

Pseudo one-sided rectangular duals

By Sander Beekhuis

Structure

Problem

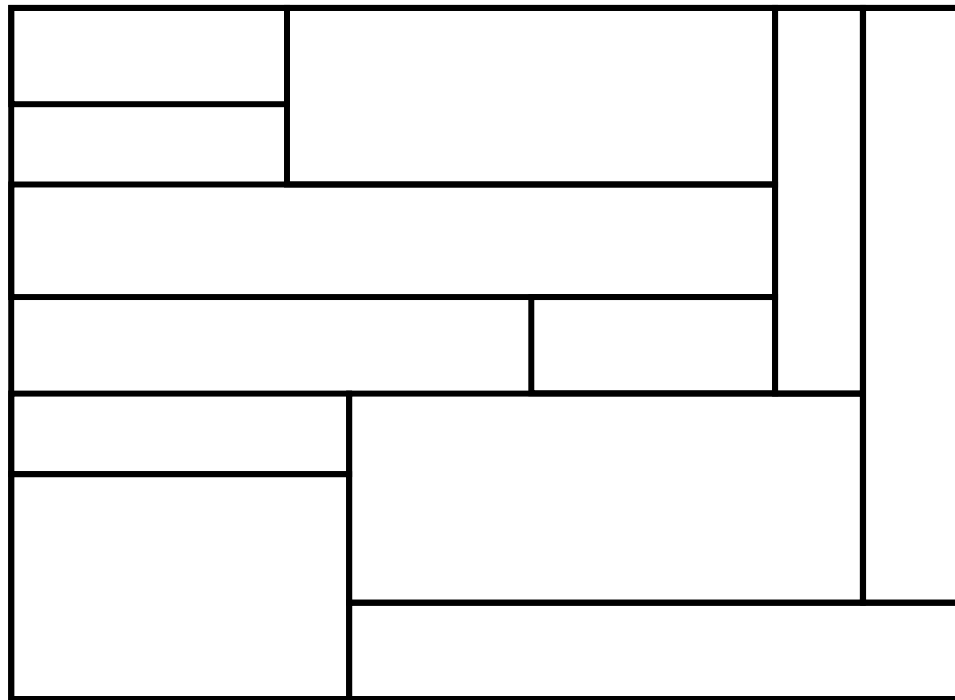
Results

Conjecture

Rectangular layout

Rectangular layout L

Partition of a rectangle into finitely many interior disjoint rectangles

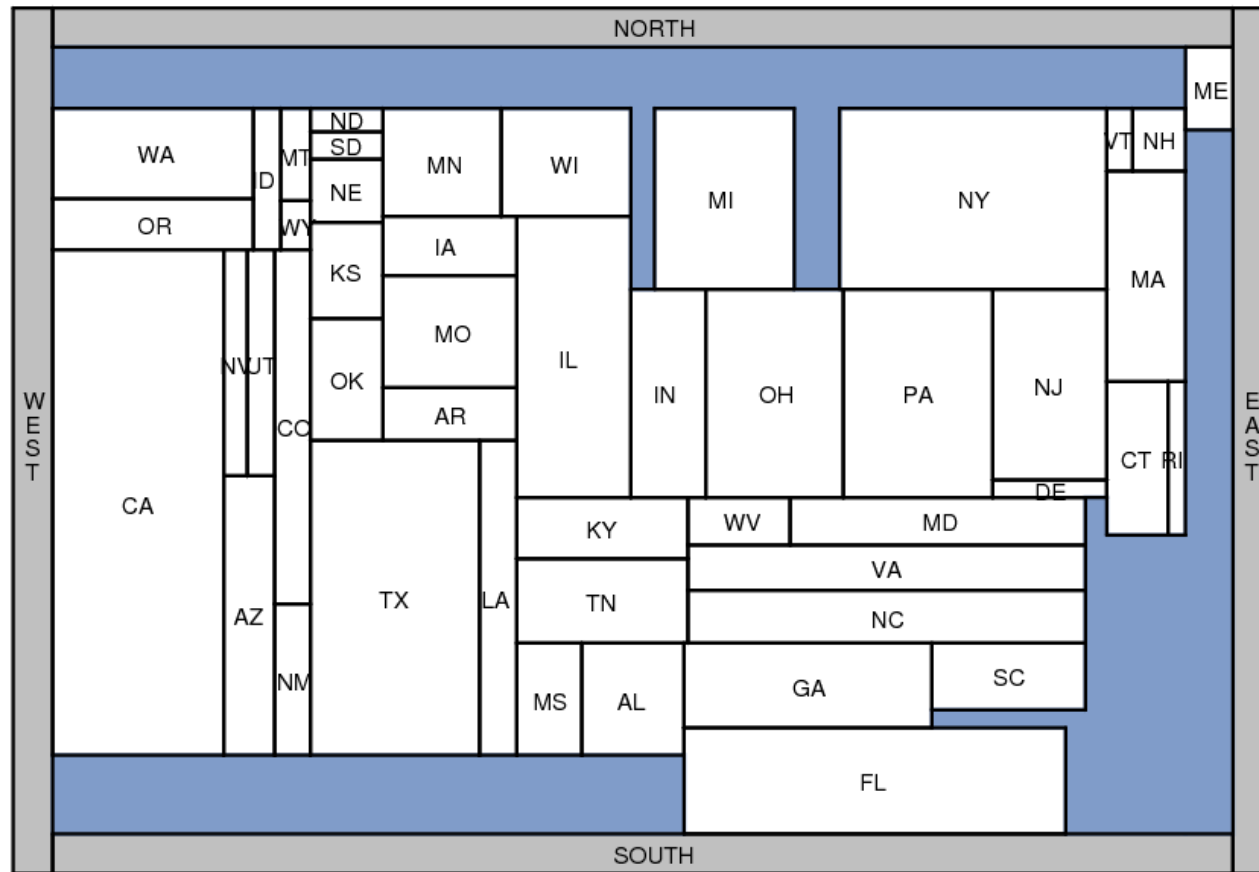


- Layouts are *equivalent* when they have the same adjacencies with the same orientation (horizontal/vertical)

Applications

Rectangular Cartograms

visualize statistical data about sets of regions; regions are rectangles; area proportional to some geographic variable



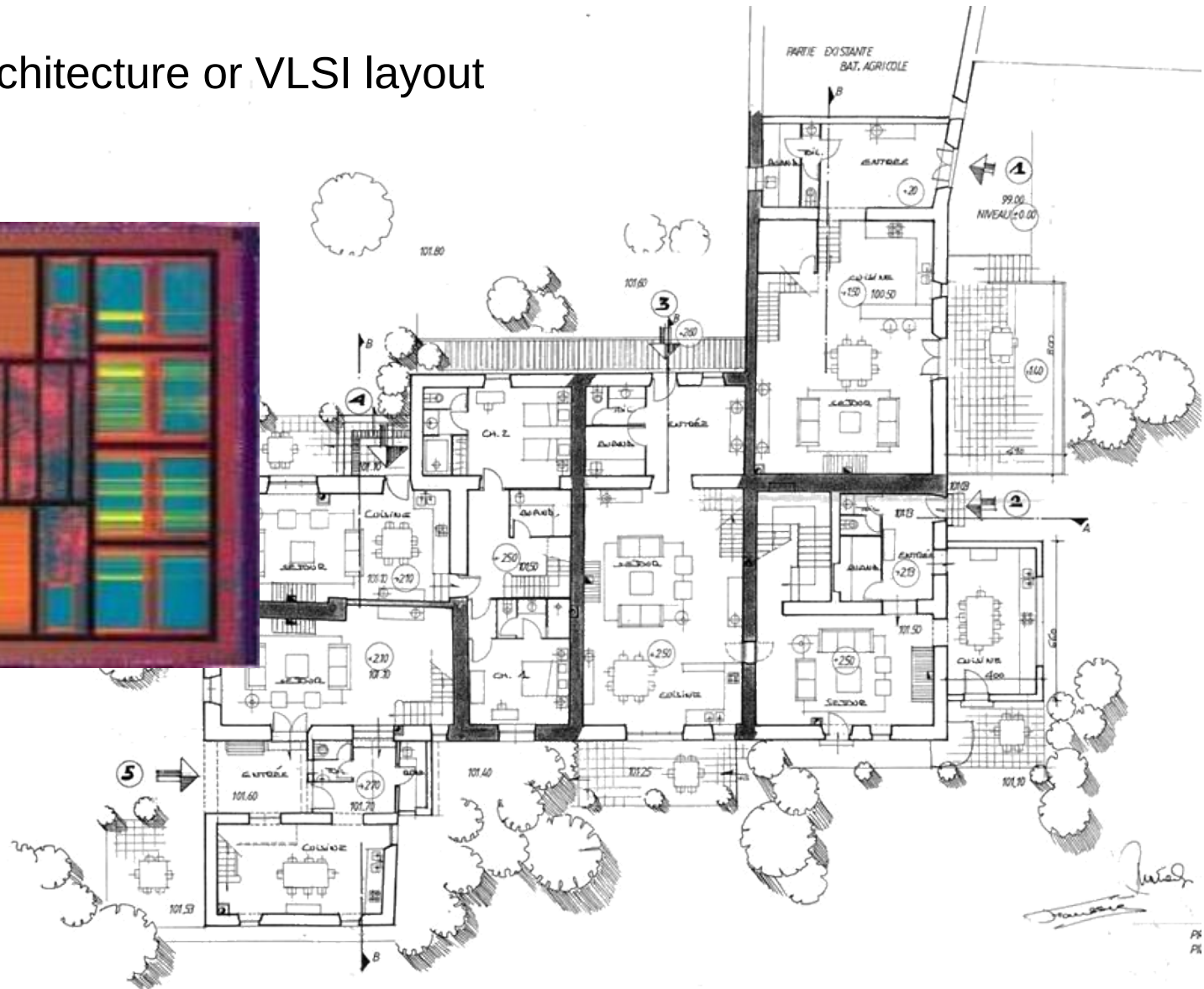
introduced by Raisz in 1934



building architecture or VLSI layout



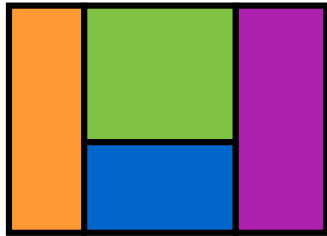
building architecture or VLSI layout



Area-Universal

Area-universal layout L

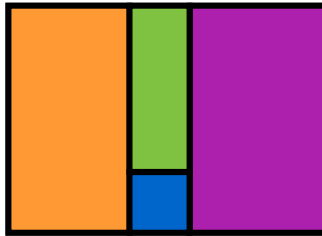
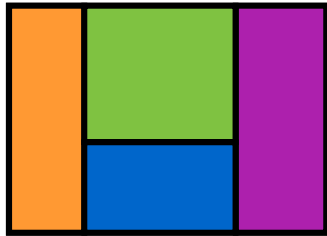
For every assignment of sizes to the areas of L there is a equivalent layout realizing these sizes



Area-Universal

Area-universal layout L

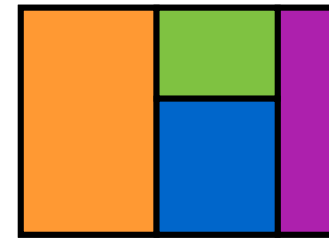
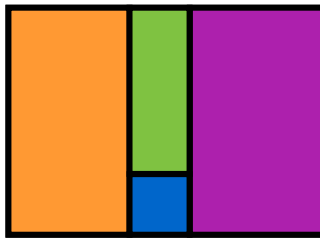
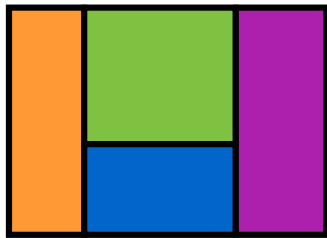
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Area-Universal

Area-universal layout L

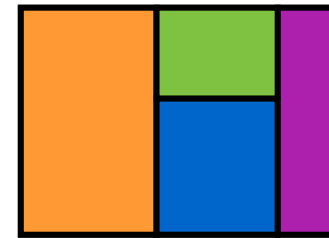
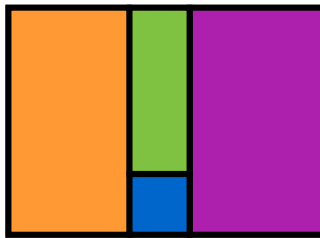
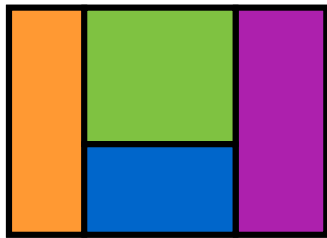
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Area-Universal

Area-universal layout L

For every assignment of sizes to the areas of L there is a equivalent layout realizing these sizes



Uses

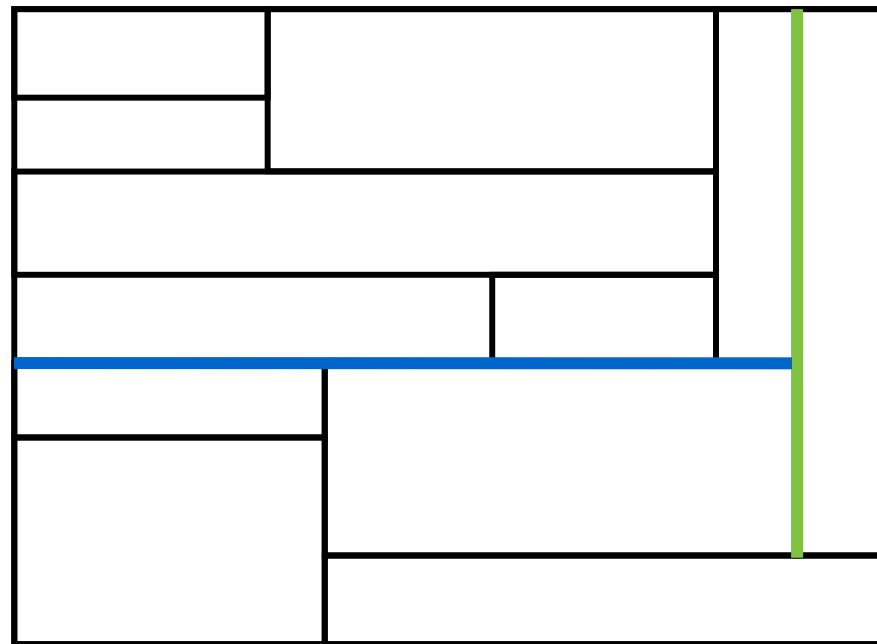
Animations; layout first – function later

One-sided

One-sided layout L

Every maximal line segment in L is the side of a rectangle

maximal
horizontal
segment

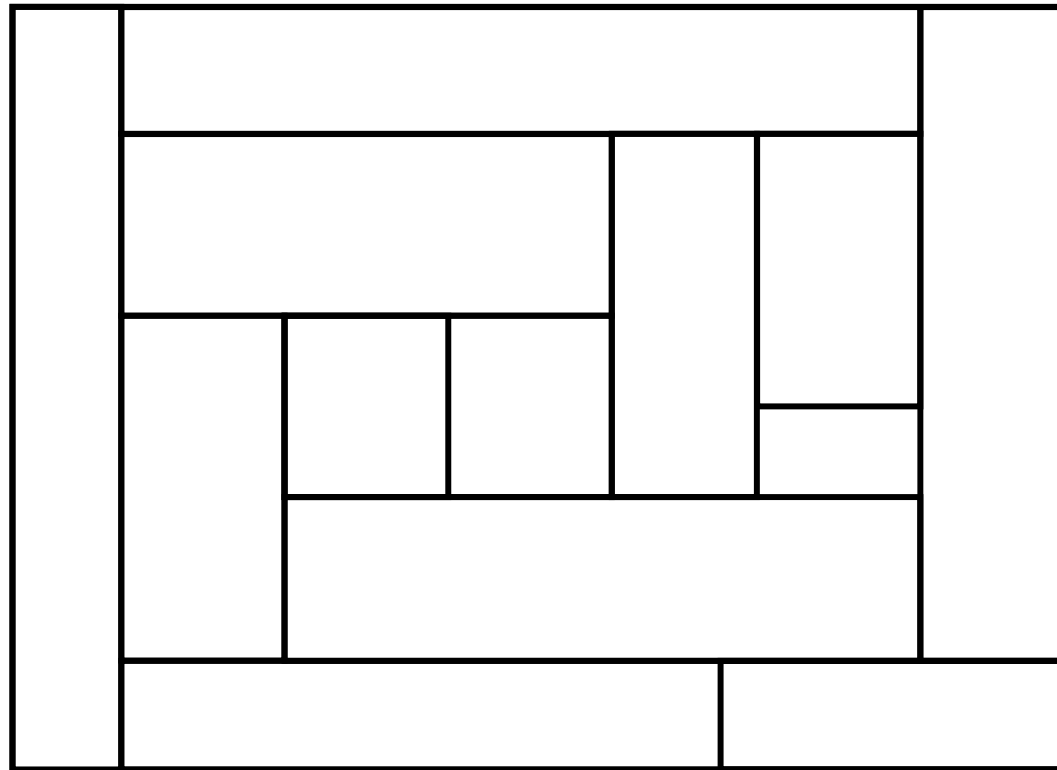


maximal
vertical
segment

One-sided

One-sided layout L

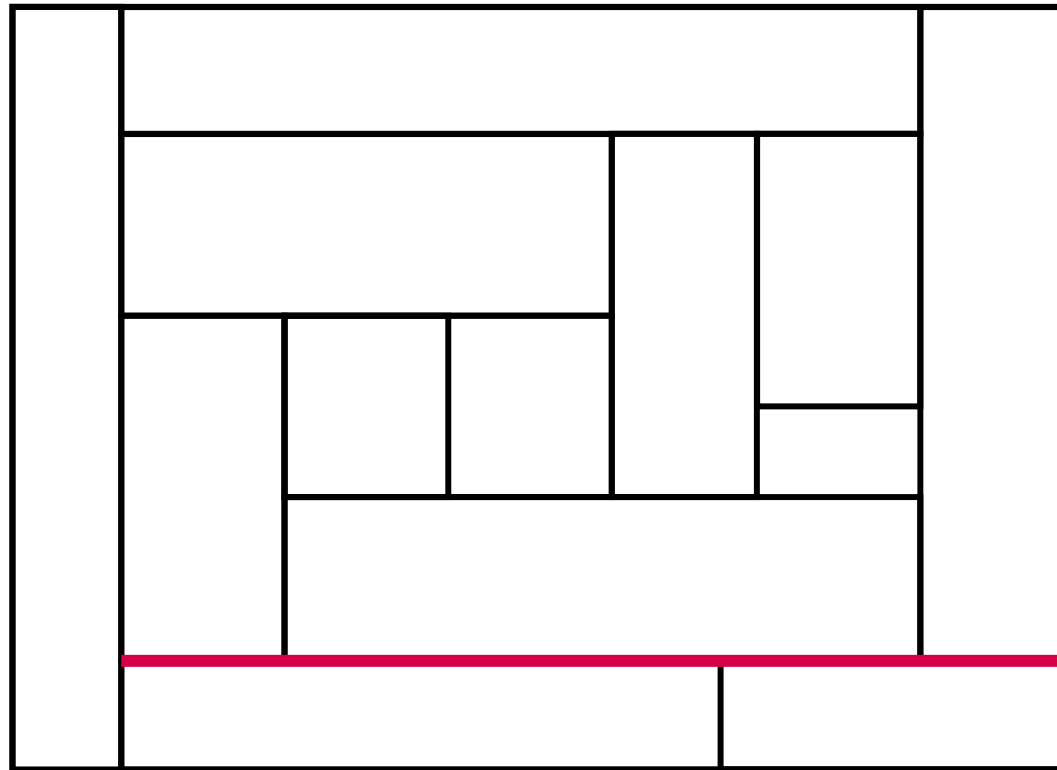
Every maximal line segment in L is the side of a rectangle



One-sided

One-sided layout L

Every maximal line segment in L is the side of a rectangle



Comparison

Area-Universal

The same layout fits
any area assignment

One-sided

Every maximal
segment is the side
of a rectangle

These are equivalent [Eppstein et al. , 2012]

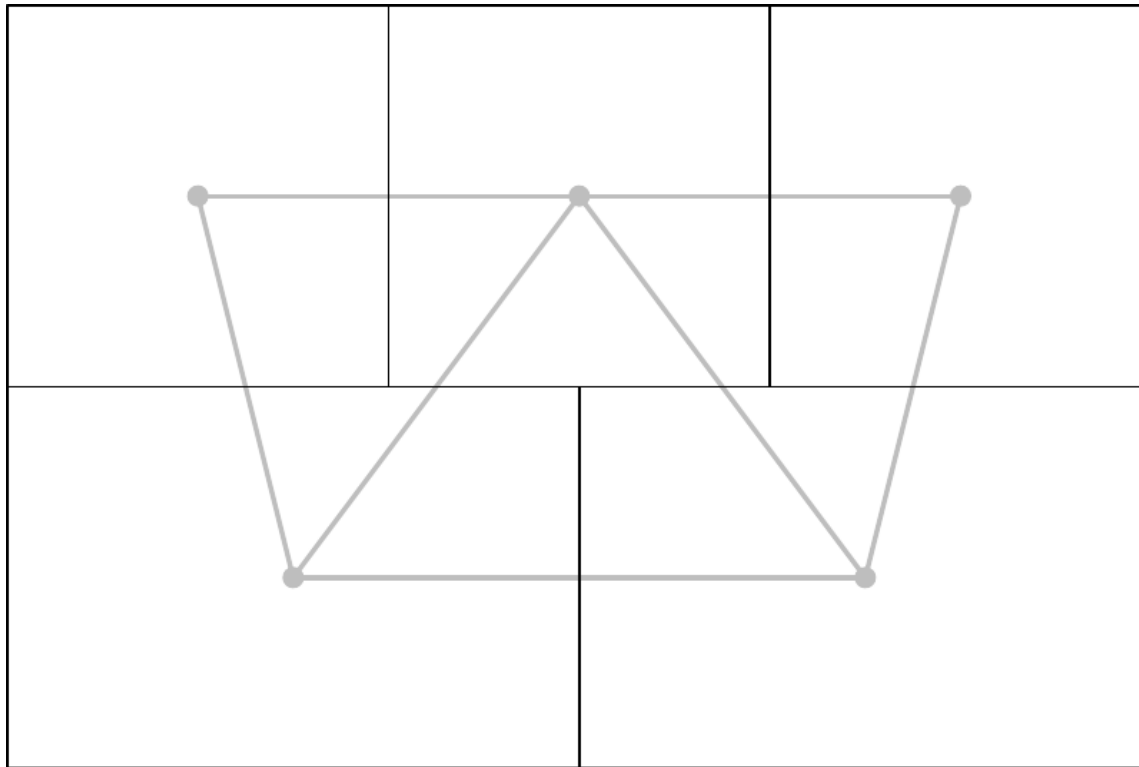
Not all graphs have an area-universal/one-sided dual

[Rinsma , 1987]

Rectangular dual

Rectangular dual of a graph G

- Rectangular layout
- Same adjacencies as G

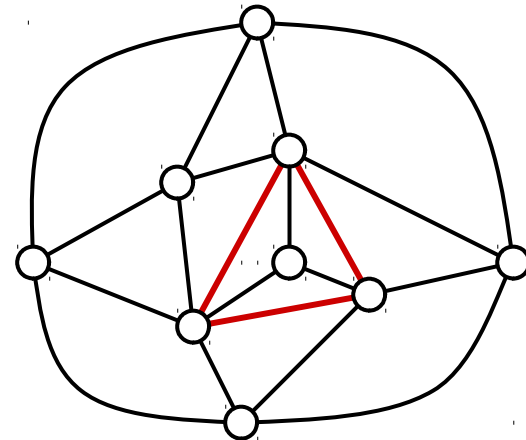
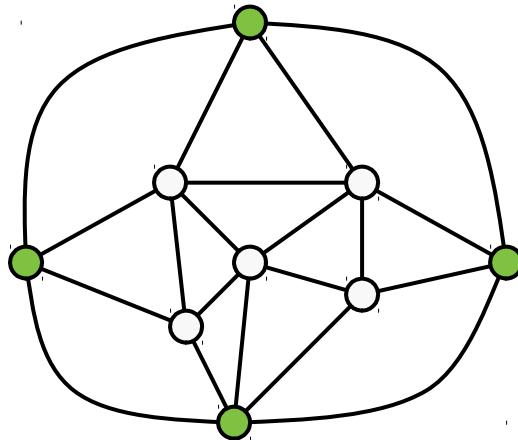
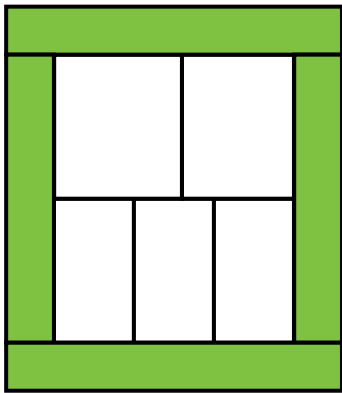


Rectangular dual

[Kozminski & Kinnen '85]

A planar graph G has a rectangular dual with 4 rectangles on the boundary if and only if

- every interior face is a triangle and the exterior face is a quadrangle
- G has no separating triangles

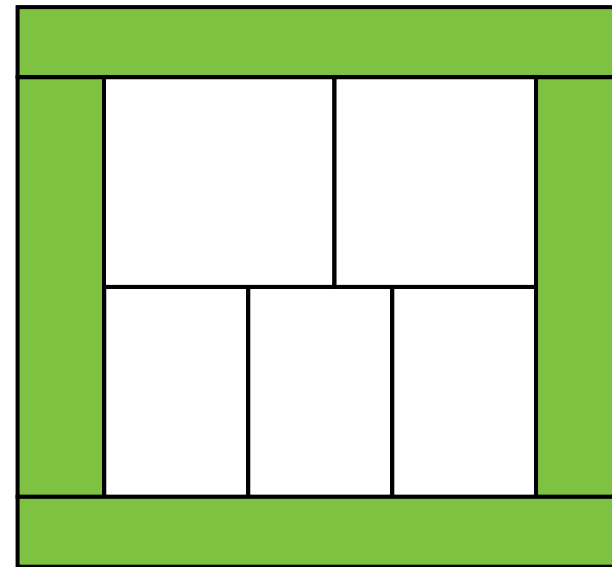
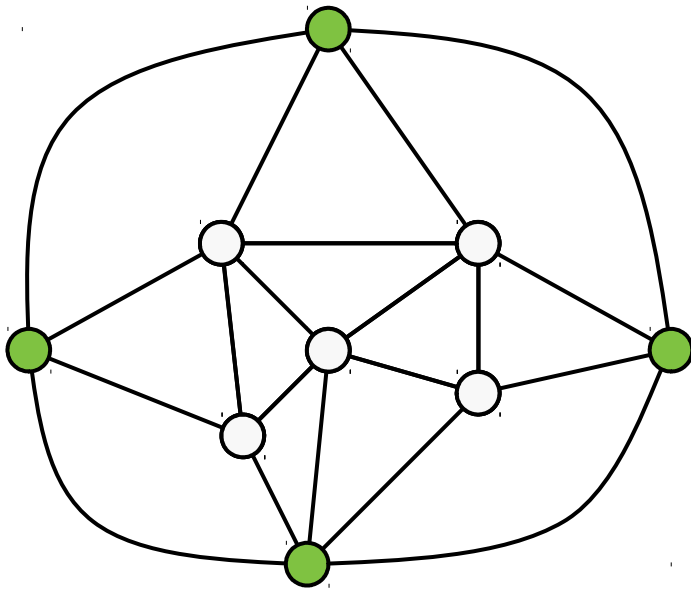
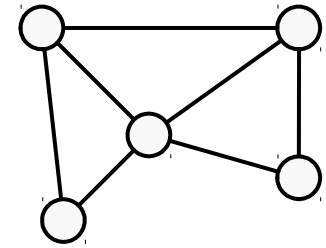


Extended Graph

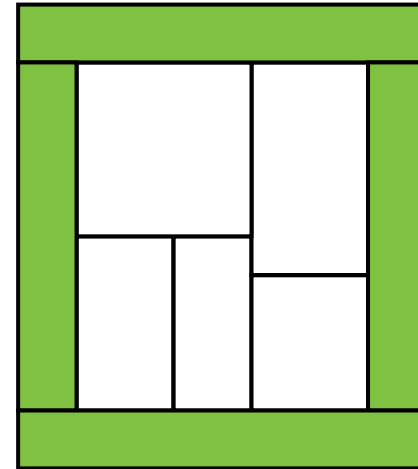
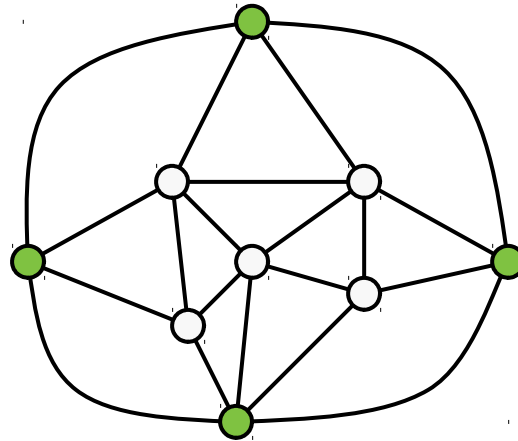
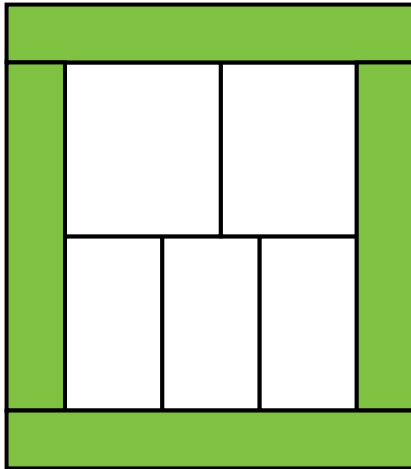
- Bring other graphs in this form
- Do this by adding 4 vertices (*poles*)

Without creating a separating triangle

- A extended graph corresponds to a certain corner assignment

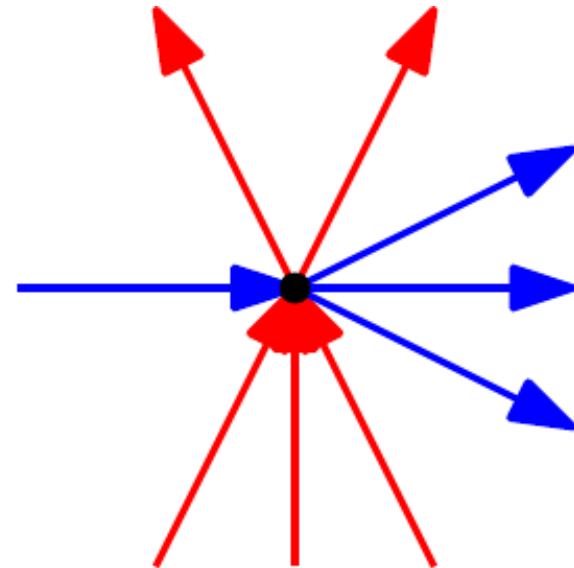
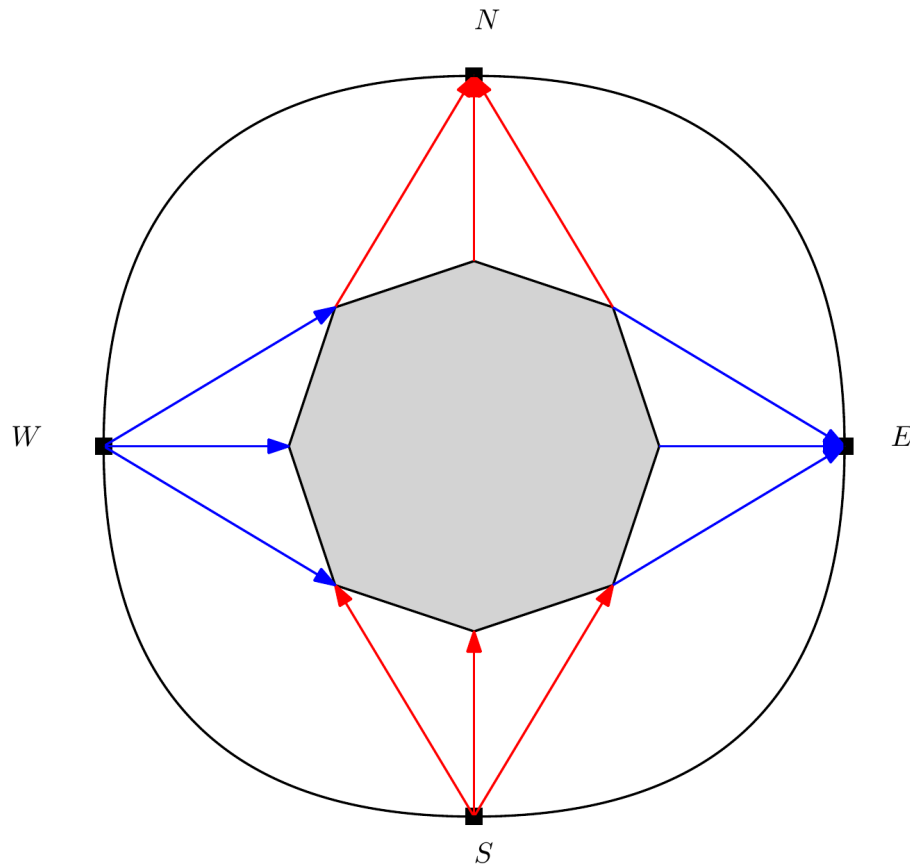


Extended Graphs do not fix layout



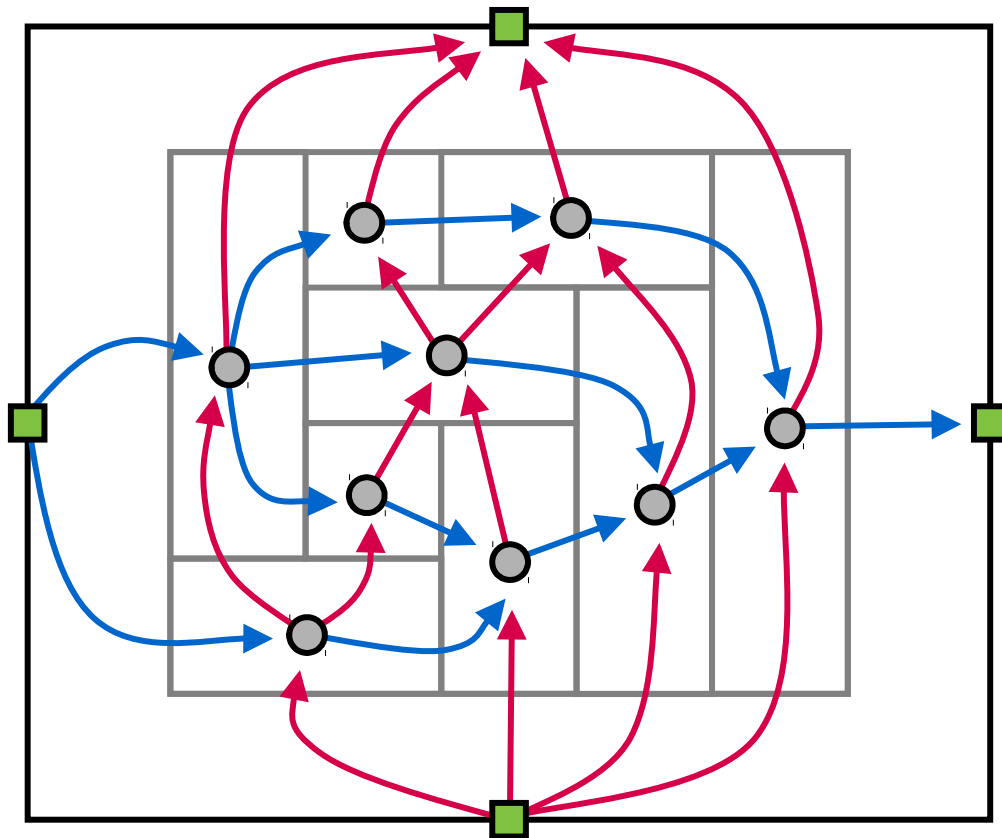
Regular edge labeling

- Oriented coloring of the extended graph
- Exterior vertex condition
- Interior vertex condition



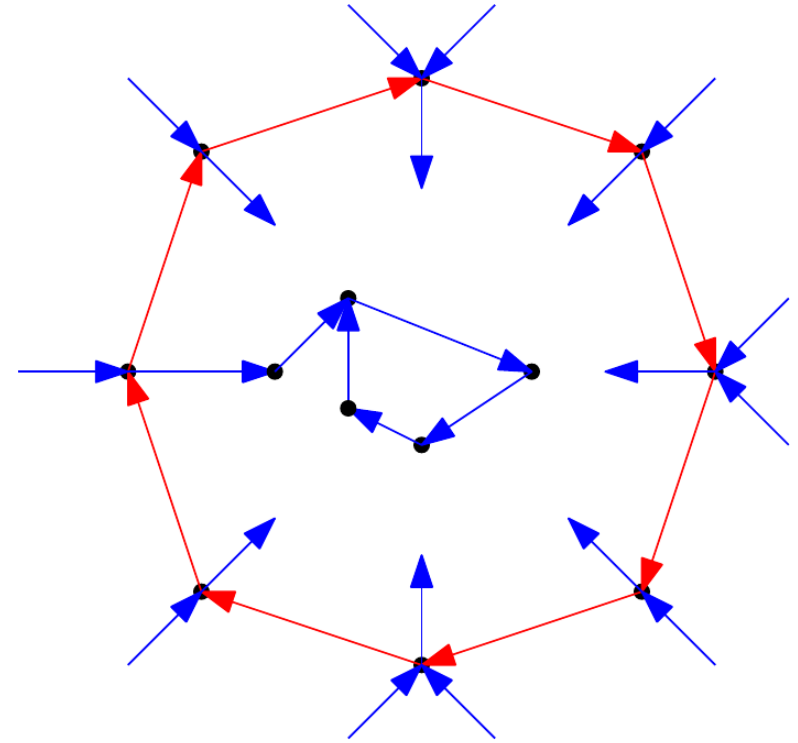
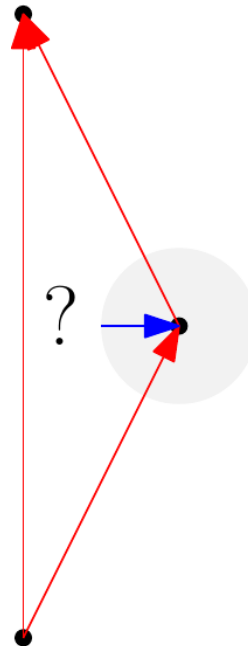
Regular edge labeling

- Corresponds to a equivalence class of layouts
 - Red: Vertical adjacency
 - Blue: Horizontal adjacency



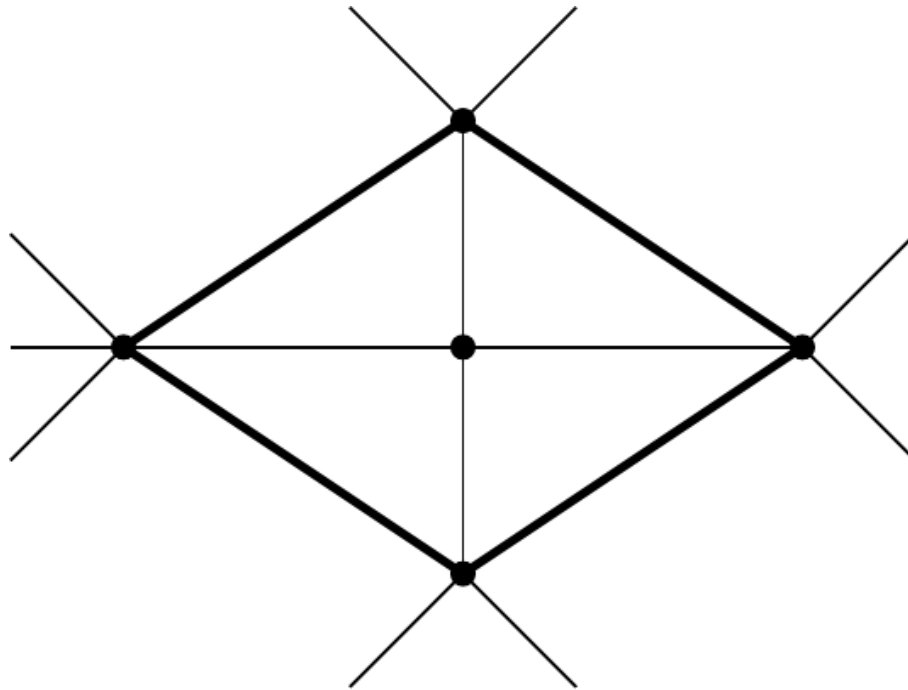
Properties of a REL

- Two acyclic flows
 - Inside a cycle there is another cycle
- No mono-colored triangles



Separating k-cycle

- A separating k-cycle is a cycle whose removal disconnects the graph

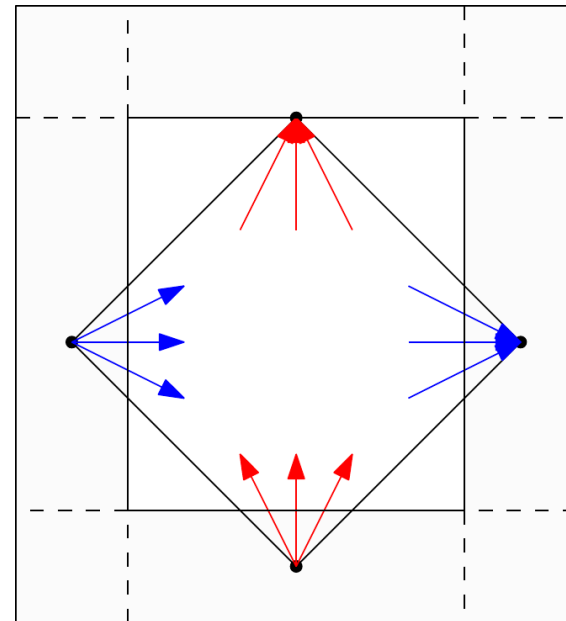
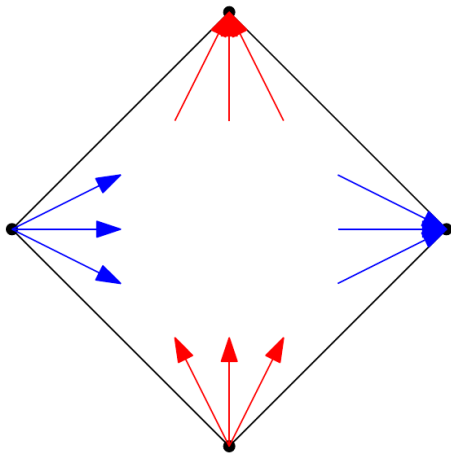


A separating 4-cycle

Properties of a REL

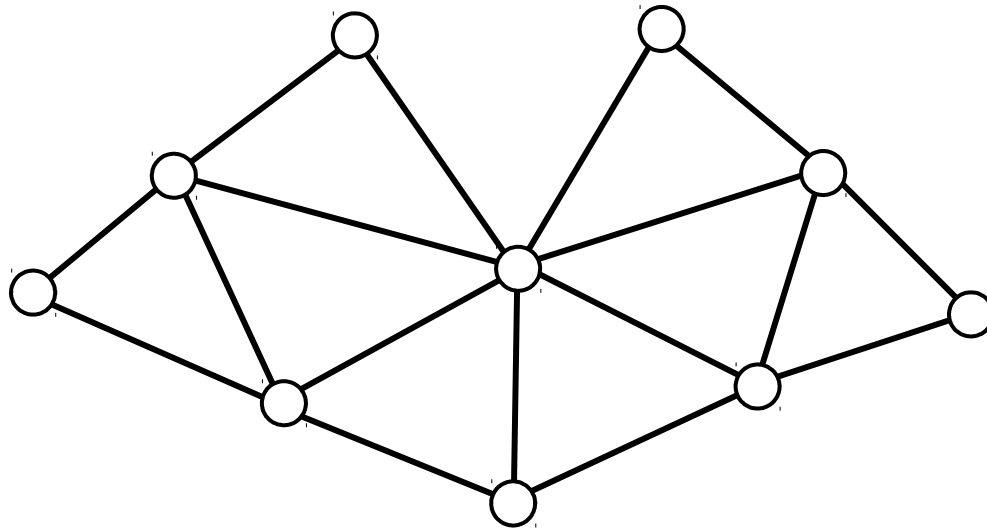
- Interior of a separating 4-cycle

The color and orientation of a single interior edge adjacent to a cycle vertex determines the color and orientation of all edges adjacent to a cycle vertex



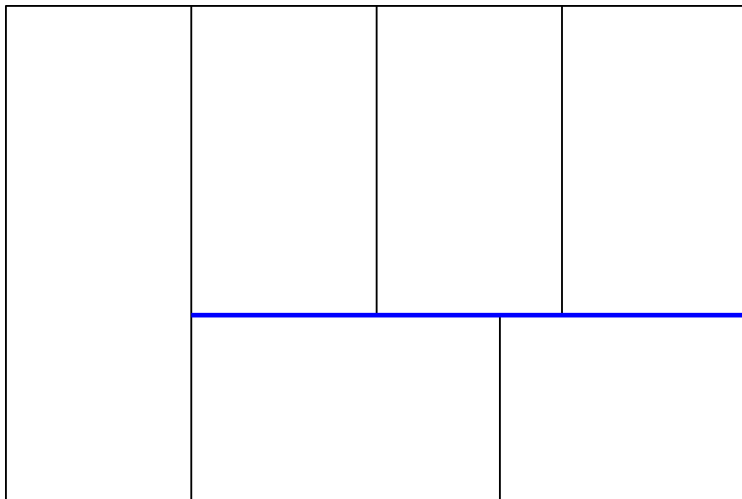
Not all graphs are one-sided

- We can show this by enumerating all possible REL's
- In the results section we will show for some graphs that they are not one-sided



So what can we do?

- We will call our dual **k-sided** if every maximal segment is the boundary of at most k adjacent rectangles all on the same side of the line

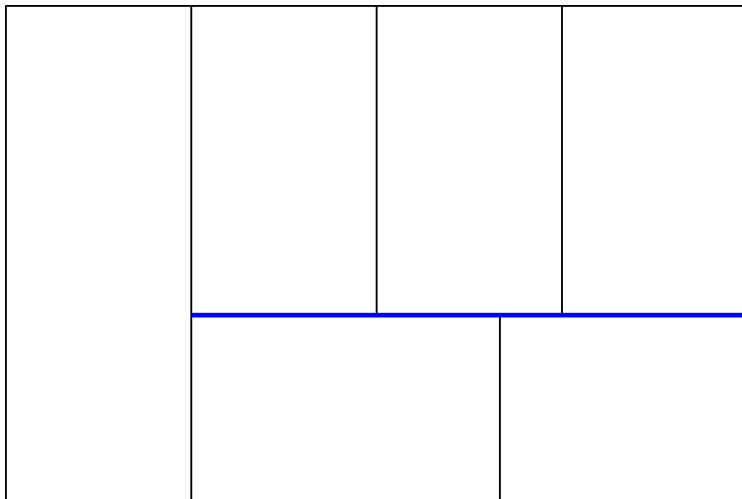


2-sided

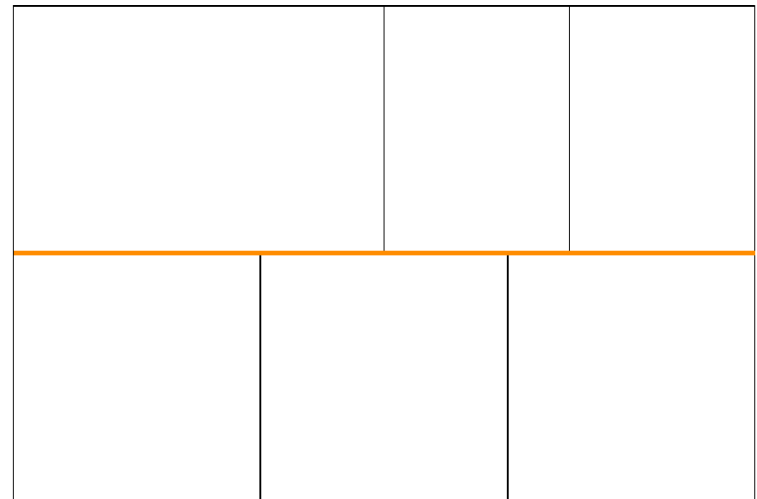
- Do graphs without an one-sided dual admit a k -sided dual for some k ? (hopefully small)

So what can we do?

- We will call our dual **k-sided** if every maximal segment is the boundary of at most k adjacent rectangles all on the same side of the line



2-sided

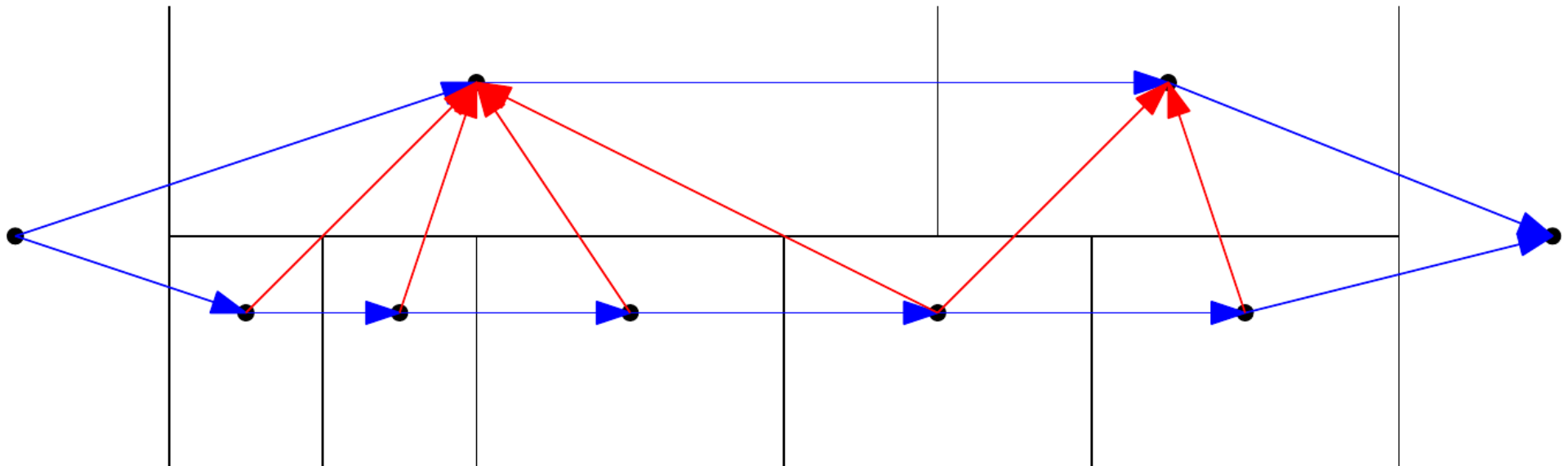


3-sided

- Do graphs without an one-sided dual admit a k -sided dual for some k ? (hopefully small)

What does k-sided look like in the REL?

- Red and blue faces with one of the two paths having at most $k+1$ vertices



Structure

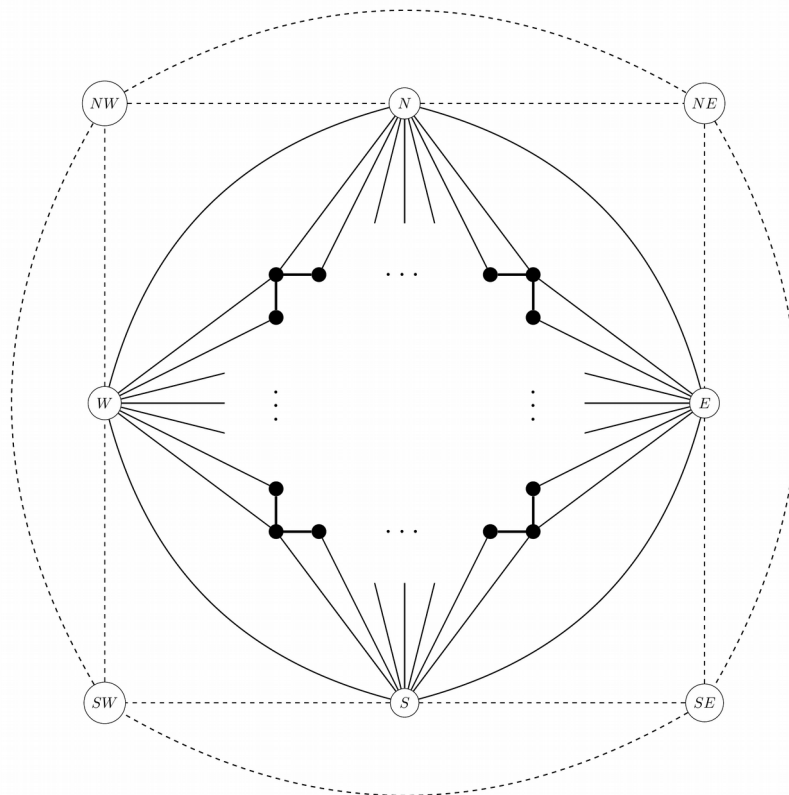
Problem

Results

Conjecture

Fixing a extended graph

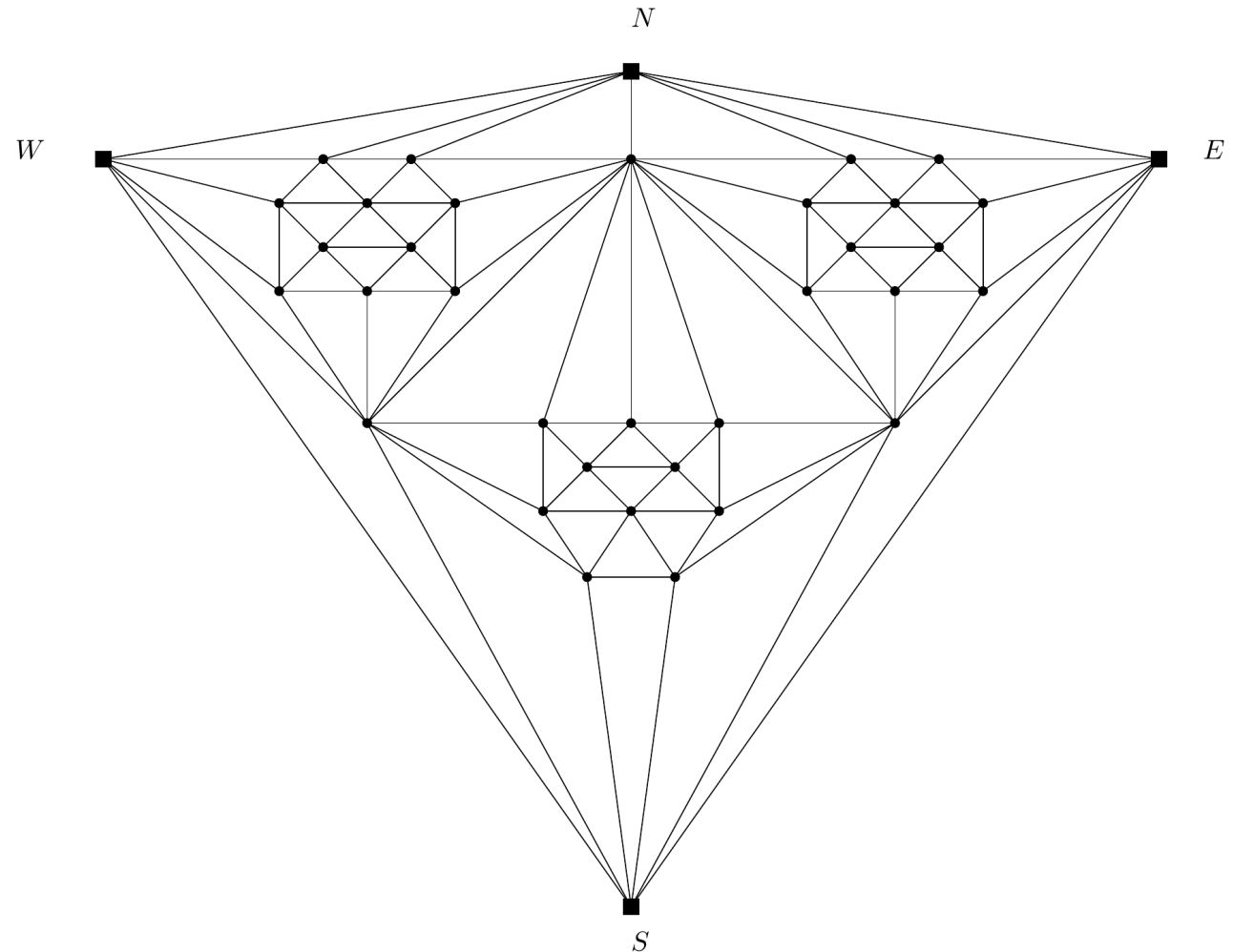
- We can consider a single extended graph of G , $E(G) = G' \dots$ by considering rectangular duals of G'
- Because then there is only one choice for $E(G')$



Note that this uses a separating 4-cycle

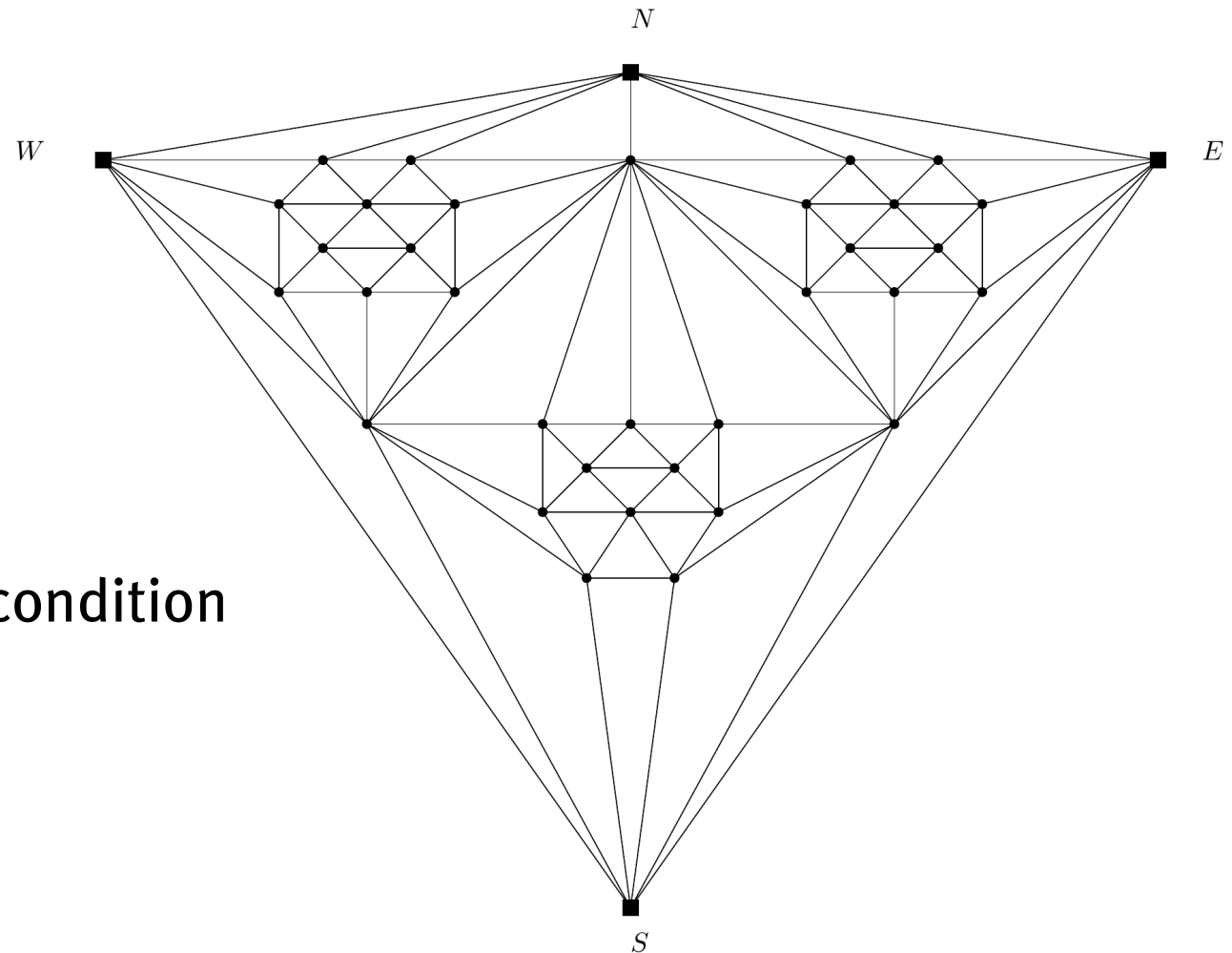
∞ -sided graphs with 4-cycles

Any extended graph with separating 4 cycles can be ∞ -sided, even if all these cycles go through a pole.



∞ -sided graphs with 4-cycles

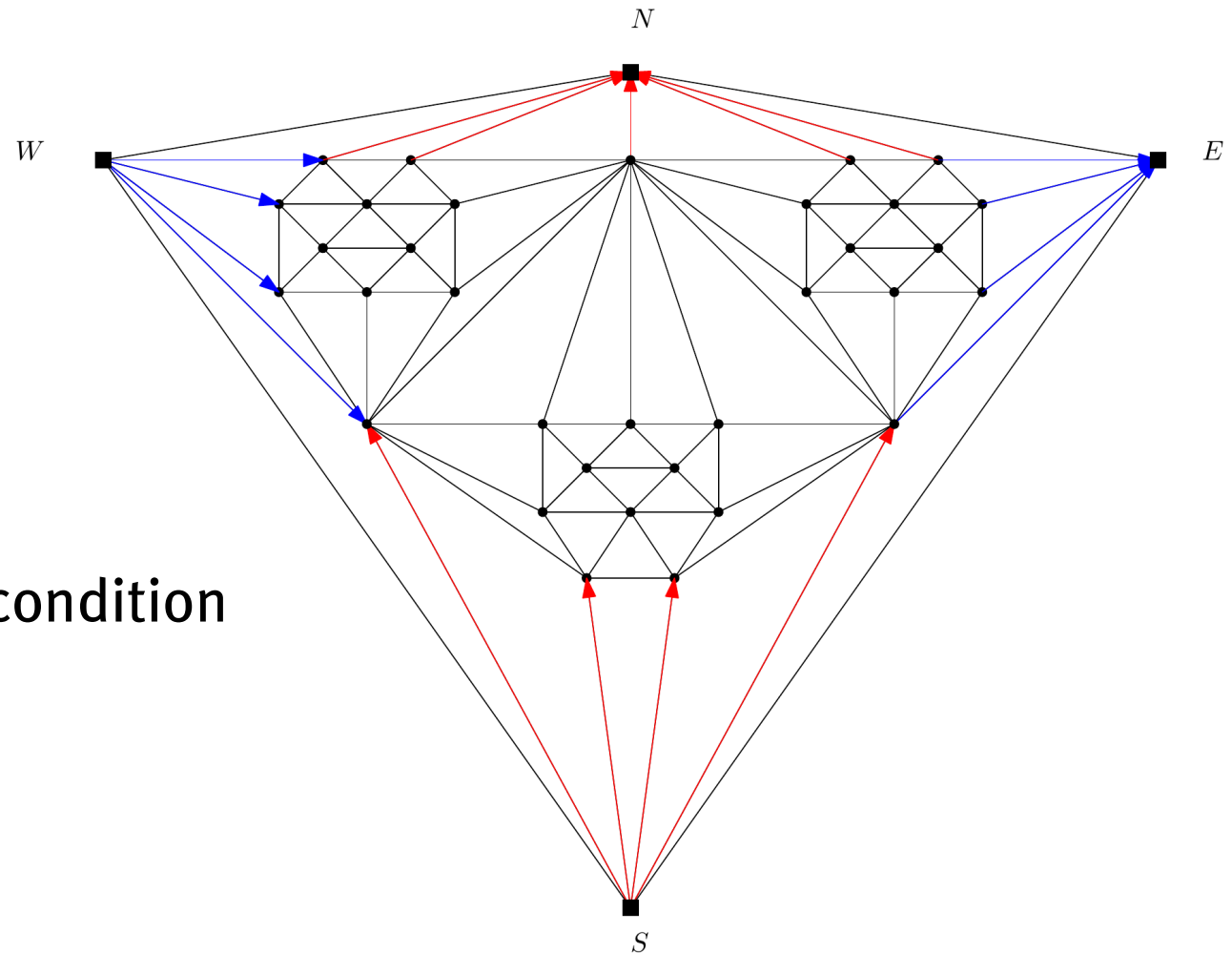
Any extended graph with separating 4 cycles can be ∞ -sided, even if all these cycles go through a pole.



- Exterior vertex condition

∞ -sided graphs with 4-cycles

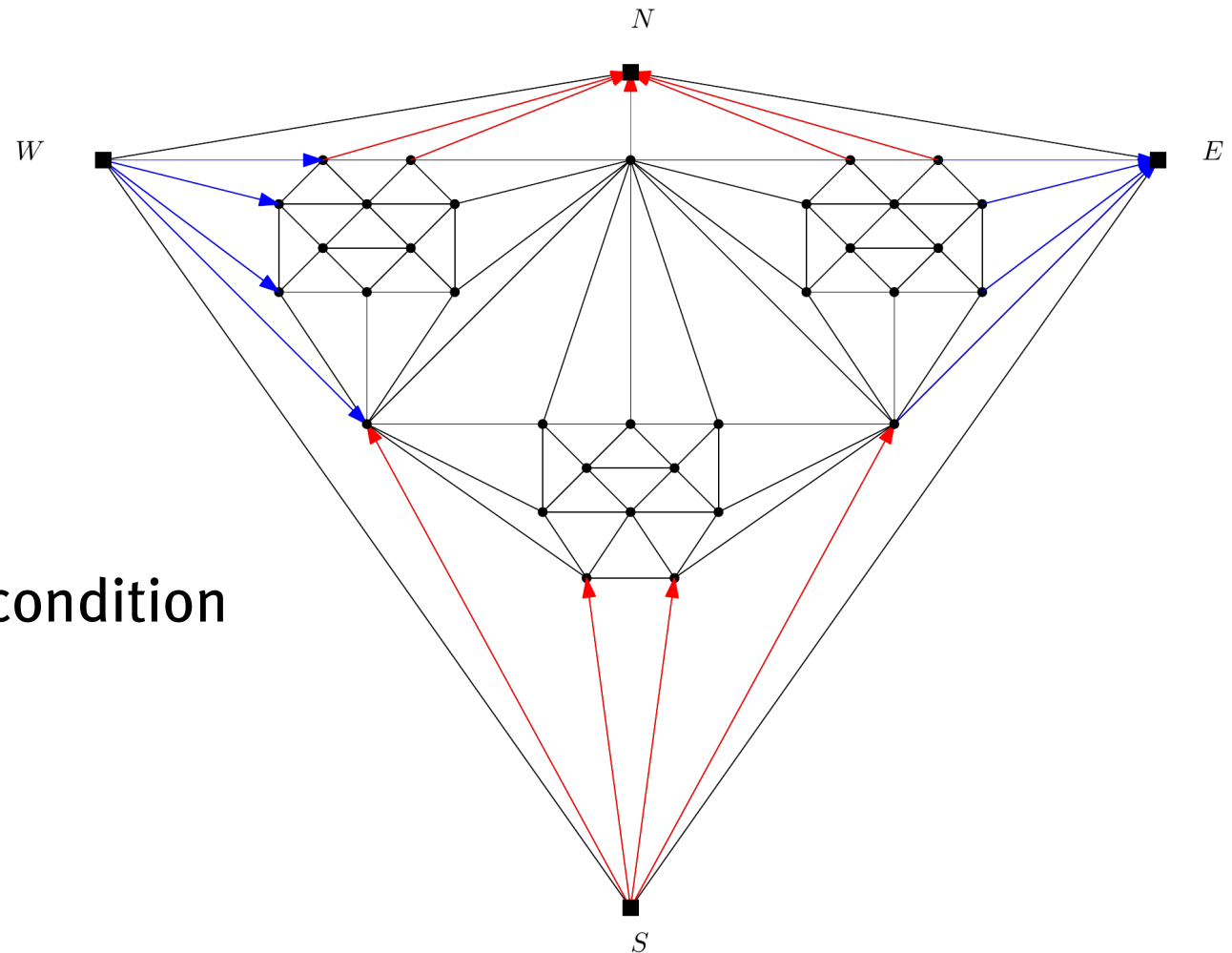
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- Exterior vertex condition

∞ -sided graphs with 4-cycles

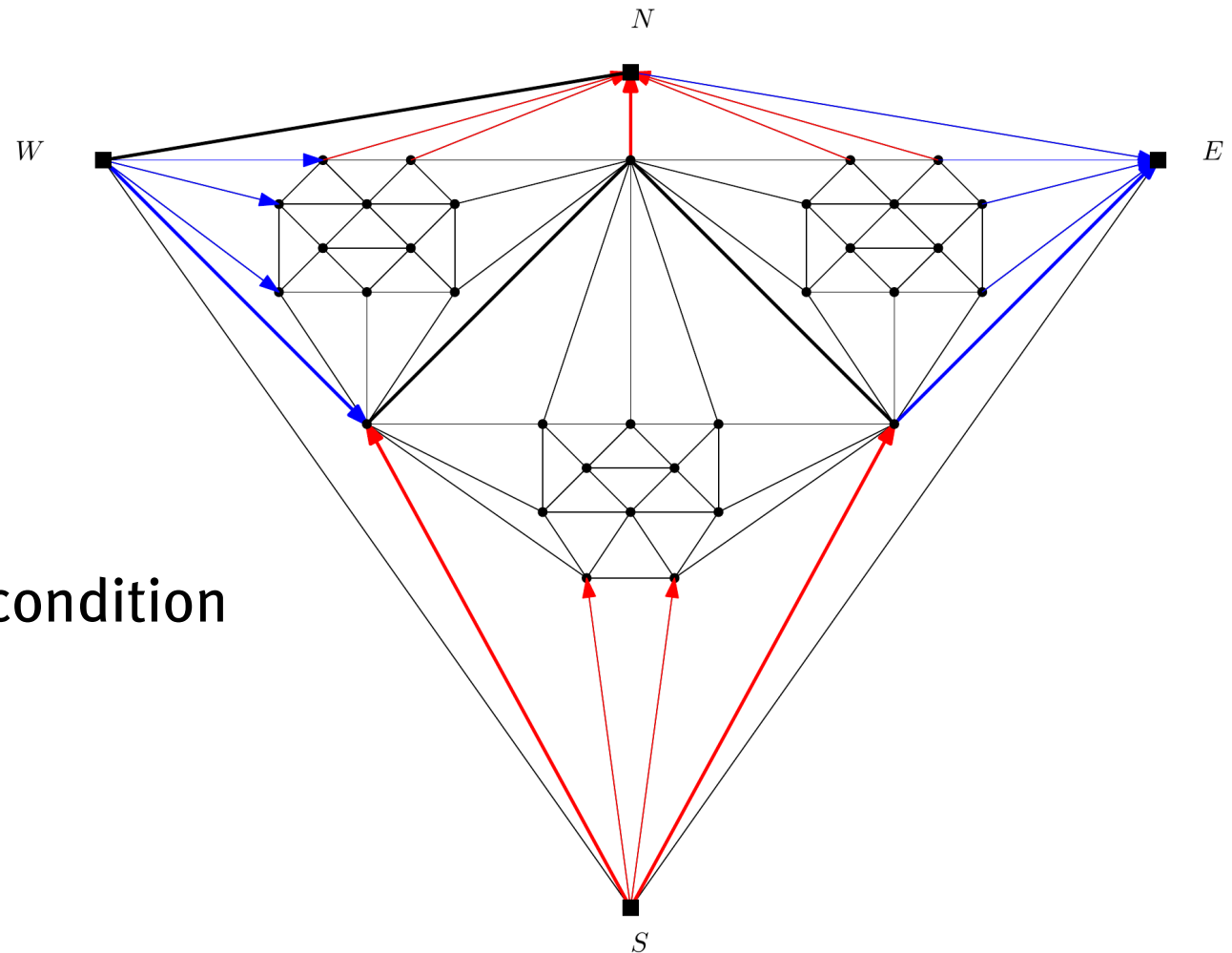
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- Exterior vertex condition
- 4-cycles ...

∞ -sided graphs with 4-cycles

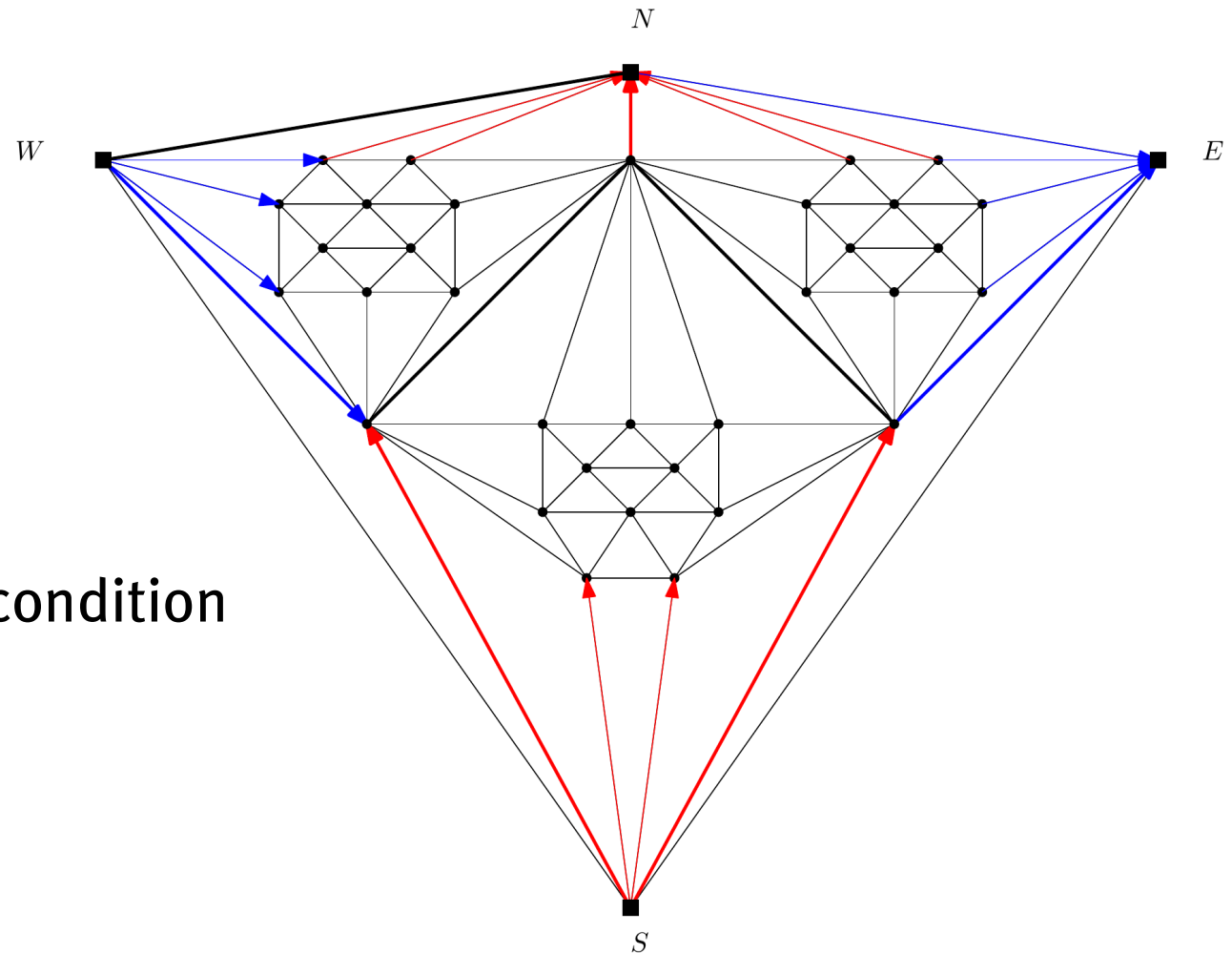
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- Exterior vertex condition
- 4-cycles ...

∞ -sided graphs with 4-cycles

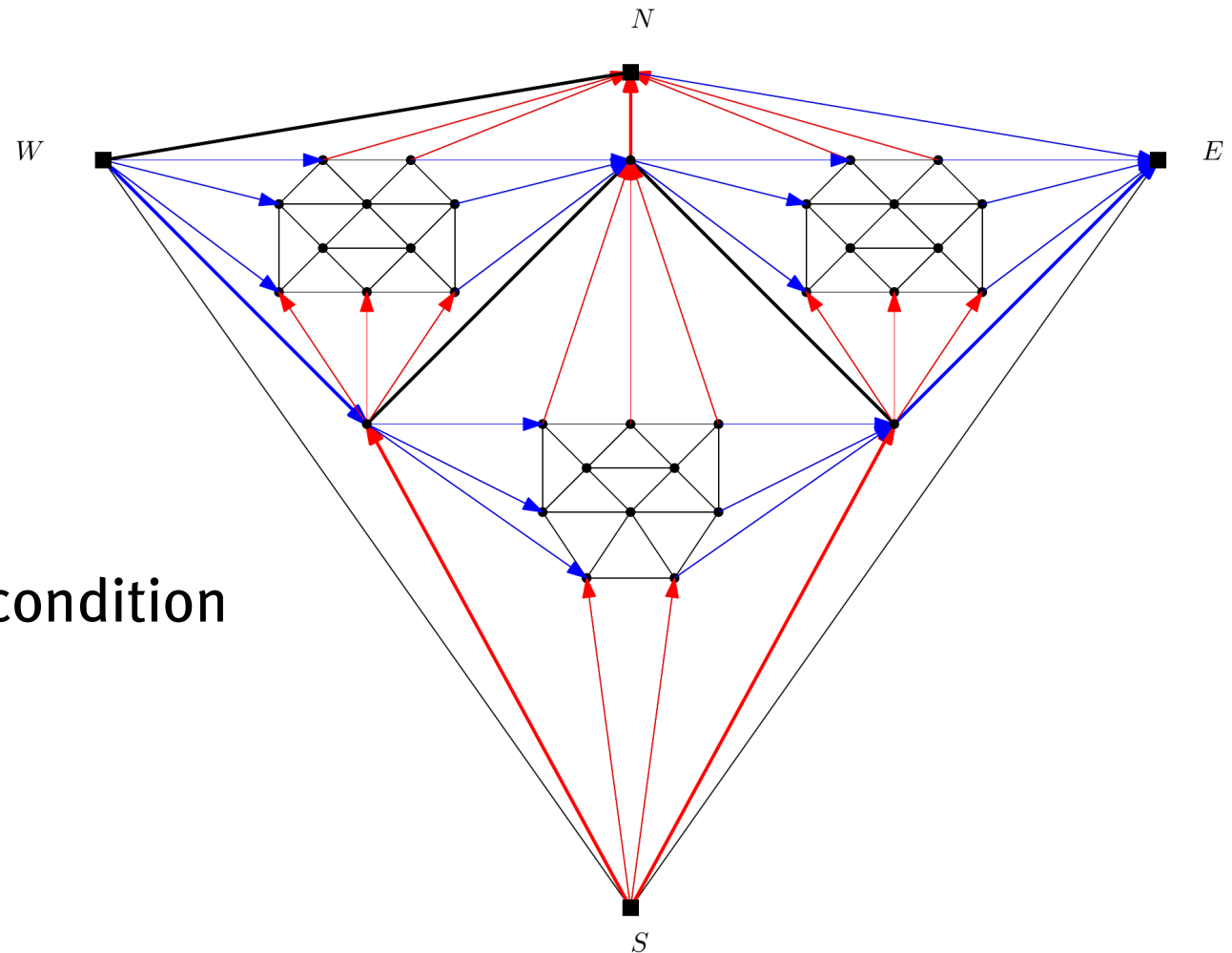
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- Exterior vertex condition
- 4-cycles ...
color inside

∞ -sided graphs with 4-cycles

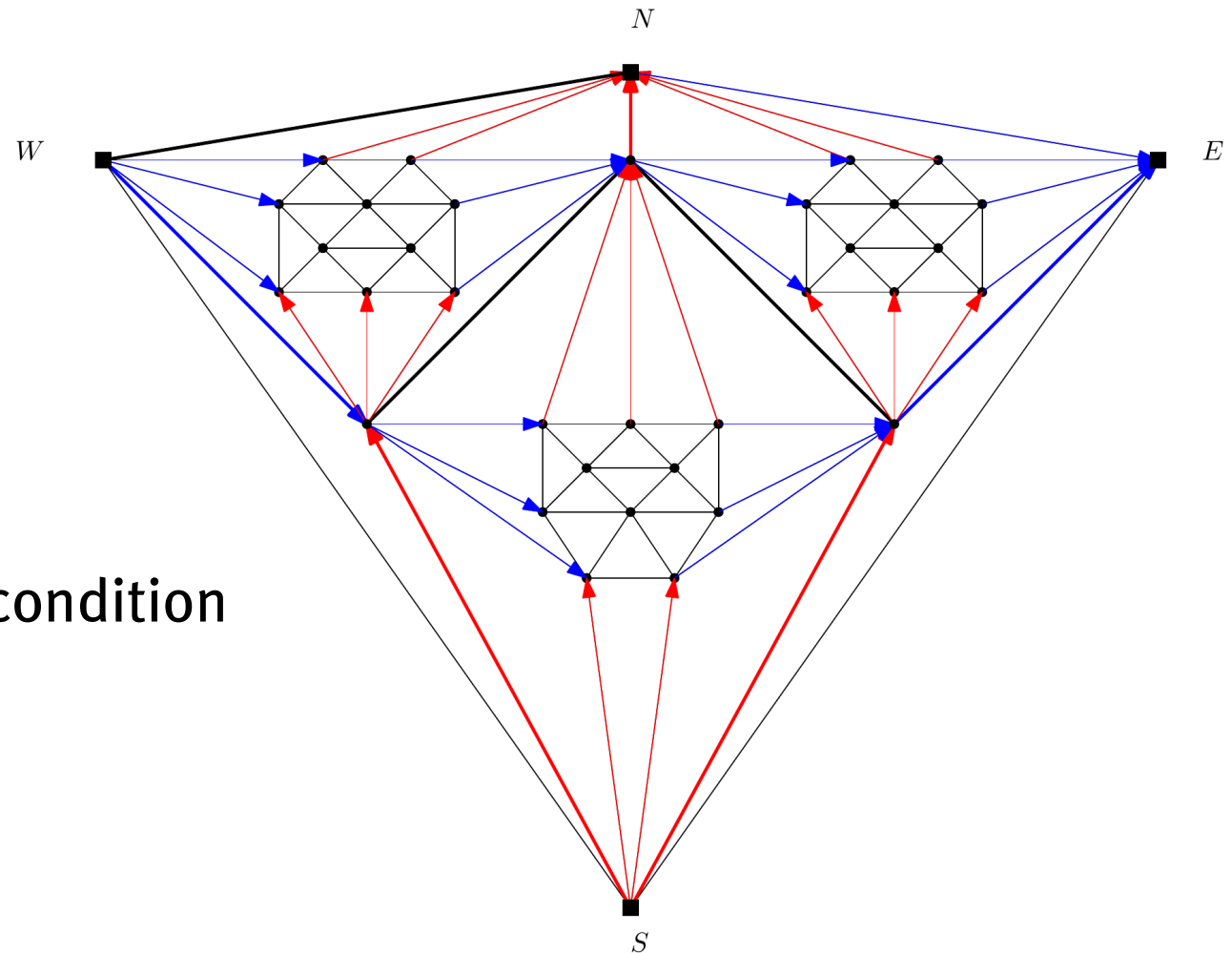
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- Exterior vertex condition
- 4-cycles ...
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∞ -sided graphs with 4-cycles

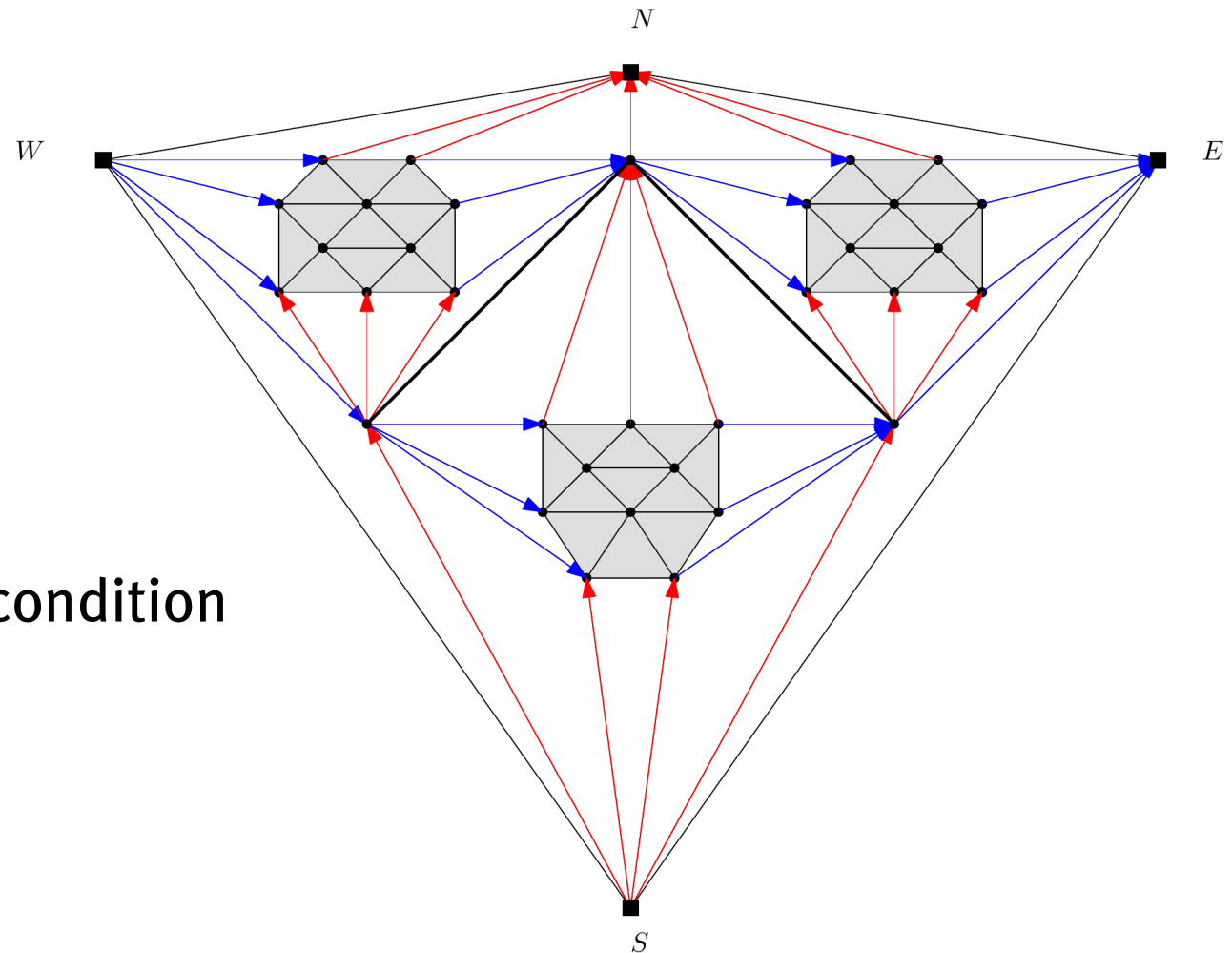
Any extended graph with separating 4 cycles can be ∞ -sided, even if all these cycles go through a pole.



- Exterior vertex condition
- 4-cycles ...
color inside
- Problem!

∞ -sided graphs with 4-cycles

Any extended graph with separating 4 cycles can be ∞ -sided, even if all these cycles go through a pole.



- Exterior vertex condition
- 4-cycles ...
color inside
- Problem!

Structure

Problem

Results

Conjecture

Conjecture

- What about extended graphs without any separating 4-cycle?

Maybe they are all 2-sided!

- The lack of 4-cycles gives a lot of freedom in most cases

But sometimes we can restrict this quite a lot

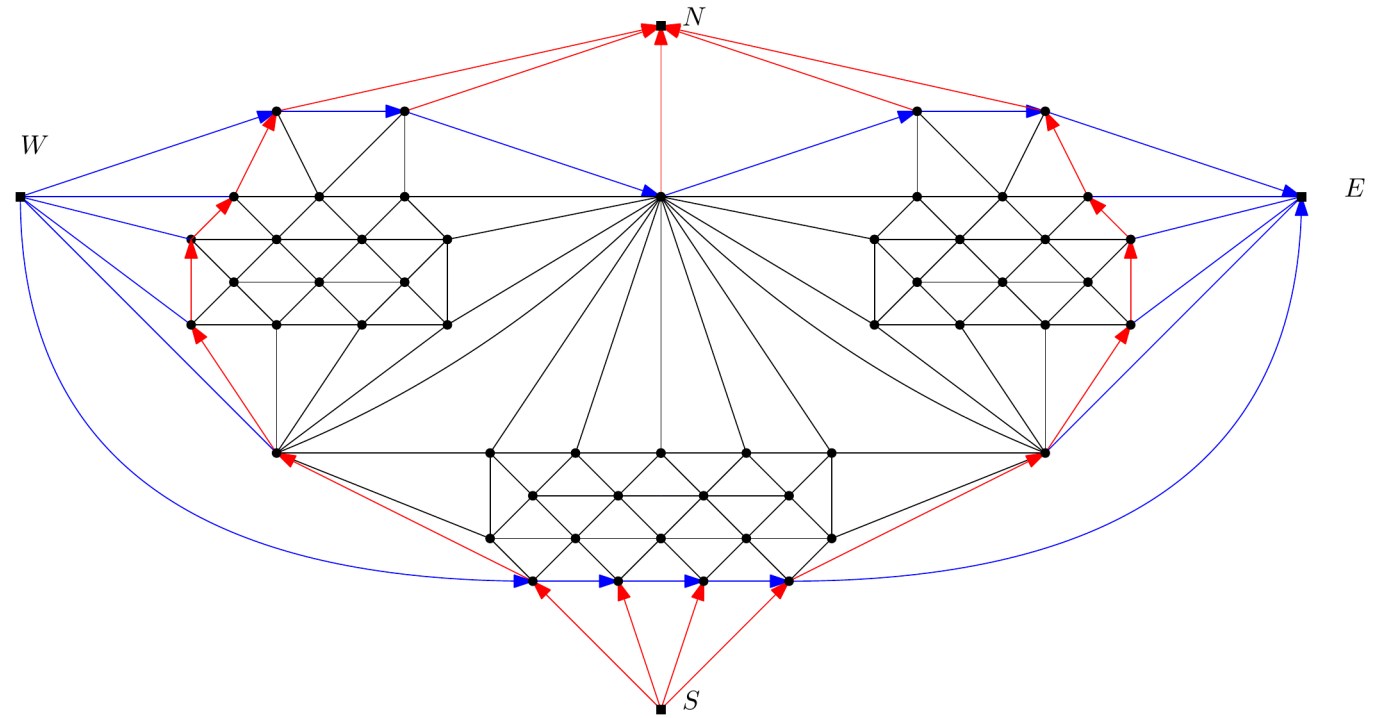
Tricky Example

Tricky Example

- Graph

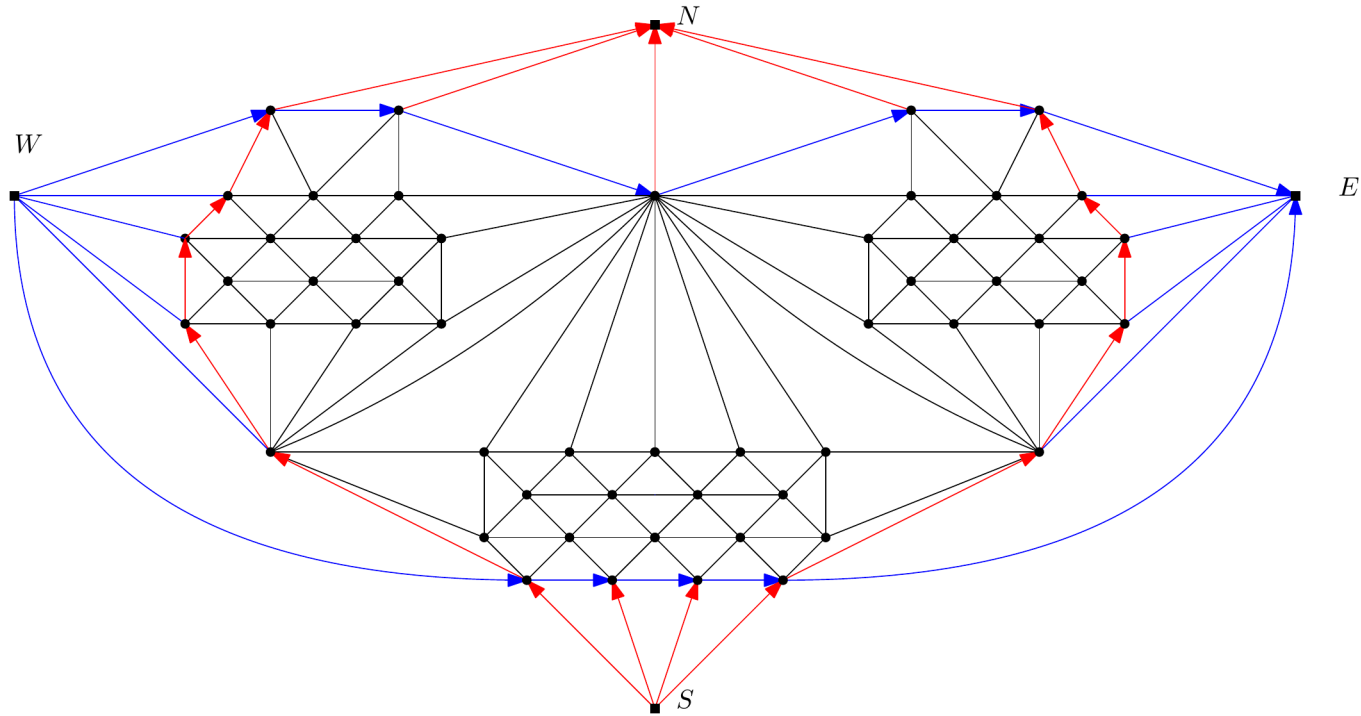
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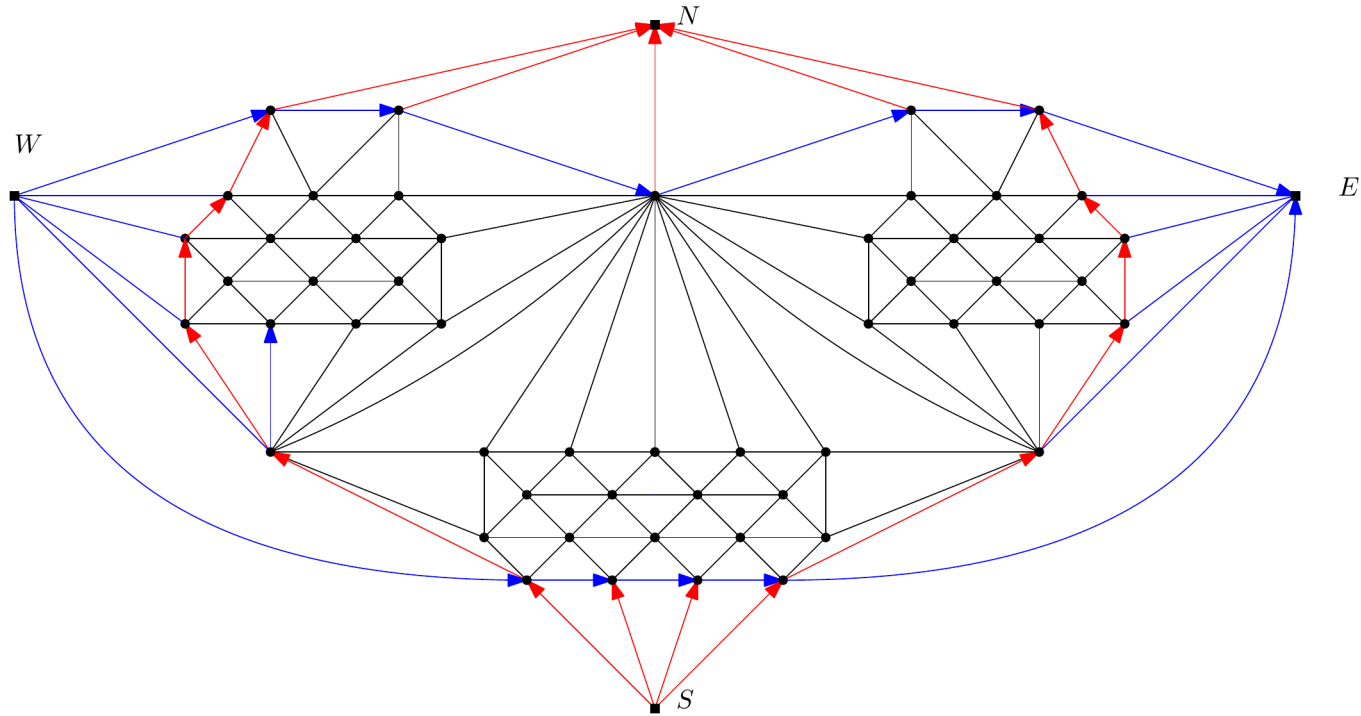
Tricky Example

- Graph
- Suppose blue
- ...



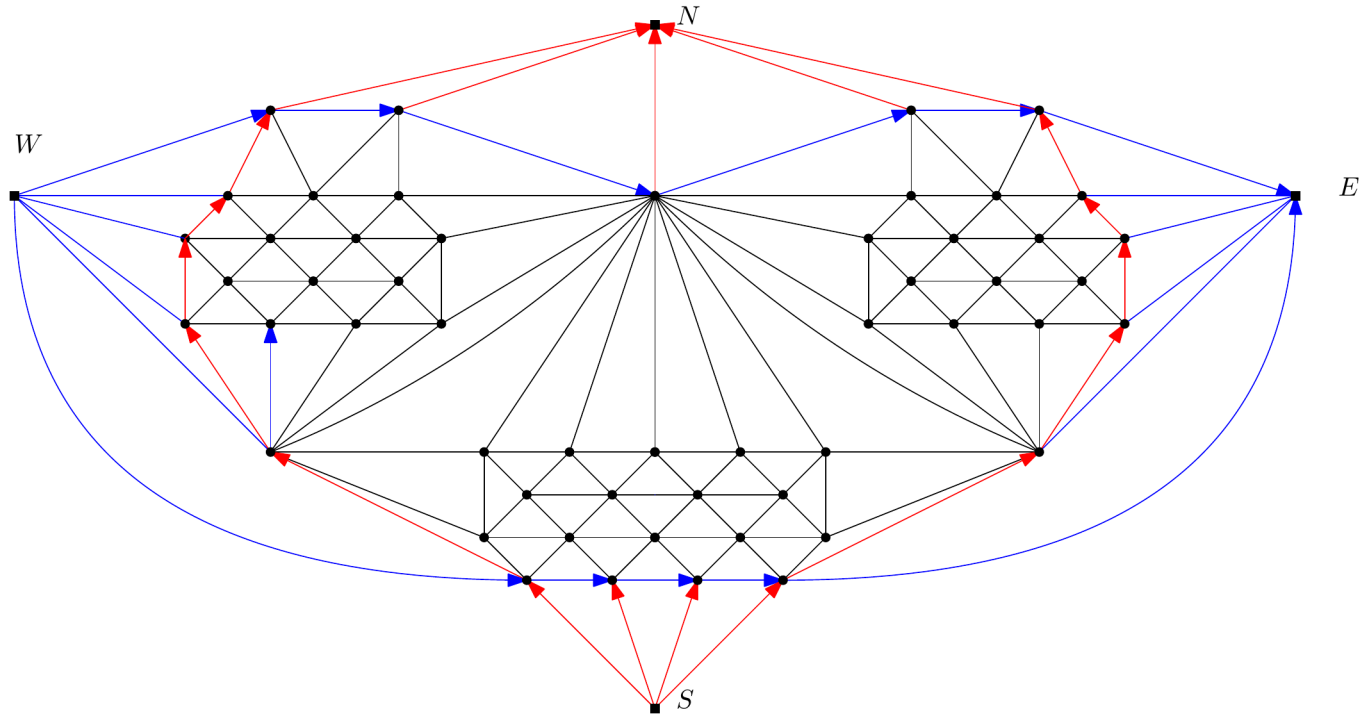
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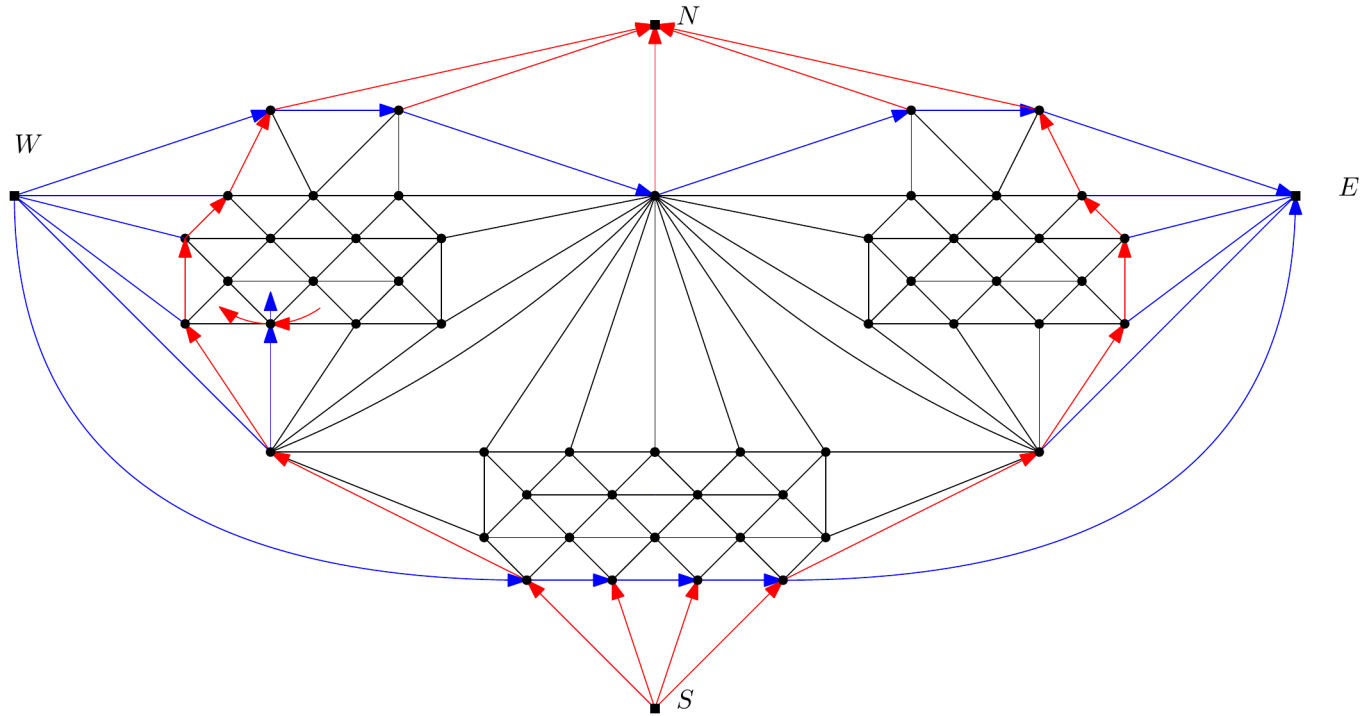
Tricky Example

- Graph
- Suppose blue ...
Incoming red ...



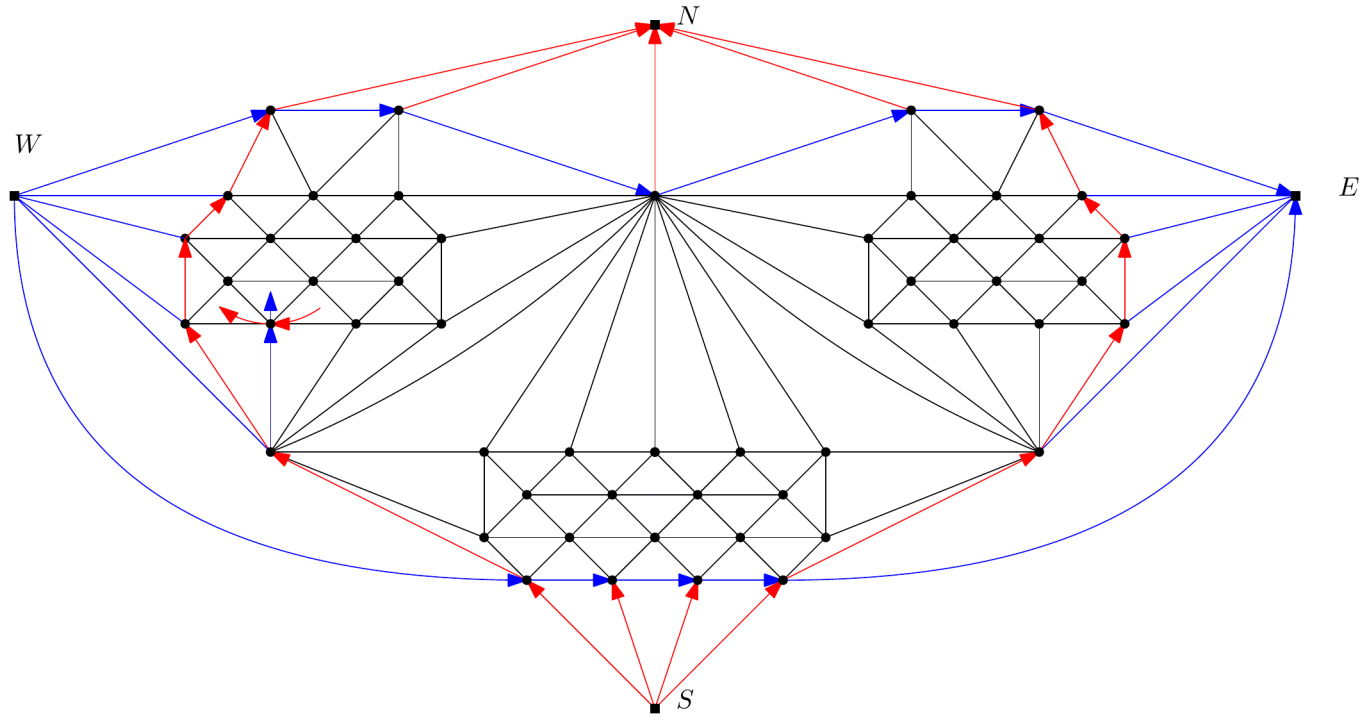
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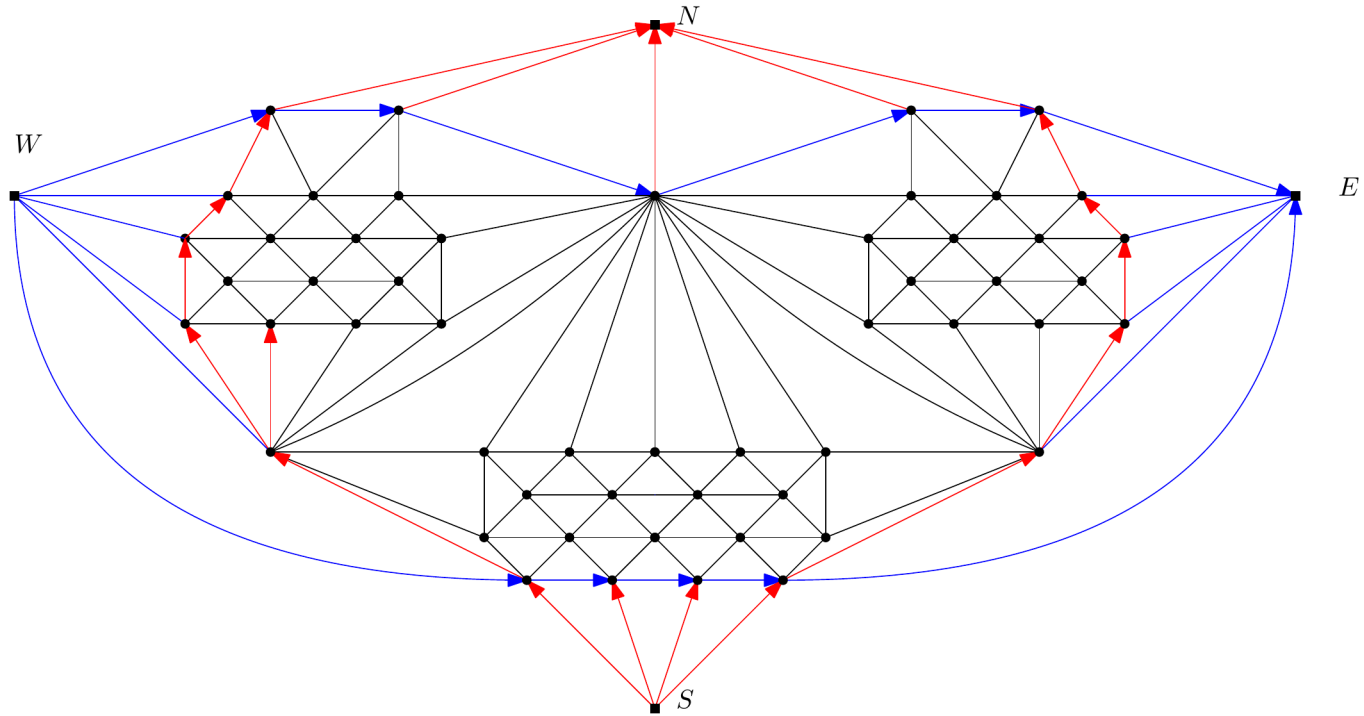
Tricky Example

- Graph
- Suppose blue ...
Incoming red ...
Contradiction



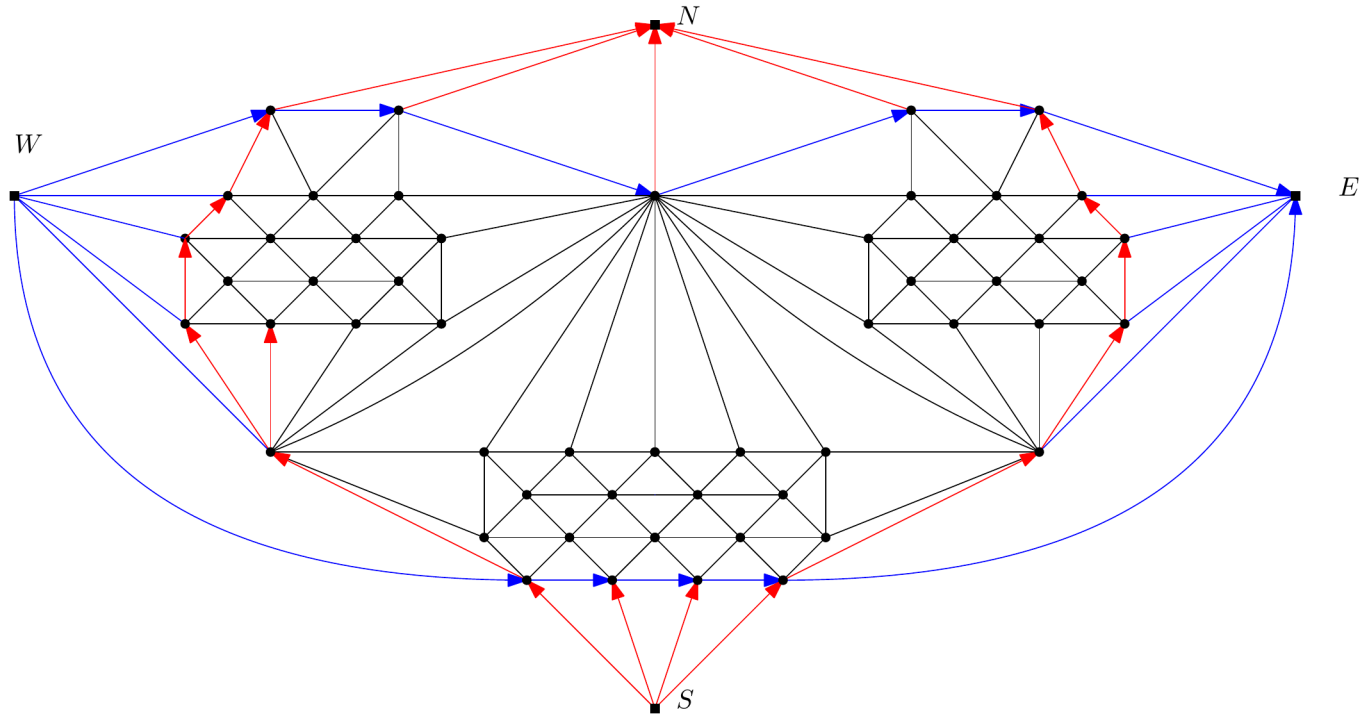
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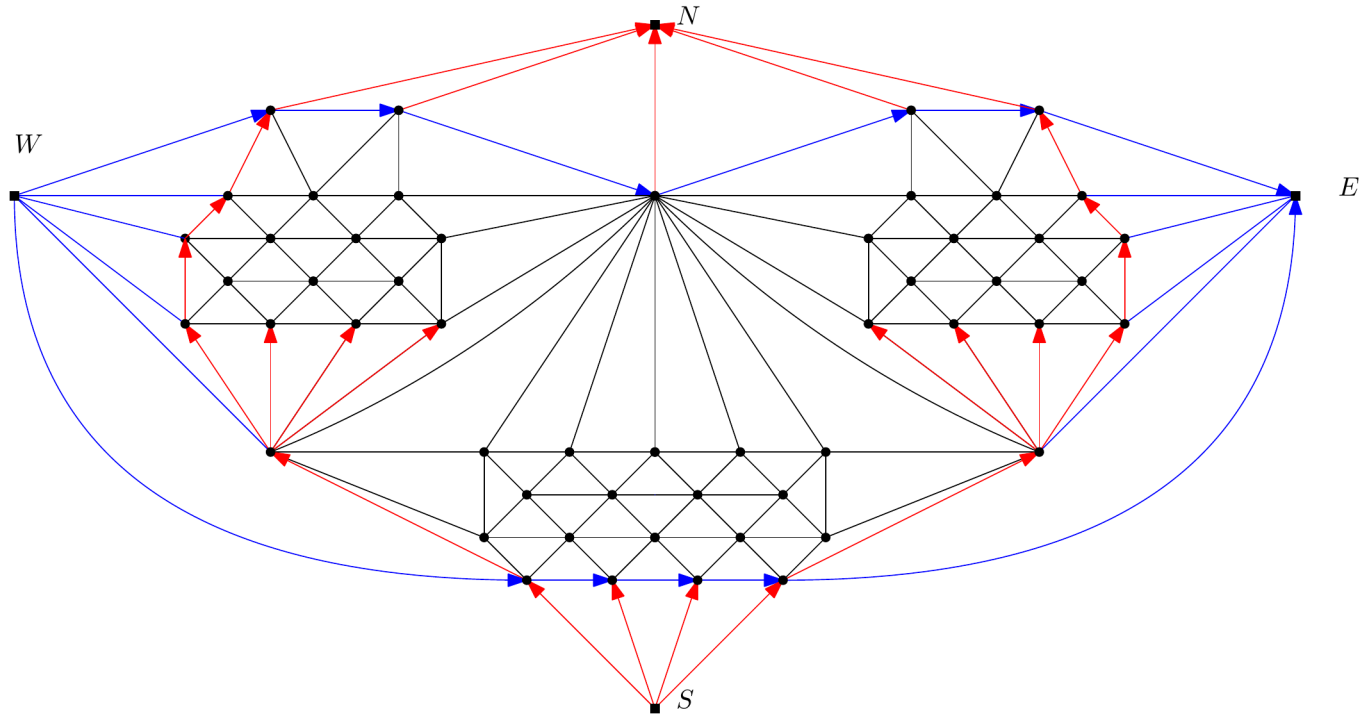
Tricky Example

- Graph
- Suppose blue ...
Incoming red ...
Contradiction
- Repeat this argument



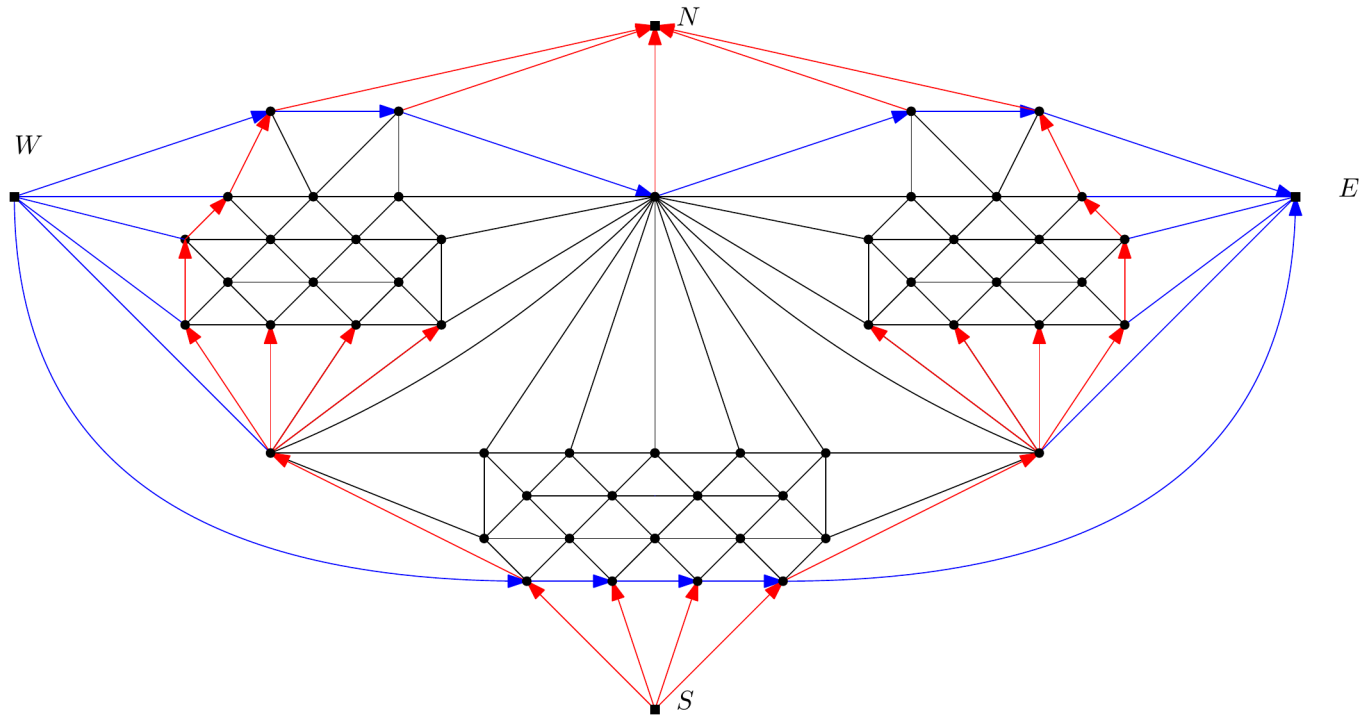
Tricky Example

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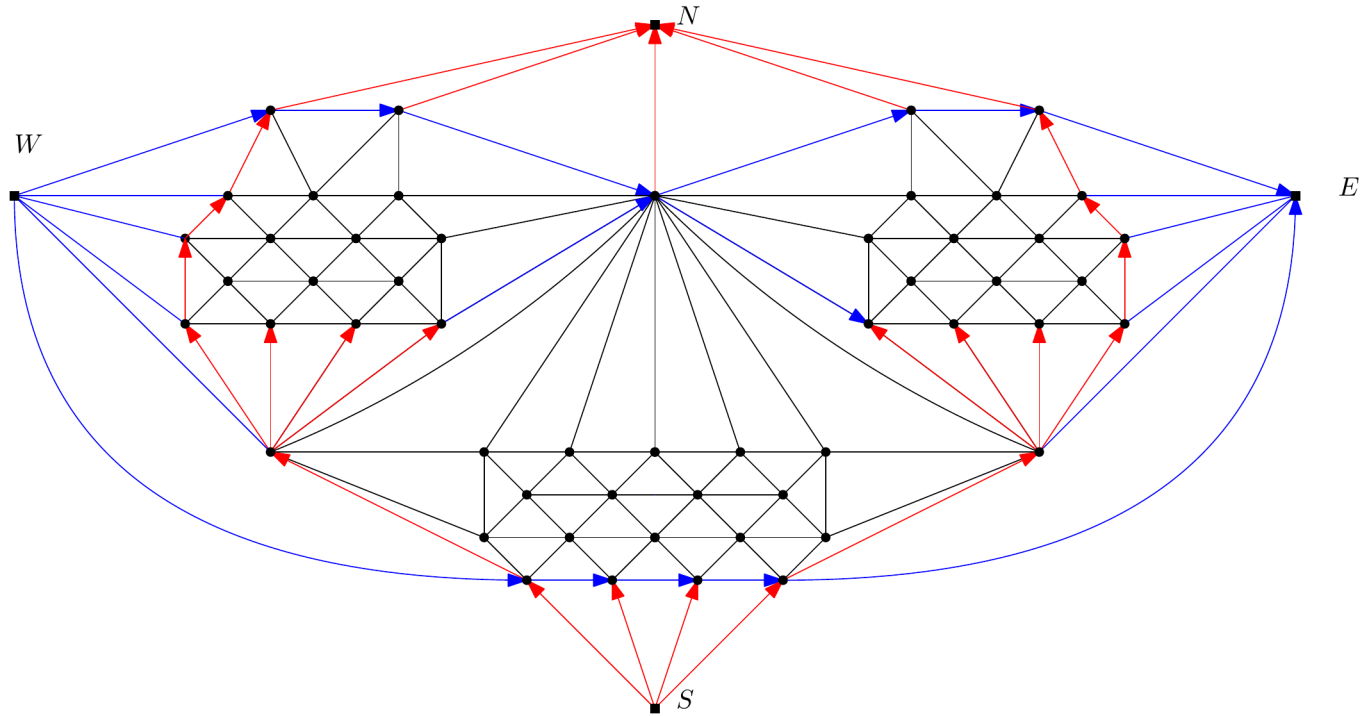
Tricky Example

- Graph
- Suppose blue ...
Incoming red ...
Contradiction
- Repeat this argument
- Color change at corner



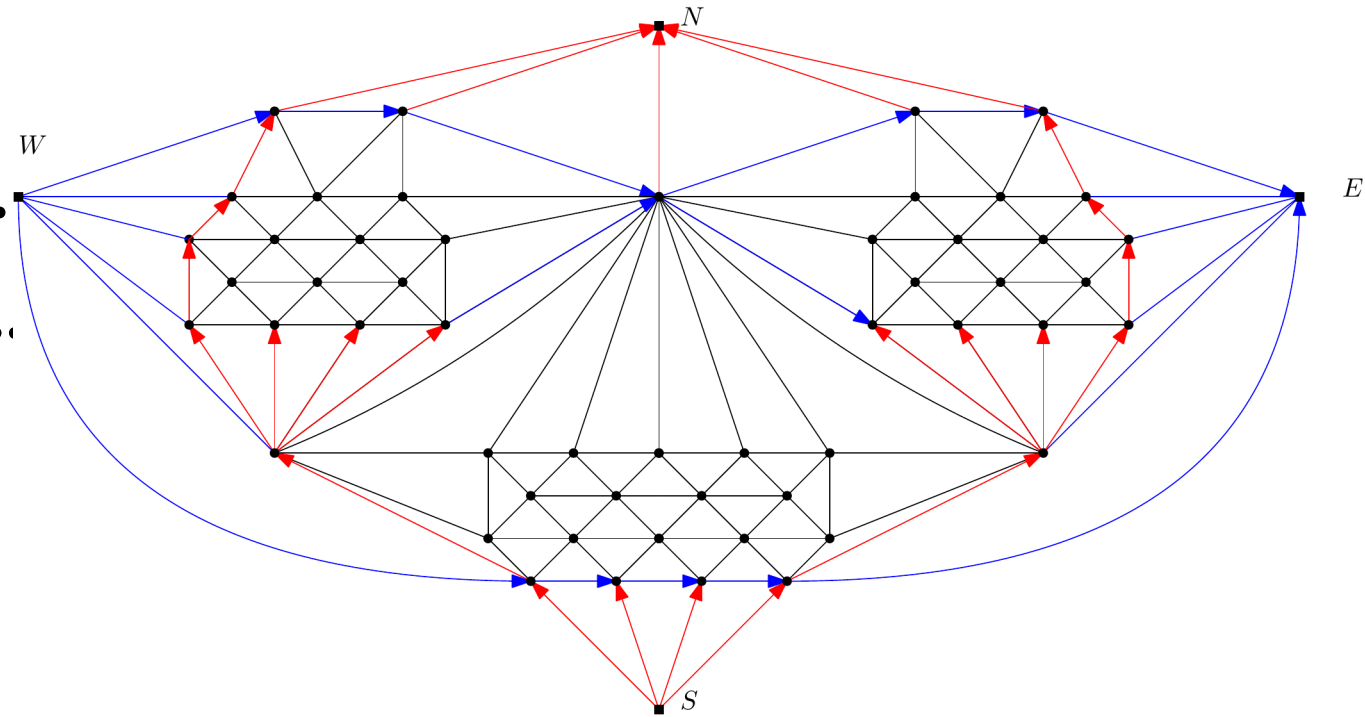
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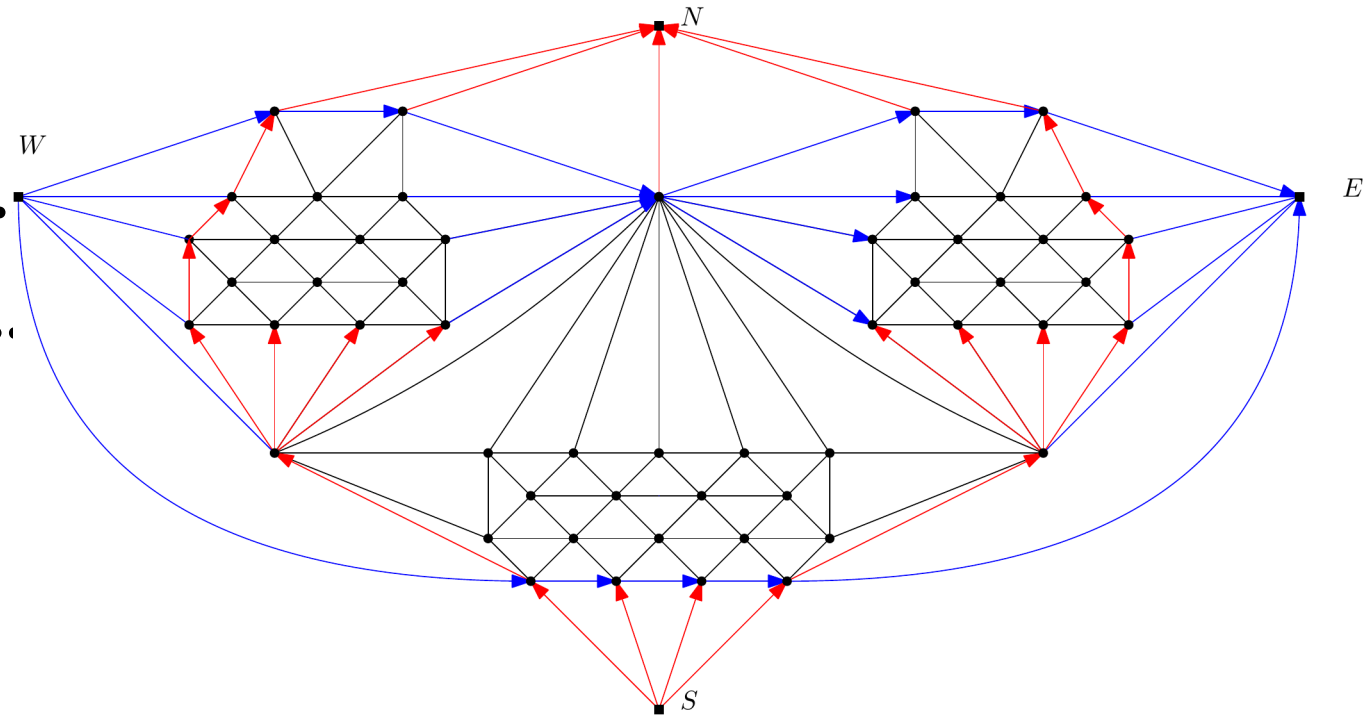
Tricky Example

- Graph
- Suppose blue .
Incoming red ...
Contradiction
- Repeat this
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- Color change at
corner
- Interior vertex
condition



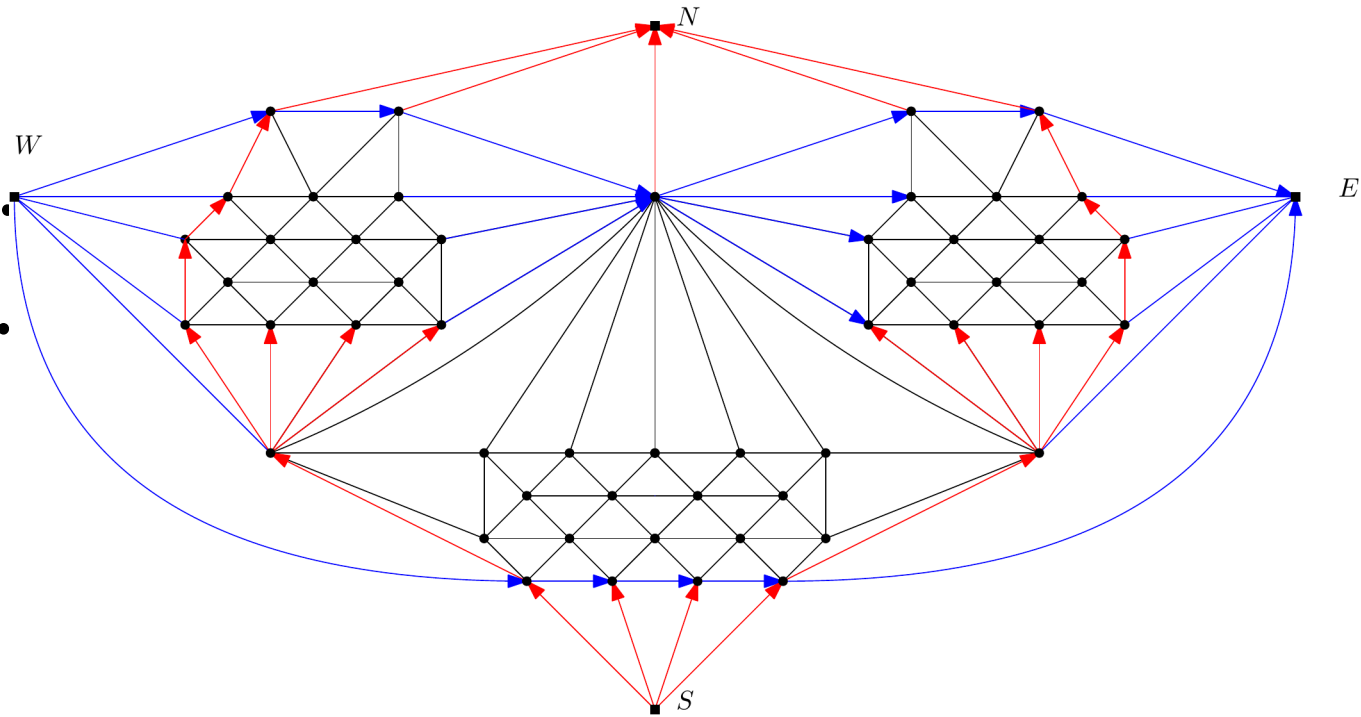
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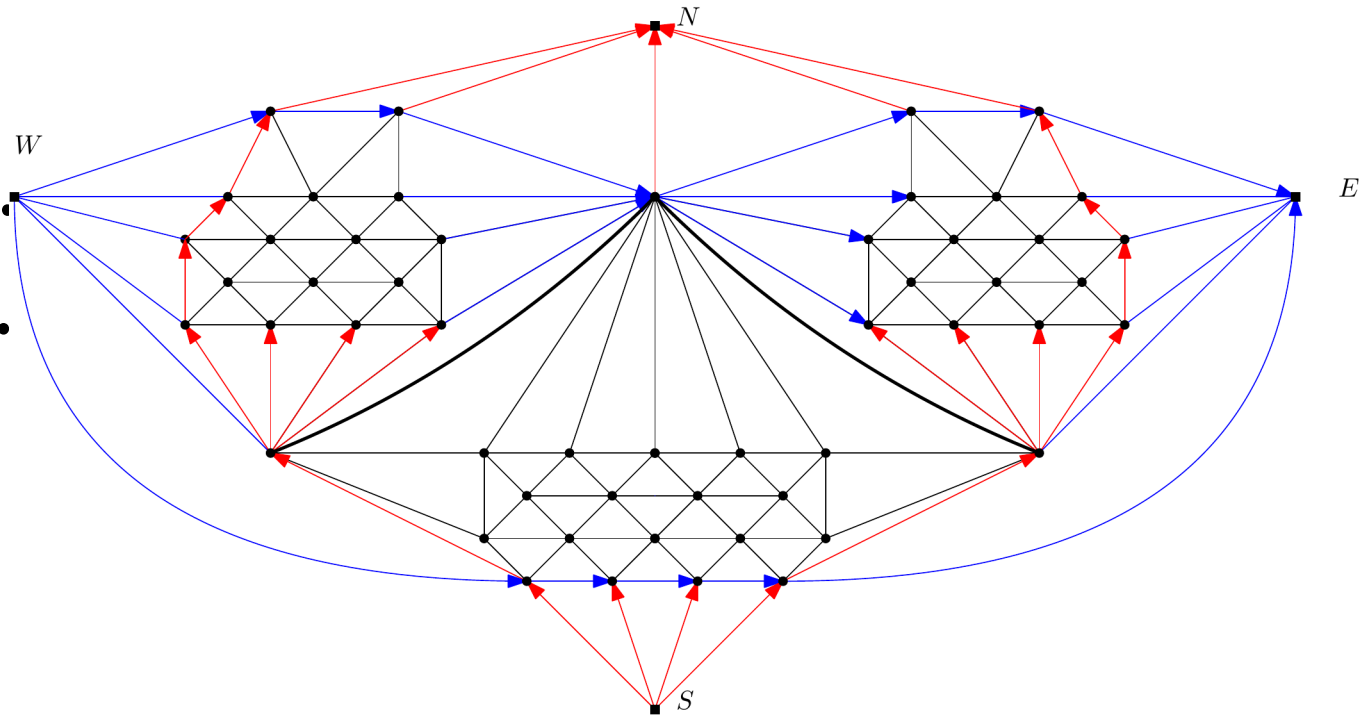
Tricky Example

- Graph
- Suppose blue ...
Incoming red ...
Contradiction
- Repeat this argument
- Color change at corner
- Interior vertex condition
- Problem ...



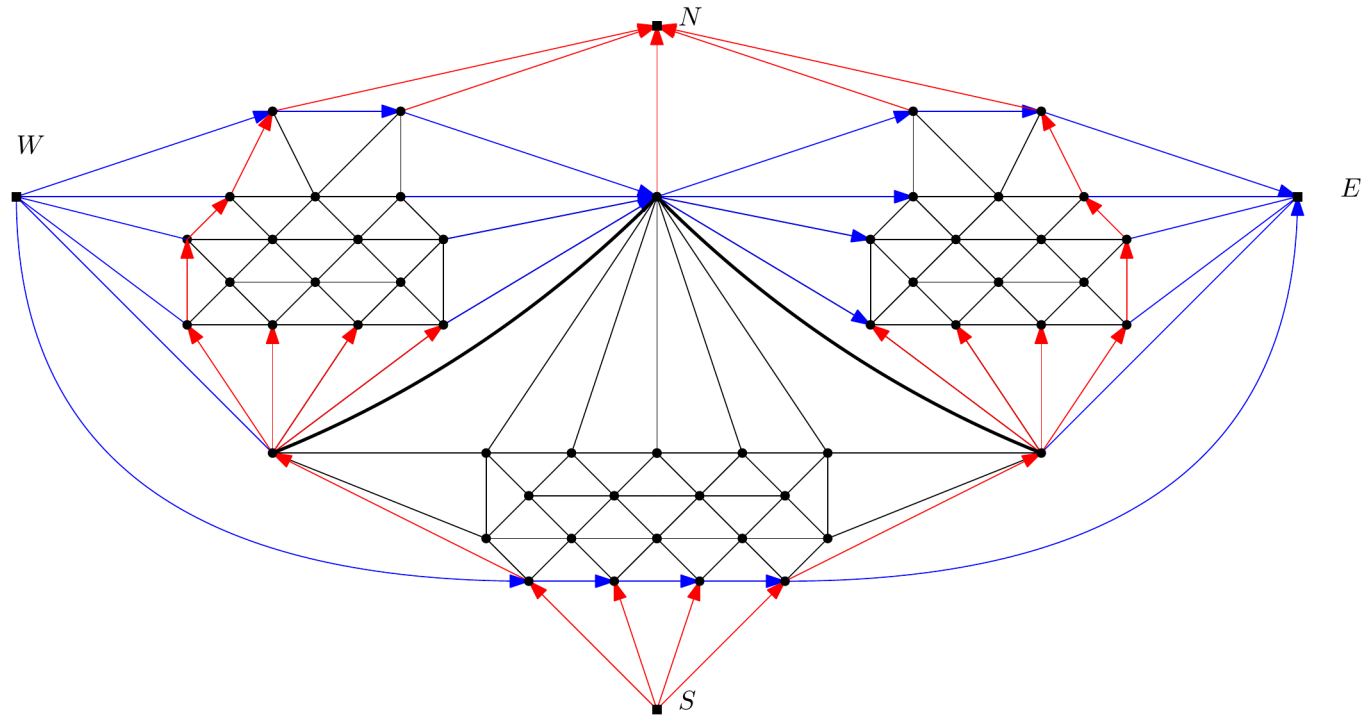
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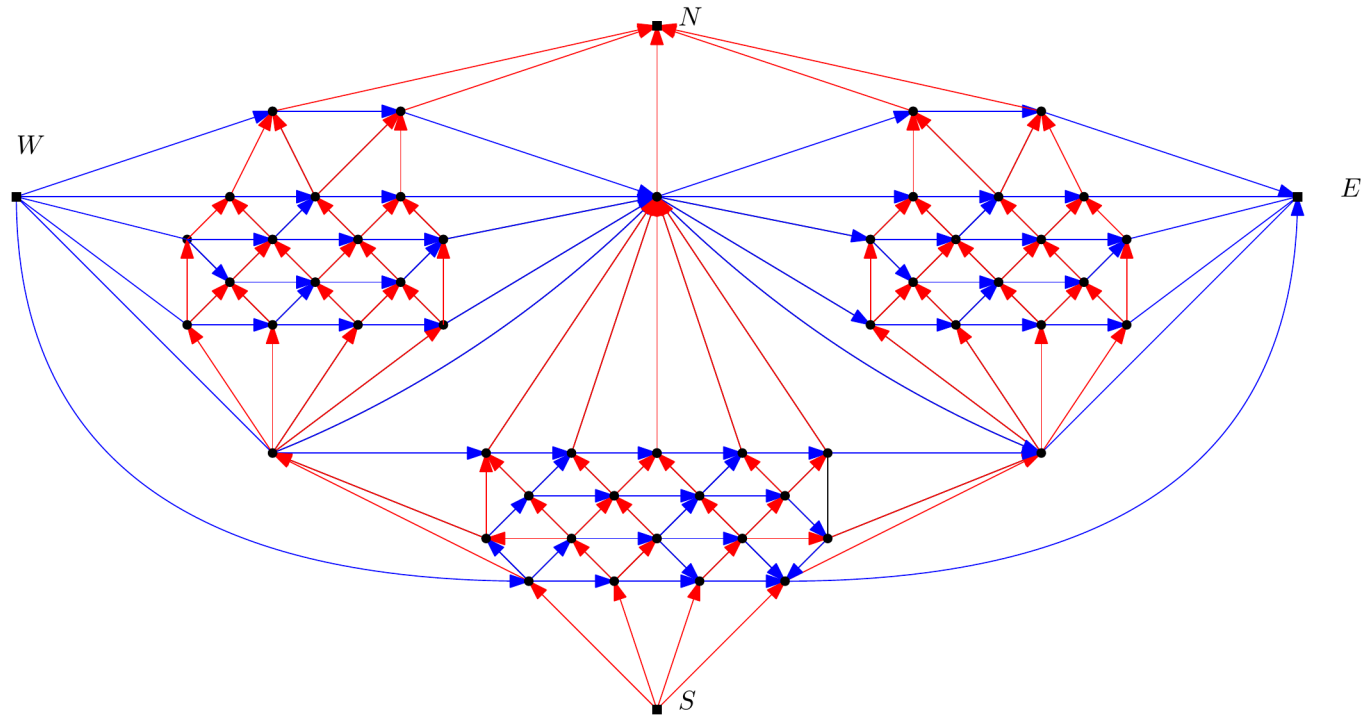
Tricky Example

- Graph
- Suppose blue ...
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- Repeat this argument
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- Problem ...
Solution!



Tricky Example

- Graph
- Suppose blue ...
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Contradiction
- Repeat this argument
- Color change at corner
- Interior vertex condition
- Problem ...
Solution!



Conclusion

- A graph has multiple extended graphs $E(G)$
 - If $E(G)$ has a separating 3-cycle this corner assignment has no dual
 - If $E(G)$ has a separating 4-cycle this corner assignment gives a rectangular dual of G . But it may be ∞ -sided
 - If $E(G)$ has neither we hopefully show that it is 2-sided
- Current approach is with a constricting sweep-cycle
 - Has to do quite specific things, see previous example.
- Questions?