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Nonexistence of a certain rectangular floorplan with specified areas and adjacency

I Rinsma
Mathematics Department, University of Canterbury, Christchurch 1, New Zealand
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Abstract. In facility layouts—the designing of floorplans with certain rooms adjacent to each other—there are often area constraints for the rooms. Robinson and Janjic showed that, if areas are specified for rooms with a given maximal outerplanar adjacency graph, then any convex polygon with the correct area can be divided into convex rooms to satisfy both area and adjacency requirements. If the perimeter and rooms must be rectangular, undimensioned floorplans can be found to fit any maximal outerplanar adjacency graph with at most four vertices of degree 2. It is shown that in some cases it is not always possible to satisfy the area constraints.

Introduction

A *rectangular floorplan* is a rectangle (the plan perimeter) with its interior divided by straightline segments (walls) into smaller rectangles called *rooms*. To each floorplan corresponds an *adjacency graph* in which the vertices represent the rooms, and two vertices are joined by an edge if the corresponding rooms have parts of their boundaries in common. That is, they share some length of wall. It is not sufficient for them to touch in a corner only.

If the exterior of the floorplan is ignored, then the adjacency graph is the dual of the floorplan treated as a graph. Earl and March (1979) call this type of adjacency graph, where only interior adjacencies between rooms are shown, the *weak dual* of the plan. Places where three rooms meet correspond to triangles, and four rooms to quadrilaterals in the adjacency graph.

An *outerplanar graph* (Harary, 1969) is a graph that can be embedded in the plane so that all its vertices lie on the exterior face. An outerplanar graph is *maximal outerplanar* if no edge can be added without losing outerplanarity. Then each interior face is a triangle. If areas are specified for rooms with a given maximal outerplanar adjacency graph, then any convex polygon with the correct area can be divided into convex rooms so that area and adjacency requirements are both satisfied (Robinson and Janjic, 1985).

Rectangular floorplans with maximal outerplanar adjacency graphs

A rectangular floorplan with an outerplanar adjacency graph implies each room has at least one wall in common with the plan boundary (Lynes, 1977). *Corner rooms* and *crossrooms* both have exactly two walls forming part of the perimeter. For corner rooms, these walls are adjacent, whereas for crossrooms they are opposite each other. An *endroom* has three walls on the boundary. A rectangular floorplan has a two-connected maximal outerplanar graph if each room meets the floorplan perimeter in one continuous length of wall and never more than three rooms meet at any point. Thus crossrooms are not allowed. Vertices of degree 2 correspond either to corner rooms or to endrooms. Clearly any maximal outerplanar adjacency graph with more than four vertices of degree 2 does not correspond to any rectangular floorplan.

Figure 1 shows a rectangular floorplan with four rooms and its associated adjacency graph, which is maximal outerplanar. The corner rooms, A and D, correspond to vertices of degree 2 in the adjacency graph.

It follows from Earl and March (1979) that every maximal outerplanar graph with at most four vertices of degree 2 is the adjacency graph of some rectangular floorplan without internal rooms.

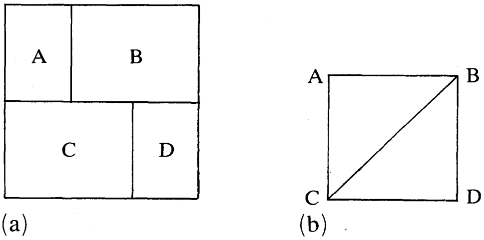

Figure 1. (a) A rectangular floorplan and (b) its maximal outerplanar adjacency graph.

Area constraints

The problem is: given a maximal outerplanar graph with at most four vertices of degree 2, and specified areas for the vertices (rooms), can a rectangular floorplan be found whose adjacency graph is the given graph, also satisfying the area requirements?

A room and its corresponding vertex are represented by the same uppercase letter, and the prescribed area of the room by the corresponding lowercase letter.

Given the adjacency graph, the nonisomorphic floorplans (under reflection or rotation) can be obtained. First, corner rooms and endrooms are chosen, where there are fewer than four vertices of degree 2. Then the edges of the adjacency graph are coloured in either of two 'colours' to specify the orientation of the corresponding walls. This colouring must obey certain rules if the plan is to be rectangular (Earl and March, 1979). The corresponding undimensioned floorplans and the necessary and sufficient conditions for each plan to be dimensioned correctly to satisfy the area requirements can then be found.

Two 'colourings' of an adjacency graph differing in the colour of one edge correspond to two nonisomorphic floorplans sharing at least one common area condition.

Figure 2 shows two different colourings for the adjacency graph of figure 1. The two floorplans have a common single area condition: $ad < bc$.

A division of a rectangle into four rectangular regions adjacent in the manner of figure 2, has a similar area condition. In general, each area condition for any plan corresponds to one such division.

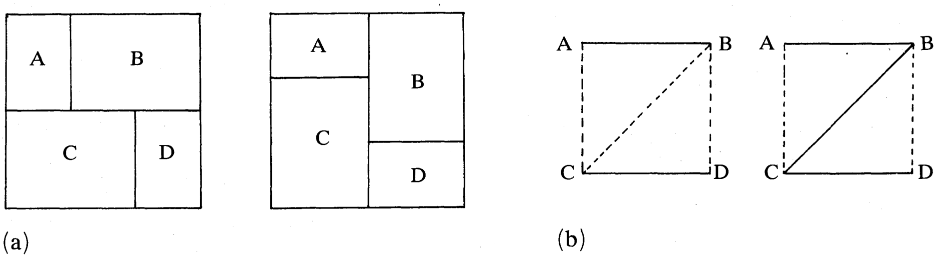

Figure 2. Two nonisomorphic rectangular floorplans (a), with the same adjacency graph (b). The plans differ in the direction of the wall joining rooms B and C. Edges of the graph are coloured. Solid lines represent walls in the vertical direction, and broken lines those in the horizontal direction. The area requirement for both floorplans is $ad < bc$.

Counterexample
Consider the maximal outerplanar graph shown in figure 3. As A, C, E, and G all have degree 2, they must correspond to corner rooms.

The sixteen nonisomorphic undimensioned rectangular floorplans with this adjacency graph are shown in figure 4.

The necessary and sufficient conditions for each plan to be correctly dimensioned to suit particular area requirements are shown in table 1. Table 1(b) shows which

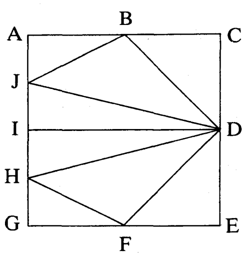

Figure 3. The adjacency graph for the counterexample.

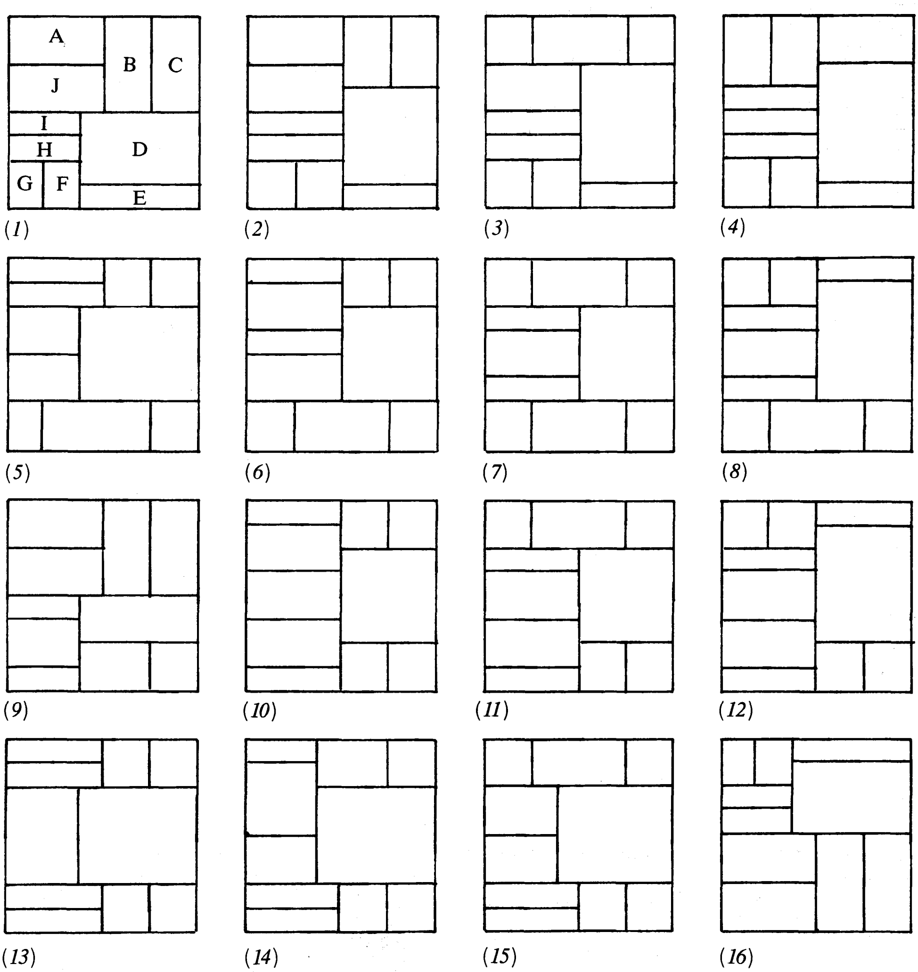

Figure 4. The sixteen undimensioned floorplans with adjacency graph that of figure 3.

of inequalities 1–24 hold for each floorplan (*1*)–(*16*). Each inequality occurs twice, in neighbouring plans.

For each plan, at least one of the area conditions has i appearing on the left-hand side of the inequality. Thus, if i is sufficiently large, each plan will be false. The same applies for variables c and e.

If the areas of the rooms are

$$a = b = c = d = e = f = g = 1, \qquad h = i = j = 2,$$

then at least one of the conditions for each plan will be false.

So, given any maximal outerplanar graph with at most four vertices of degree 2, it is not always possible to find a rectangular floorplan satisfying adjacency and area conditions.

Table 1. The necessary and sufficient conditions for a plan to be correctly dimensioned to suit particular area requirements. Part (b) shows which of the twenty-four inequalities in part (a) are required to dimension each of the sixteen plans from figure 4.

(a) Conditions

1	$(b+c)(f+g+h+i) < (a+j)(d+e)$	13	$(e+f)(a+i+j) < (b+c)(g+h)$
2	$e(h+i) < d(f+g)$	14	$c(f+g+h+i+j) < (a+b)(d+e)$
3	$(b+c)(h+i) < d(a+j)$	15	$e(h+i+j) < d(f+g)$
4	$dg < (e+f)(h+i)$	16	$c(h+i+j) < d(a+b)$
5	$(b+c)(g+h+i) < (a+j)(d+e+f)$	17	$dg < (e+f)(h+i+j)$
6	$i(e+f) < d(g+h)$	18	$c(g+h+i+j) < (a+b)(d+e+f)$
7	$i(b+c) < d(a+j)$	19	$(e+f)(i+j) < d(g+h)$
8	$a(d+e) < (b+c)(f+g+h+i+j)$	20	$ad < (b+c)(i+j)$
9	$e(a+h+i+j) < (f+g)(b+c+d)$	21	$c(i+j) < d(a+b)$
10	$ad < (b+c)(h+i+j)$	22	$e(a+b+h+i+j) < (f+g)(c+d)$
11	$g(b+c+d) < (e+f)(a+h+i+j)$	23	$(e+f)(a+b+i+j) < (c+d)(f+g)$
12	$a(d+e+f) < (b+c)(g+h+i+j)$	24	$g(c+d) < (e+f)(a+b+h+i+j)$

(b) Plan conditions

(*1*)	1, 2	(*7*)	10, 15, 16, 17	(*12*)	18, 23, 24
(*2*)	1, 8, 9	(*8*)	16, 22, 24	(*13*)	6, 7
(*3*)	8, 14, 15	(*9*)	4, 5, 6	(*14*)	7, 13, 20
(*4*)	14, 22	(*10*)	5, 11, 12, 13	(*15*)	19, 20, 21
(*5*)	2, 3, 4	(*11*)	12, 17, 18, 19	(*16*)	21, 23
(*6*)	3, 9, 10, 11				

Remarks

It can be shown that this is the smallest number of vertices for a maximal outerplanar graph for which no rectangular floorplan is possible.

Conclusions

It has been shown that no rectangular floorplan can be drawn suiting both area and adjacency requirements for certain maximal outerplanar graphs.

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