Pseudo one-sided rectangular duals

By Sander Beekhuis

Structure

Problem

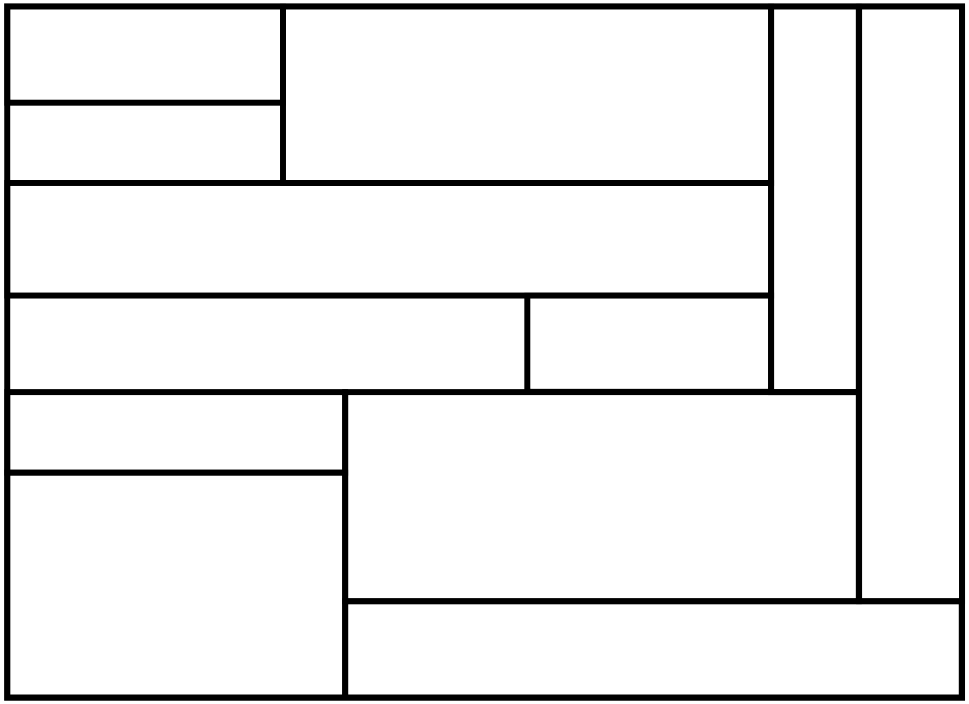
Results

Conjecture

Rectangular layout

Rectangular layout L

Partition of a rectangle into finitely many interior disjoint rectangles

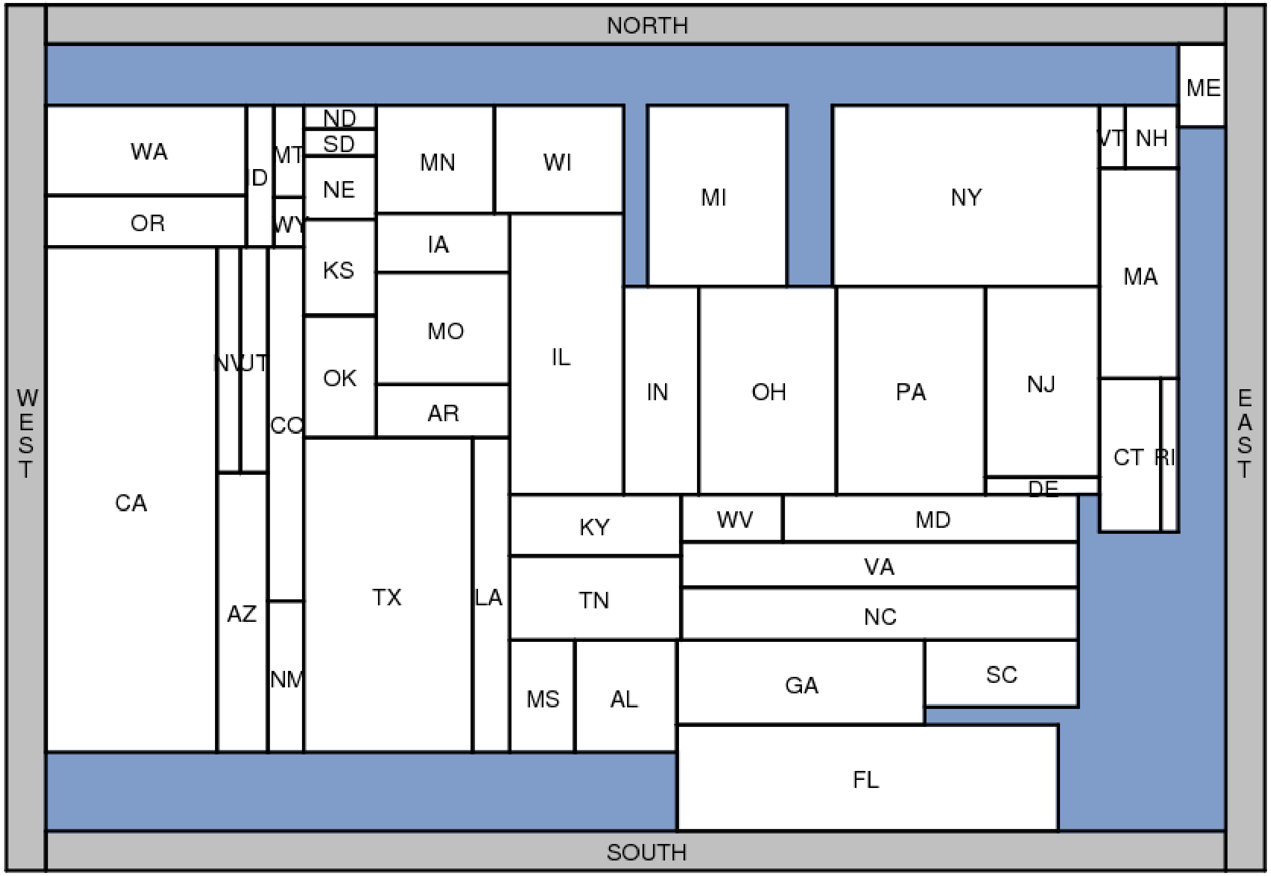


- Layouts are *equivalent* when they have the same adjacencies with the same orientation (horizontal/vertical)

Applications

Rectangular Cartograms

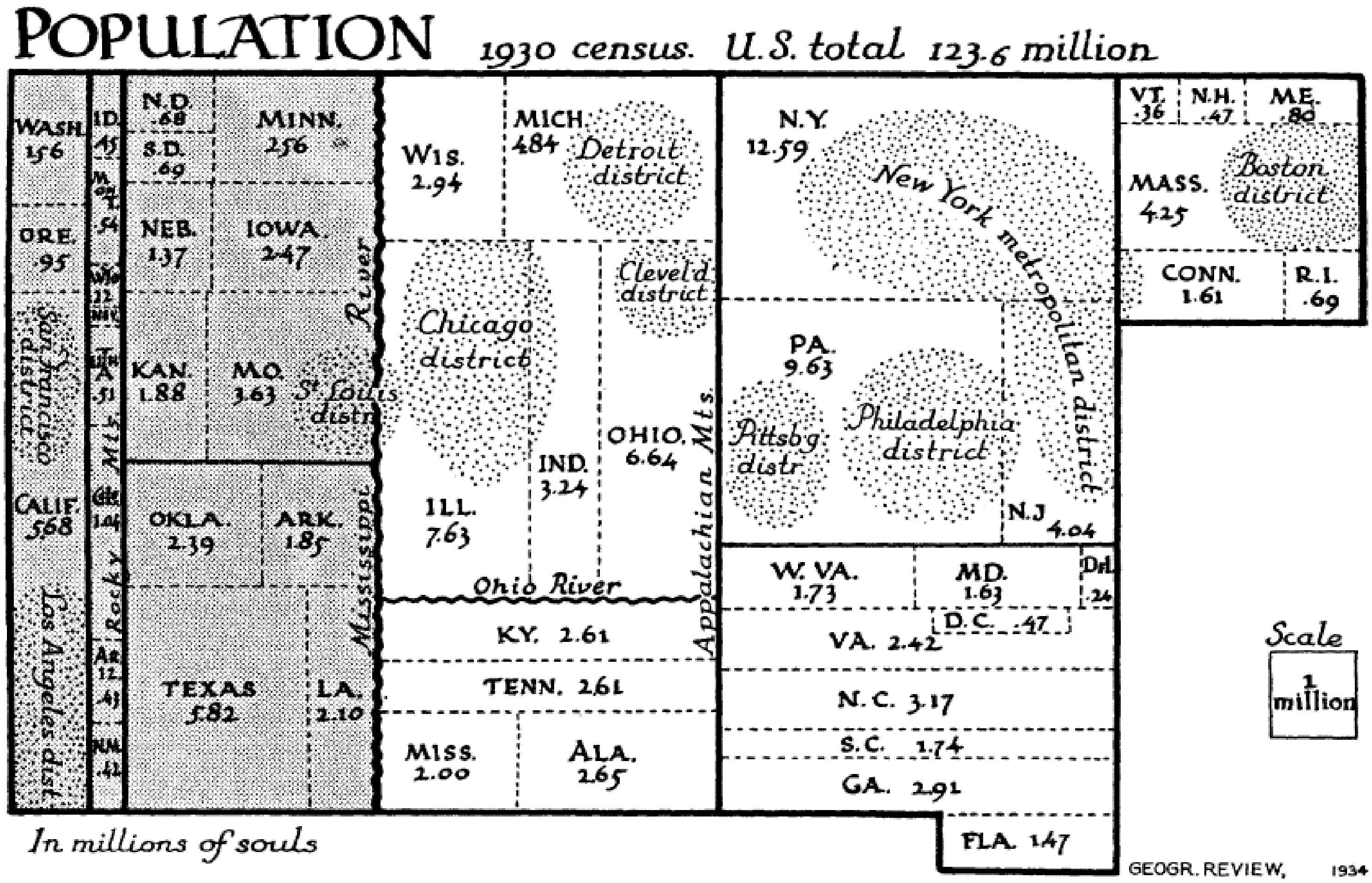
visualize statistical data about sets of regions; regions are rectangles; area proportional to some geographic variable



Applications

Rectangular Cartograms

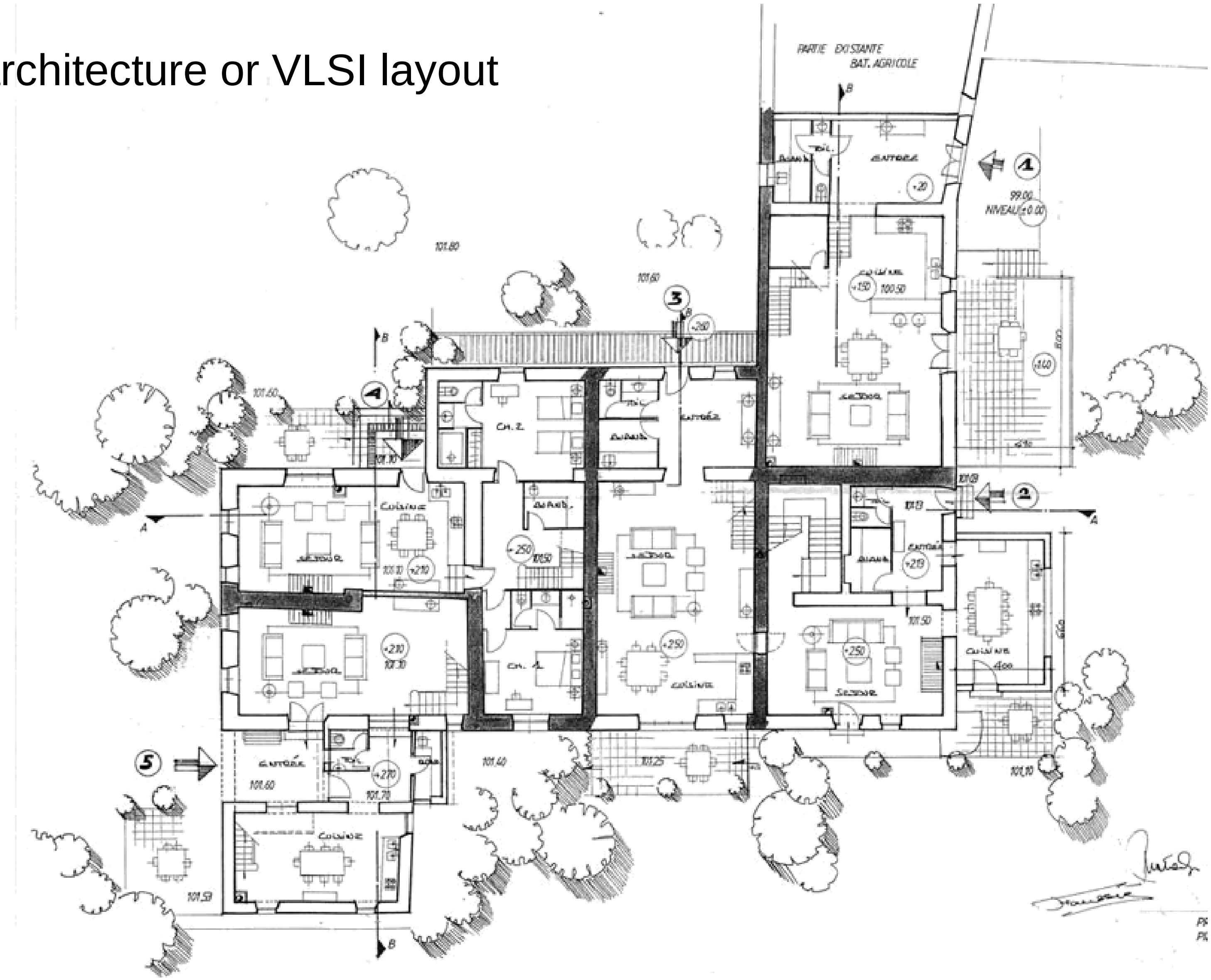
introduced by Raisz in 1934



Applications

Floorplans

building architecture or VLSI layout



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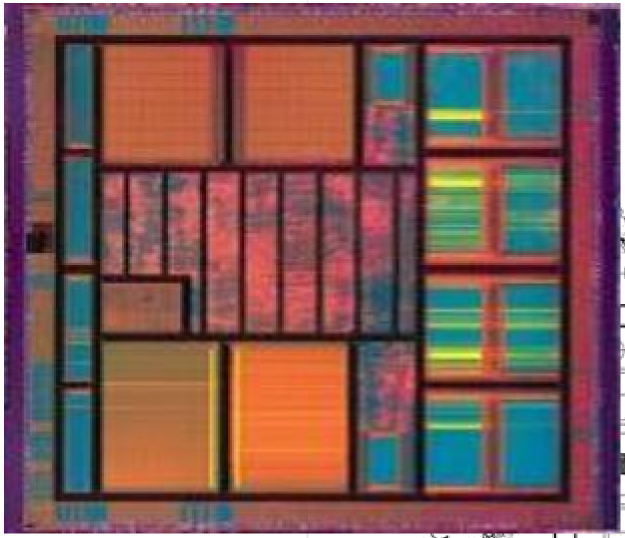
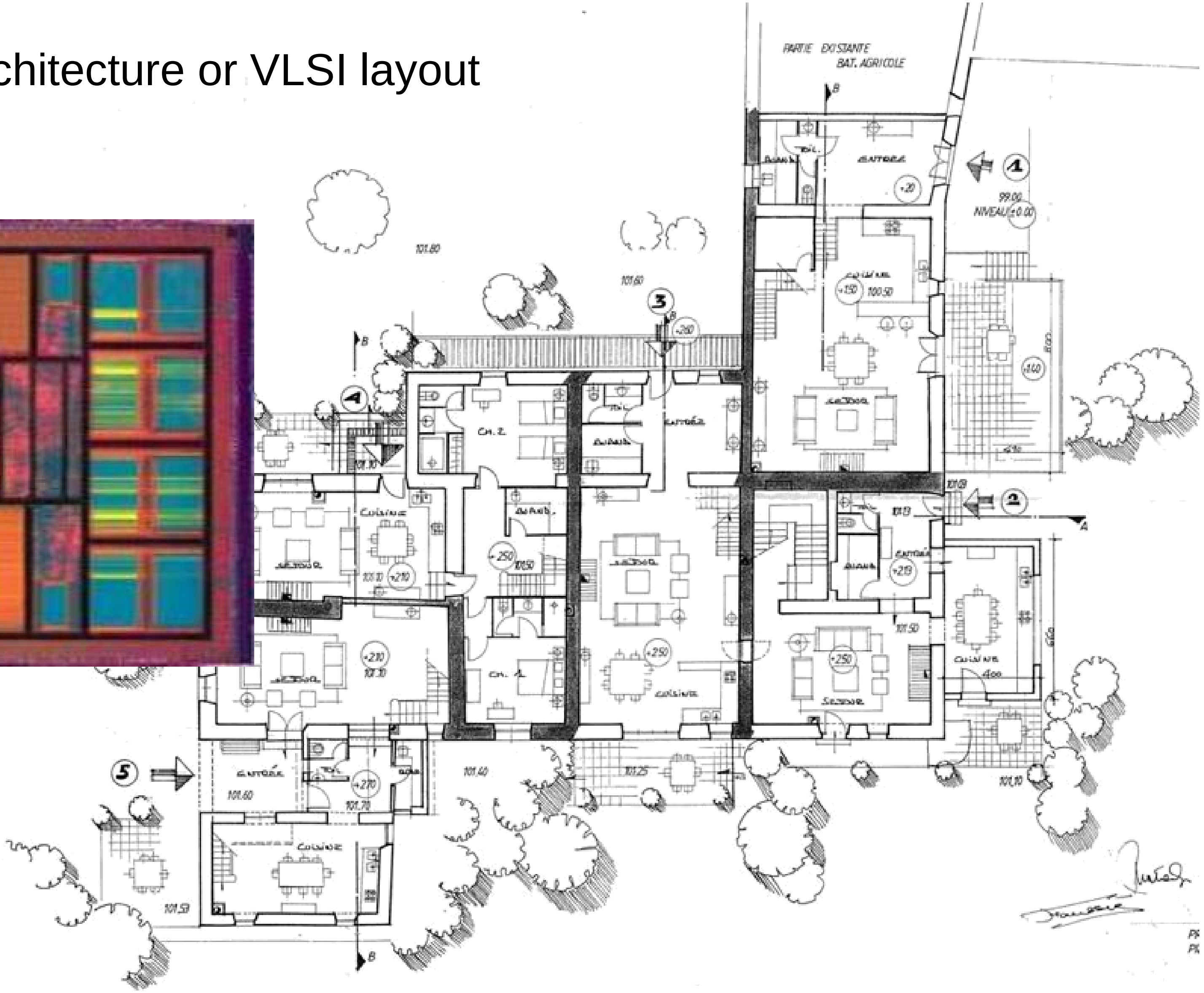
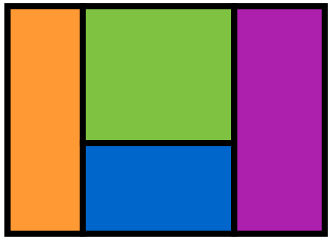




Area-Universal

Area-universal layout L

For every assignment of sizes to the areas of **L** there is a equivalent layout realizing these sizes



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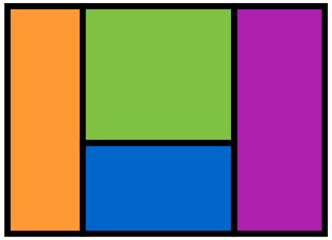

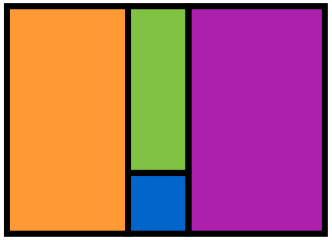

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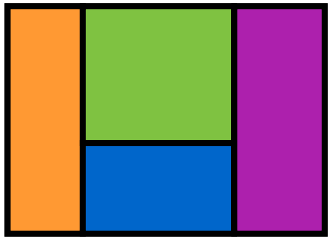

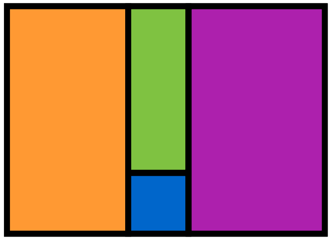

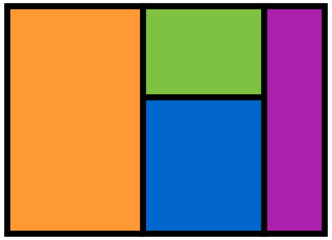

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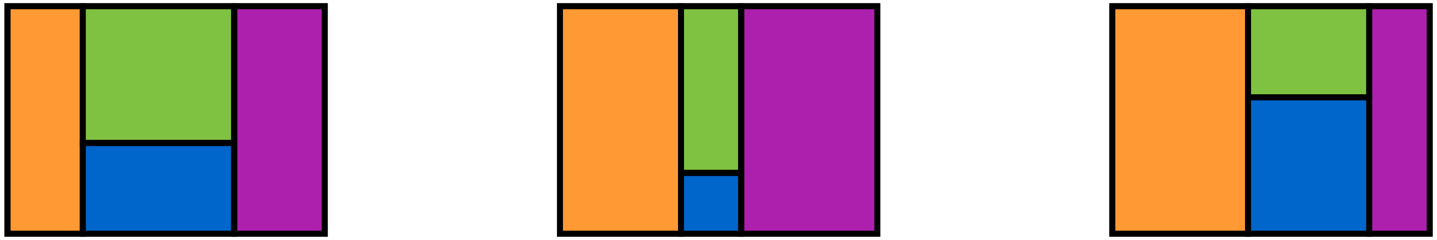


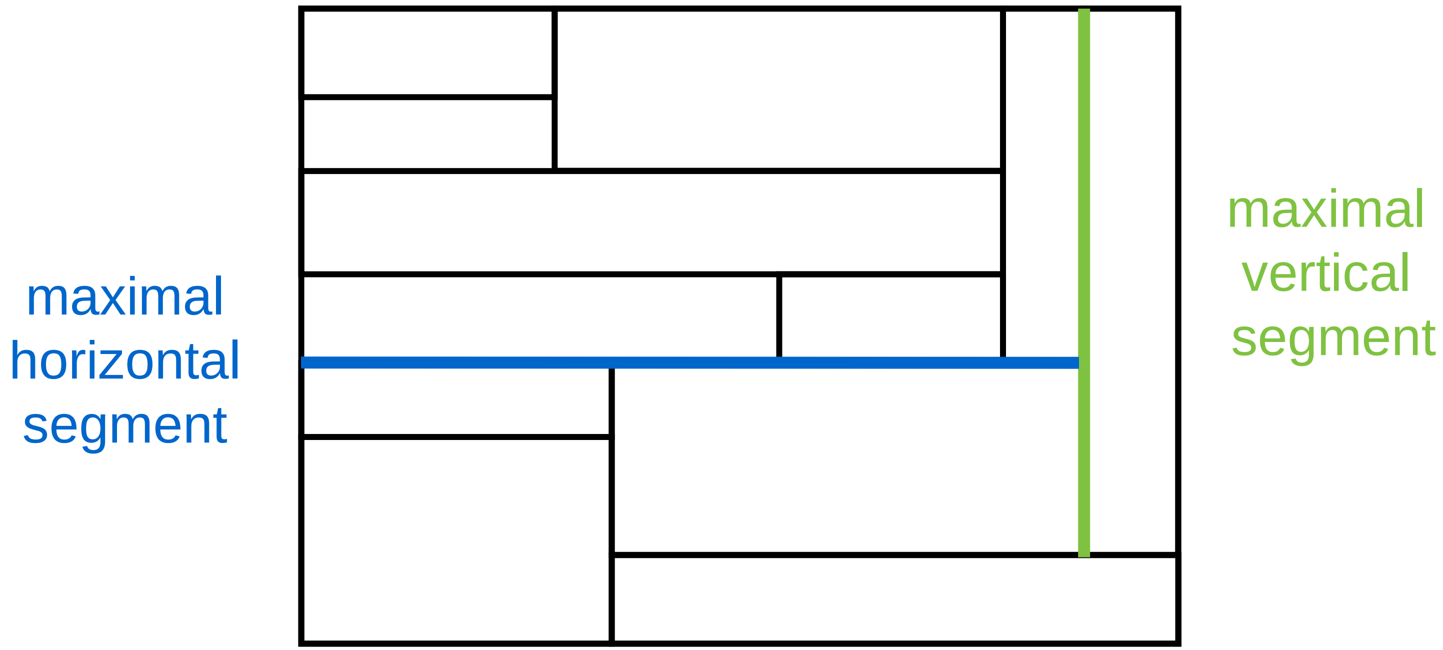
Uses

Animations; layout first – function later

One-sided

One-sided layout L

Every maximal line segment in L is the side of a rectangle



One-sided

One-sided layout **L**

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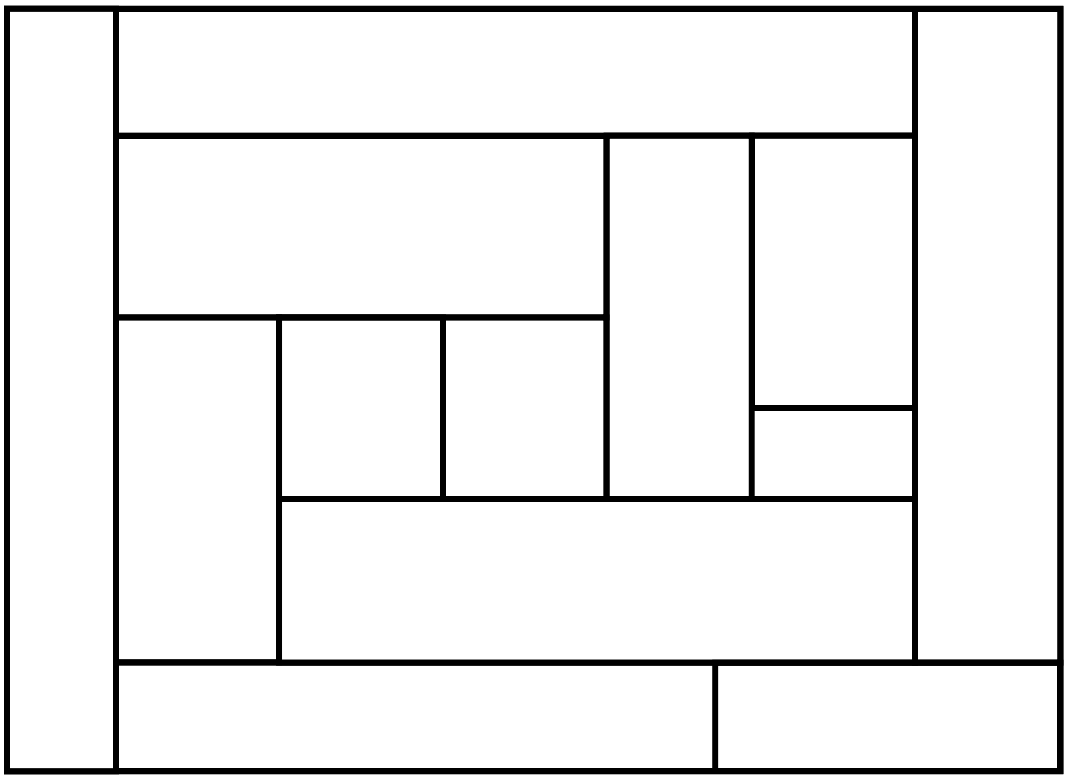

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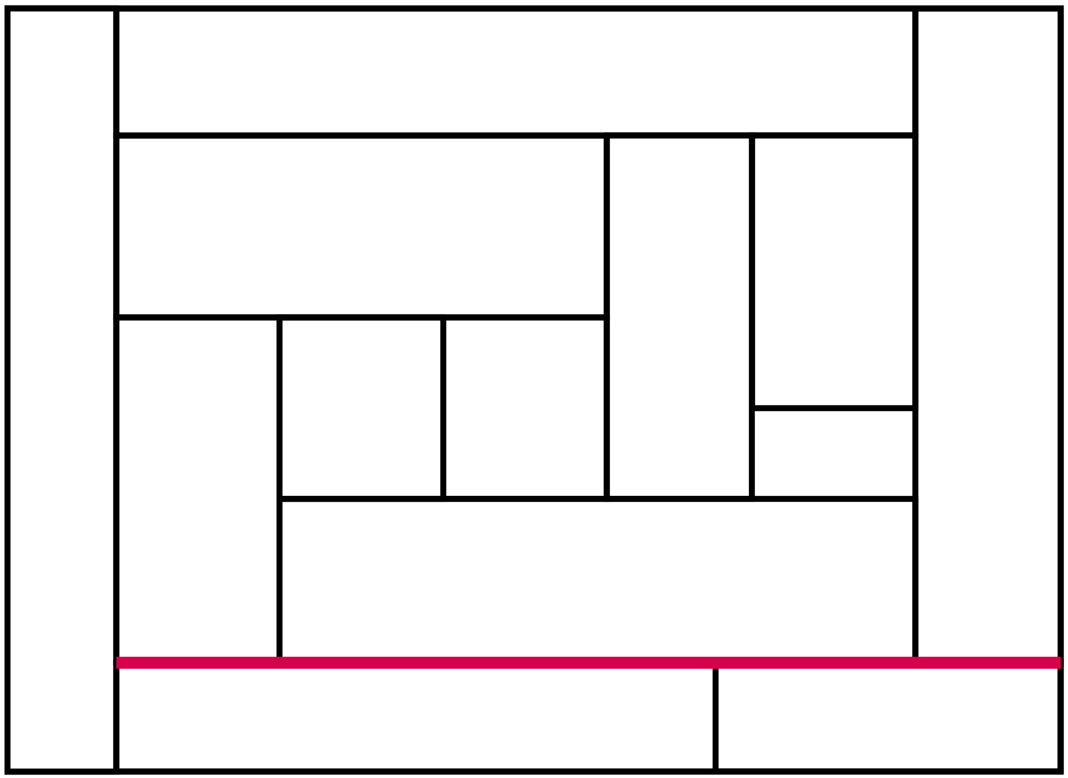


Comparison

Area-Universal

The same layout fits any area assignment

One-sided

Every maximal segment is the side of a rectangle

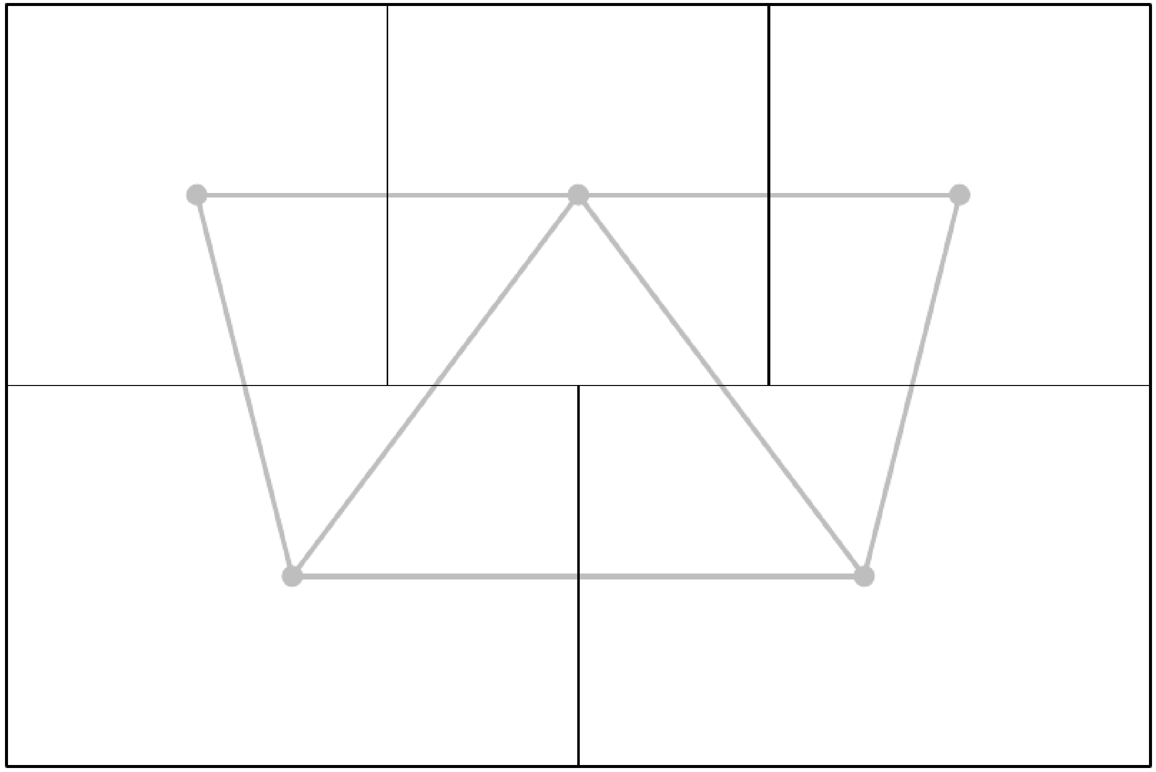
These are equivalent [Eppstein et al. , 2012]

Not all graphs have an area-universal/one-sided dual [Rinsma , 1987]

Rectangular dual

Rectangular dual of a graph G

- Rectangular layout
- Same adjacencies as G

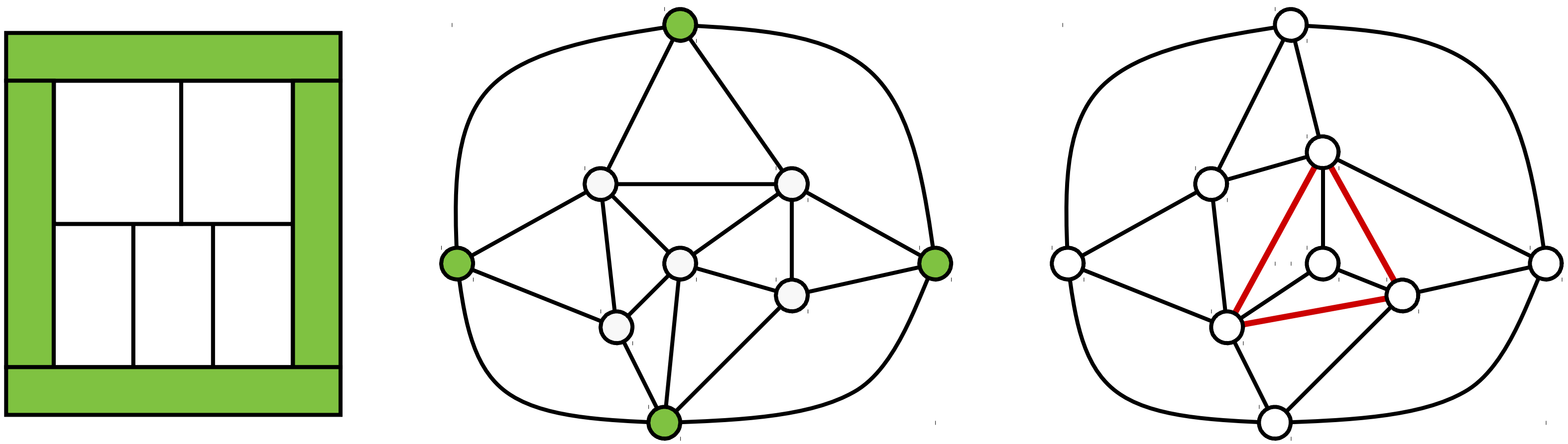


Rectangular dual

[Kozminski & Kinnen ’85]

A planar graph G has a rectangular dual with 4 rectangles on the boundary if and only if

- every interior face is a triangle and the exterior face is a quadrangle
- G has no separating triangles

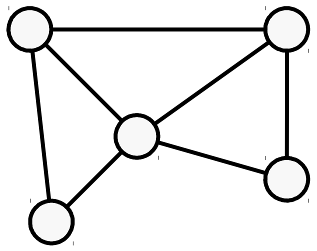


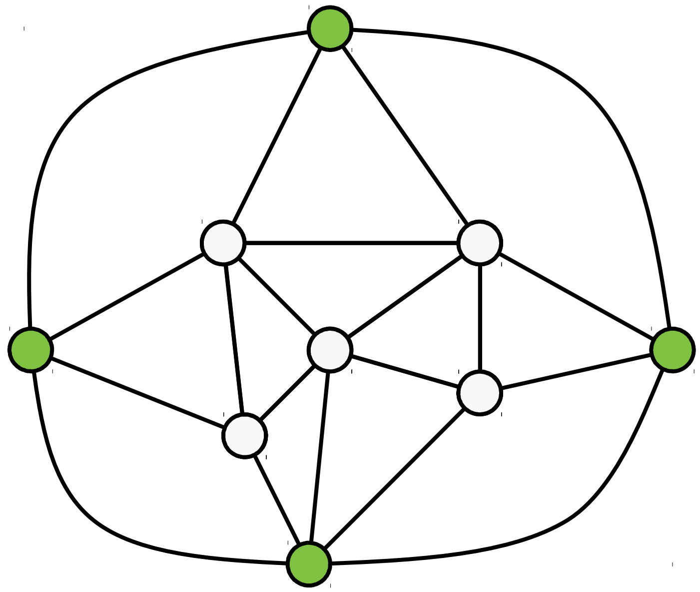
Extended Graph

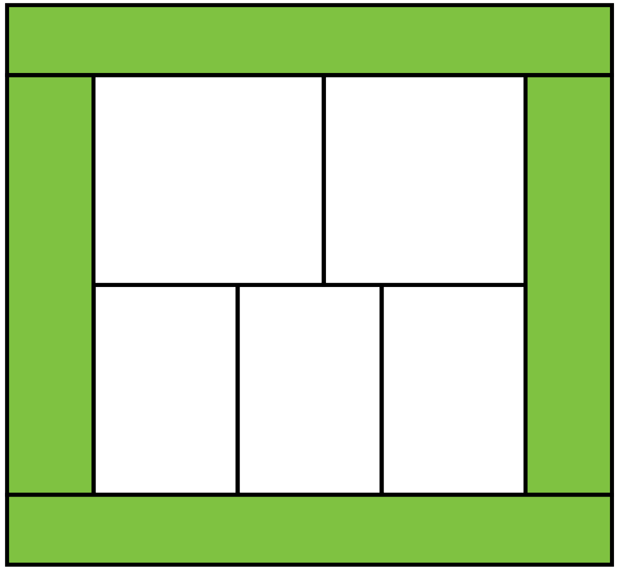
- Bring other graphs in this form
- Do this by adding 4 vertices (*poles*)

  Without creating a separating triangle

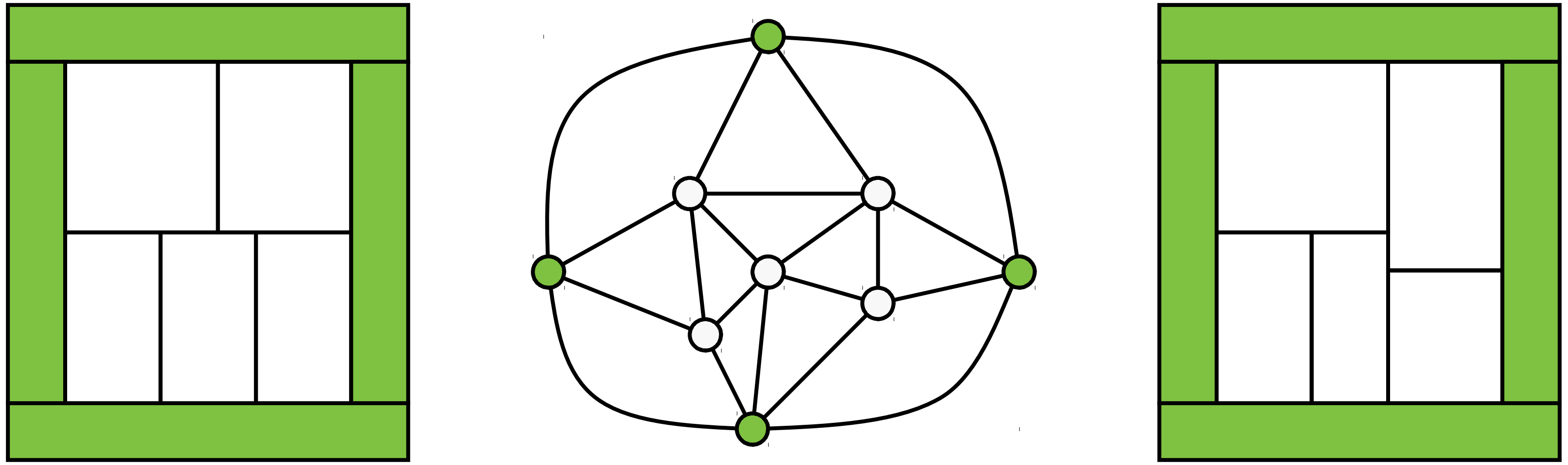
- A extended graph corresponds to a certain corner assignment





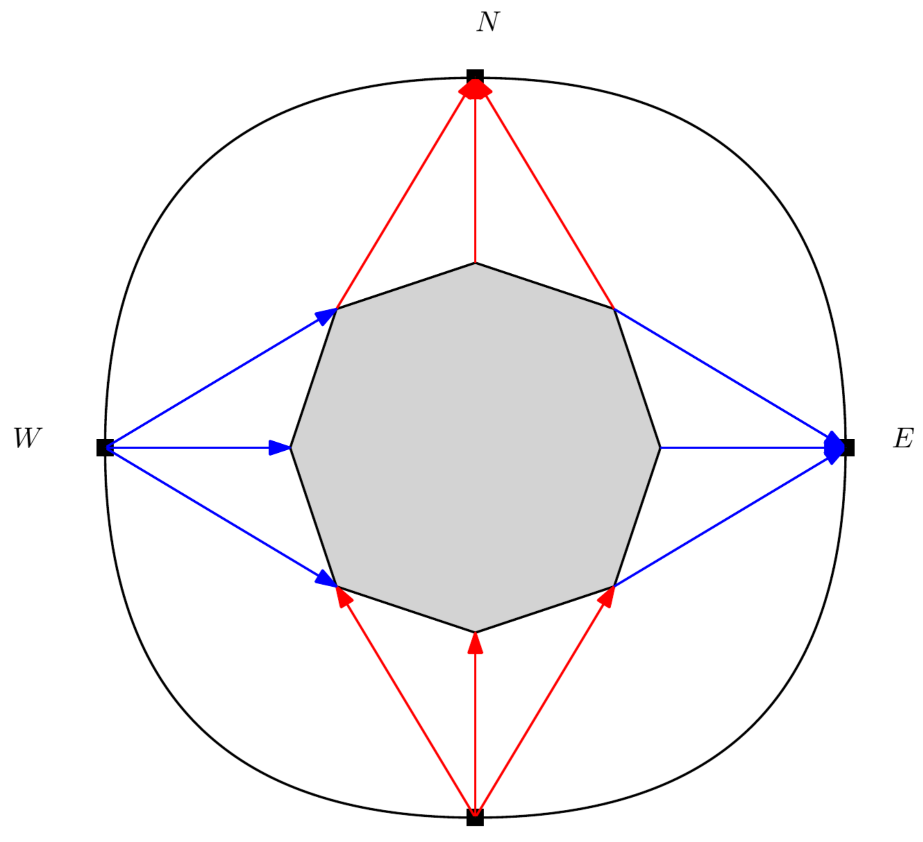


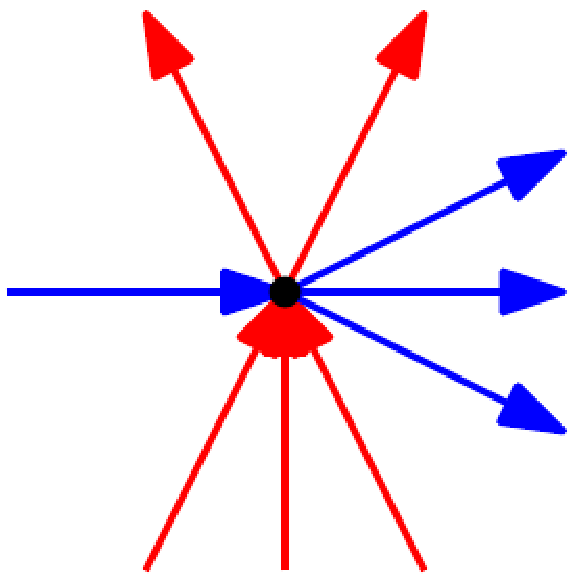
Extended Graphs do not fix layout



Regular edge labeling

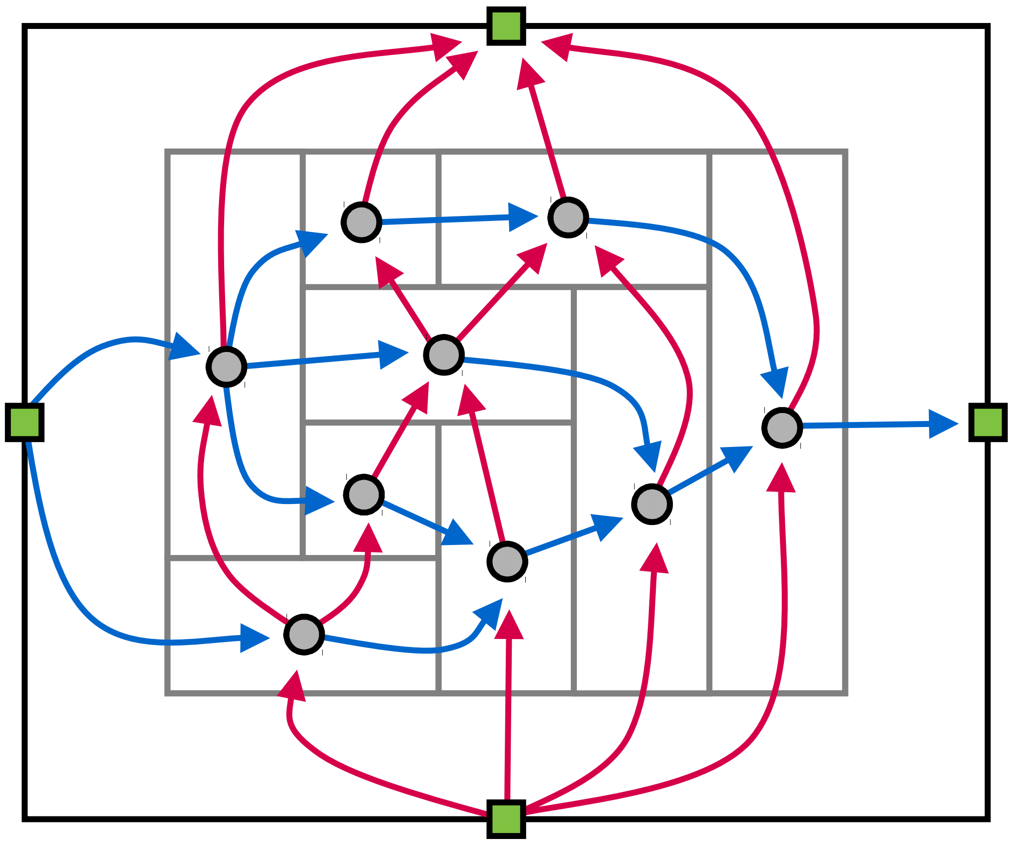
- Oriented coloring of the extended graph
- Exterior vertex condition
- Interior vertex condition





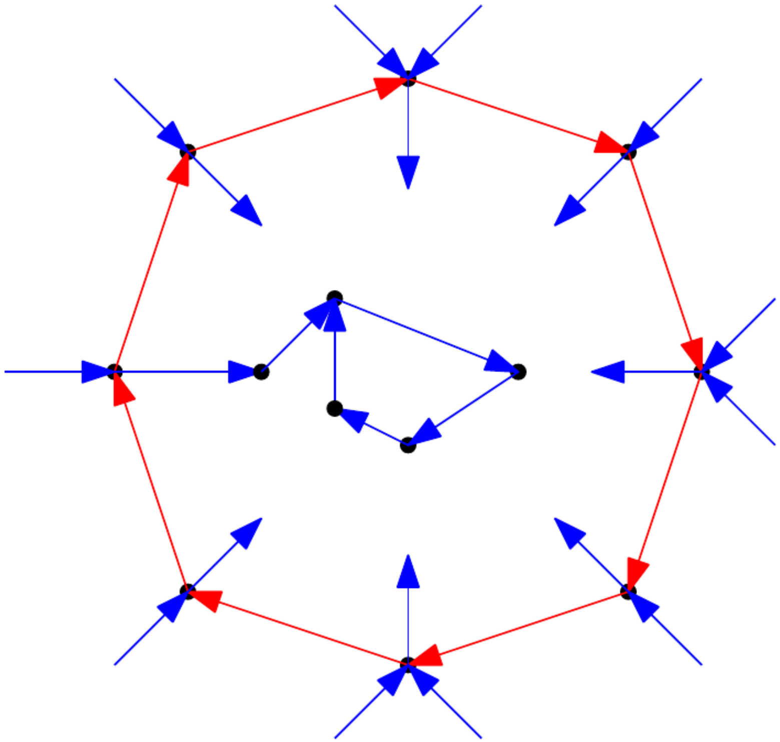
Regular edge labeling

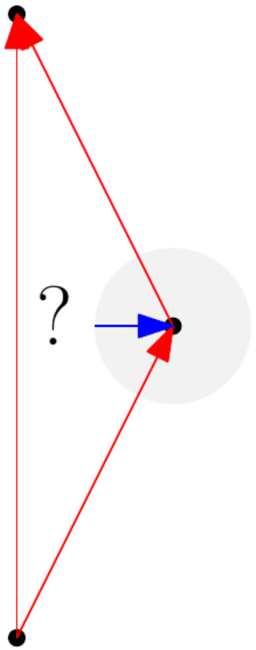
- Corresponds to a equivalence class of layouts
  - Red: Vertical adjacency
  - Blue: Horizontal adjacency



Properties of a REL

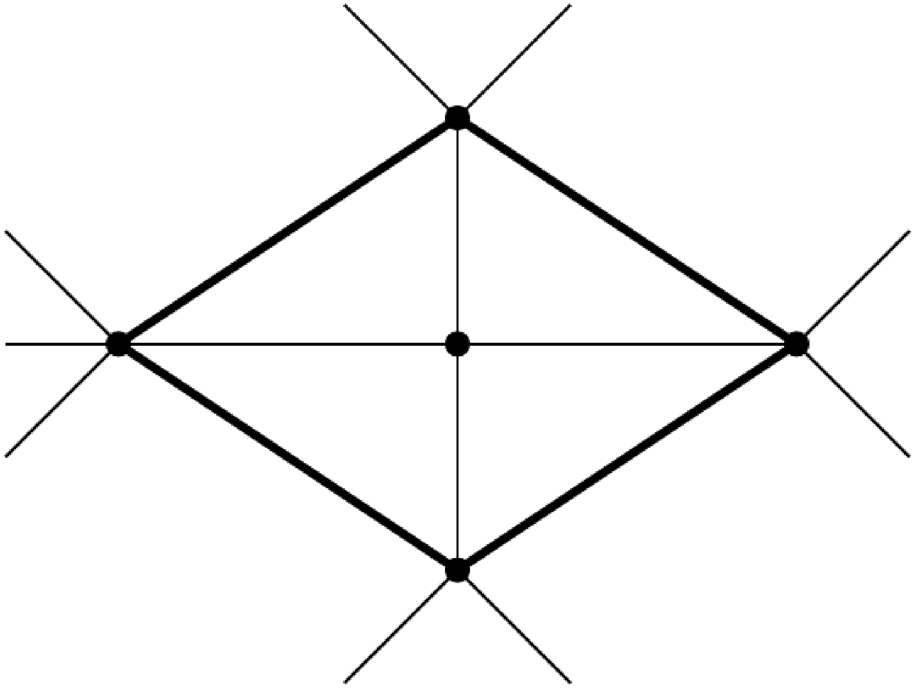
- Two acyclic flows
  - Inside a cycle there is another cycle
- No mono-colored triangles





Separating k-cycle

- A separating k-cycle is a cycle whose removal disconnects the graph

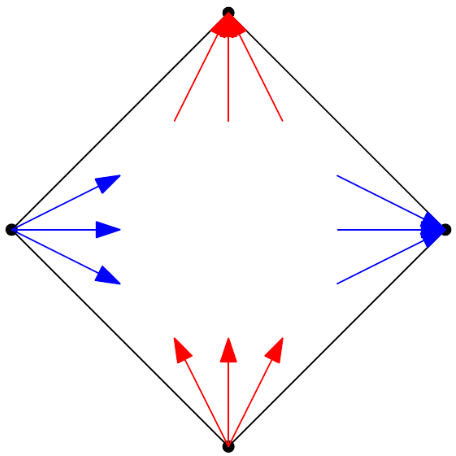


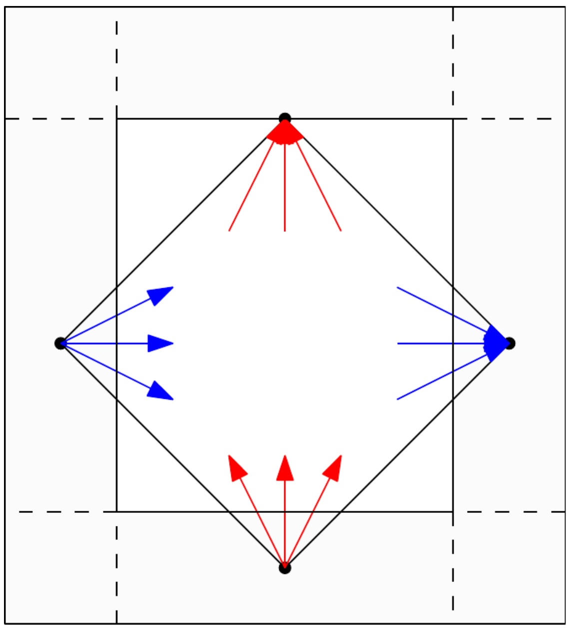
A separating 4-cycle

Properties of a REL

- Interior of a separating 4-cycle

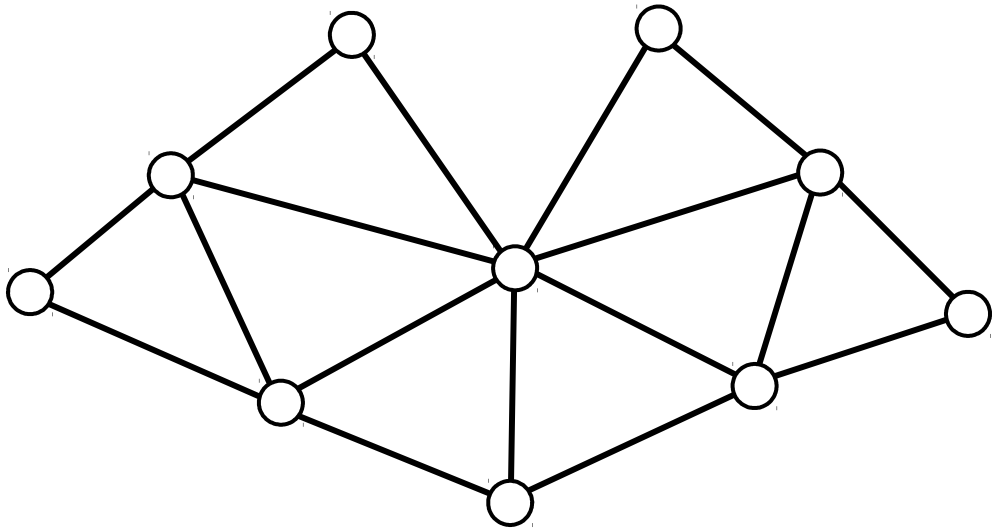
  The color and orientation of a single interior edge adjacent to a cycle vertex determines the color and orientation of all edges adjacent to a cycle vertex





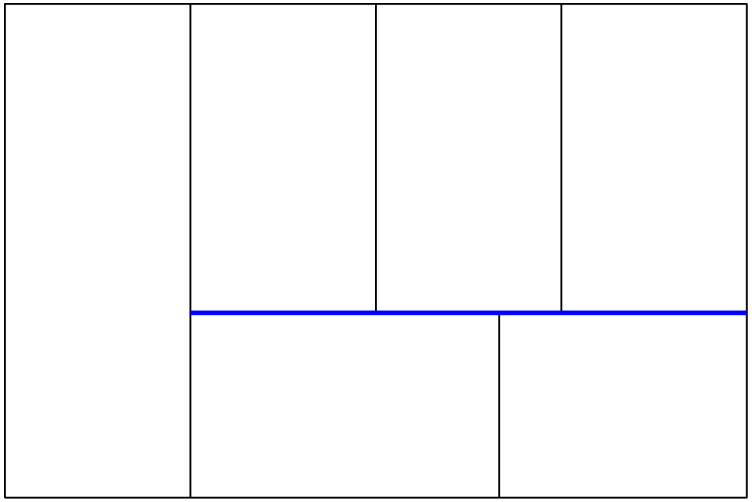
Not all graphs are one-sided

- We can show this by enumerating all possible REL's
- In the results section we will show for some graphs that they are not one-sided



So what can we do?

- We will call our dual **k-sided** if every maximal segment is the boundary of at most k adjacent rectangles all on the same side of the line



2-sided

- Do graphs without an one-sided dual admit a k-sided dual for some k? (hopefully small)

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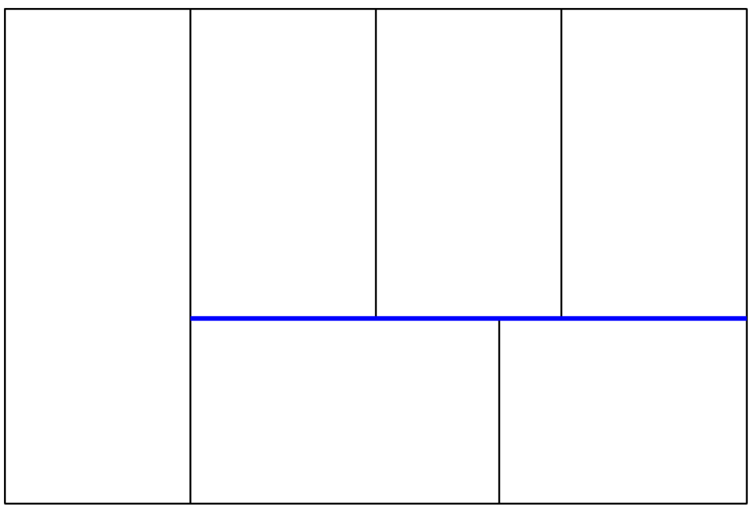

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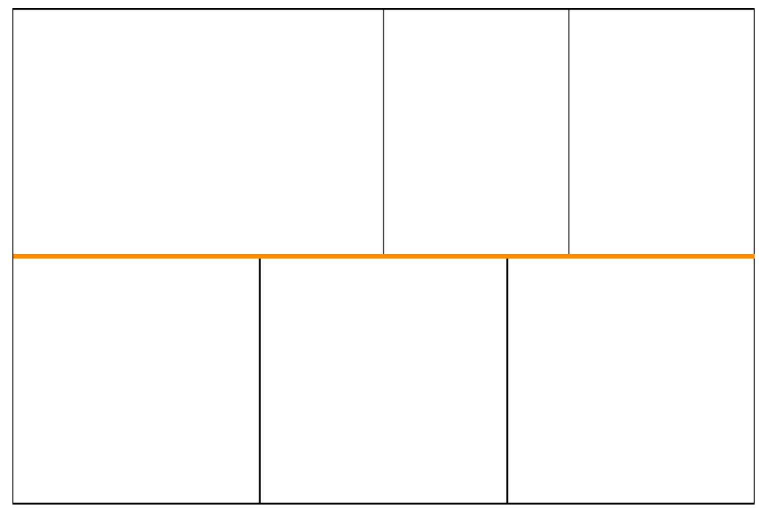
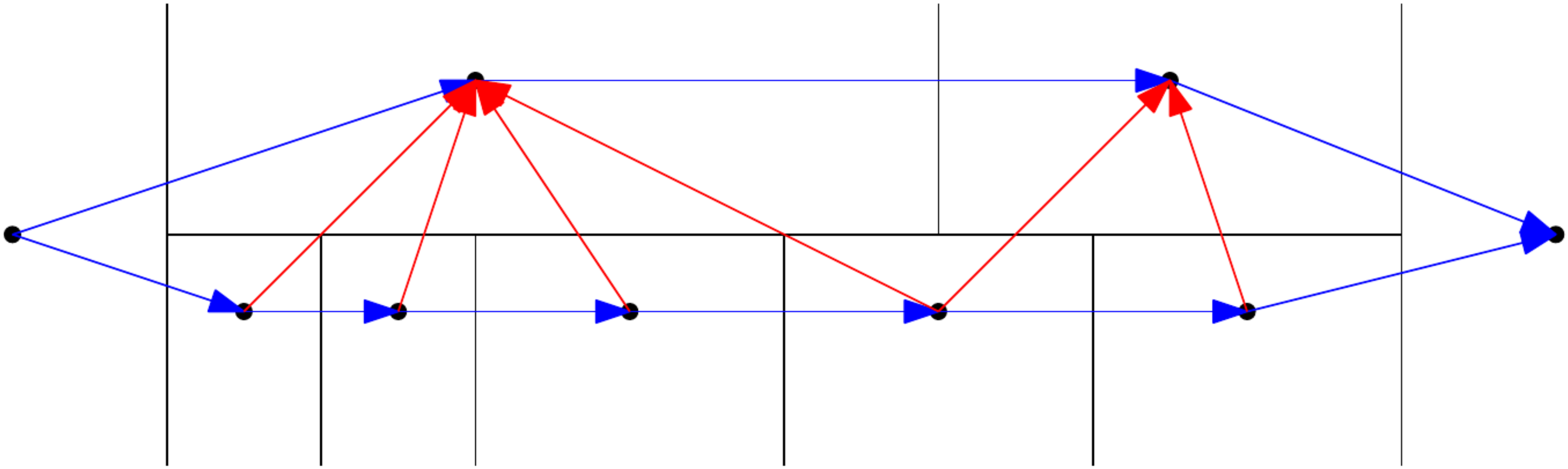


3-sided

- Do graphs without an one-sided dual admit a k-sided dual for some k? (hopefully small)

What does k-sided look like in the REL?

- Red and blue faces with one of the two paths having at most k+1 vertices



Structure

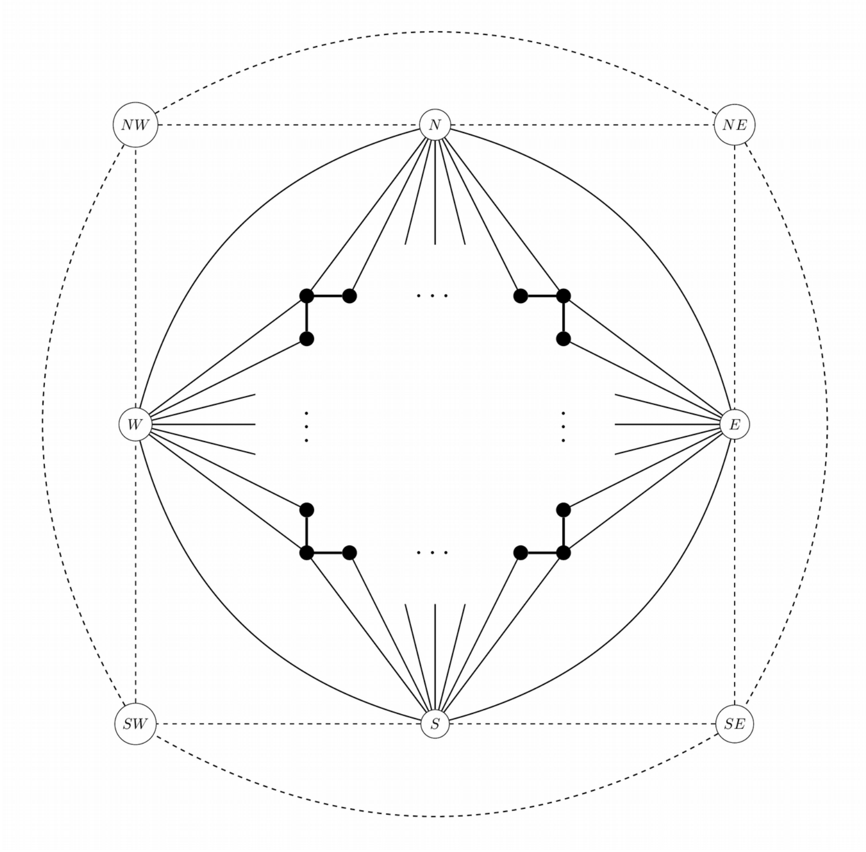
Problem

Results

Conjecture

Fixing a extended graph

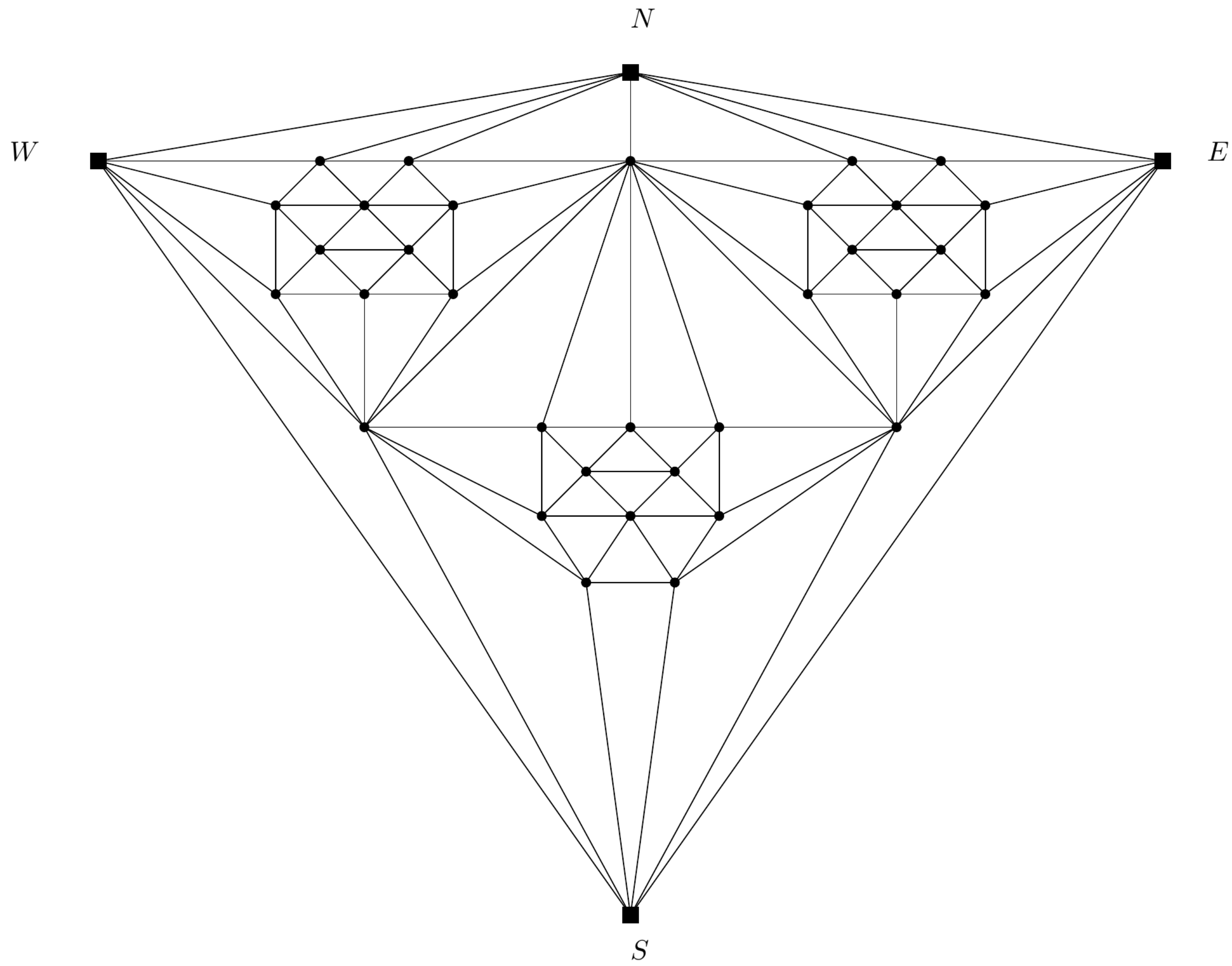
- We can consider a single extended graph of **G**, **E(G) = G'** ...
  by considering rectangular duals of **G'**
- Because then there is only one choice for **E(G')**



Note that this uses a separating 4-cycle

∞-sided graphs with 4-cycles

Any extended graph with separating 4 cycles can be ∞-sided, even if all these cycles go trough a pole.



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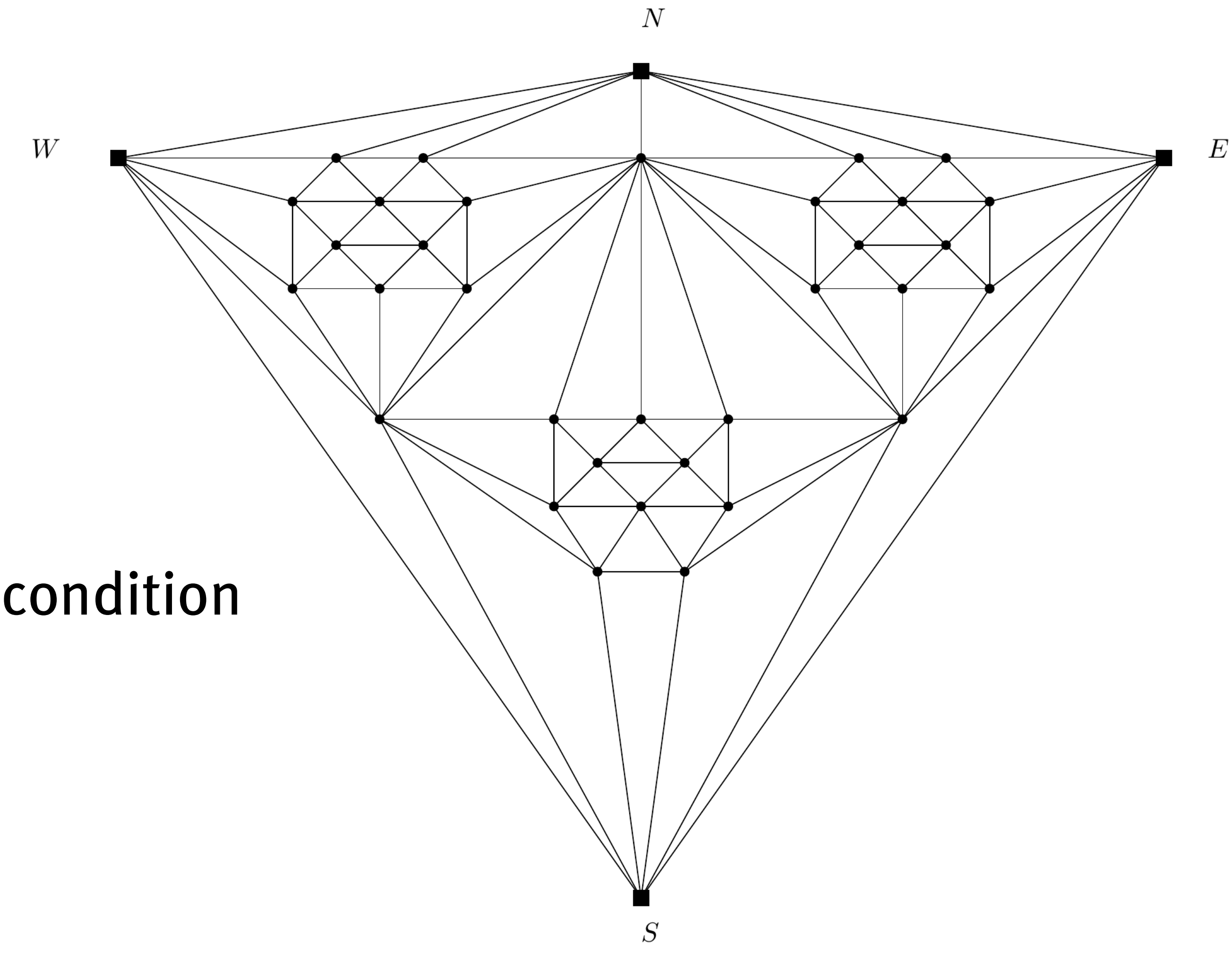

- Exterior vertex condition

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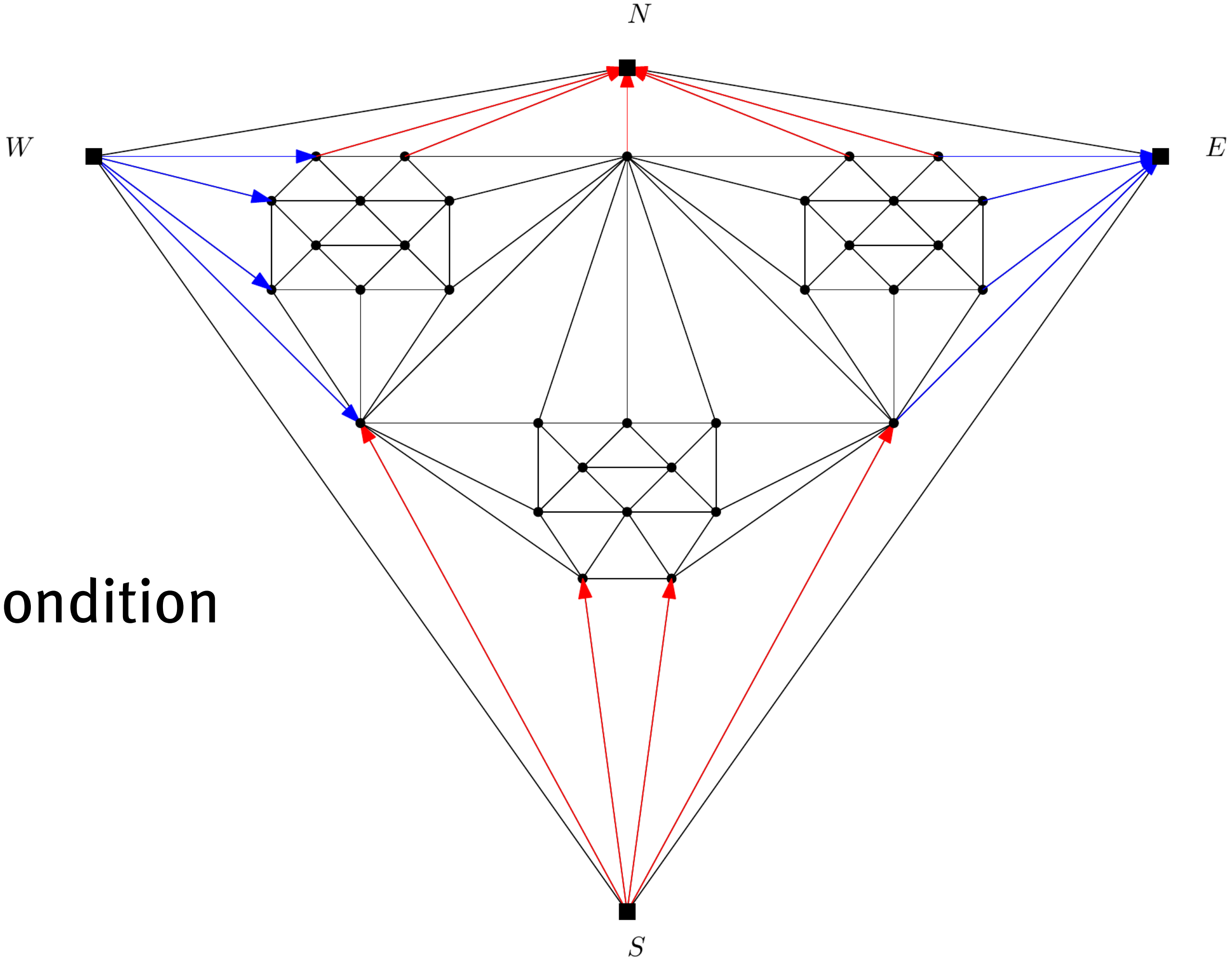

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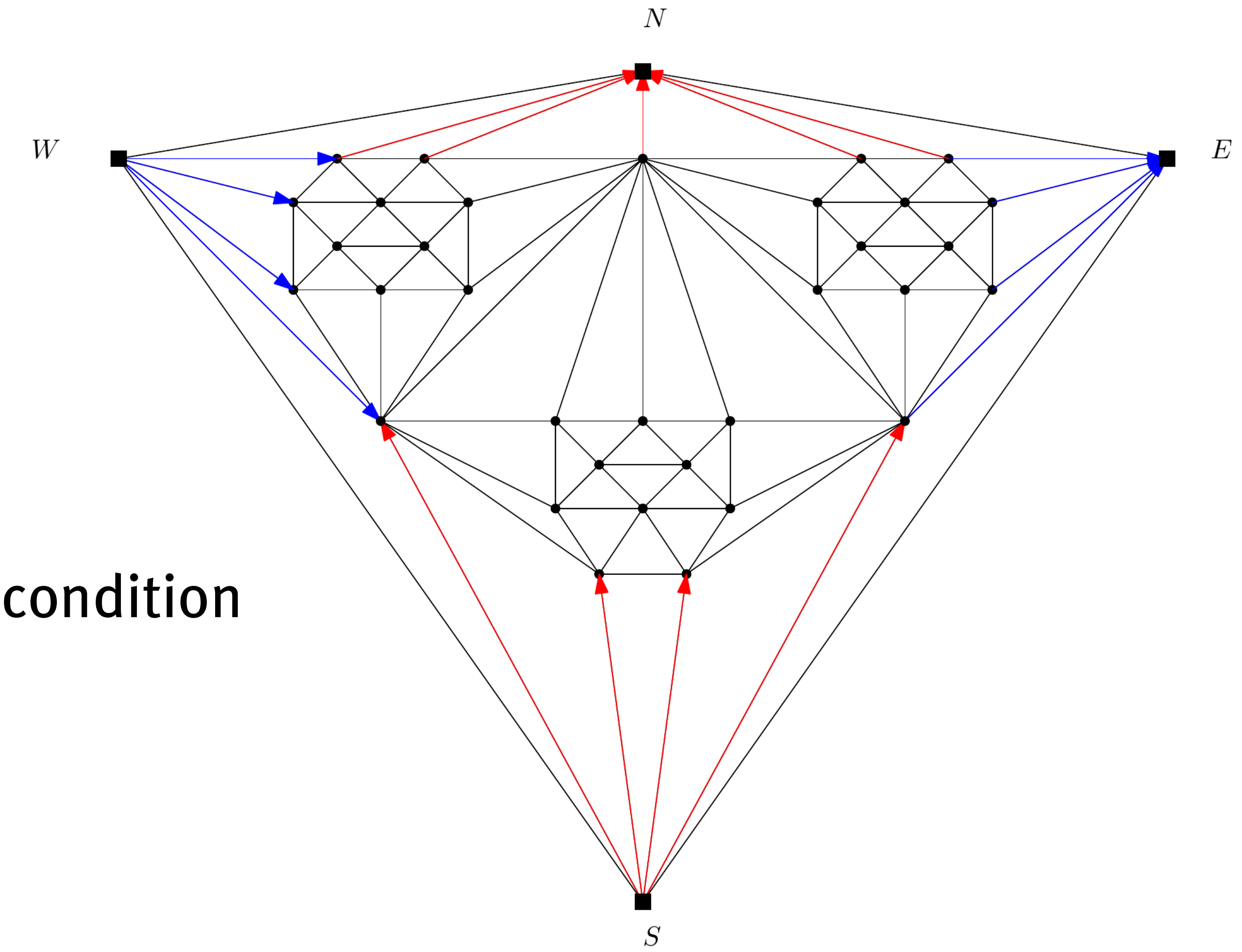

- Exterior vertex condition
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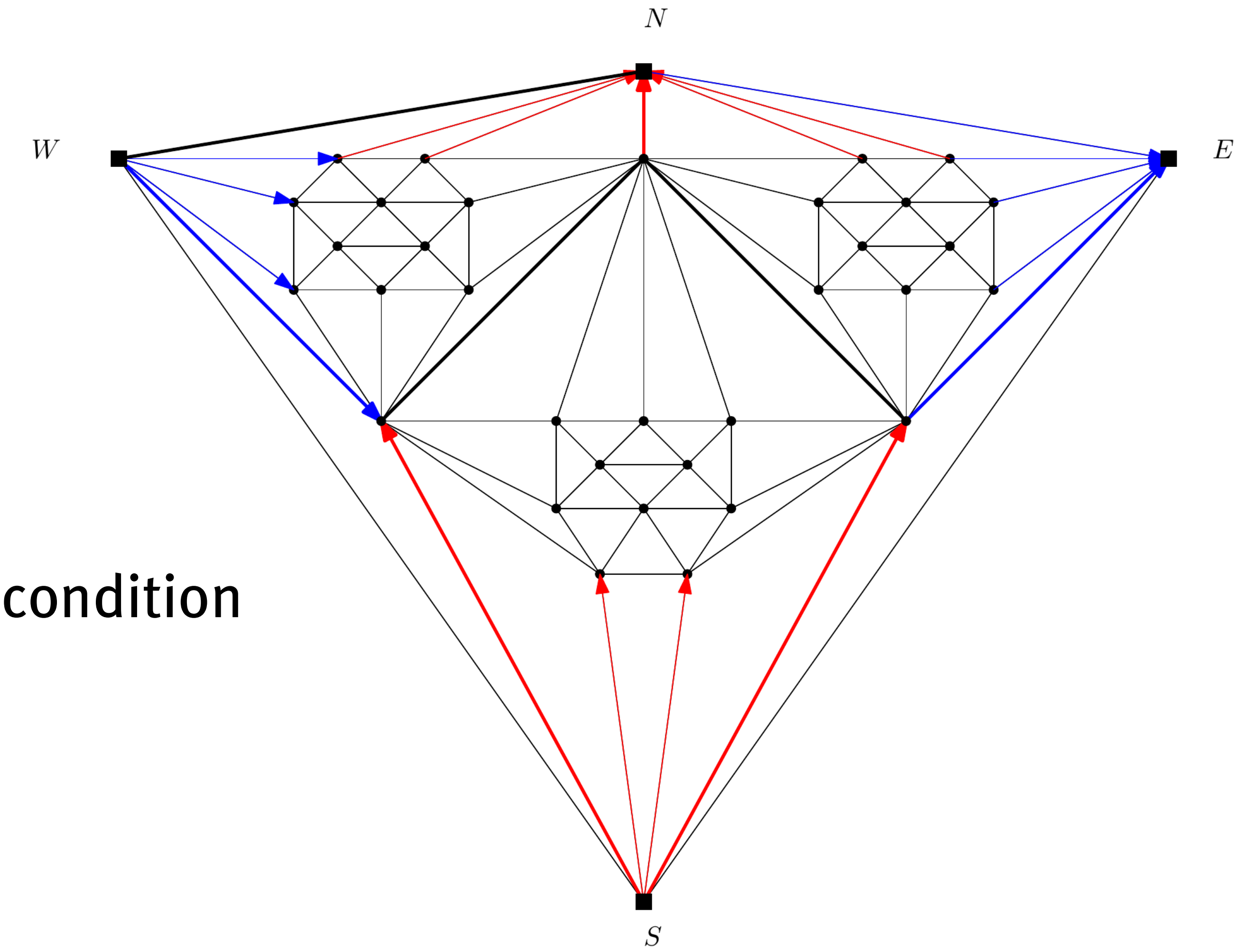

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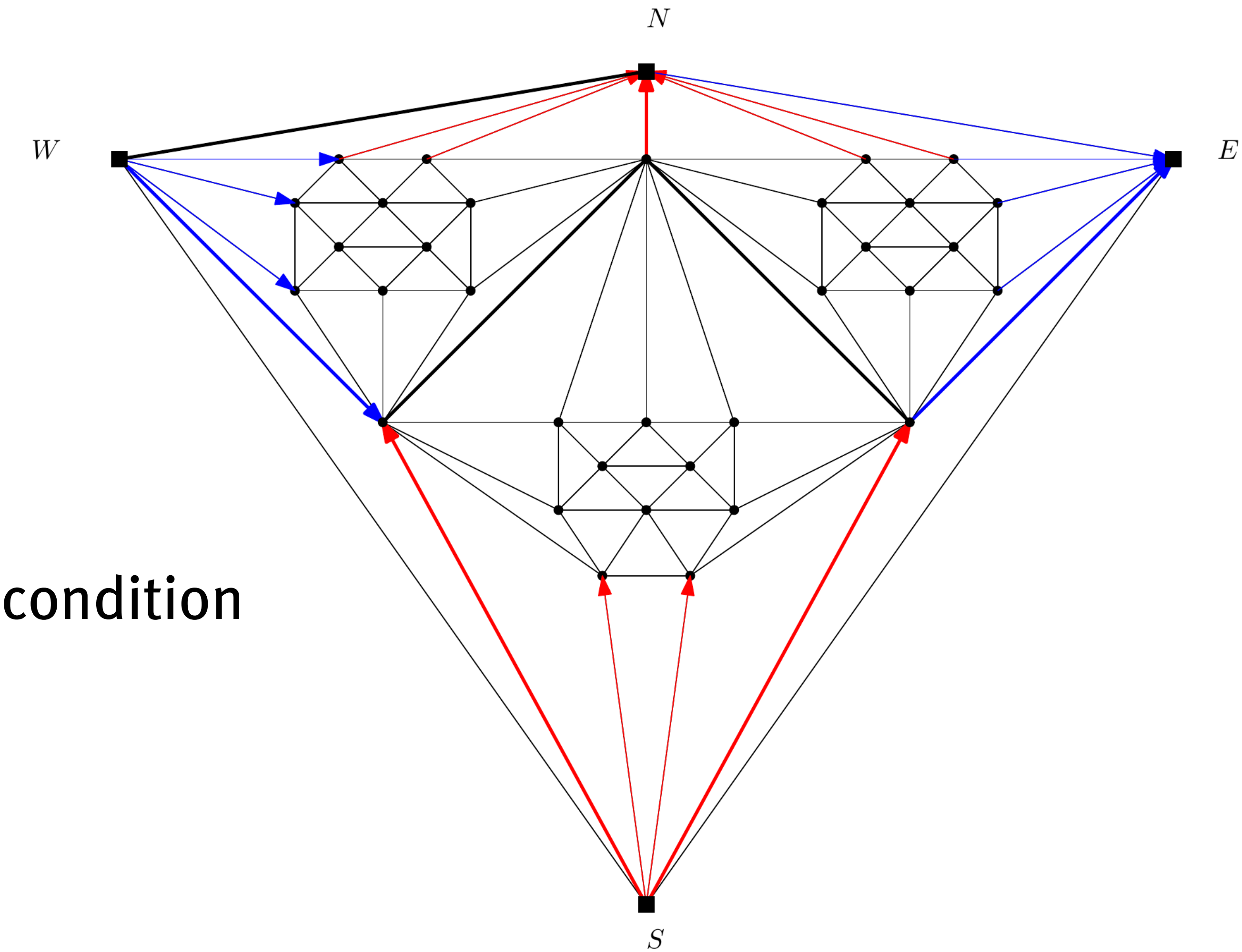

- Exterior vertex condition
- 4-cycles ...
  color inside

∞-sided graphs with 4-cycles

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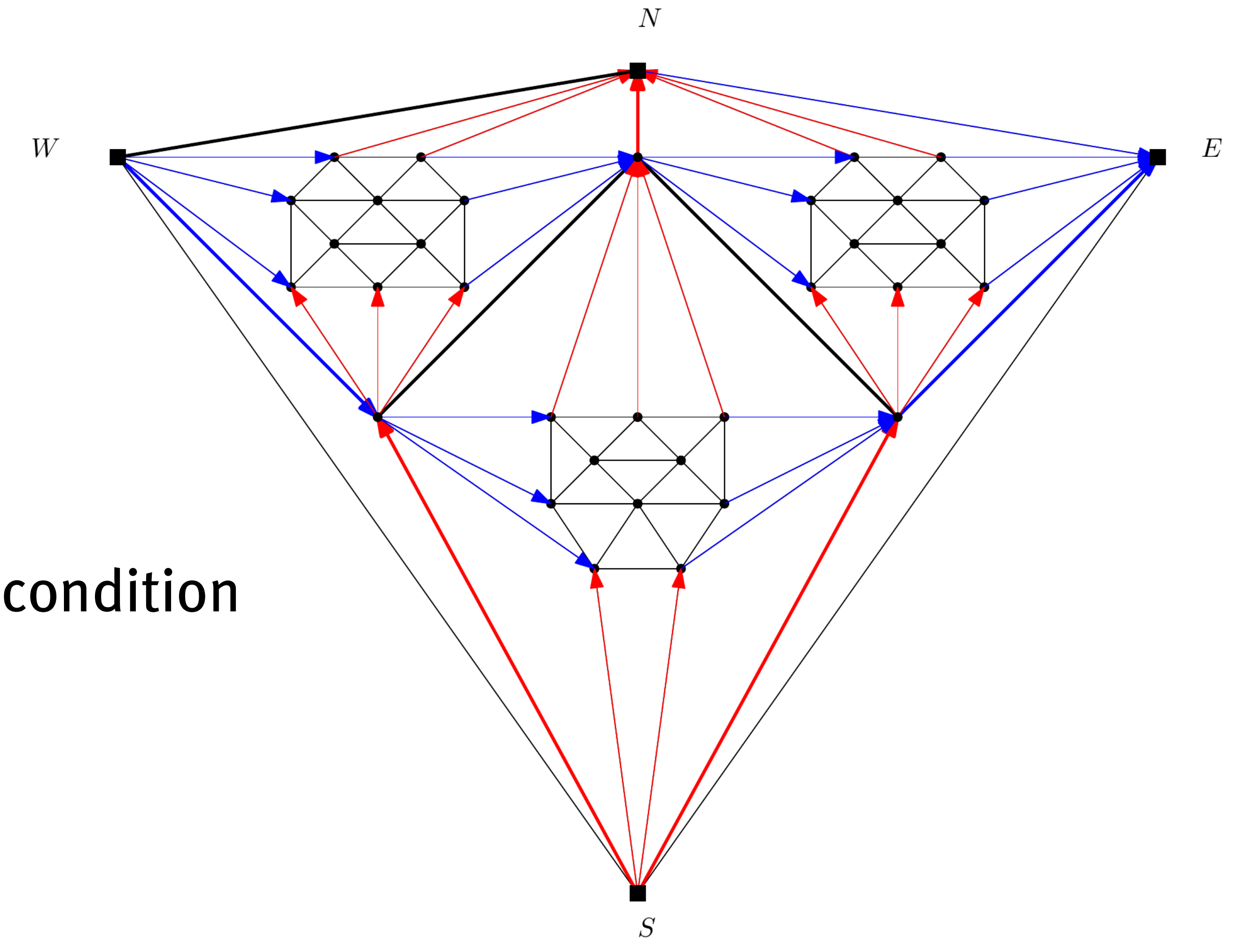

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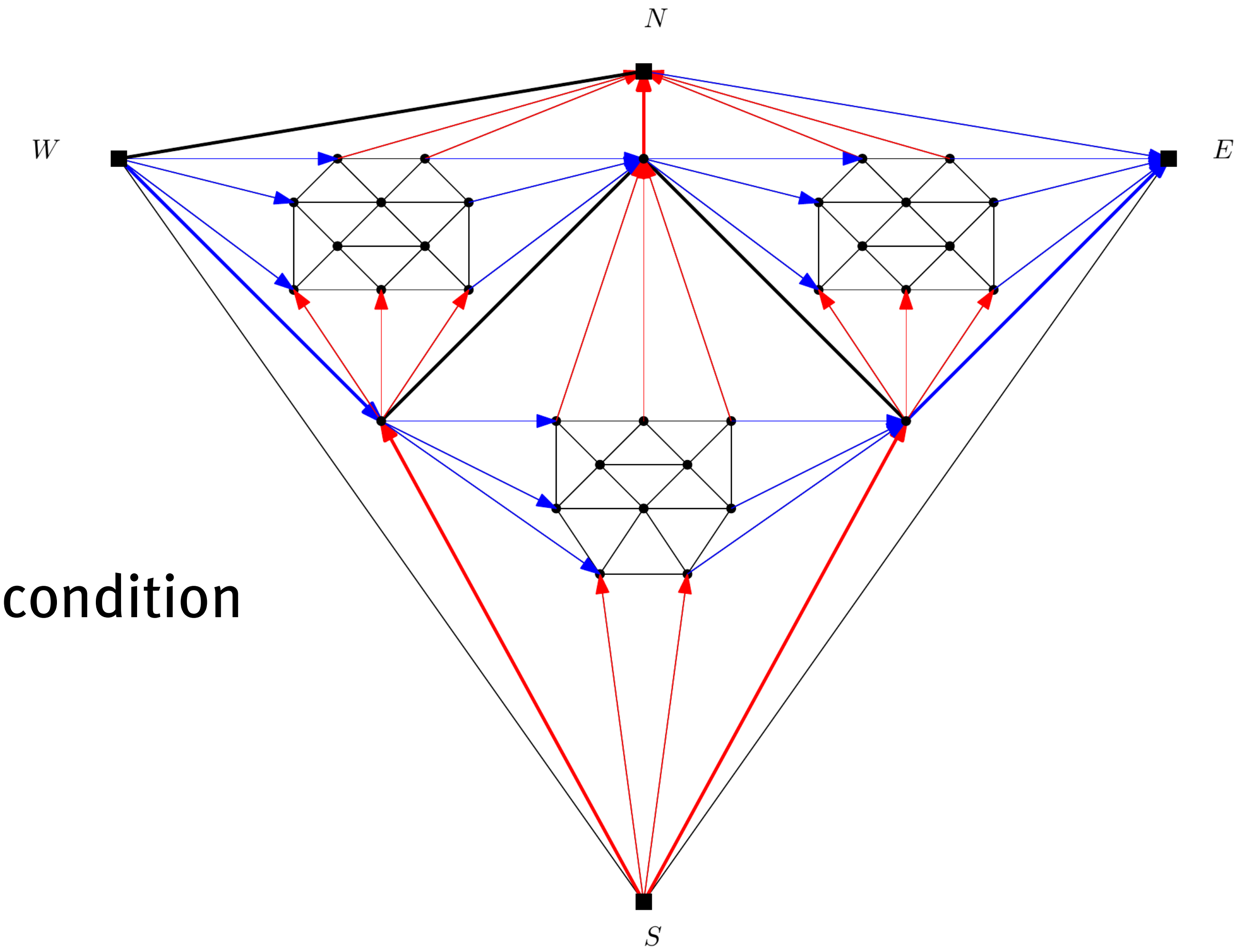

- Exterior vertex condition
- 4-cycles ...
  color inside
- Problem!

∞-sided graphs with 4-cycles

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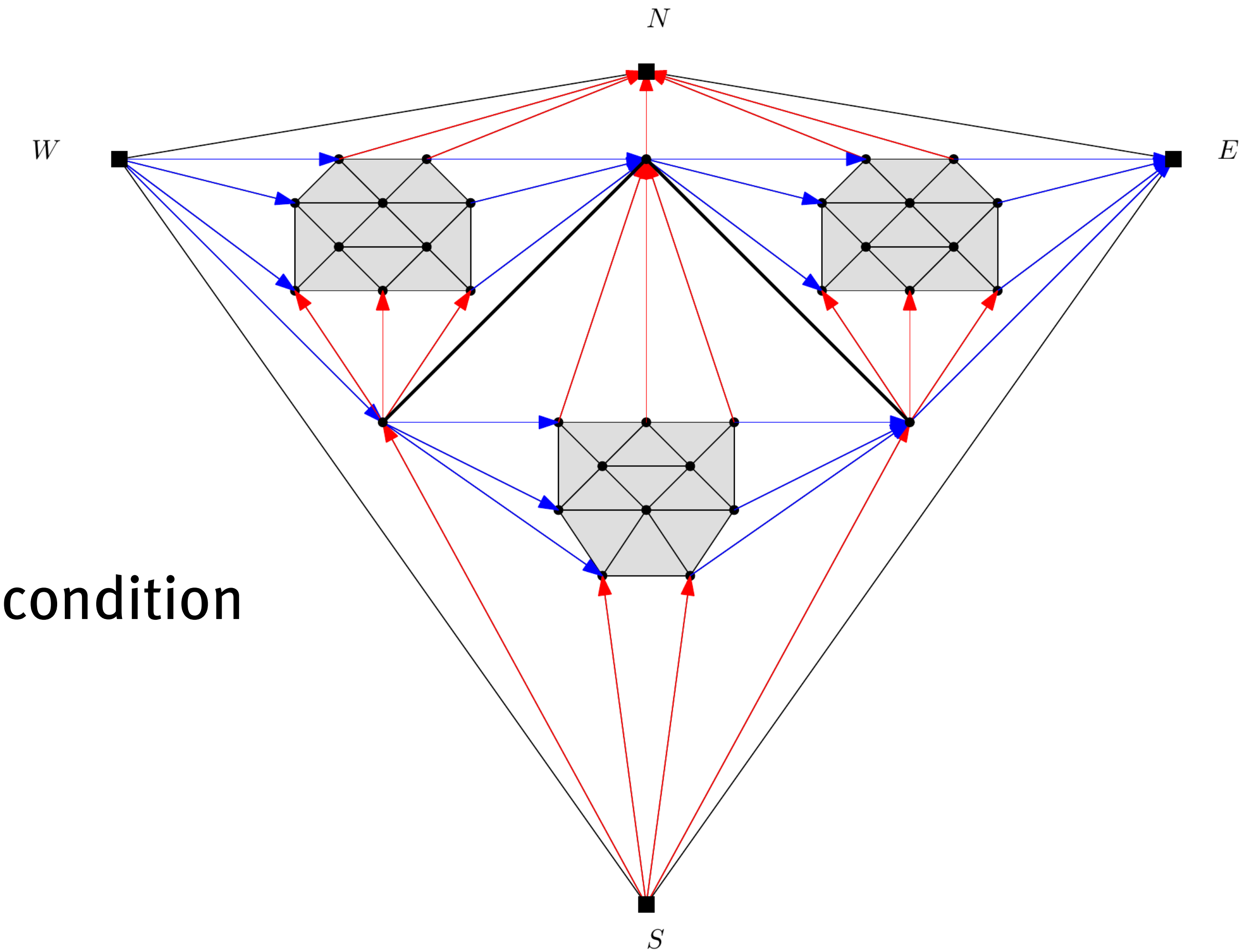


- Exterior vertex condition
- 4-cycles ...
  color inside
- Problem!

Structure

Problem

Results

Conjecture

Conjecture

- What about extended graphs without any separating 4-cycle?

  Maybe they are all 2-sided!

- The lack of 4-cycles gives a lot of freedom in most cases

  But sometimes we can restrict this quite a lot

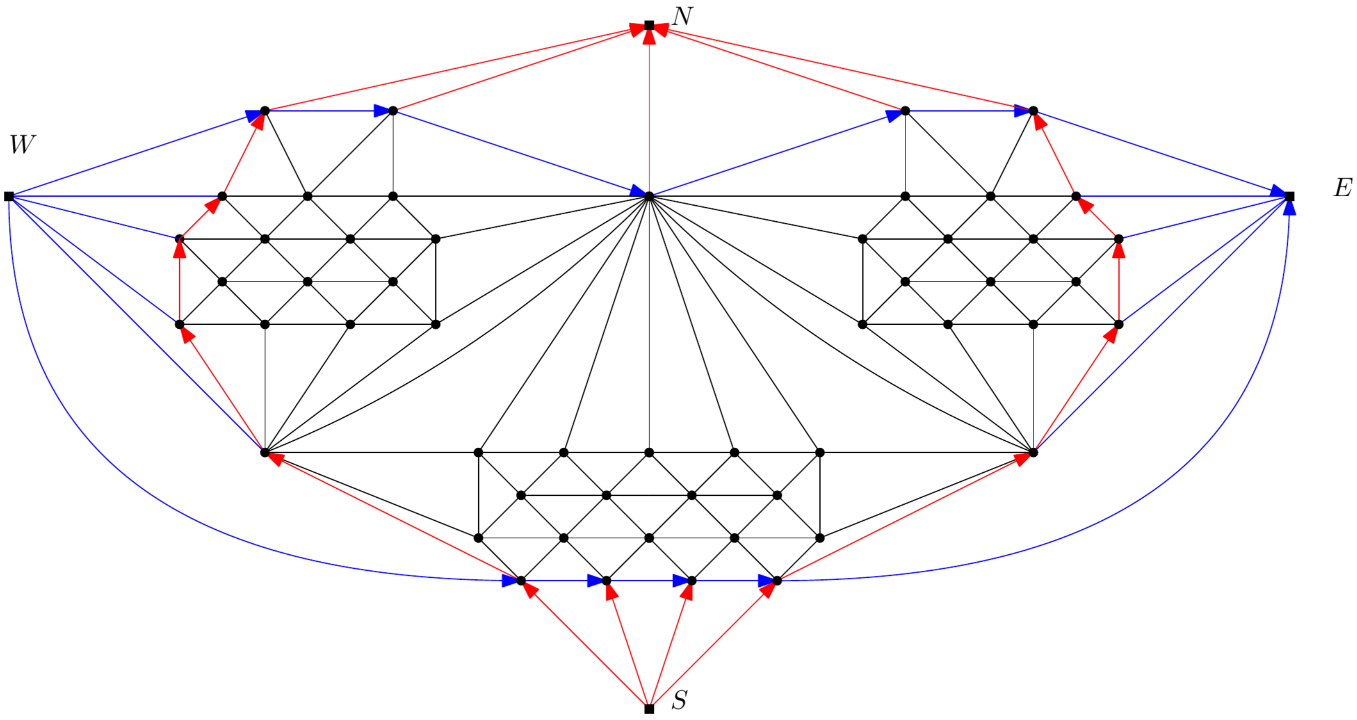
Tricky Example

Tricky Example

- Graph

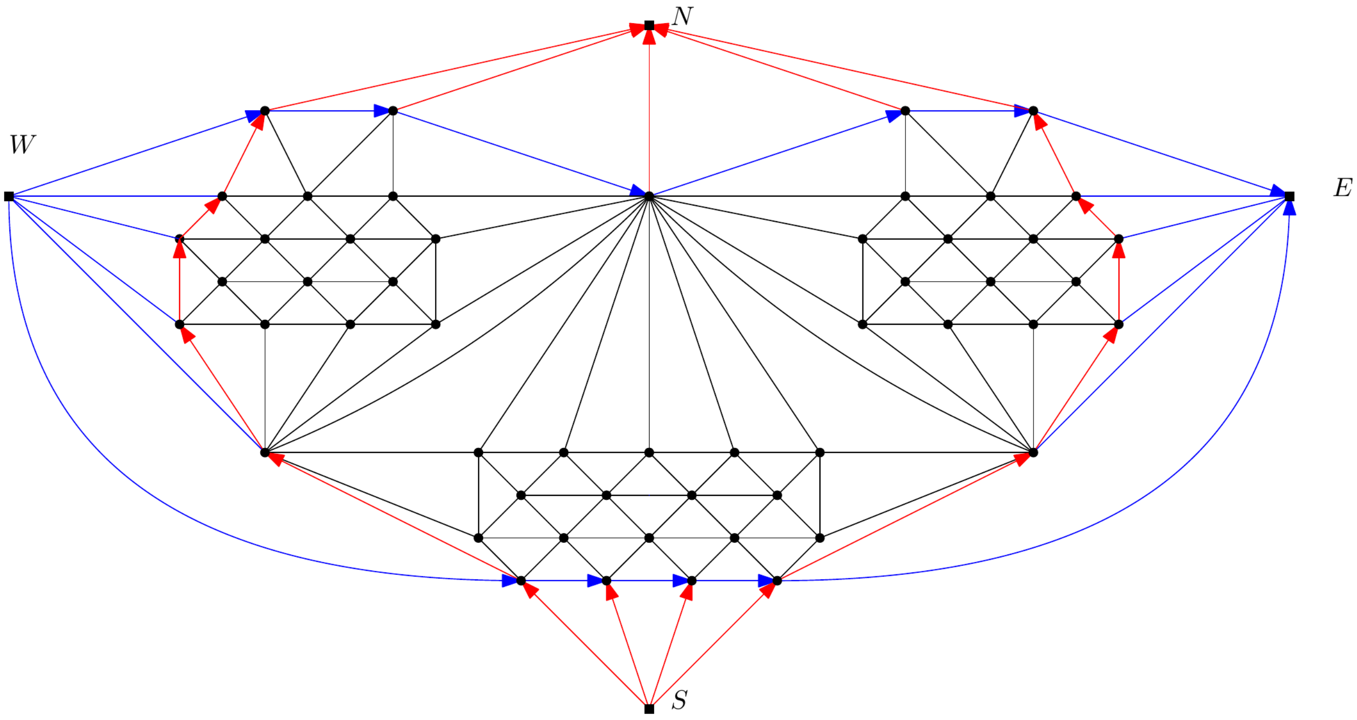
Tricky Example

- Graph



Tricky Example

- Graph
- Suppose blue …



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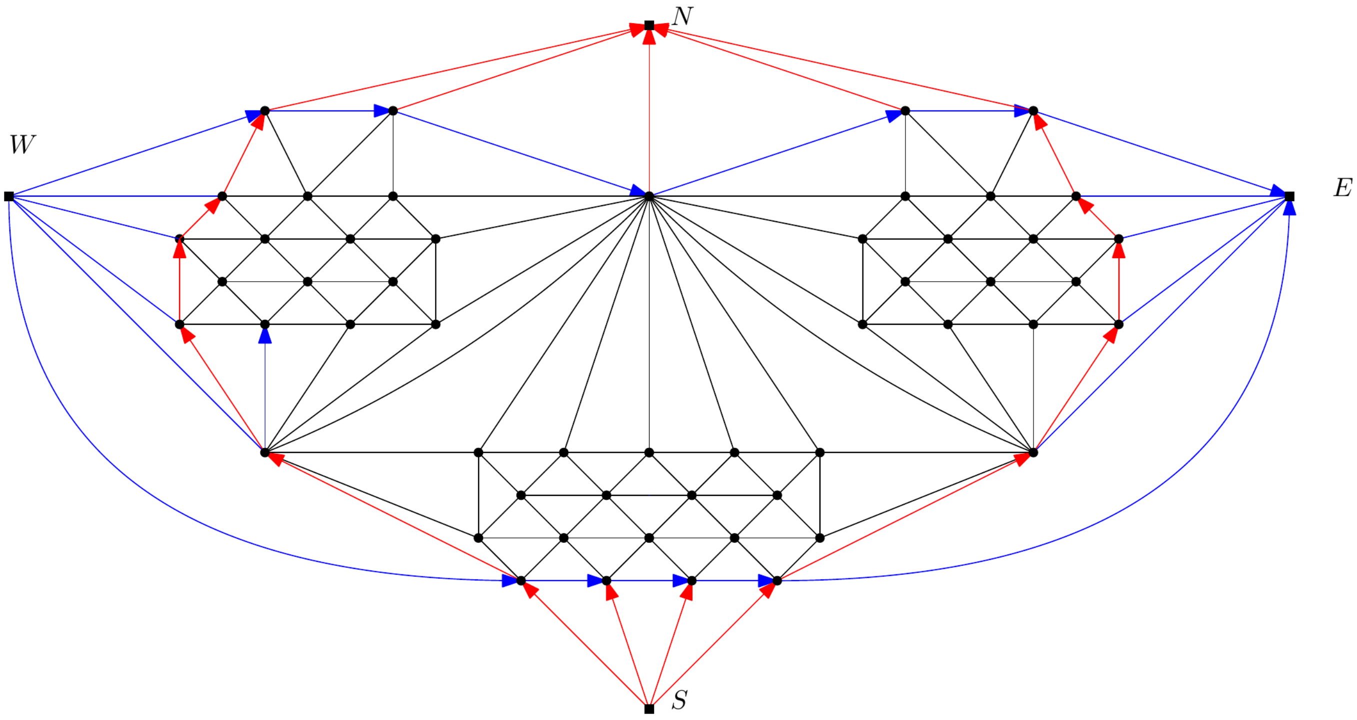

Tricky Example

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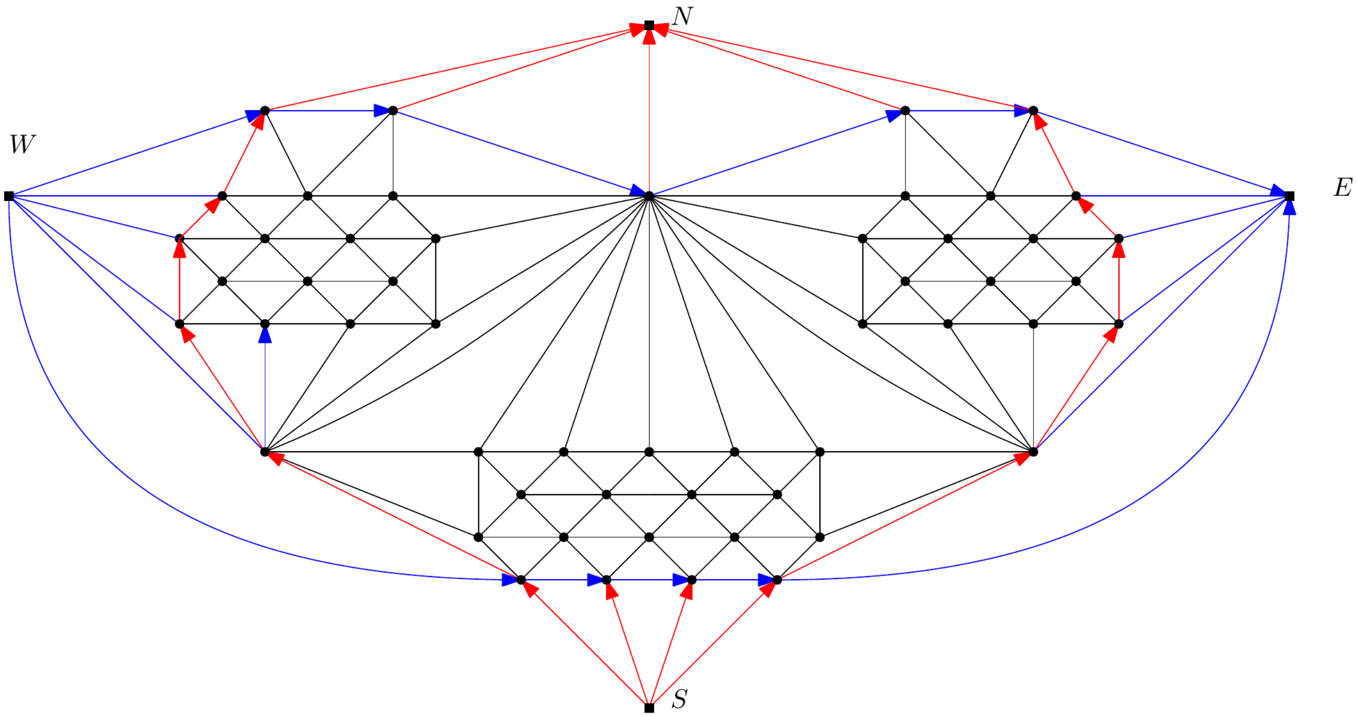
  ...

  Incoming red

  ...



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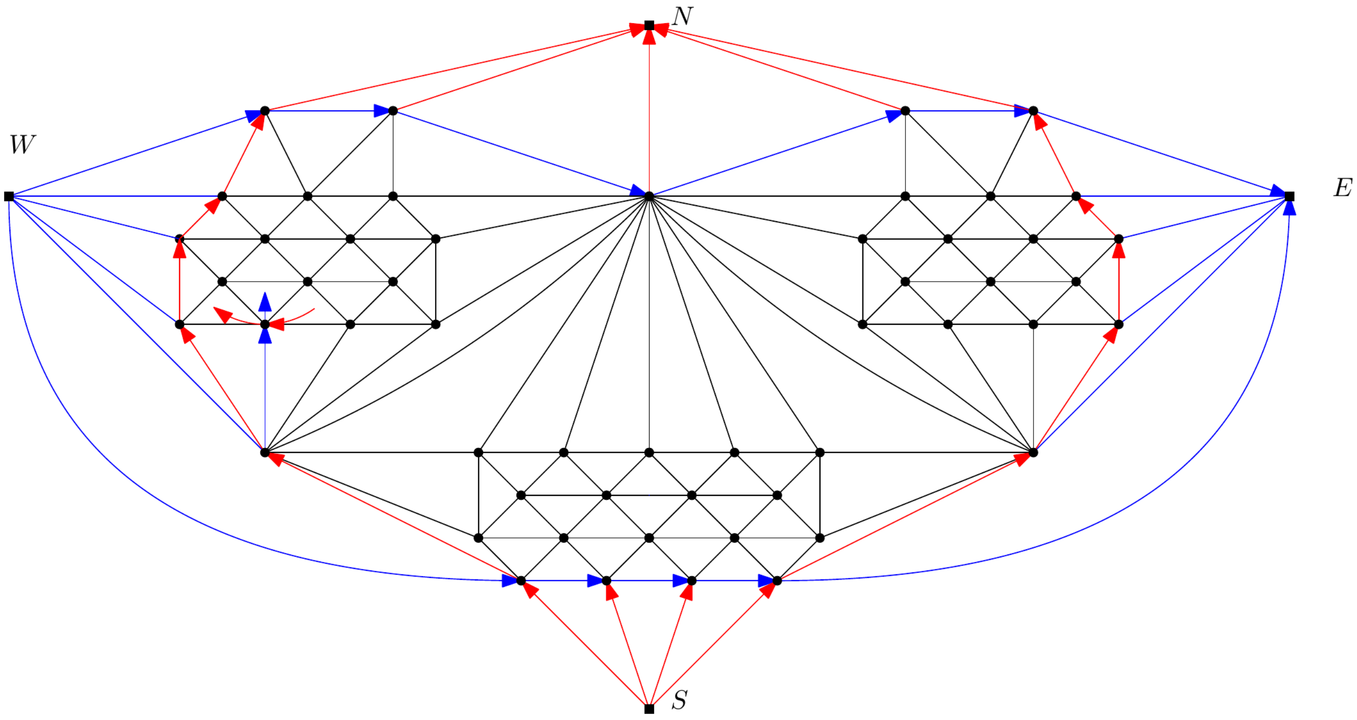

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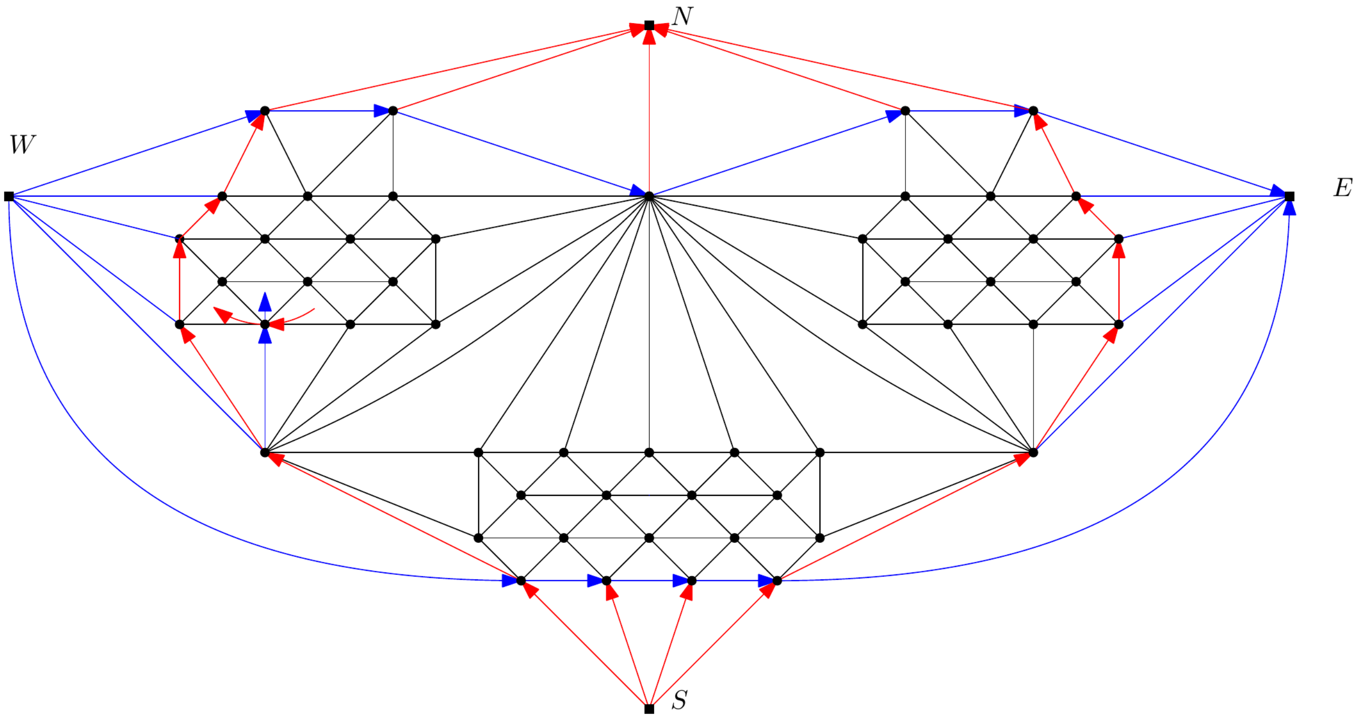

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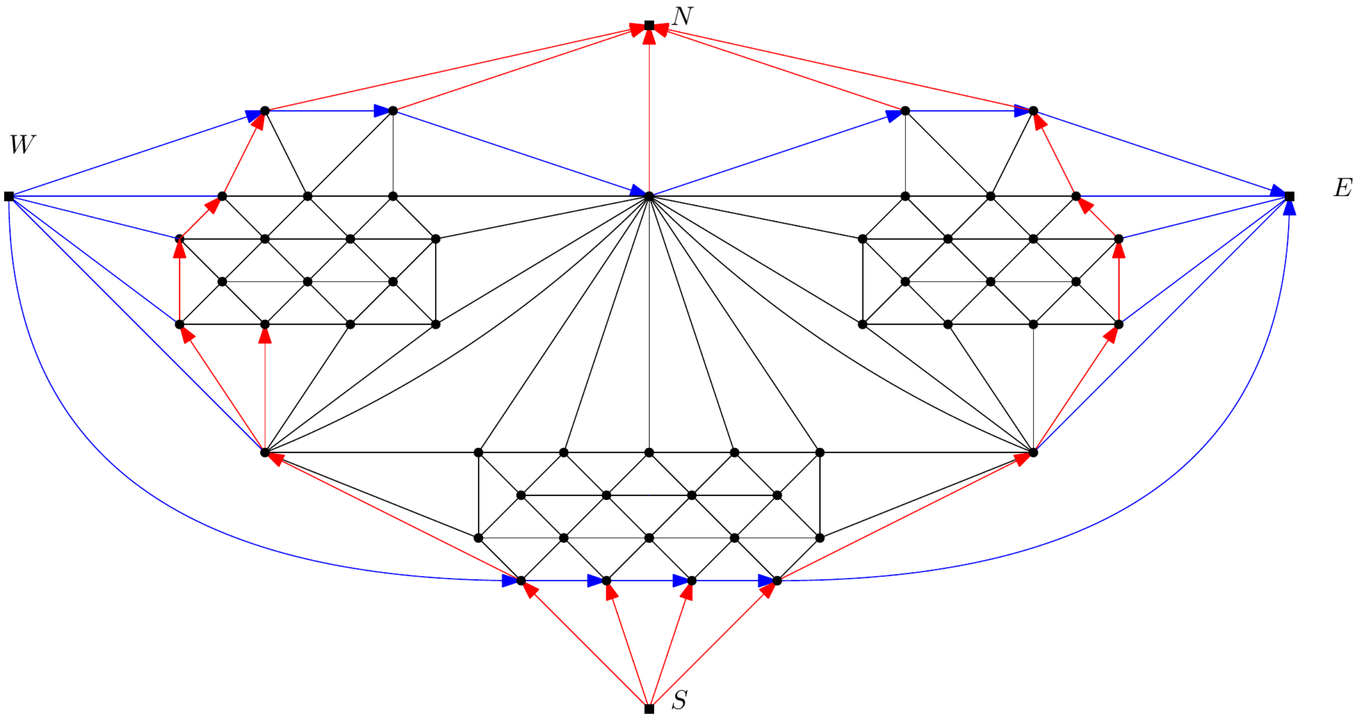

Tricky Example

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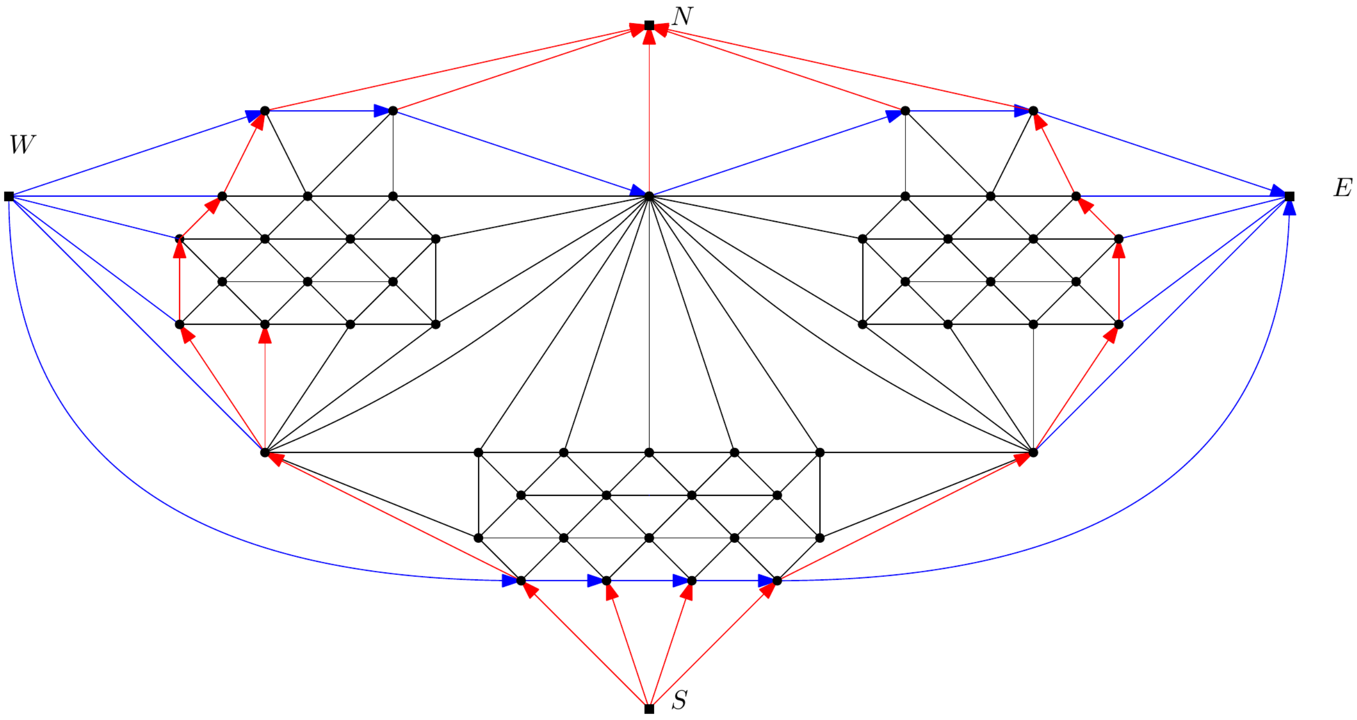
  Contradiction
- Repeat this argument



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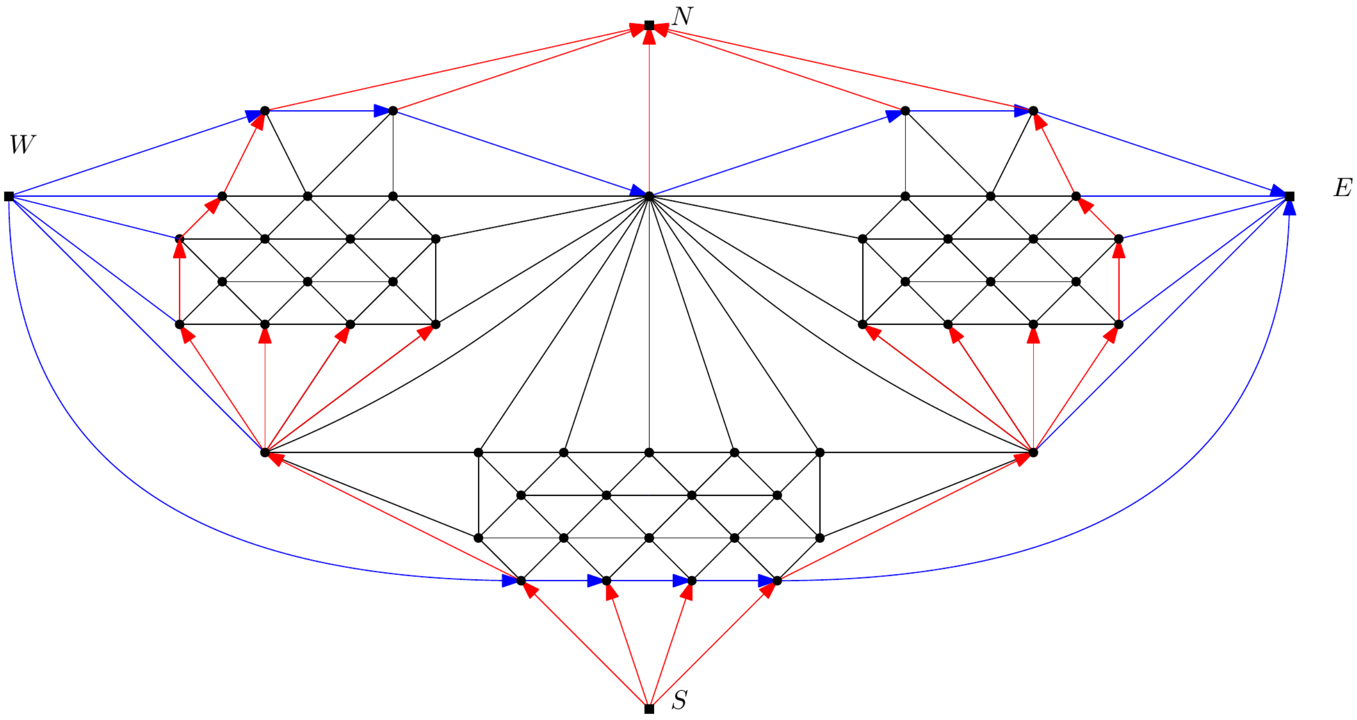

Tricky Example

- Graph
- Suppose blue

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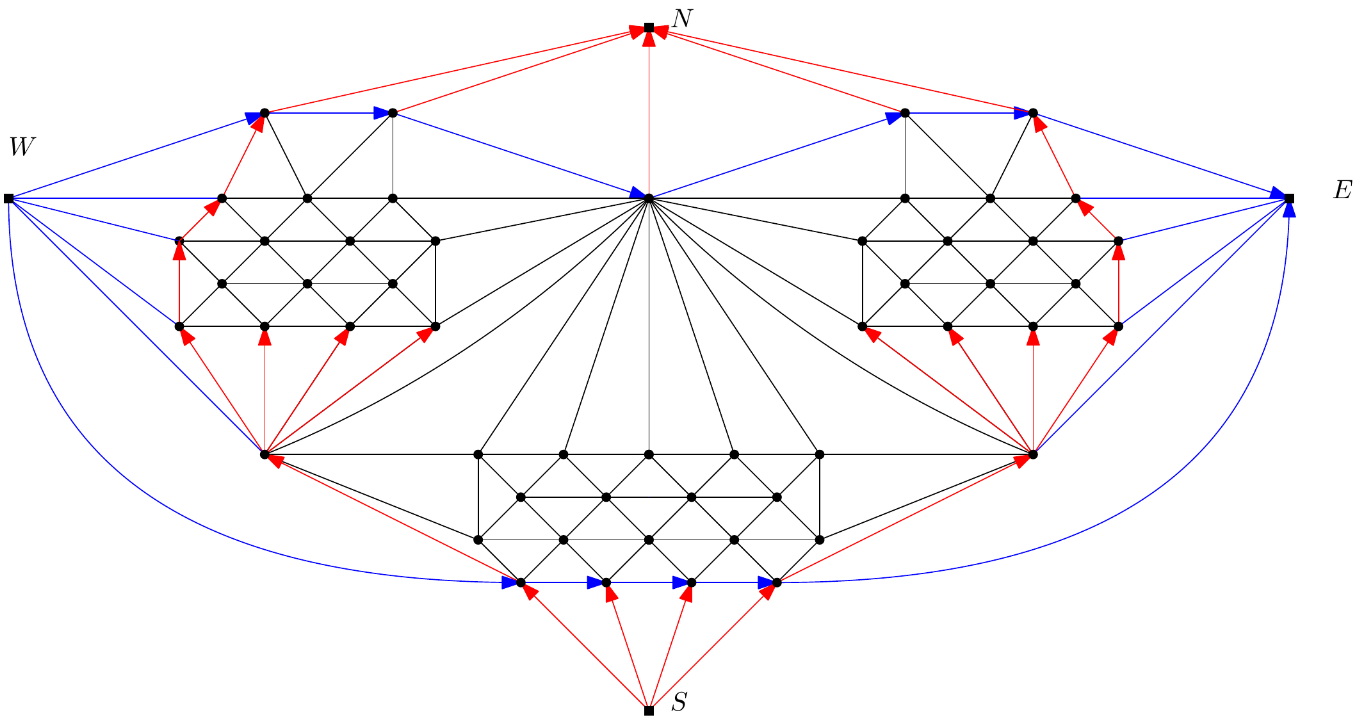
  Incoming red

  ...

  Contradiction
- Repeat this argument
- Color change at corner



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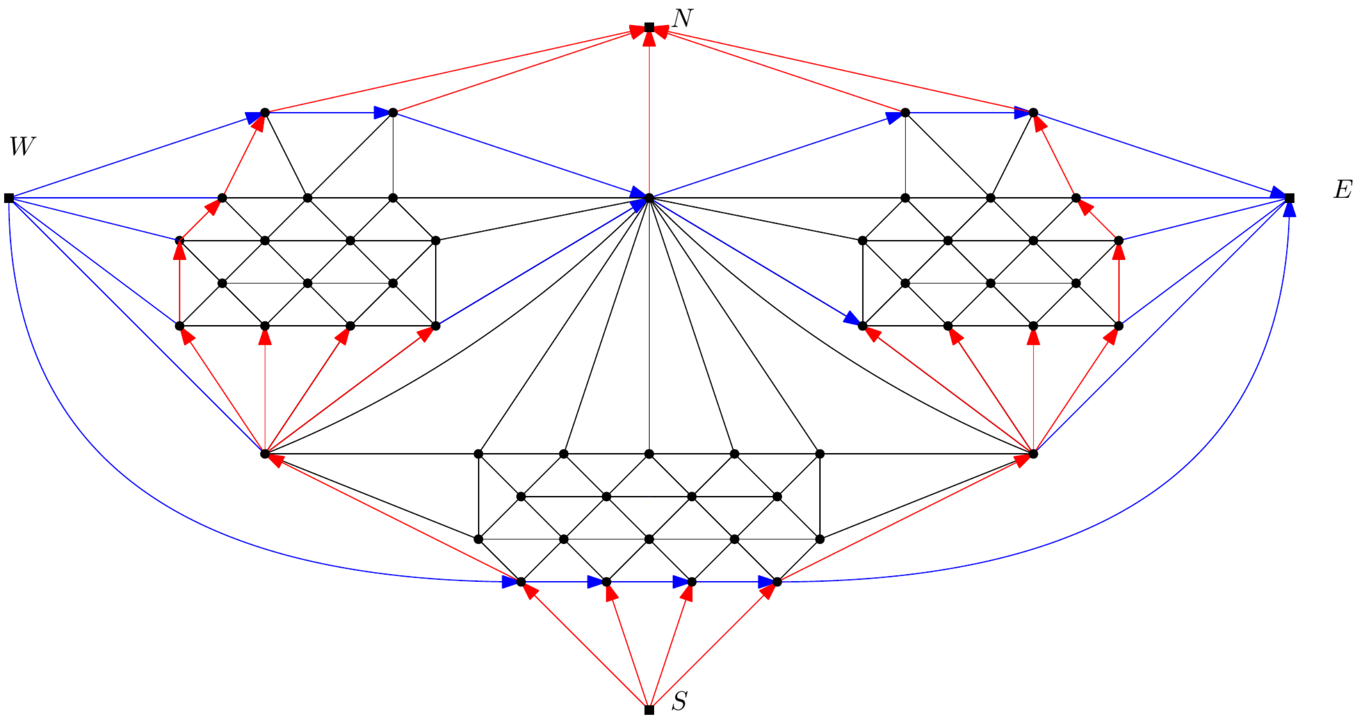
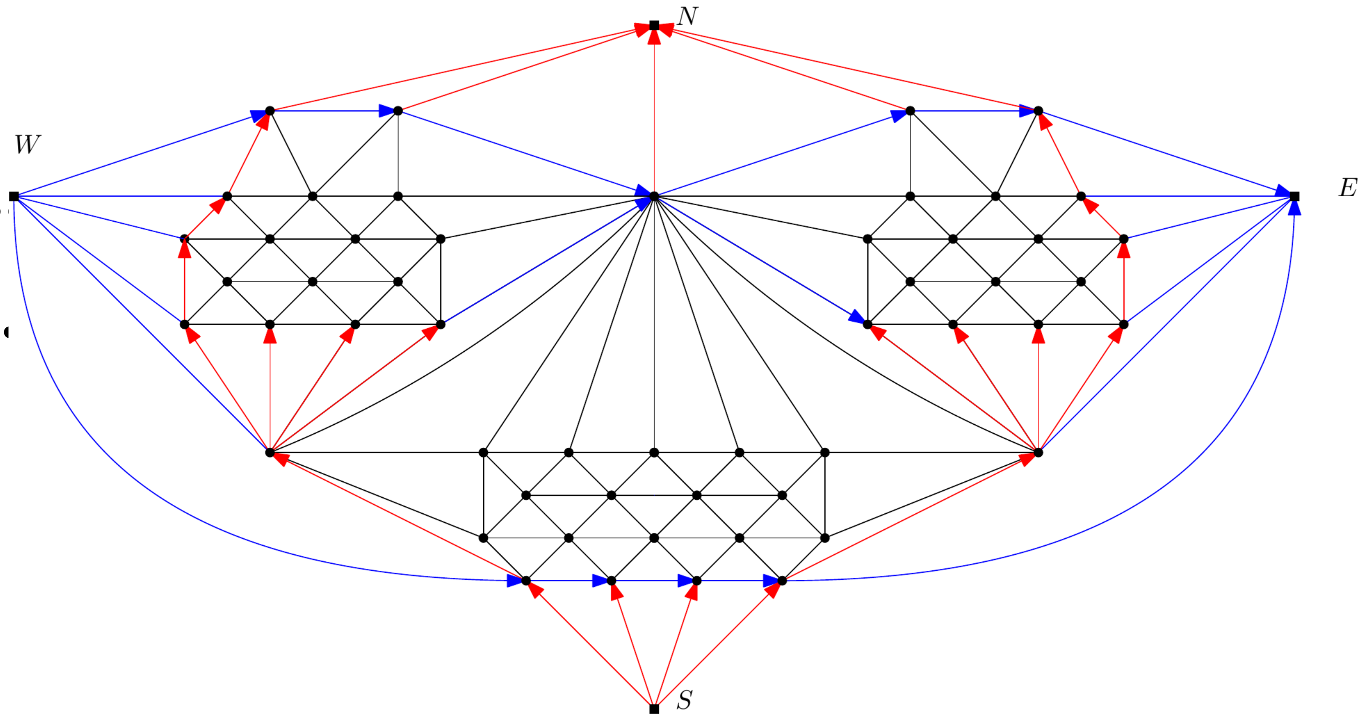


Tricky Example

- Graph
- Suppose blue ...

  Incoming red ...

  Contradiction
- Repeat this argument
- Color change at corner
- Interior vertex condition



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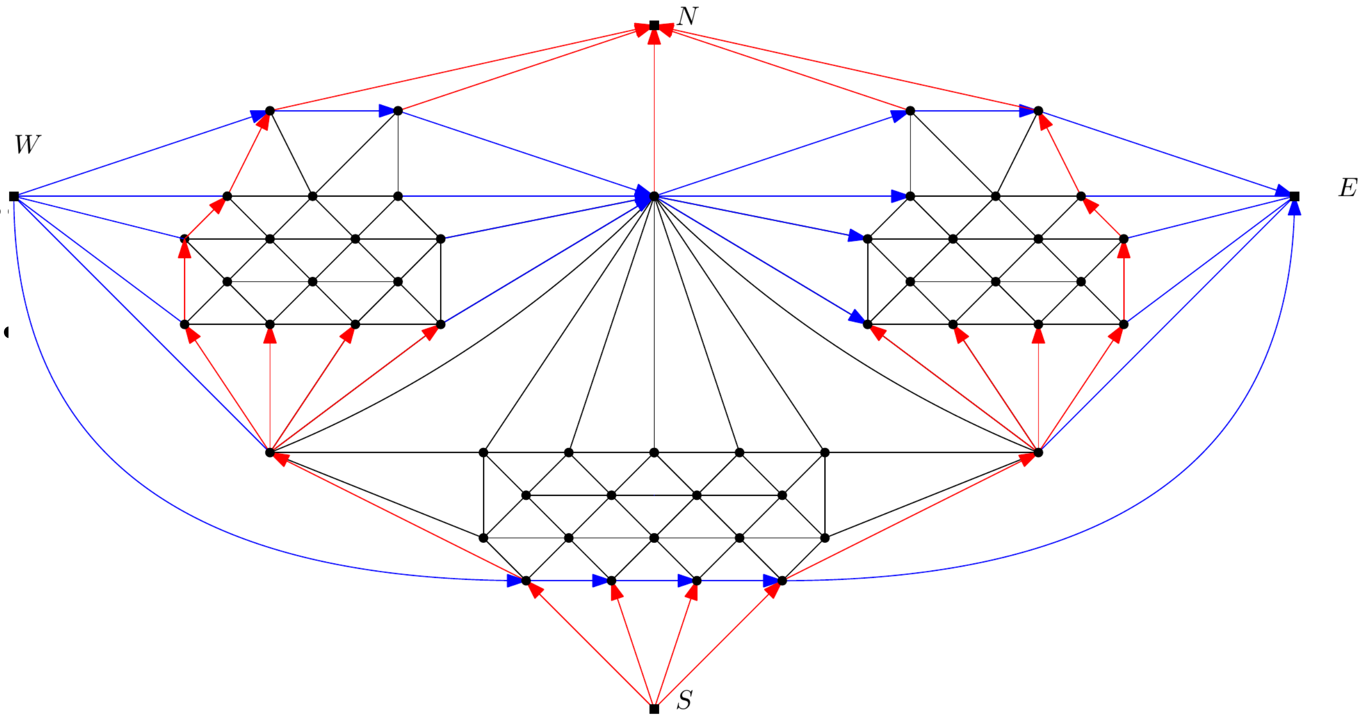
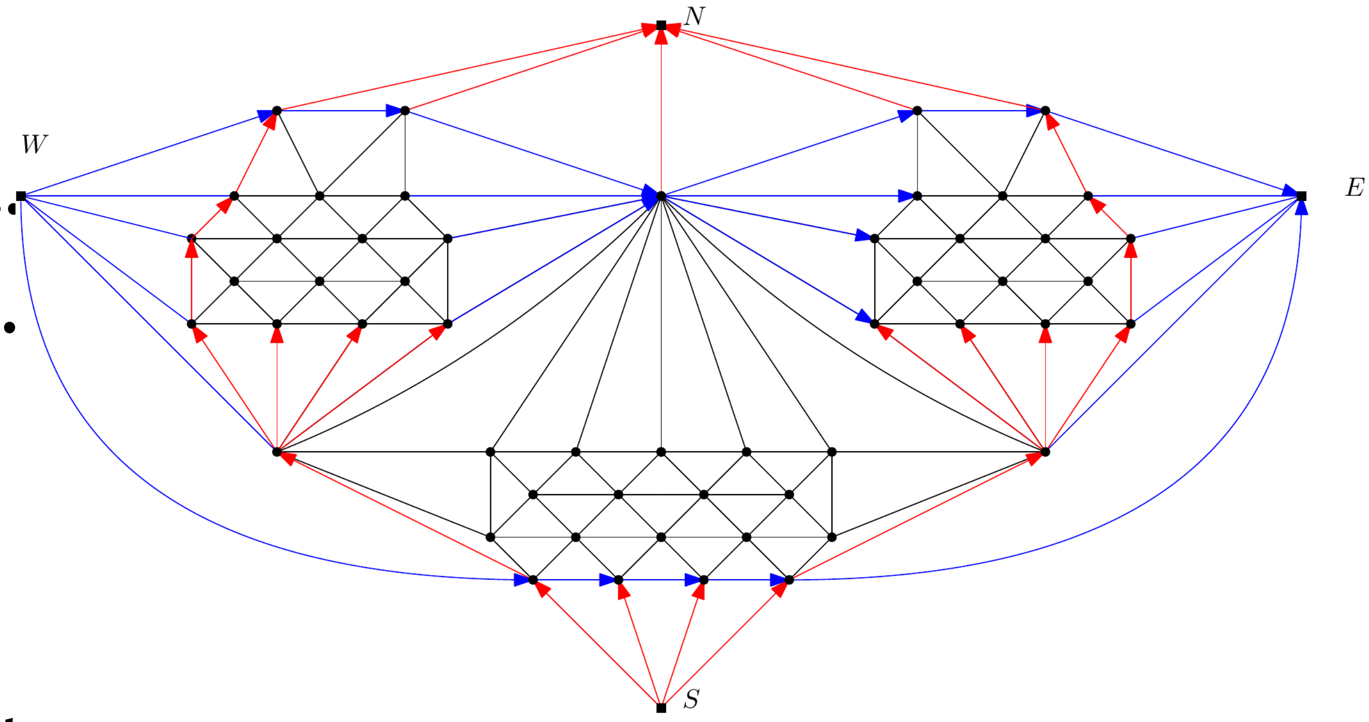


Tricky Example

- Graph
- Suppose blue ...

  Incoming red ...

  Contradiction
- Repeat this argument
- Color change at corner
- Interior vertex condition
- Problem ...



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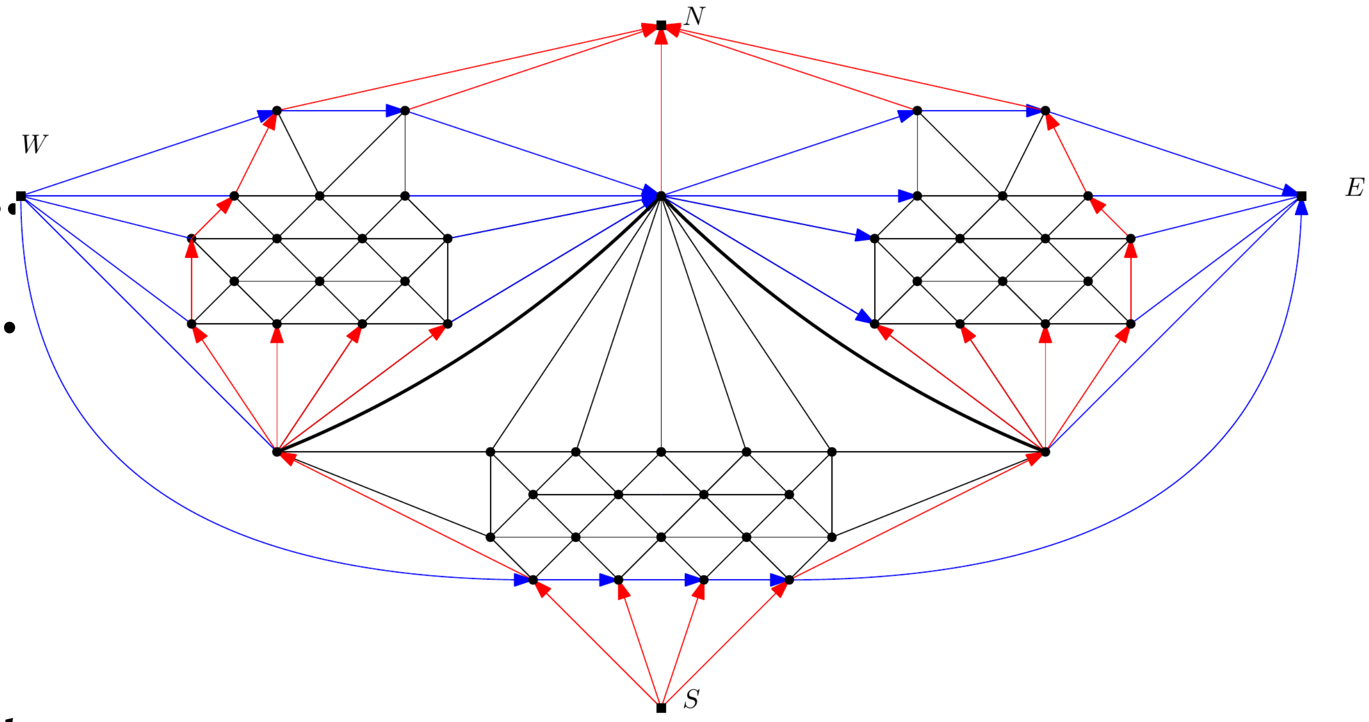

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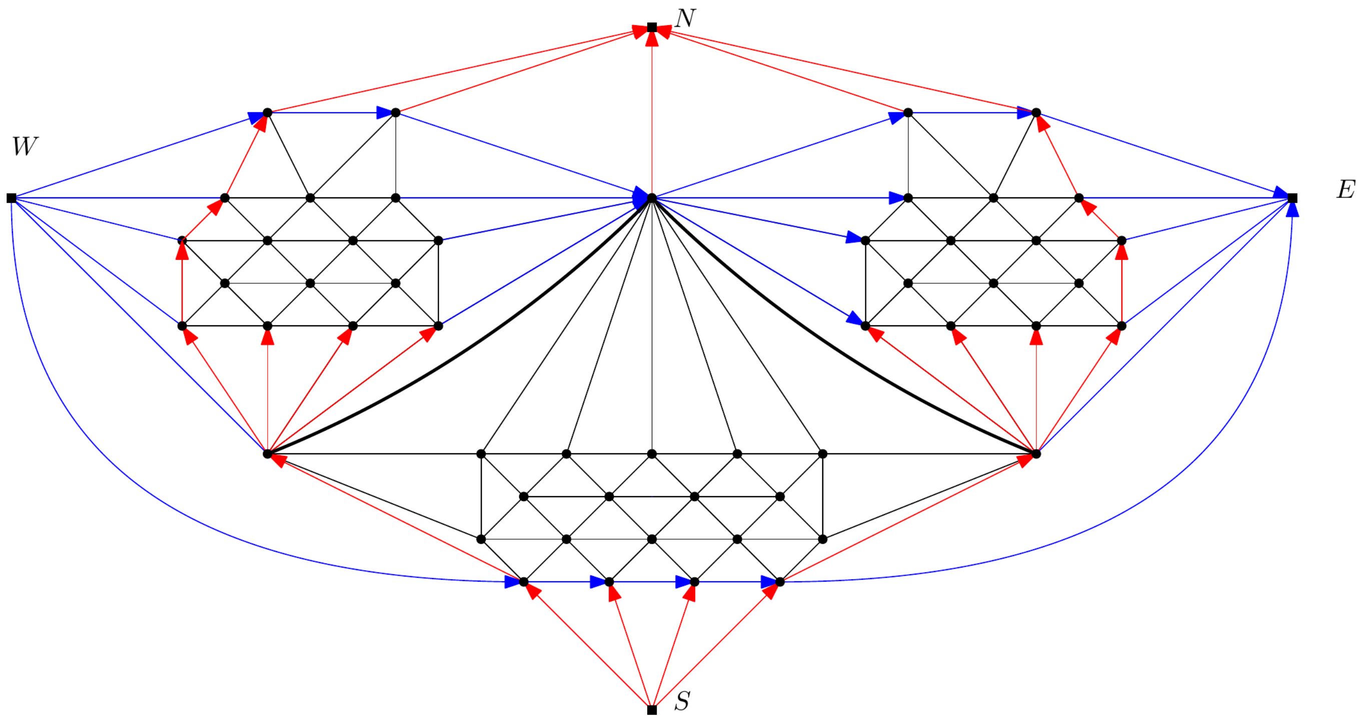

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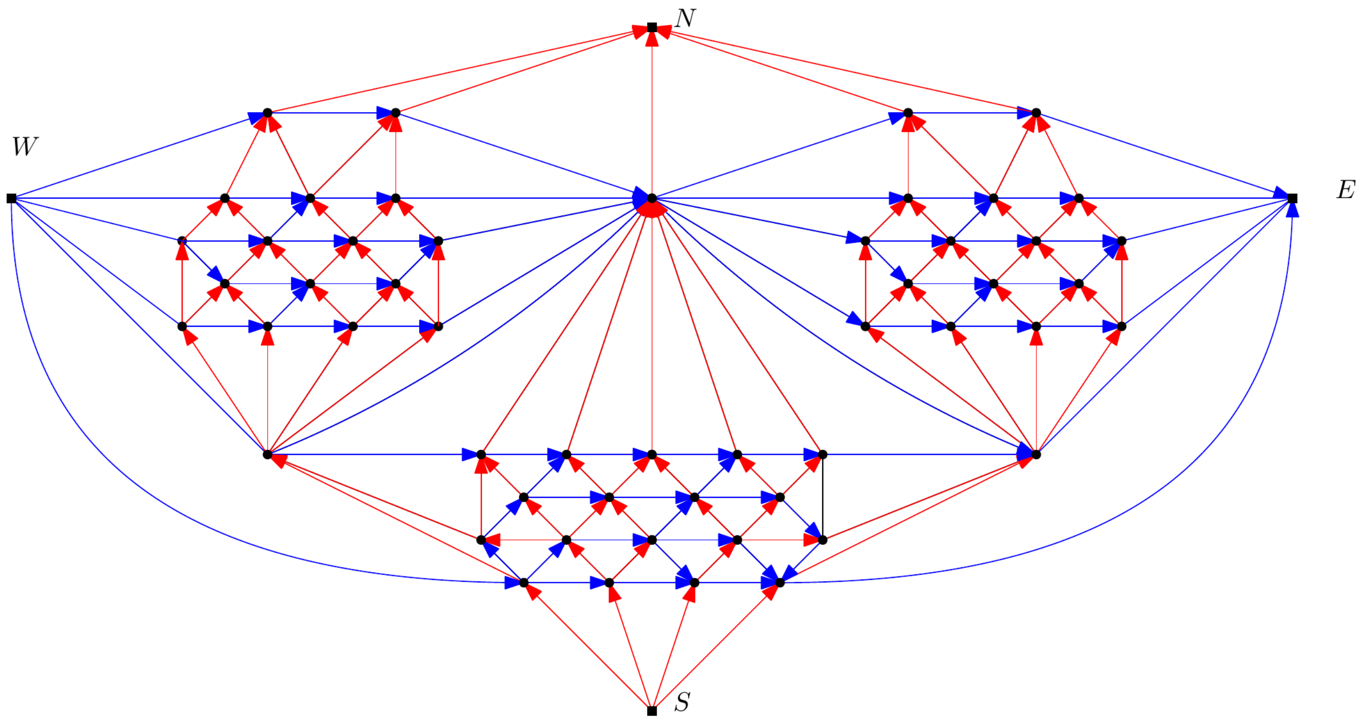

Conclusion

- A graph has multiple extended graphs **E(G)**
  - If **E(G)** has a separating 3-cycle this corner assignment has no dual
  - If **E(G)** has a separating 4-cycle this corner assignment gives a rectangular dual of **G**. But it may be ∞-sided
  - If **E(G)** has neither we hopefully show that it is 2-sided
- Current approach is with a constricting sweep-cycle
  - Has to do quite specific things, see previous example.
- Questions?