# Irreducible triangulations of the 4-gon and 4-connectedness

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$\mathcal{C}|_{\mathcal{W}}$ versus $\mathcal{C}_{\mathcal{W}}$ And then $\mathcal{C}|_{\mathcal{W}}|\mathcal{W}$ versus $\mathcal{C}_{\mathcal{W}}|\mathcal{W}$

**Notational concerns** We will use $\mathcal{C}$ to indicate the current sweep line cycle. Note that we can consider the path $\mathcal{C} \setminus \mathrm{S}$. We will order it from W to E.

We will let $\mathcal{W}$ denote a interior walk . Given such a walk of $k$ vertices we index it's nodes $w_1, \ldots, w_k$ in such a way that $w_1$ is closer to $W$ then $w_k$ is (and thus that $w_k$ is closer to $E$ then $w_1$ is).

FiXme: have i defined this already

Then $w_1$ and $w_k$ indicate the two unique vertices of the walk that are also part of the cycle. We will then let $\mathcal{C}|_{\mathcal{W}}$ denote the part of $\mathcal{C} \setminus \mathrm{S}$ that is between $w_1$ and $w_k$ (including). $\mathcal{C}_{\mathcal{W}}$ will denote the closed walk formed when we paste $\mathcal{C}|_{W}$ and $\mathcal{W}$.

Since paths are a subclass of walks all of the above notation can also be used for a path $\mathcal{P}$. Note that the closed walk $\mathcal{C}_{\mathcal{P}}$ in this case will actually be a cycle.

**prelim** *nondistinct corner.*

## 1 Outline

We will show that there is a algorithm if there are no 4 cycles.

If graph $G$ has non-distinct corners we remove them.

The main algorithm will recieve as input a extended graph $\bar{G}$ without non-distinct corners and will return a regular edge labeling such that all red faces are $(1-\infty)$ using a sweepcycle approach inspired by Fusy [**?**].

FiXme: spelling Fusy and cite

We will start by creating a walk $W$. This walk may not be a valid path, it doesn't even have to be a path. During the algorithm we will make a number of moves that will turn this candidate walk into a valid path. In each move we shrink $C$ by employing a valid path and change the candidate walk..

One invariant we will always maintain is that the area bounded by $\mathcal{C}_{\mathcal{W}}$ will never have interior vertices. .

FiXme: What is exactly the area bounded by a closed walk

### 1.1 The initial candidate walk

We define the *level* of a vertex of $G$ as the distance of this vertex to the cycle minus S $\mathcal{C} \setminus \{\mathrm{S}\}$. Let $v_i$ denote all the level 0 vertices in $G$ in the order that they occur on $\mathcal{C}$ That is $\mathcal{C} \setminus \{\mathrm{S}\}$ is given by $\mathrm{W}v_1 \ldots v_n\mathrm{E}$. As candidate walk we will start with $W$, we will then take the level 1 vertices adjacent to $v_1$ in clockwise order, followed by the level 1 vertices adjecent to $v_2$ in clockwise order (in so far they didn't already occur) and so further until we add the level 1 vertices adjacent to $v_n$ and finish with $E$.

FiXme: To proof:This is a walk

## 1.2 moves

The candidate walk can have two kinds of problems. It either is non-simple or it has chords. Otherwise it is a valid path.

FiXme: cf Kusters. Where there are also two problems for a proper boundary path