# Introduction

### 1. Rectangular layout

My thesis is on rectangular layouts. A rectangular layout is a subdivision of a rectangle in smaller interior-disjoint rectangles

### 2. Applications

Why do we care about such subdivisons?

They are useful in a number of fields. For example in architecture in making a floorplan of a building or in the design of computer chips, when we want to plan where to place the different parts of a computer chip.

### 3.Applications

Another application is in cartography. Here we can make rectangular cartograms, this are diagrams where the regions from an ordinary map are replaced with rectangles whose size conveys some meaning. We use this to visually convey information.

For example, on this slide we see a cartogram displaying the population of the 48 mainland states in 1930. The rectangular cartogram clearly see the north east of the unides staste is the most populous. For example the area around New York. And we can also see in which states almost nobody lives for example the states near the rocky mountains.

### 4. Rectangular duals

Most of the time we don’t want any rectangular layout, we want a layout where certain adjacencies hold. The visualization we just saw for example would be less useful if all the states would be entirely jumbled. In that case we wouldn’t be able to reference with the shape we know the US has to see which areas are more populated than their area would suggest.

Another example of the importance of adjacencies is in the architecture case. Here we might want the bathroom, bedroom and living room all adjacent to each other, while the living room should also be adjacent to, say, the hallway and the kitchen.

We are now going to formalize the concept of having certain adjacencies.

On this slide we see a blow-up of a part of the cartogram of the previous slide. All rectangles in this figure have certain adjacencies. We can represent these adjacencies by a graph. Each rectangle then becomes a vertex and each adjacency is represented by an edge.

We say that a rectangular layout with the same adjacencies as a certain graph is a rectangular dual of that graph. So the cartogram on this slide is a rectangular dual of the graph on this slide.

//Note that in this example Raisz did not maintain adjacencies between states, for example Delaware and Pennsylvania.

### 5. Rectangular duals

The natural first question is whether every graph has a rectangular dual?

It turns out this is not the case. Kozminski and Kinnen found that a graph has a rectangular dual with 4 rectangles on the outside exactly when it satisfies two requirements. A graph satisfying these requirements is a valid graph.

So the middle graph has the left layout as rectangular dual and the graph on the right has no rectangular dual. Because of of the separating trinagle. This would namely be a rectangle sourounded by only three rectangles, whihch is impossible.

### 6. Polar vertices

We will call the 4 outer vertices the poles. If this are not the only vertices the will **they bound an area bounded by 4 rectangles. This area must be a rectangle itself, since every inner corner must be created by bordering to two different rectangles. And the rectangle is the only shape made horizontal and vertical line segments with just 4 corners.**

### **7. Rectangular duals are not unique**

Although not every graph a rectangular dual, the graph that do have a rectangular dual often have more than one rectangular dual. In the left one A and B share a vertical adjacency; A is below B. while in the right dual they share a horizontal adjacency; A is left of B.

### 8. Area-universal layouts

But sometimes finding one single layout is not enough. Sometimes we want to have a layout that is the same for all sizes we can assign to the rectangles. For example when we want to depict the population of country at different moments in time. While the sizes of the regions then, of course, change. It is easiest to recognize the same structures if the adjacencies do not change.

Of course this is not the only place where an area-universal layout is useful, also in example for chip design we could make a single design having several editions where different components have different sizes, leading to different stats.

Hence we want to find an area-universal layout as rectangular dual of a certain graph G. So, we want to answer the question: “which graphs have an area-unversal dual”

### 9. Maximal segments

To answer his question we have to take a look at maximal segments.

[Explain maximal segments]

### 10. One-sided layouts

A layout is one-sided when every maximal segement is the sided of a single rectangle.

Is this layout one-sided?

No, the blue maximal segment has two rectangles below and three above.

### 11. One-sided layouts

Is this one one-sided?

Nope, red segement

Rember that we were looking for area-unversal layouts? Eppstein and his colleagues have shown that a layout is only area-universal when it is one-sided.

### 12. One-sided duals?

And in a certain sense this result is unfortunate, because we know not every graph that has a rectangular dual has a one-sided dual. For example the graph on this slide found by rinsma. This graph has 16 rectangular duals, as can be seen in the background.

### 13. One-sided duals?

So unfortunately not all graphs that have an rectangular dual have an area-universal dual. In my thesis we started exploring these graphs. Posing ourselves the question, what can we do for those graphs without an area-universal dual?

Because we have no area-universal dual we have no layout working for EVERY choice of sizes, but can we construct a layout that works for A LOT of size choices?

### 14. k-sided duals

Here the concept of k-sided segments comes in. We say a segment is k-sided for a certain integer k when the maximal segment has at most k rectangles on one side. Below we see a layout with a 2-sided maximal segment and a layout with a 3-sideded maximal segment. A layout is k-sided if all maximal segments are k-sided Hence

2-sided means lot of size choices work compared to 10-sided? [new slide??]

Hence our research question becomes: “Do graphs without a one-sided dual admit a k-sided dual for some k?”

### 15. Separating 4-cycles

Before finally stating my results we need to introduce one final concept. That of a separating 4-cycle.

### 16. Results

Staigtforward slide

# Regular Edge Labeling

### 17. Title slide

Most of the time we won’t reason on these rectangular duals directly. Instead we will use an coloring and orientation of the graphs. We call such a coloring and orientation a regular edge labeling. Each different regular edge labeling corresponds to a different layout.

We see separating 4-cycles influence such a regular edge labeling deeply. Thus is logical they influence the results.

### 20. Interior vertex condition

Any labeling satisfying these two conditions is already equivalent to a rectangular layout. As can be seen by an algorithm by Kant and He.

But we will expolere some more properties that follow form the previous two conditions…

### 21. No mono-colored triangles

… for example that we can’t have monocolored triangles. That is, a triangle with the same color on all three edges.

At least one vertex has in and outgoing blue. This one skips a red edges in the interior vertex condition.

### 22-23. Separating 4-cycle

And also the separating 4-cycles we just introduced come back. Since a 4-cycle is represented by 4 rectangles the interior of such a 4-cycle must be represented by a rectangle, just like the exterior vertex condition. So we now the color and orientation of the edges inside this cycle that are still adjacent to this cycle.

### 24. k-sided regular edge labeling

Let us look at when an edge labeling is k-sided

# Result 1

### 25. Title

Now we can prove result 1. We will, for any integer k, provide a graph that in every rectangular dual has a face that doesn’t correspond to a k-sided segment. We will do this by taking a number of forced steps using the properties and conditions we just proved.

### 26-28. Dual that is not k-sided

Straightforward stepping trough steps.

# Result 2

### 29. Title

In the previous proof we used separating 4-cycle to show that for every k there is a graph that is not k-sided. We will now show that if a graph doesn’t have such separating 4-cycels we can show it is at most d-1 sided where d is the maximal degree of a vertex that is not a pole.

I show this in my thesis using an algorithm. In this presentation we can only offer an high-level overview.

### **30. Algorithm**

**The proposed algorithm has 4 steps. In each step we slightly improve our current layout. So that in the end we can show we have a d-1 – sided layout for every graph that has no separating 4-cycle.**

### **31. Step 1: Sweepcycle**

**The first step is finding a baisic rel we can further improve upon during the rest of the algoritm. For this step we use an algorithm that is simmlar to an algorithm by Fusy developed by Fusy in 2006.**

**He designed a sweepcycle algorithm. We can visualize this as some sort of rubber band….**

**We maintain invariants on the rubber band (what is an invariant) even more the Fusy**

**Every step we find an updating step.**

### **33. Step 2:Flip Blue Z’s**

**Explain why?**

**State that we suspect this is not necessary but that this step is necessary for the proof.**

**The problem is now large blue faces.**

### **35. Step 3: Flip Topfans**

**We want to deal with top fans because we this makes it easier**

### **39. Step 4: Subdivide blue faces**

**After returning to our example we see we end up with not only a d-1 sided (whih in this case would be 5-sided) but in face a 2-sided layout.**

# **Tijdsplanning**

Intro (20)

Rel (10)

Result 1 (5)

Result 2 (10)

Totaal 45