Investigations

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Form all cornerassignments without any seperating 4-cycle a onesided dual.

False

Then it also follows that there are no onesided colorings of this graph.

Any valid exteneded graph is a 5-connected traingulation minus one edge.

Lemma 1. *For plane triangulations having no separating 4-cycle is the same as being 5-connected*

Proof. We will show this by contraposition (i.e. having a separating 4-cycle is equivalent to not being 5-conencted). If a traingulation G has a separating 4-cyle then the nodes of this 4-cycle are a 4-cutset and G is not 5-connected. On the other hand, if a triangulation is not 5 connected there is a cutset X of size at most 4. Removing X splits G into several connected components. By the property that G maximally planar the nodes in X must form a cycle. (They should form a closed curve preventing edges from the one component to the other one.) □

When we remove an edge from a 5-connected triangulation we get a valid extended graph without any seperating 4-cycle.

Proof. Let G be any 5 connected plane triangulation. By Lemma 1 we know that G has no separating 4-cycle. If we consider any edge e and $\tilde{G} = G \setminus e$ then certainly $\tilde{G}$ also has no seperting 4-cycle. Since any separting 4-cycle of $\tilde{G}$ would have already been one of G. □

Note that a 5-conneted graph minus one edge is not necessarily again 5-connected.

Unfortunatly not all extended graphs without seperating 4-cyles can be generated this way. Conside for example the graph in Figure 1. This example has been checked by hand and python on not containing 4cycles.

1 5-connected traingulations

5 connected triangulations can be created iteratively using a number of moves.

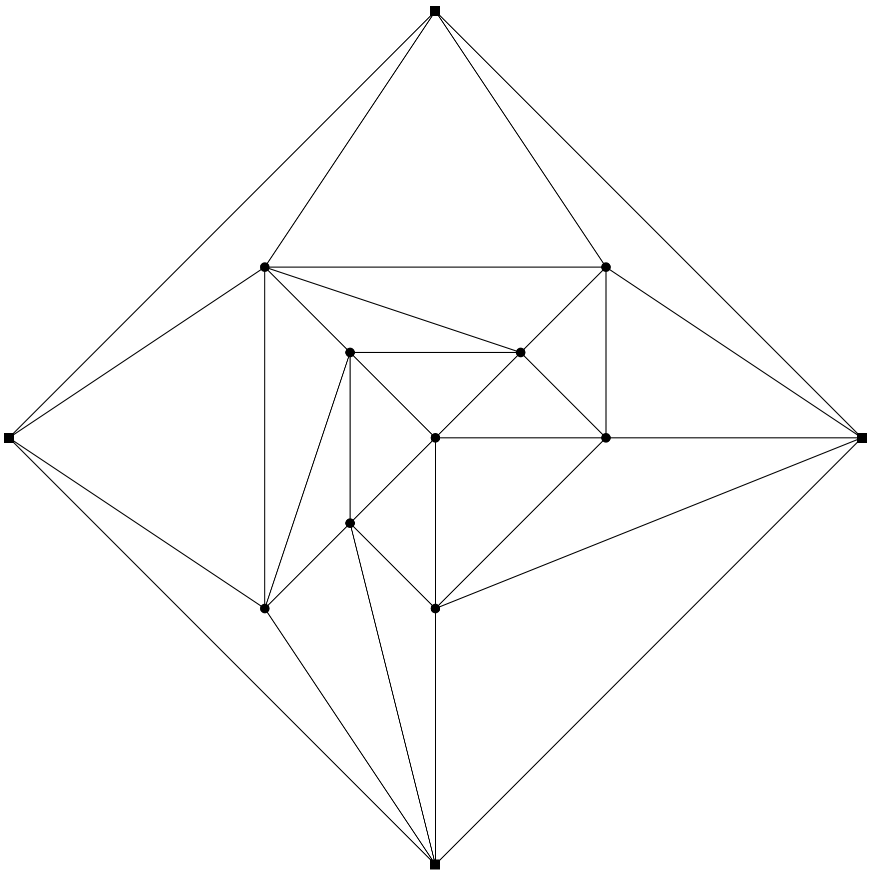

Figure 1