Irreducible triangulations of the 4-gon and 4-connectedness

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$\mathcal{C}|_{\mathcal{W}}$ versus $\mathcal{C}_{\mathcal{W}}$ And then $\mathcal{C}|_{\mathcal{W}}|\mathcal{W}$ versus $\mathcal{C}_{\mathcal{W}}|\mathcal{W}$

Notational concerns We will use $\mathcal{C}$ to indicate the current sweep line cycle. We will repeatedly only consider the path $\mathcal{C} \setminus \{\mathrm{S}\}$. In that case we will always order it from W to E.

We will let $\mathcal{W}$ denote a interior walk . Given such a walk of k vertices we index it's nodes $w_1, \dots, w_k$ in such a way that w_1 is closer to W then w_k is (and thus that w_k is closer to E then w_1 is).

FiXme: have i defined this already

Then w_1 and w_k indicate the two unique vertices of the walk that are also part of the cycle. We will then let $\mathcal{C}|_{\mathcal{W}}$ denote the part of $\mathcal{C} \setminus \mathrm{S}$ that is between w_1 and w_k (including). $\mathcal{C}_{\mathcal{W}}$ will denote the closed walk formed when we paste $\mathcal{C}|_{W}$ and $\mathcal{W}$.

Since paths are a subclass of walks all of the above notation can also be used for a path $\mathcal{P}$. Note that the closed walk $\mathcal{C}_{\mathcal{P}}$ in this case will actually be a cycle.

prelim *nondistinct corner.*

1 Outline

We will show that there is a algorithm if there are no 4 cycles.

If graph G has non-distinct corners or cutvertices we treat them separately.

The main algorithm will recieve as input a extended graph $\bar{G}$ without non-distinct corners and no separating 4 cycles and will return a regular edge labeling such that all red faces are $(1-\infty)$ using a sweepcycle approach inspired by Fusy [**?**].

FiXme: spelling Fusy and cite

We will start by creating a walk W. This walk may not be a valid path, it doesn't even have to be a path. During the algorithm we will make a number of moves that will turn this candidate walk into a valid path. In each move we shrink C by employing a valid path and change the candidate walk.

One invariant we will always maintain is that the area bounded by $\mathcal{C}_{\mathcal{W}}$ will never have interior vertices. .

FiXme: What is exactly the area bounded by a closed walk

1.1 The initial candidate walk

Let v_i denote all the vertices of $\mathcal{C} \setminus \{\mathrm{W}, \mathrm{S}, \mathrm{E}\}$ in the order that they occur on $\mathcal{C} \setminus \{\mathrm{S}\}$. That is $\mathcal{C} \setminus \{\mathrm{S}\}$ is given by $\mathrm{W}v_1 \dots v_n\mathrm{E}$. As candidate walk we will

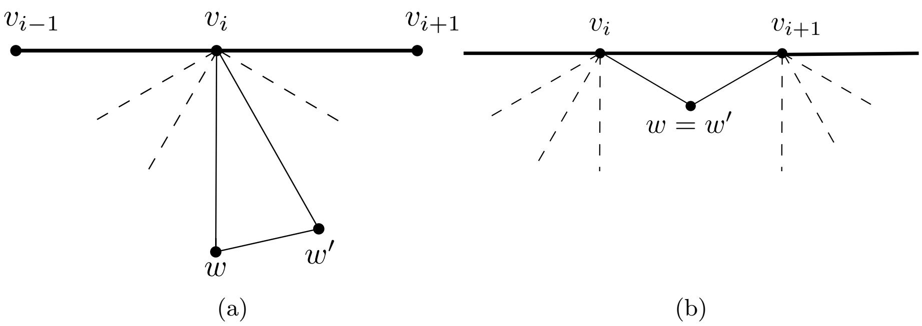

Figure 1: The two main cases of the proof showing that W is a walk after removing duplicates.

start with W, we will then take the vertices adjacent to v_1 between E and v_2 in clockwise order (exclusive), followed the vertices adjacent to v_2 between v_1 and v_3 in clockwise order and so further until we finally add the vertices adjacent to v_n between v_{n-1} and E in clockwise order and finally we finish by adding E.

Lemma 1. *After removing subsequent duplicates the collection W described above is indeed a walk.*

FiXme: introduce a term for "edges subsequent to each other in clockwise order around v"

Proof. To show that W is a walk it's sufficient to show that every vertex is adjacent to the next vertex. Let us suppose that w and w' are two subsequent vertices in W, we will show that they are connected if $\{w, w'\} \cap \{\text{W}, \text{E}\} = \emptyset$ after that we will consider this edge case. There are then two main case for w, w'. Either (a) w and w' are vertices adjecent to some v_i subsequent in clockwise order or (b) w was the last vertex adjecent to some v_i and thus w' is the first vertex adjacent to v_{i+1}.

The following two situations can also be seen in Figure 1.

In case (a) we note that $v_i w$ and $v_i w'$ are edges next to each other in clockwise order around v_i. Since every interior face of $\bar{G}$ is a triangle ww' must be an edge. We thus see that w, w' are adjacent and not duplicates.

In case (b) we note that $v_i w$ and $v_i v_{i+1}$ are edges subsequent in clockwise order, hence wv_{i+1} is also an edge. Hence w is the first vertex adjacent to v_{i+1} after v_i in clockwise order. Thus $w = w'$, they are duplicates and we will remove w.

Now for the edge cases: W and w_1 are vertices adjacent to v_1 subsequent in clockwise order, and hence connected. w_m and E are vertices adjacent to v_n subsequent in clockwise order and hence connected. □

1.2 Porperties the walk already satisfies

Lemma 2. *$\mathcal{C}_\mathcal{W}$ has no interior vertices.*

Lemma 3. $\mathcal{C}|_\mathcal{W}$

1.3 Moves

The candidate walk can have two kinds of problems. It either is non-simple or it has chords. Otherwise it is a valid path.

FiXme: cf Kusters. Where there are also two problems for a proper boundary path