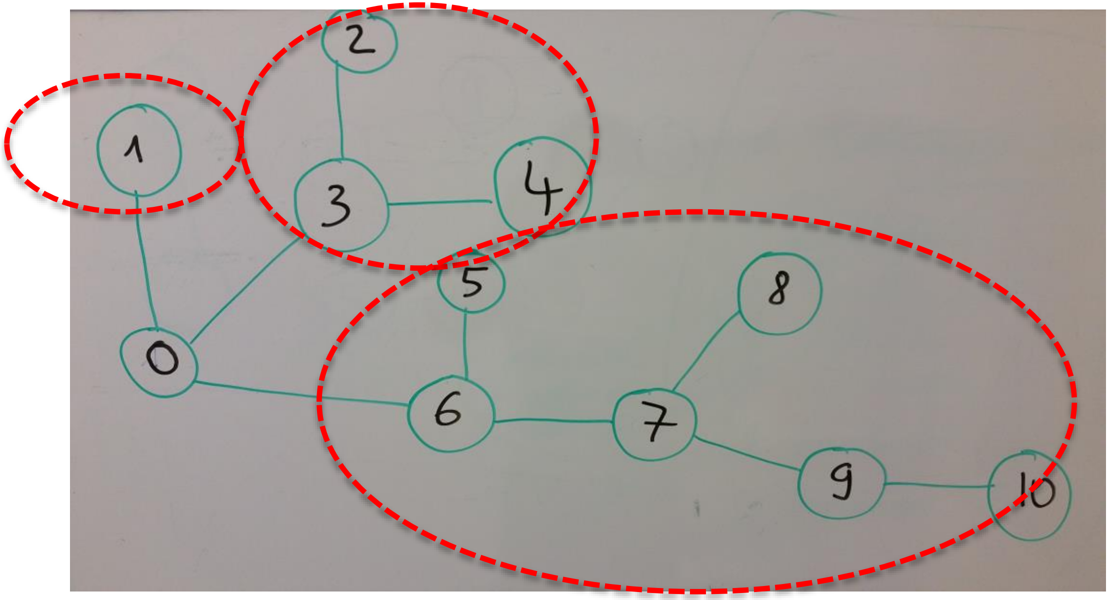
Breadth-First Search (BFS): Tree



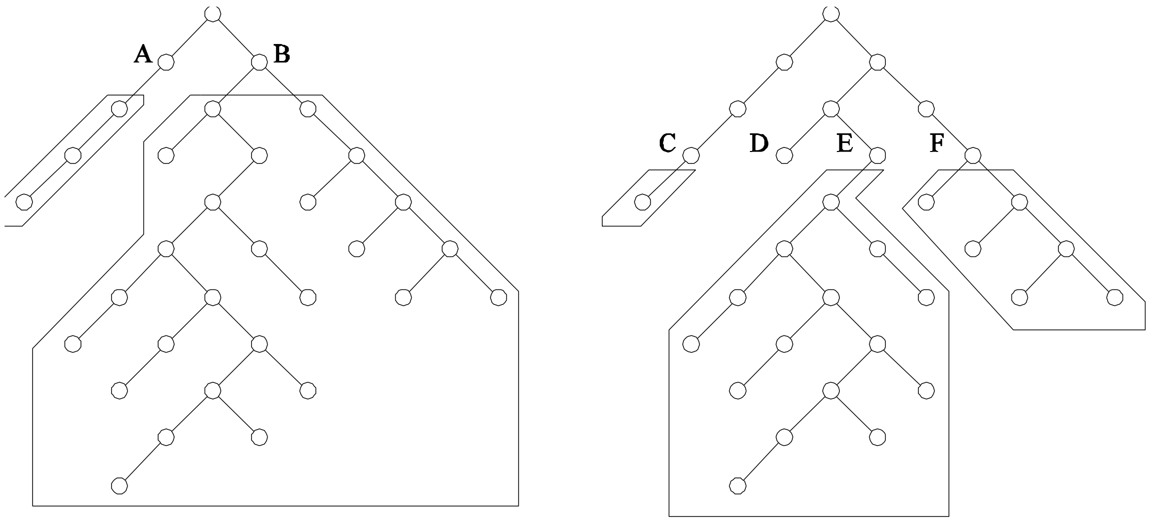
Breadth-First Search (BFS): Tree



BSF



DFS



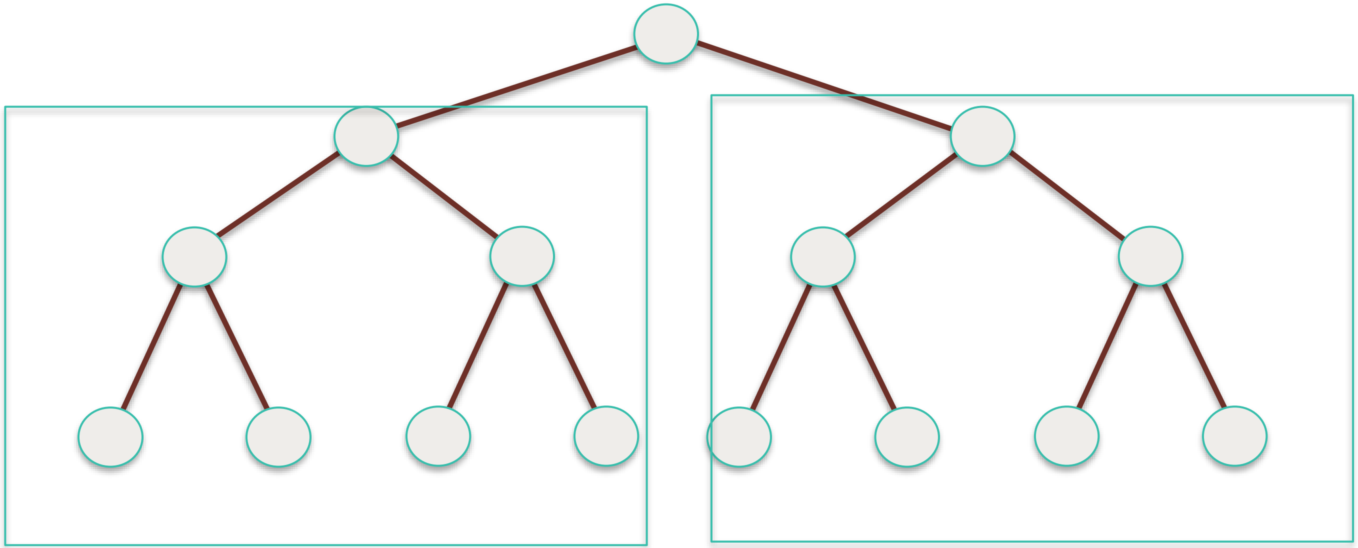
(a) (b)

The unstructured nature of tree search and the imbalance resulting from static partitioning

BSF Pseudo-code (parallel)

- Ideas:
- Process level in parallel
  - Synchronization is needed at each level
- Process an entire level in parallel
  - Load balancing is needed

Parallel Depth-First Search

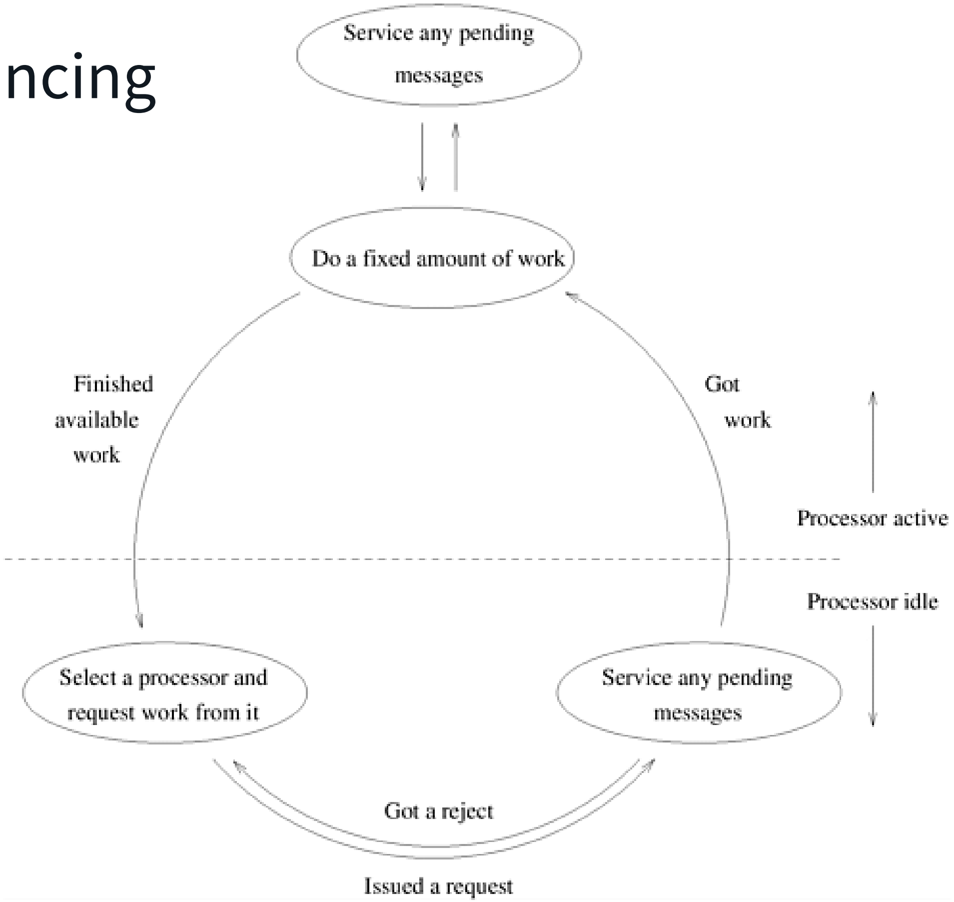


DFS: Dynamic Load Balancing

- When a processor runs out of work, it gets more work from another processor
- This is done using work requests and responses in message passing machines and locking and extracting work in shared address space machines
- On reaching final state at a processor, all processors terminate
- Unexplored states can be conveniently stored as local stacks at processors
- The entire space is assigned to one processor to begin with

DFS: Dynamic Load Balancing

Applied with distributed memory system

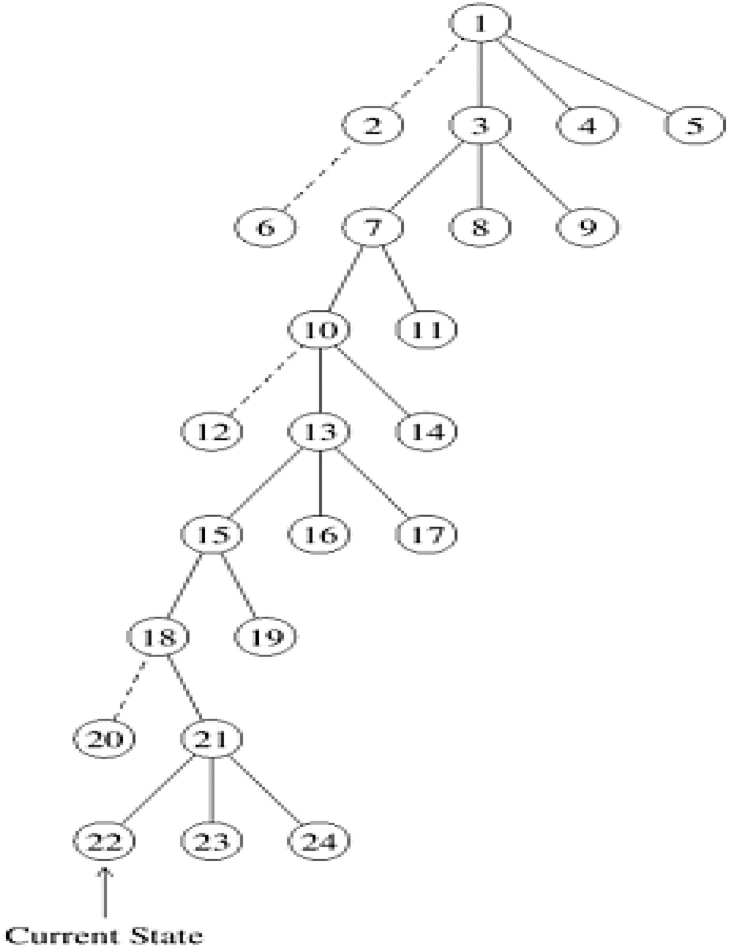


Parallel DFS: Work splitting

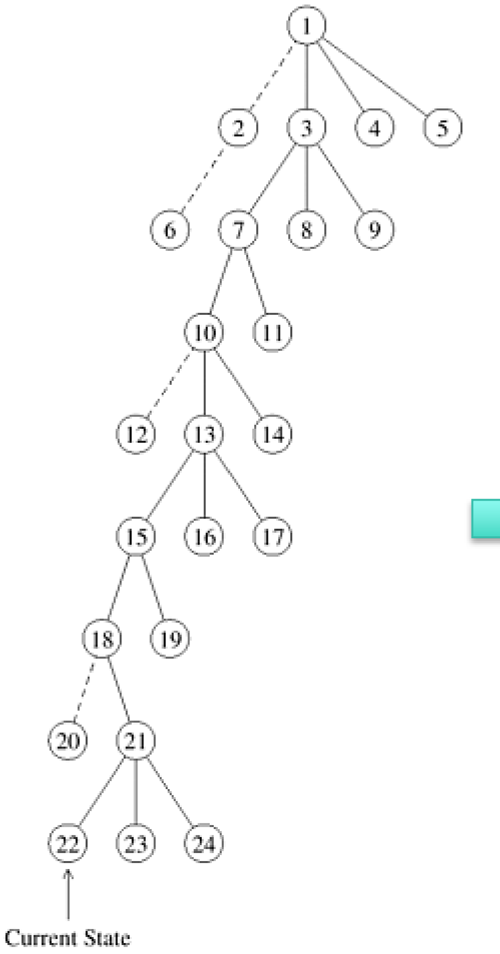
- Work is split by splitting the stack into two
- Ideally, we do not want either of the split pieces to be small
- Select nodes near the bottom of the stack (node splitting)
- Select some nodes from each level (stack splitting).
- The second strategy generally yields a more even split of the space

Parallel DFS: Work splitting

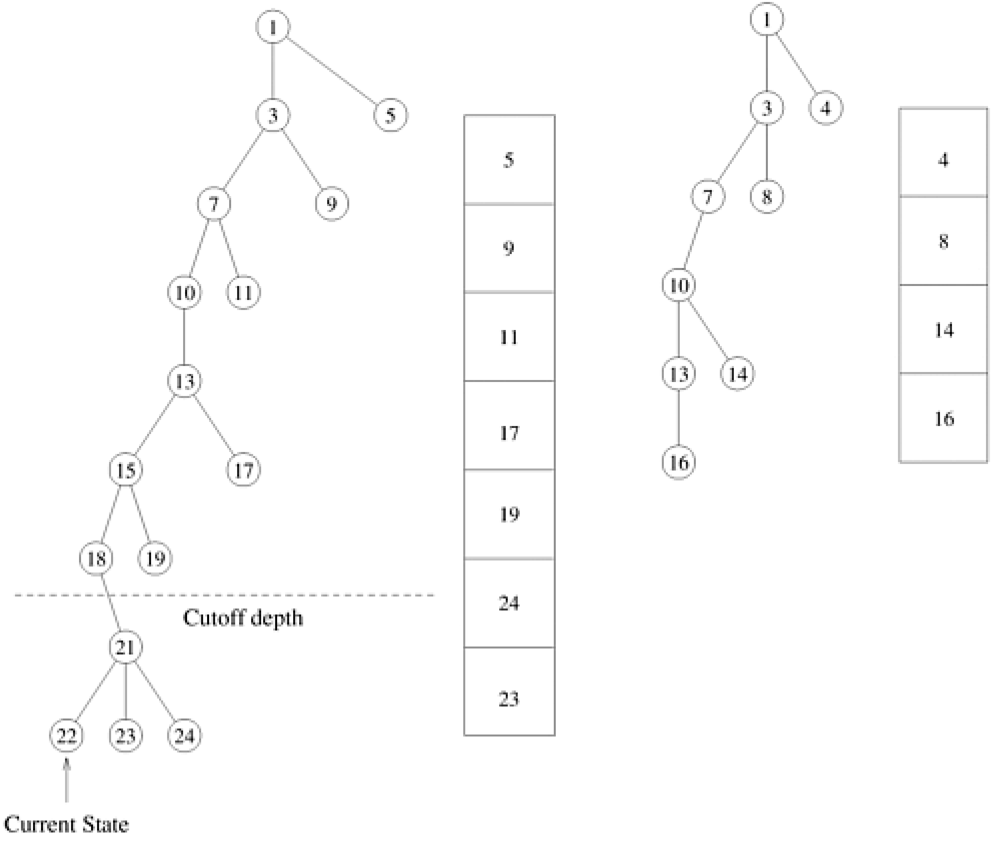
- Original works



(a)



(a)



(a) (b)

Splitting the DFS tree: the two subtrees along with their stack representations are shown in (a) and (b).

Parallel Depth-First Search: Pseudocode

```
DFS-recursive(G, s):
    mark s as visited
    for all neighbours w of s in Graph G:
        if w is not visited:
            DFS-recursive(G, w)
```

References

[1] *Introduction to Parallel Computing,* Ananth Grama, George Karypis, Vipin Kumar, Anshul Gupta, Addison Wesley, 2003.


Q&A