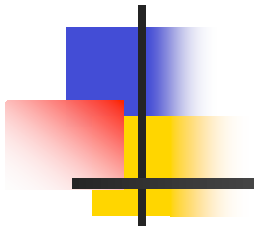

Statistical Methods for determining optimal rifle cartridge dimensions

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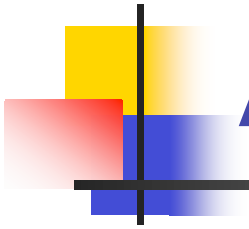

Acknowledgments

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I would like to thank my professors for advising me in all aspects of the project:

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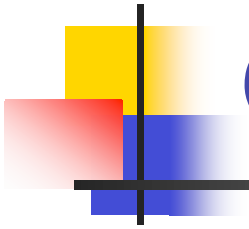
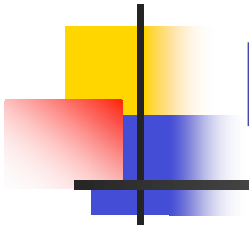


Overview

- Rifle Precision is dependent on characteristics of the individual firearm and the ammunition.
- The acceleration of the bullet passing through the barrel causes a three dimensional sine wave vibration.
- The characteristics of this vibration determine the precision of the rifle.

Overview - Continued

- The vibration (i.e. the rifle precision) depend on the dimensions of the cartridge and the chamber.
- The dimensions of the chamber are fixed.
- However, many marksmen produce their own ammunition with specific cartridge dimensions.

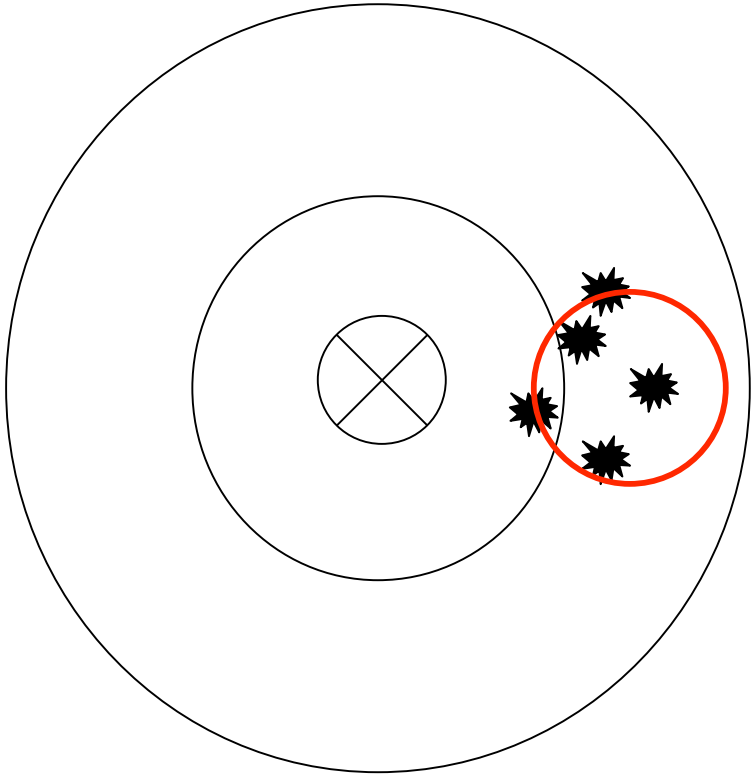


Motivation

- Surprisingly, a systematic method for determining these optimal characteristics using design of experiment techniques has not been published.
- Competitive marksmen use their intuition, experience and trial and error to determine the optimal cartridge dimensions for their rifle. They typically do not use statistical methodology.
- We set out to design and conduct an experiment to determine such optimization.

The Response Variables

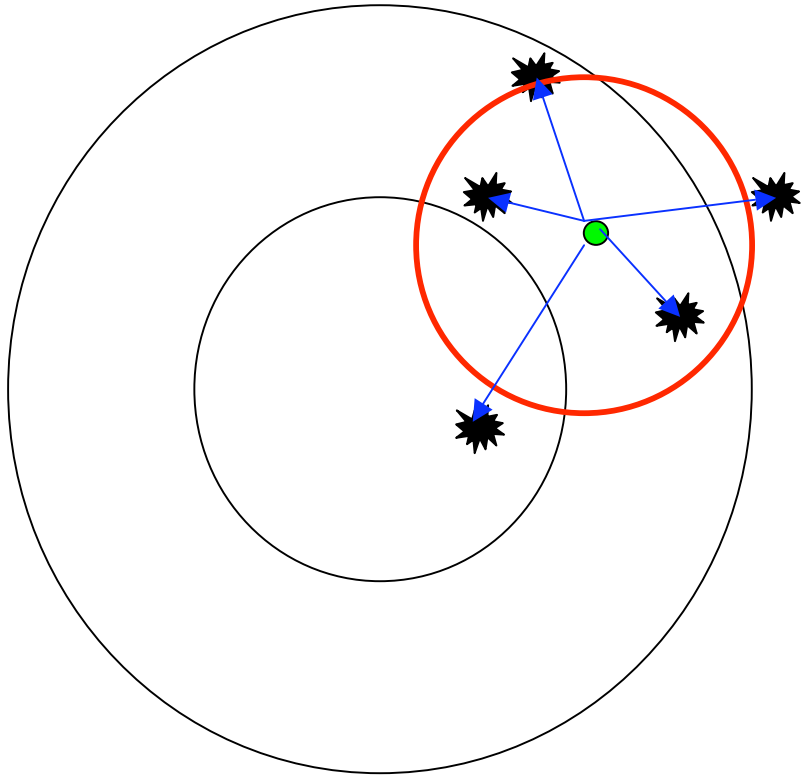
- Measure of Precision
- Characteristic of a group of shots
- There are various ways to measure the precision
- We chose the Mean Group Radius for this study



Precision Measure

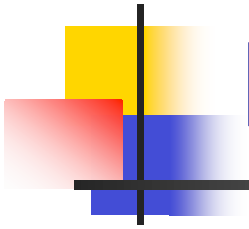
- **Mean Group Radius**
- Measure of the average Euclidean Distance from the group center

$$MR = \frac{\sum_{i=1}^{n} \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}{n}$$



Main Factors

- Seating Depth
  - How far the bullet is seated into the case.
  - 6 levels
  - 0.005 to 0.030 inches incremented by 0.005 inches
- Powder Charge
  - Amount of powder used in the cartridge by weight
  - 10 levels
  - 25.3 to 26.2 grains incremented by 0.1 grains



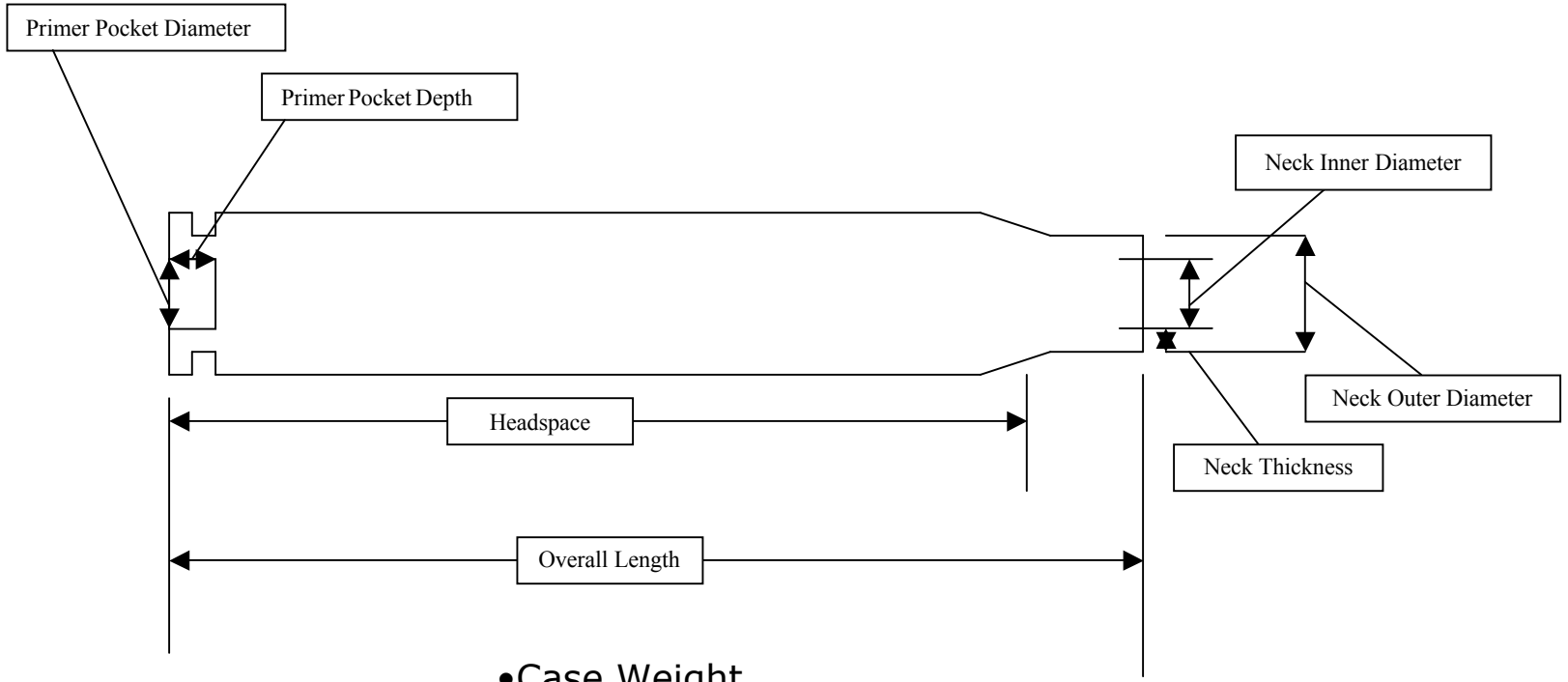
Experimental Design

- Full Factorial Design with two factors
- 6 seating depths * 10 charges = 60 levels
- We identified 13 covariates that describe the cartridge dimensions.
- Cases were prepare in 4 lots of 100 cases, for a total of 400 experimental units.

Blocking Factor

- As we prepared the cases, we have found statistically significant differences in several covariates between lots.
- Therefore, we decided to use the lots as blocks.
- The randomization across lots:
  - I randomly assigned a case from each lot to each experimental level for a total of 60 cases per lot
  - I then took the remaining 40 cases per lot and randomly assigned them an experimental level
  - Each level ended up having between 5 and 8 cases.
- Each case was numbered.

The 13 Covariates



- Case Weight
- Case Volume by Weight
- Overall Bullet Length
- Bullet Weight
- Primer Weight
- Case Mouth Square

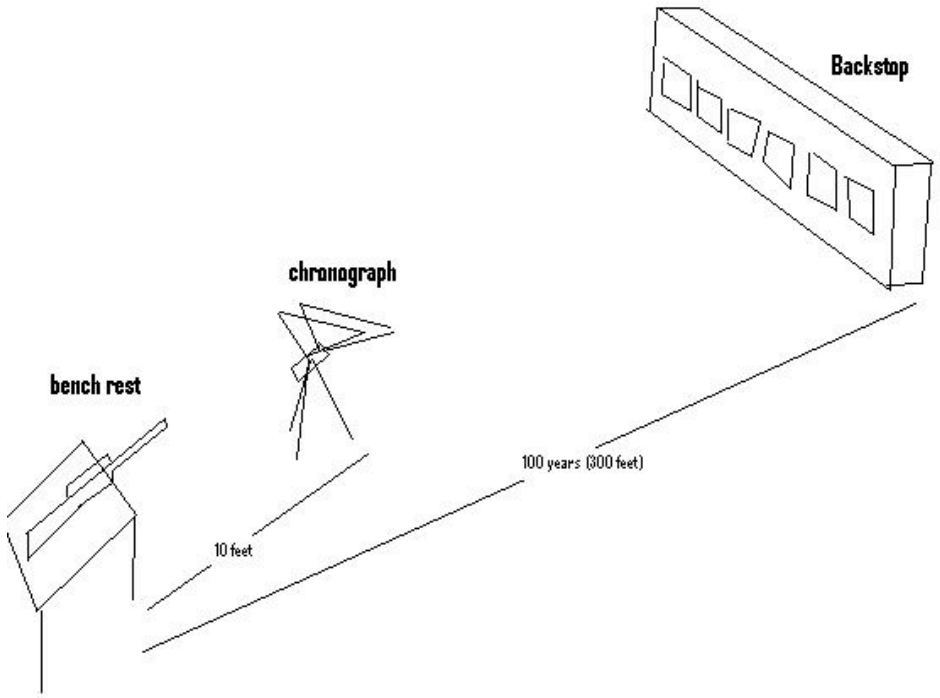
The Cartridge Components

- The brass cartridge cases were “once-fired” military rounds made by Winchester
- A 29 Step process was followed to prepare the brass to ensure uniformity between the brass cases.
- Bullets used were copper jacketed, polymer tipped, 55 grain Hornady VMAX
- Primers used were Winchester Small Rifle Primers
- Powder used was Hodgdon H4895

Experimental Details

- The rifle was a Savage 10FLP chambered in .223 Remington.
- Distance from rifle to target is 100 yards.
- Each shot was fired into a separate target.
- Barrel was cleaned after every 20 shots (except for the last 40).
- We also used a chronograph to measure the velocity of the each bullet. However, the velocity was not measured for some shots, due to the shooting angle.

Experimental Setup



The shooting

- Shots were fired from a sitting benchrest position
- The rifle is supported on a bench platform with the shooter sitting beside it
- The fore end was supported by a tripod with shot bag
- The stock was supported by a contoured shot bag

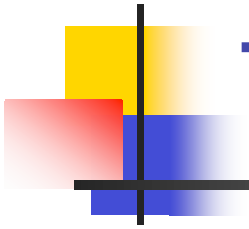
The shooting position

Environmental Conditions

- Shots were spread over two days of shooting
- Day 1 Ranged from 60-74 degrees F and Day 2 Ranged form 50 - 55 degrees F.
- Wind was blowing at 10-20 mph
- Over cast with intermittent rain

Nuisance & Bad Data Points

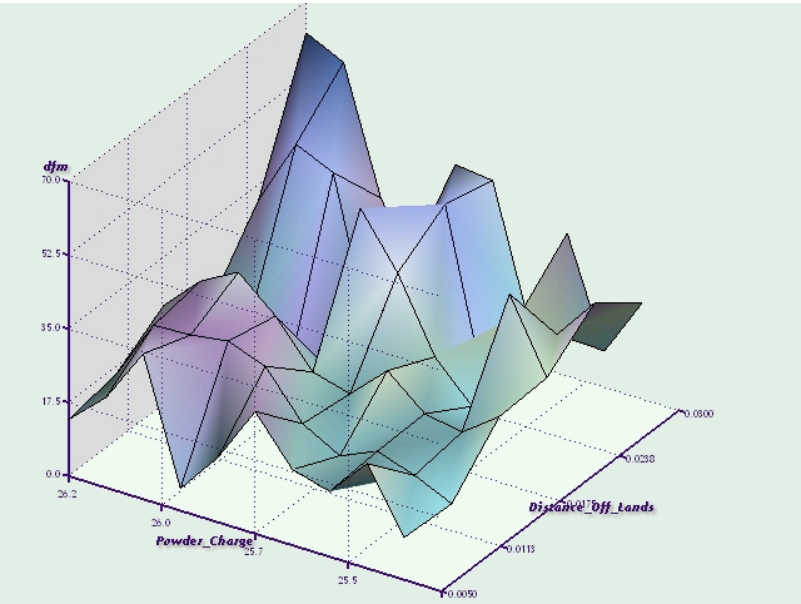
- Wind was blowing most of the time
- Human Error
  - 5 pulled to right
  - 5 pulled to left
- Copper Fouling in barrel
  - 4 tumbled and hit target sideways
- 3 where shot into wrong target, so I could not identify which shot was which
  - 6 shots total were invalidated due to this
- All together: 20 shots were invalidated

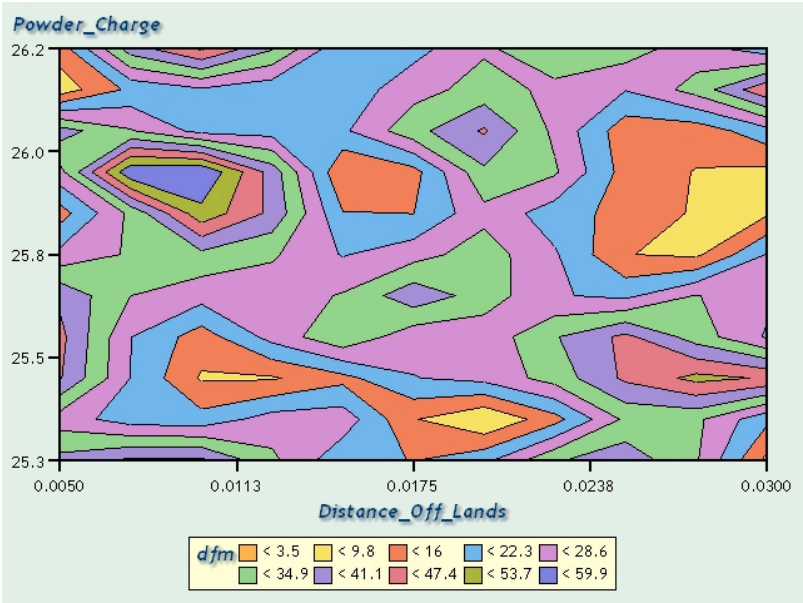


The Precision Data

- Upon completing the data collection, I computed the group centers $(\bar{x}, \bar{y})$ for each of the factor-level combinations.
- For each shot I computed the Euclidean distance from it's factor level group center
- The average of these distances over the factor-level combinations is refereed to as the Mean Group Radius.
- I then obtained the "Response Surface" over the level combinations using the Mean Group Radius.

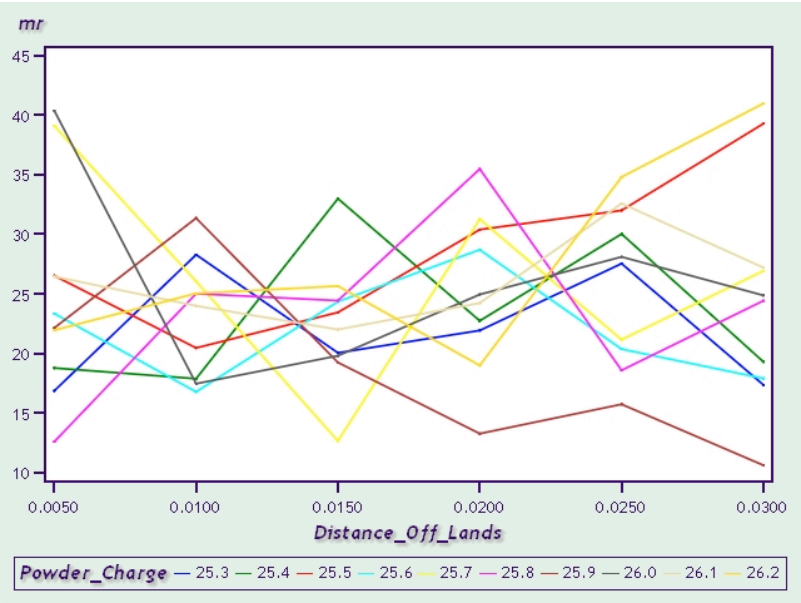
The Group Mean Radius

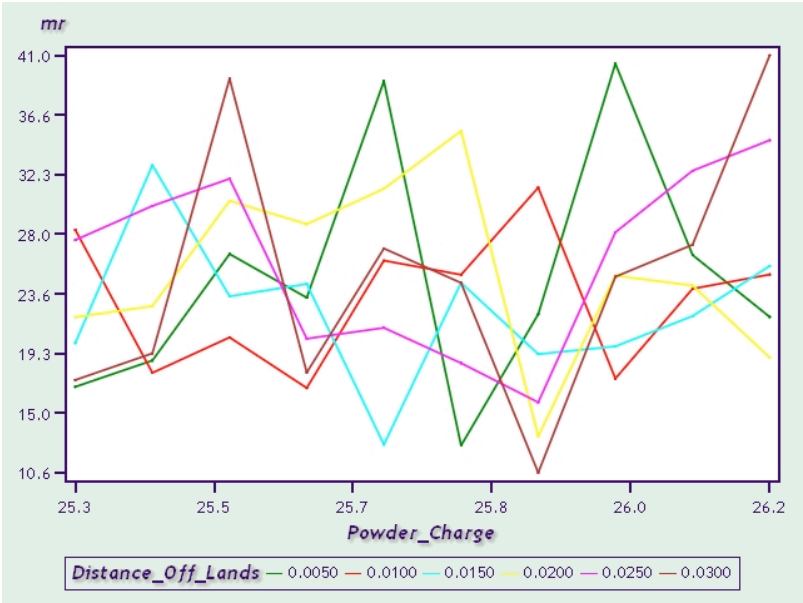




Mean Group Radius (mm)

Mean Group Radius





Mean Group Radius (mm)

Analysis of Variance

- We ran PROC GLM in SAS for the Shot Distances from the Group Centers
  - P-Value for the overall model was less than 0.001
  - No Covariates were found to be significant

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	62	17752.73721	286.33447	1.78	0.0008
Error	317	51124.82714	161.27706		
Corrected Total	379	68877.56436			

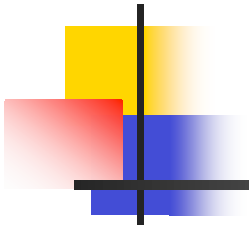
Source	DF	Type I SS	Mean Square	F Value	Pr > F
lot	3	109.68557	36.56186	0.23	0.8778
Seating Depth	5	614.93659	122.98732	0.76	0.5774
Powder Charge	9	2917.0905	324.12117	2.01	0.0378
Seating Depth * Powder Charge	45	14111.02455	313.57832	1.94	0.0006

The Optimal Cartridge Dimensions

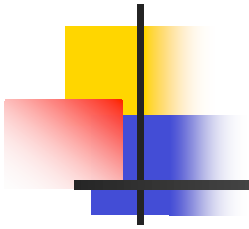
- To identify the minimal mean response for the factor-levels, we tabulated and sorted the mean radius, and identified the “best” ones and their corresponding factor-levels.
- Mean Group Radius
  - MR=10.6214 Powder Charge=25.9 Seating Depth=0.030
  - MR=12.5963 Powder Charge=25.8 Seating Depth=0.005
- Maximum for comparison
  - MR=40.96598 Powder Charge=26.2 Seating Depth=0.030

Future Work

- We have identified two optimal levels of seating depth and powder charge.
- Are the two minimum levels statistically different?
- Load several lots of each optimal level and test. A good reason to go shooting again!



Thank you !!!



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