Homework 2

February 14, 2022

Transistor DC behavior. Base design and SiGe HBT.

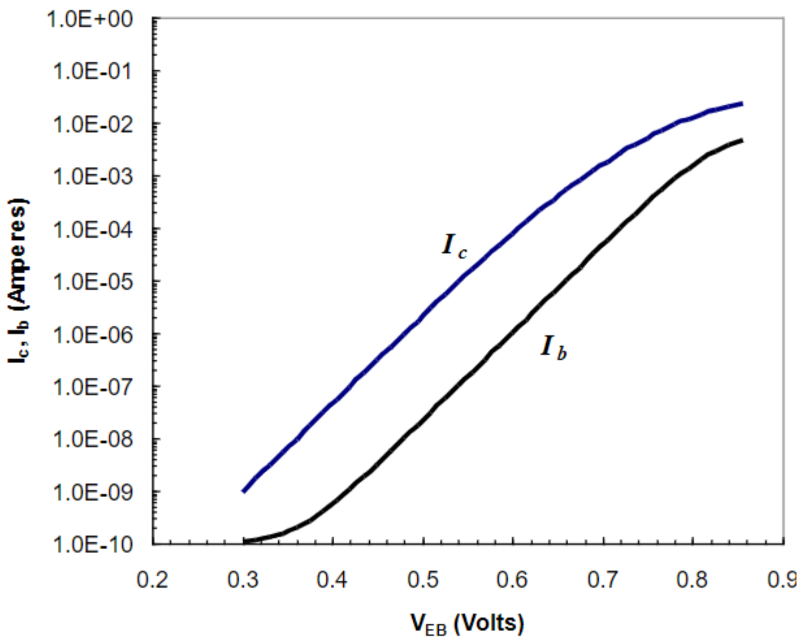

Figure 1: Graph given from Problem 1

1. (a) The current gain of a common emitter current is simply $\frac{I_c}{I_b}$. In this case, each value of β can be estimated by tracing the values of I_b and I_c, and dividing.
   At $V_{EB} = 0.3, I_b$ appears to be 1.0E-10, and I_c appears to be 1.0E-09, giving a β of 10.
   At $V_{EB} = 0.4, I_b$ appears to be 4.5E-10, and I_c appears to be 5.0E-08, giving a β of about 110.
   At $V_{EB} = 0.5, I_b$ appears to be 2.0E-8, and I_c appears to be 2.0E-06, giving a β of 100.
   At $V_{EB} = 0.6, I_b$ appears to be 1.0E-6, and I_c appears to be 9.0E-05, giving a β of 90.
   At $V_{EB} = 0.7, I_b$ appears to be 4.0E-5, and I_c appears to be 1.9E-03, giving a β of about 50.
   At $V_{EB} = 0.8, I_b$ appears to be 1.8E-3, and I_c appears to be 1.3E-02, giving a β of about 7.

   (b) See the graph in figure 2.

   (c) Emitter current I_e is simply base current added to collector current, which is a property of Kirchhoff's law. It is shown in figure 2, as well as current gain β.

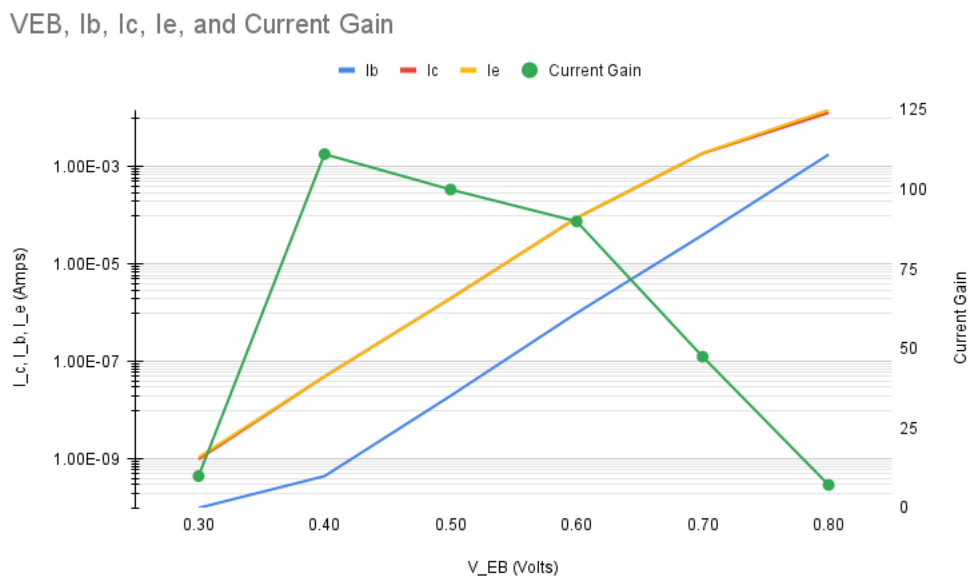

Figure 2: Graph comparing V_{EB}, I_b, I_c, I_b, and β.

2. For the small signal model, the voltage gain is given as $g_m R_L$, or $\frac{I_C}{V_T} R_L$. The voltage gain in this case would be:

$$\frac{100\mu\text{A}}{26\text{mV}} * 50\Omega = \frac{0.0001\text{A}}{0.026\text{V}} * 50\Omega = 0.192\text{V}$$

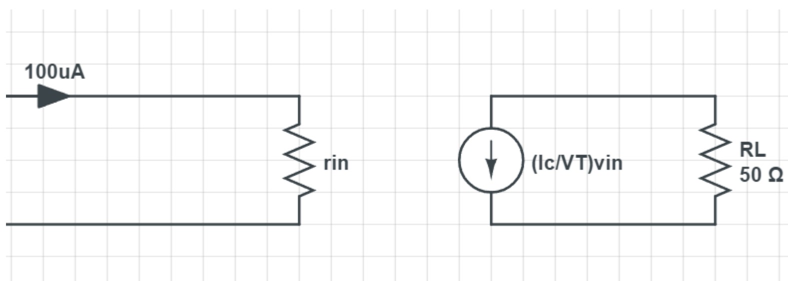

Figure 3: Common Emitter Small Signal Model for Problem 1

3. (a) The load line is drawn in orange in figure 4 below.

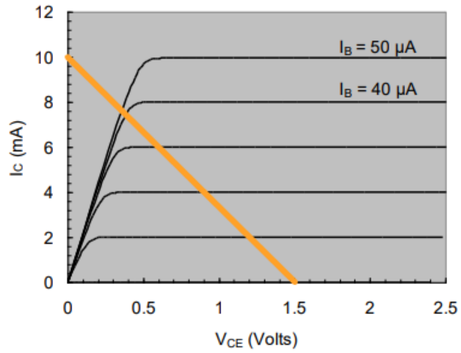

Figure 4: Transistor I_C/V_{CE} curve with the load line drawn in orange.

(b) If V_{CC} is 1.5V and load is 150Ω, that means the voltage at the collector terminal (by Ohm's law) will be 1.5V $-$ ($I_C * 150\Omega$). Since the emitter is grounded, this is the formula for V_{CE}. When I_B is 30µA, I_C is about 6mA. This means:

$$V_{CE} = 1.5 - (0.006)(150) = 0.6\text{V}$$

(c) For the above characteristics, the transistor is in an Active region (although barely). The point (0.6, 6) on the 30 µA curve is on a linear portion.

(d) If the base current is reduced to 15µA, I_C would be about 3mA. In this case, V_{CE} would be:

$$V_{CE} = 1.5 - (0.003)(150) = 1.05\text{V}$$

Since this point, (1.05, 3) on the graph (which would fall on an undrawn 15µA curve) would be on a flat point, this would fall into an Active region.

(e) If the base current is increased to 50µA, I_C would be about 10mA. In this case, V_{CE} would be:

$$V_{CE} = 1.5 - (0.01)(150) = 0\text{V}$$

Since the voltage is 0 at this point, the transistor would be in Saturation mode (or, simply, off).

4. Under normal circumstances, the electric field in an n-region is given as:

$$E(\text{n-region}) = -\frac{kT}{q}\frac{1}{n_n}\frac{dn_n}{dx}$$

However, as the textbook states, this does not include the effect of nonuniform energy bandgap. (i.e. the equation above assumes that the doping is uniform everywhere). For arbitrary doping levels, Equation (6.25) gives:

$$J_n(x) = qD_n\frac{n_{ie}^2}{p_p}\frac{d}{dx}(\frac{n_p p_p}{n_{ie}^2})$$

This can be broken down into drift and diffusion components, which in general is given by Equation (6.5):

$$J_n(x) = qn\mu_n E + qD_n\frac{dn}{dx}$$

Therefore, solving for E and substituting J_n leads to:

$$E(\text{n-region}) = -\frac{kT}{q}(\frac{1}{n_n}\frac{dn_n}{dx} - \frac{1}{n_{ie}^2}\frac{dn_{ie}^2}{dx})$$

Q.E.D.

5. Equation (6.90) gives:

$$\alpha_R I_{RO} = \alpha_F I_{FO}$$
$$\therefore \frac{I_{FO}}{I_{RO}} = \frac{\alpha_F}{\alpha_R}$$

Equation (6.142) gives:

$$I_{CBO} = I_{F0}(1 - \alpha_R \alpha_F)$$

And Equation (6.143) gives:

$$I_{CBO} = I_{R0}(1 - \alpha_R \alpha_F)$$

Therefore, $\frac{I_{CBO}}{I_{EBO}}$ evaluates to:

$$\frac{I_{CBO}}{I_{EBO}} = \frac{I_{F0}}{I_{R0}} = \frac{\alpha_F}{\alpha_R}$$

Now, the two equations given in the problem statement can be solved for V'_{BE} and V'_{BC}:

$$V'_{BE} = \frac{kt}{q} \ln \left[1 - \frac{I_E + \alpha_R I_C}{I_{EBO}}\right]$$

$$V'_{BC} = \frac{kt}{q} \ln \left[1 - \frac{I_E + \alpha_F I_E}{I_{CBO}}\right]$$

Using Kirchoff's Law modeling a BJT as a node, it can be determined that the voltage drop from collector to emitter is the difference between the voltage drop between the base and emitter, and the base and collector. That is,

$$V_{CE} = V_{BE} - V_{BC}$$

Therefore:

$$V'_{CE} = \frac{kt}{q} \ln \left[1 - \frac{I_E + \alpha_R I_C}{I_{EBO}}\right] - \frac{kt}{q} \ln \left[1 - \frac{I_E + \alpha_F I_E}{I_{CBO}}\right]$$
$$V'_{CE} = \frac{kT}{q} \ln \left[1 - \frac{I_E + \alpha_R I_C}{I_{EBO}} - 1 + \frac{I_E + \alpha_F I_E}{I_{CBO}}\right]$$

Simplify the argument by combining 1 into each fraction:

$$V'_{CE} = \frac{kT}{q} \ln \left[\frac{I_{EBO} - I_E - \alpha_R I_C}{I_{EBO}} - \frac{I_{CBO} - I_C - \alpha_F I_E}{I_{CBO}}\right]$$
$$= \frac{kT}{q} \ln \left[\frac{I_{CBO}(I_{EBO} - I_E - \alpha_R I_C)}{I_{EBO}(I_{CBO} - I_C - \alpha_F I_E})\right]$$

And since it was already determined that $\frac{I_{CBO}}{I_{EBO}} = \frac{\alpha_F}{\alpha_R}$:

$$V'_{CE} = \frac{kT}{q} \ln \left[\frac{\alpha_F(I_{EBO} - I_E - \alpha_R I_C)}{\alpha_R(I_{CBO} - I_C - \alpha_F I_E)}\right]$$

Q.E.D.

6. Setting I_{EBO} and I_{CBO} to 0 in the solution for problem 5 leads to:

$$V_{CE} = \frac{kT}{q} \ln \left[\frac{\alpha_F(-I_E - \alpha_R I_C)}{\alpha_R(-I_C - \alpha_F I_E)}\right]$$

Which can be rearranged (using definition of terminal currents) as:

$$V_{CE} = \frac{kT}{q} \ln \left[\frac{I_B + I_C(1 - \alpha_R)}{\alpha_R[I_B - I_C(1 - \alpha_F)/\alpha_F]}\right]$$

Figure 5 shows a simple model of a BJT with parasitic resistances. By Kirchhoff's Law, $I_e = I_b + I_c$.

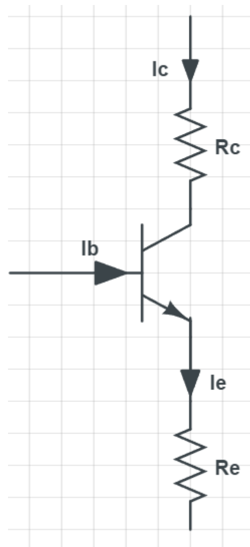

Figure 5: Simple model of a BJT with parasitic resistances R_c and R_e on the collector and emitter, respectively.

The total voltage drop including these parasitic elements, then, is:

$$V_{CE} = \frac{kT}{q} \ln \left[\frac{I_B + I_C(1-\alpha_R)}{\alpha_R[I_B - I_C(1-\alpha_F)/\alpha_F]} \right] + r_e(I_b + I_c) + r_c I_c$$

Q.E.D.

7. Because we can consider only diffusion current, we can say that the current density is simply:

$$J_n(x) = \frac{dn_p}{dx}$$

Since the problem statement gives n_p, we can take the derivative of it to find the current density:

$$J_n(x) = n_{p0}[\exp{(\frac{qV_{BE}}{kT})} - 1] \frac{\cosh{(\frac{W_B - x}{L_{nB}})}}{\sinh{(\frac{W_B}{L_B})}} (\frac{-1}{L_{nB}})$$

Therefore, for $J(x = W_B)$:

$$J_n(x = W_B) = n_{p0}[\exp{(\frac{qV_{BE}}{kT})} - 1] \frac{\cosh{(0)}}{\sinh{(\frac{W_B}{L_B})}} (\frac{-1}{L_{nB}})$$

And for $J(x = 0)$:

$$J_n(x = W_B) = n_{p0}[\exp{(\frac{qV_{BE}}{kT})} - 1] \frac{\cosh{(\frac{W_B}{L_B})}}{\sinh{(\frac{W_B}{L_B})}} (\frac{-1}{L_{nB}})$$

Finally, substitute these values into the given α_T:

$$\alpha_T = \frac{n_{p0}[\exp{(\frac{qV_{BE}}{kT})} - 1] \frac{\cosh{(0)}}{\sinh{(\frac{W_B}{L_B})}} (\frac{-1}{L_{nB}})}{n_{p0}[\exp{(\frac{qV_{BE}}{kT})} - 1] \frac{\cosh{(\frac{W_B}{L_B})}}{\sinh{(\frac{W_B}{L_B})}} (\frac{-1}{L_{nB}})}$$

The first and third terms cancel out. Both sinhs cancel out, and $\cosh(0) = 1$, so:

$$\alpha_T = \frac{1}{\cosh\left(\frac{W_B}{L_{nB}}\right)}$$

Q.E.D.

In figure 2.24(c), at $N_B = 1 * 10^{18}/cm^3$, the diffusion length appears to be about 20 micrometers. Therefore:

$$\begin{aligned}\alpha_T &= \frac{1}{\cosh\left(\frac{100\text{nm}}{20\mu\text{m}}\right)}\\ &= \frac{1}{\cosh(0.05)}\\ &= \frac{1}{1.00125026044}\\ &= 0.99875130075\end{aligned}$$

8. From the graph given in Lecture 8:
   (a) About 1.25.
   (b) About 0.88.
   (c) About 0.50.
   (d) (Seemingly) Exactly 0.

9. We are given:

$$R_{Sbi} = \left(q\int_0^{W_b} p_p(x)\mu_p(x)\,dx\right)^{-1}$$

It can be approximated, at low currents, that $p_p(x) \approx N_b(x)$. However, I am unsure how to involve W_B in this equation.

10. Electron diffusion is:

$$D_{nB} = \frac{kT\mu_{nB}}{q}$$

Therefore, the base transit time can be written as:

$$t_b \approx \frac{W_B^2 q}{kT\mu_{nB}}$$

Equation (2.140) gives a relationship between the minority-carrier lifetime μ_n and the base doping concentration N_b:

$$\mu_n = 232 + \frac{1180}{1 + (\frac{N_b}{8} * 10^{16})}$$

Therefore:

$$t_b \approx \frac{W_B^2 q}{kT(232 + \frac{1180}{1+(\frac{N_b}{8}*10^{16})})}$$

This could then be solved for various doping concentrations and base widths as found on Figure 2.24 of the textbook, however, I'm unsure of the relationship between base width and doping concentration. I was unable to find an equation relating the two, so I am simply at a loss as to which values to extract off the table to plug into my above equation.