Analytic LQR Design for Spacecraft Control System Based on Quaternion Model

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Abstract: In this paper, linear-quadratic regulator (LQR) method is proposed for the design of nonlinear spacecraft control systems. The proposed design is based on the linearized spacecraft model that involves only three components of the quaternion. By using a simple and special structure of the linearized reduced quaternion model, an analytical solution for the controller. It is shown that the analytical solution of the state feedback matrix is an explicit function of the cost matrices Q and R. The analytic solution makes it convenient in design practice to tune the feedback matrix and the cost matrices Q and R to balance the requirements between response performance and fuel consumption. It is shown that the designed controller globally stabilizes the nonlinear spacecraft system, whereas it locally optimizes the spacecraft performance. A design example is provided to show the effectiveness of the design method. **DOI: 10.1061/(ASCE)AS.1943-5525.0000142.**

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Introduction

Quaternion has been used for decades to represent the spacecraft attitude (Shuster 1993). It has several advantages over the Euler angle representation. First, quaternion representation does not have any singular point at any attitude. Second, quaternion representation does not depend on any rotational sequence. Therefore, control design methods based on quaternion spacecraft model have been investigated for decades. For example, Wen and Kreutz-Delgado (1991) introduced Lyapunov function to design model independent control law, model dependant control law, and adaptive control law; Boskovic and Mehra (2001) and Wallsgrove and Akella (2005) used Lyapunov functions to design control algorithms for spacecraft systems with control input saturation. Although Lyapunov functions are powerful in global stability analysis, Paielli and Bach (1993) noted that it is difficult to find a stable control law and associated Lyapunov functions for the general nonlinear systems because the design process is postulated by intuition. Moreover, the design is focused on the global stability; it does not address the performance of the control system, which is important in practical system design. By using the classical frequency domain method, Paielli and Bach (1993) adopted quaternion-based linear error dynamics to get the desired performance for the attitude control system. Wie et al. (1989) showed that there exists some state feedback that globally stabilizes the nonlinear spacecraft system, and the feedback matrix assigns the closed loop poles for the dynamics described by the rotational angle about the rotational axis. Zhou and Colgren (2005) discussed a more general problem from a different point of view. They obtained a linearized model with all components of the quaternion in the state variables from the nonlinear spacecraft system. However, this linearized system with all quaternion components is not fully controllable. This means that many powerful design methods in linear control system theory such as pole assignment, linear-quadratic regulator (LQR) control, and H_∞ control cannot be directly applied to the spacecraft control system design if a full quaternion-based linearized model is used.

On the other hand, although the Euler angle representation has a singular point and the representation depends on the rotational sequence, the linearized Euler angle–based spacecraft model was proved to be fully controllable. Therefore, all linear system design methods can be directly applied to the spacecraft control system design for the Euler angle model, and these methods are described in many standard text books, for example, Wie (1998), Wertz (1978), and Sidi (1997). More importantly, there are many successful applications of using these powerful control design methods, for example, Stoltz et al. (1998).

Recently, Yang (2010) showed that the reduced quaternion model that uses only vector components of the quaternion is fully controllable. Moreover, the linearized reduced quaternion model has a simple and special structure. In this paper, the results obtained by Yang (2010) will be extended. The linearized reduced quaternion model will be used for the spacecraft systems, and an analytical formula for LQR optimal control that is explicitly related to the cost matrices Q and R will be derived. Moreover, it will be shown that under some mild restriction, the LQR feedback controller globally stabilizes the original nonlinear spacecraft; n addition, the LQR controller has a diagonal structure in the state feedback matrices D and K.

The rest of the paper is organized as follows: the second section discusses the reduced nonlinear and linearized quaternion spacecraft model and reviews some results in Yang (2010) that will be used in this paper. The third section discusses the LQR optimal control design method. The analytical formulas of the LQR controller will be analyzed to show that the LQR controller globally stabilizes the original nonlinear spacecraft system. The relationship of the LQR design and the closed loop eigenvalues will be given. The fourth section provides some design example to demonstrate the excellent performance of the LQR design method. Conclusions are summarized in the last section.

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Reduced Quaternion Spacecraft Model

Let

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \tag{1}$$

be the constant inertia matrix of the spacecraft, and let

$$\omega = (\omega_1, \omega_2, \omega_3)^T$$

be the angular velocity vector of the spacecraft with respect to the inertial frame, represented in the spacecraft body frame. Let

$$u = (u_1, u_2, u_3)^T$$

be the control torques in body frame. Denote

$$\omega \times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

as a matrix operator that maps the cross product $\omega \times x$ to a matrix product $\omega \times x$. It is well-known that the spacecraft dynamics system equation is given by (Wertz 1978)

$$J\dot{\omega} = -\omega \times (J\omega) + u \tag{2}$$

It is assumed that the constant inertia matrix J is a diagonal matrix because this is approximately correct in most real spacecraft designs. Let

$$q_0 = \cos\left(\frac{\alpha}{2}\right), \qquad q = [q_1, q_2, q_3]^T = \hat{e}^T \sin\left(\frac{\alpha}{2}\right)$$

and

$$\bar{q} = [q_0, q^T]^T = \left[\cos\left(\frac{\alpha}{2}\right), \hat{e}^T \sin\left(\frac{\alpha}{2}\right)\right]^T \tag{3}$$

be the quaternion that represents the rotation of the body frame relative to the inertial frame, where $\hat{e}$ = unit rotational axis; and α = rotational angle about the rotational axis. The nonlinear spacecraft kinematics equations of motion can be represented by the quaternion (Wie 1998; Wertz 1978; Sidi 1997) as follows:

$$\begin{cases} \dot{q} = -\frac{1}{2}\omega \times q + \frac{1}{2}q_0\omega \\ \dot{q}_0 = -\frac{1}{2}\omega^T q \end{cases} \tag{4}$$

Yang (2010) showed that the nonlinear spacecraft kinematics equations of motion [Eq. (4)] can be replaced by a set of independent nonlinear spacecraft kinematics equations of motion that leads to a controllable linearized quaternion model.

Lemma 1 (Yang 2010): For any given q, if $\alpha \neq \pi$, there exists a one-to-one mapping between ω and $\dot{q}$. Moreover, let

$$f(q) := q_0 = \sqrt{1 - q_1^2 - q_2^2 - q_3^2}$$

and

$$\Omega = \begin{bmatrix} f(q) & -q_3 & q_2 \\ q_3 & f(q) & -q_1 \\ -q_2 & q_1 & f(q) \end{bmatrix}$$

then, the one-to-one mapping is given by

$$\omega = 2\Omega^{-1}\dot{q} \tag{5}$$

Therefore, (4) can be replaced by

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} f(q) & -q_3 & q_2 \\ q_3 & f(q) & -q_1 \\ -q_2 & q_1 & f(q) \end{bmatrix}\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \frac{1}{2}\Omega\omega = g(q_1, q_2, q_3, \omega) \tag{6}$$

The linearized spacecraft system can be derived from Eqs. (2) and (6) by using the first order Taylor expansion around the stationary point as follows:

$$\dot{\omega} = J^{-1}u, \qquad \left.\frac{\partial g}{\partial \omega}\right|_{\substack{\omega = 0 \\ q_1 = q_2 = q_3 = 0}} = \frac{1}{2}I_3,$$

$$\left.\frac{\partial g}{\partial q}\right|_{\substack{\omega = 0 \\ q_1 = q_2 = q_3 = 0}} = 0_3$$

Therefore

$$\begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0_3 & 0_3 \\ \frac{1}{2}I_3 & 0_3 \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} J^{-1} \\ 0_3 \end{bmatrix}u = Ax + Bu \tag{7}$$

where

$$A = \begin{bmatrix} 0_3 & 0_3 \\ \frac{1}{2}I_3 & 0_3 \end{bmatrix}, \qquad x = \begin{bmatrix} \omega \\ q \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} J^{-1} \\ 0_3 \end{bmatrix} \tag{8}$$

It is easy to verify that this linearized spacecraft system equation is controllable. It will be shown in the next section that there exist diagonal matrices D and K such that the state feedback

$$u = -[D \quad K]x = -Gx \tag{9}$$

is the LQR design for the linearized quaternion system Eq. (7), and the LQR design globally stabilizes the nonlinear spacecraft system described by Eqs. (2) and (4) .

LQR Design

Because the linearized spacecraft system (7) is fully controllable, LQR design can be directly obtained (Athans and Falb 1966). The advantage of the LQR design is obvious because we can find optimal K and D to achieve other design goals, such as minimizing the control energy (required because of the restrictions on control authority and saturation) and optimizing the response performance. Instead of solving nonlinear Lyapunov matrix equation to get Q and R matrices, analytical feedback formulas can be found because of the simple and special structure of the linearized reduced quaternion spacecraft model.

For the linearized reduced quaternion spacecraft system Eq. (7) the linear quadratic cost function to be minimized is

$$L = \frac{1}{2}\int_0^\infty [x^T Qx + u^T Ru]dt \tag{10}$$

where Q and R are positive definite matrices. The optimal control is uniquely given by (Athans and Falb 1966)

$$u(t) = -R^{-1}B^T Fx(t) = -Gx \tag{11}$$

where F = constant positive definite matrix, which is the solution of the Lyapunov matrix algebraic equation

$$-FA - A^T F + FBR^{-1}B^T F - Q = 0 \tag{12}$$

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In the rest of the discussion, it is assumed that J, Q, and R are all diagonal matrices because J is always designed to be approximately diagonal in real spacecrafts; Q and R are oftentimes selected to be diagonal in engineering design practice. With these assumptions, the problem can greatly be simplified. It is well-known that the LQR feedback Eq. (11) will guarantee the stability of the linearized closed loop system and minimize the cost function of Eq. (10) that is a combined cost of cumulative control system error and the control energy.

For the linearized spacecraft model described by Eq. (7), an analytical solution F of Eq. (12) is derived as follows. Let

$$F = \begin{bmatrix} F_{11} & F_{12} \\ F_{12}^T & F_{22} \end{bmatrix}, \qquad Q = \begin{bmatrix} Q_1 & 0_3 \\ 0_3 & Q_2 \end{bmatrix} \tag{13}$$

where the elements of F and Q in Eq. (13) = 3 by 3 matrices. Substituting (8) and (13) into Eq. (12) and using simple manipulations yield

$$\begin{bmatrix} F_{11}J^{-1}R^{-1}J^{-1}F_{11} & F_{11}J^{-1}R^{-1}J^{-1}F_{12} \\ F_{12}^T J^{-1}R^{-1}J^{-1}F_{11} & F_{12}^T J^{-1}R^{-1}J^{-1}F_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(F_{12}^T + F_{12}) + Q_1 & \frac{1}{2}F_{22} \\ \frac{1}{2}F_{22} & Q_2 \end{bmatrix} \tag{14}$$

Because Q and R are positive definite and $F_{12}^T = F_{12}$, comparing the (2,2) block on both sides of Eq. (14) yields

$$F_{12} = JR^{1/2}Q_2^{1/2} \tag{15}$$

Because J, Q, and R are diagonal, substituting Eq. (15) into the (1,1) block of Eq. (14) gives

$$F_{11} = JR^{1/2}\left(Q_1 + \frac{1}{2}(JR^{1/2}Q_2^{1/2} + Q_2^{1/2}R^{1/2}J)\right)^{1/2} \tag{16}$$

Substituting Eqs. (15) and (16) into the (2,1) block of Eq. (14) gives

$$F_{22} = 2Q_2^{1/2}\left(Q_1 + JR^{1/2}Q_2^{1/2}\right)^{1/2} \tag{17}$$

Eqs. (15)–(17) give a complete solution of Lyapunov matrix Eq. (12). Therefore, Eq. (11) can be rewritten as

$$\begin{aligned} u(t) &= -R^{-1}B^T F x(t) == -[R^{-1}J^{-1}F_{11}, R^{-1}J^{-1}F_{12}]x \\ &= -[D, K]x \end{aligned} \tag{18}$$

Under some additional conditions, the LQR optimal control given by Eq. (18) globally stabilizes the nonlinear system described by Eqs. (2) and (4). Let

$$P = Q_2^{-1/2}R^{1/2}J$$

and the Lyapunov function be

$$V = \frac{1}{2}\omega^T P\omega + q_1^2 + q_2^2 + q_3^2 + (1 - q_0)^2 \tag{19}$$

Then, the derivative of the Lyapunov function along the trajectory described by the nonlinear system Eqs. (2) and (4) is given by

$$\begin{aligned} \frac{dV}{dt} &= \omega^T P\left(-J^{-1}\omega\times J\omega - J^{-1}R^{-1}[J^{-1} \quad 0]\begin{bmatrix} F_{11} & F_{12} \\ F_{12}^T & F_{22} \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix}\right) \\ &\quad + \omega^T q \\ &= -\omega^T PJ^{-1}\omega\times J\omega - \omega^T PJ^{-1}R^{-1}J^{-1}F_{11}\omega \\ &\quad - \omega^T PJ^{-1}R^{-1}J^{-1}F_{12}q + \omega^T q \\ &= -\omega^T PJ^{-1}\omega\times J\omega \\ &\quad - \omega^T Q_2^{-1/2}\left(Q_1 + \frac{1}{2}(JR^{1/2}Q_2^{1/2} + Q_2^{1/2}R^{1/2}J)\right)^{1/2}\omega \\ &= -\omega^T Q_2^{-1/2}R^{1/2}\omega\times J\omega \\ &\quad - \omega^T Q_2^{-1/2}\left(Q_1 + \frac{1}{2}(JR^{1/2}Q_2^{1/2} + Q_2^{1/2}R^{1/2}J)\right)^{1/2}\omega\psi \end{aligned} \tag{20}$$

Because P, J, Q, and R are all diagonal positive definite matrices, the second term of the last expression is negative definite. If $Q_2^{-1}R = cI$, i.e., $R = cQ_2$; or $Q_2^{-1}R = cJ$, i.e., $R = cQ_2J$, where c is a constant, then the first term vanishes, therefore, dV/dt is negative semidefinite, and the nonlinear system described by nonlinear system Eqs. (2) and (4) is globally stable with the optimal controller given by Eq. (18). Actually, it can be shown that the closed loop nonlinear system is asymptotically stable. Let $S = \{x|\dot{V}(x) = 0\}$. Because D and K are full rank matrices, clearly $S = \{x = (\omega, q) = (0, q)\}$. From Eq. (2) because $u = -D\omega - Kq \neq 0$ if $q \neq 0$, no solution can always stay in S except $x = (\omega, q) = (0, 0)$. By using a well-known result in Khalil 1992 (Corollary 3.2), the origin is globally asymptotically stable. By definition (Khalil 1992, p. 111), the region of attraction of the nonlinear system is the whole space spanned by x.

In system design practice, if the performance and the local stability are the only design considerations, Q and R can be chosen without any restriction; if the global stabilization is also required for nonlinear spacecraft system, some restriction, though it is mild, has to be placed on Q and R, i.e., either $R = cQ_2$ or $R = cQ_2J$, where c is any positive constant.

To establish the relationship between the closed loop poles and the design matrices Q, and R, Eq. (18) can be simplified further as follows. Let $Q_1 = \mathrm{diag}(q_{1i})$; $Q_2 = \mathrm{diag}(q_{2i})$; and $R = \mathrm{diag}(r_i)$, the matrices D and K can be further simplified as

$$\begin{aligned} D &= R^{-1/2}\left(Q_1 + \frac{1}{2}(JR^{1/2}Q_2^{1/2} + Q_2^{1/2}R^{1/2}J)\right)^{1/2} = \mathrm{diag}(d_i) \\ &= \mathrm{diag}\left(\sqrt{\frac{q_{1i}}{r_i} + J_{ii}\sqrt{\frac{q_{2i}}{r_i}}}\right) \end{aligned} \tag{21}$$

with

$$d_i = \sqrt{\frac{q_{1i}}{r_i} + J_{ii}\sqrt{\frac{q_{2i}}{r_i}}} \tag{22}$$

and

$$K = R^{-1/2}Q_2^{1/2} = \mathrm{diag}(k_i) = \mathrm{diag}\left(\sqrt{\frac{q_{2i}}{r_i}}\right) \tag{23}$$

with

$$k_i = \sqrt{\frac{q_{2i}}{r_i}} \tag{24}$$

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Therefore, Eq. (18) becomes

$$u(x) = -[D,K]x = -\begin{bmatrix} \sqrt{\frac{q_{11}}{r_1} + J_{11}\sqrt{\frac{q_{21}}{r_1}}} & 0 & 0 & \sqrt{\frac{q_{21}}{r_1}} & 0 & 0 \\ 0 & \sqrt{\frac{q_{12}}{r_i} + J_{22}\sqrt{\frac{q_{22}}{r_2}}} & 0 & 0 & \sqrt{\frac{q_{22}}{r_2}} & 0 \\ 0 & 0 & \sqrt{\frac{q_{13}}{r_3} + J_{33}\sqrt{\frac{q_{23}}{r_3}}} & 0 & 0 & \sqrt{\frac{q_{23}}{r_3}} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

It is straightforward to write the closed loop system as follows:

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{dq}{dt} \end{bmatrix} = \begin{Bmatrix} -J^{-1}R^{-1/2}\left[Q_1 + \frac{1}{2}(JR^{1/2}Q_2^{1/2} + Q_2^{1/2}R^{1/2}J)\right]^{1/2} & -J^{-1}R^{-1/2}Q_2^{1/2} \\ \frac{1}{2}I_3 & 0_3 \end{Bmatrix} \begin{bmatrix} \omega \\ q \end{bmatrix} \quad (25)$$

Let the linear matrix transformation $T_{ij}(C)$ be a matrix with the following properties: (1) the (i,j) element of $T_{ij}(C)$ is C, (2) the diagonal elements are ones, (3) all the remaining elements are zeros. It is well-known that the inverse of $T_{ij}(C)$ is $T_{ij}^{-1}(C) = T_{ij}(-C)$. For $i = 1$, 2, and 3, let

$$C_i = \frac{\frac{d_i}{J_{ii}} \pm \sqrt{\left(\frac{d_i}{J_{ii}}\right)^2 - 2\frac{k_i}{J_{ii}}}}{2\frac{k_i}{J_{ii}}} = \frac{s_i \pm \sqrt{s_i^2 - 2t_i}}{2t_i} \quad (26)$$

By using the transformation of $T_{36}(C_3)T_{25}(C_2)T_{14}(C_1)(A - B[D,K])T_{14}(-C_1)T_{25}(-C_2)T_{36}(-C_3)$, the closed loop eigenvalues of linear system (7) using the LQR design are given by, for $i = 1$, 2, and 3

$$\lambda_{2i-1}, \lambda_{2i} = \frac{-\sqrt{\frac{1}{J_{ii}}\sqrt{\frac{q_{2i}}{r_i}} + \frac{q_{1i}}{J_{ii}^2 r_i}} \pm \sqrt{\frac{q_{1i}}{J_{ii}^2 r_i} - \frac{1}{J_{ii}}\sqrt{\frac{q_{2i}}{r_i}}}}{2} \quad (27)$$

Eq. (27) provides a lot of useful information for the LQR design. First, as $r_i \to 0$, the corresponding pair of eigenvalues go to minus infinity of the complex plane; as $r_i \to \infty$, the corresponding pair of eigenvalues go to origin of the complex plane. Second, as long as $q_{1i} > \sqrt{q_{2i} r_i} J_{ii}$, the corresponding pair of eigenvalues are real and unequal because $d_i/J_{ii} > \sqrt{(d_i/J_{ii})^2 - 2k_i/J_{ii}}$, these two eigenvalues are always negative. Third, if $q_{1i} = \sqrt{q_{2i} r_i} J_{ii}$, these are two equal real negative eigenvalues. Fourth, if $q_{1i} < \sqrt{q_{2i} r_i} J_{ii}$, this is a pair of complex eigenvalues with a negative real part. Therefore, increasing q_{1i} and decreasing q_{2i} will increase the damping ratio; otherwise, it will decrease the damping ratio. Finally, increasing q_{2i} and decreasing r_i will increase the natural frequency; otherwise, it will decrease the natural frequency. This information can be useful in spacecraft system design.

By using the LQR design, the closed loop poles are implicitly designed as defined by (27) and the requirements on accumulative control error and power consumption (both are important in practical design) can be balanced.

The LQR design procedure is summarized in Fig. 1. The relation of the designed controller and the original nonlinear spacecraft system is depicted in Fig. 2. The effects of the designed LQR controller on the nonlinear structure are twofold. First, the LQR controller globally asymptotically stabilizes the nonlinear spacecraft system, i.e., for any initial attitude and for any initial angular rate, the LQR controller will bring the nonlinear spacecraft to the origin $(\omega, q) = 0$. Second, when the nonlinear spacecraft is close to the origin, the linearized model is a very good approximation of the nonlinear spacecraft system; because LQR controller is an optimal design for the linearized system, therefore it is a suboptimal control for the original nonlinear spacecraft system. To the best of the author's knowledge, there is no any other spacecraft controller design method that satisfies simultaneously these two properties.

Design Examples

In this section, an example studied by Zhou and Colgren (2005) is used to illustrate the design method. The spacecraft inertial matrix is given by

$$J = \begin{bmatrix} 1{,}200 & 100 & -200 \\ 100 & 2{,}200 & 300 \\ -200 & 300 & 3{,}100 \end{bmatrix} \quad (28)$$

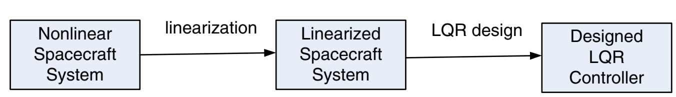

Fig. 1. LQR controller design is based on the linearized spacecraft model

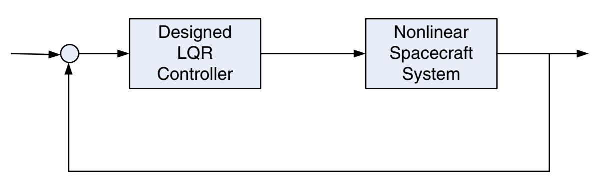

Fig. 2. LQR is suboptimal control and globally stabilizes the nonlinear spacecraft system

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It is clear that the diagonal elements of the matrix are significantly larger than off-diagonal elements. Assume that the spacecraft inertial matrix can be approximated by a diagonal matrix whose diagonal elements are equal to the diagonal elements of J, let $Q = \text{diag}(5, 5, 5, 5, 5, 5)$ and $R = \text{diag}(8, 8, 8)$, the closed loop poles are then given as

$$-0.01273212110421 + / - 0.01272387326295i;$$
$$-0.00798572833825 + / - 0.00798369205833i;$$
$$-0.00947996395486 + / - 0.00947655794419i$$

and the feedback matrix D and K are obtained from Eqs. (21) and (23) as follows:

$$D = \begin{bmatrix} 31.06637549427606 & 0 & 0 & \\ 0 & 41.71184140136478 & 0 & 0 \\ 0 & 0 & 49.51151569716377 & \end{bmatrix},$$
$$K = \begin{bmatrix} 0.79056941504209 & 0 & 0 \\ 0 & 0.79056941504209 & 0 \\ 0 & 0 & 0.79056941504209 \end{bmatrix}$$

The designed feedback controller is applied to the nonlinear spacecraft system described by Eqs. (2) and (4). A simulation test is conducted with the full Monte Carlo perturbation model described as follows: (1) in inertia matrix J, the off-diagonal elements are randomly selected between [0, 310], (2) the initial Euler angle errors of the nonlinear spacecraft system are randomly selected between $[0, \pi]$, and these initial Euler angles are converted into quaternions, and (3) the initial angular rates are randomly selected between [0, 0.1] degrees per second. For 300 Monte Carlo simulation runs they are all asymptotically stable. This result is shown in Fig. 3.

Conclusion

In this paper, a fully controllable linearized spacecraft model represented by three quaternion components is proposed. It is shown that an analytical LQR controller can easily be obtained for this quaternion-based model, and the LQR controller is a diagonal proportional and differential (PD) controller. The relationship between the LQR design parameters and the closed loop eigenvalues are established, which provides a lot of useful information that will benefit design practice. Finally, under some mild restriction, the LQR design globally stabilizes the nonlinear spacecraft system.

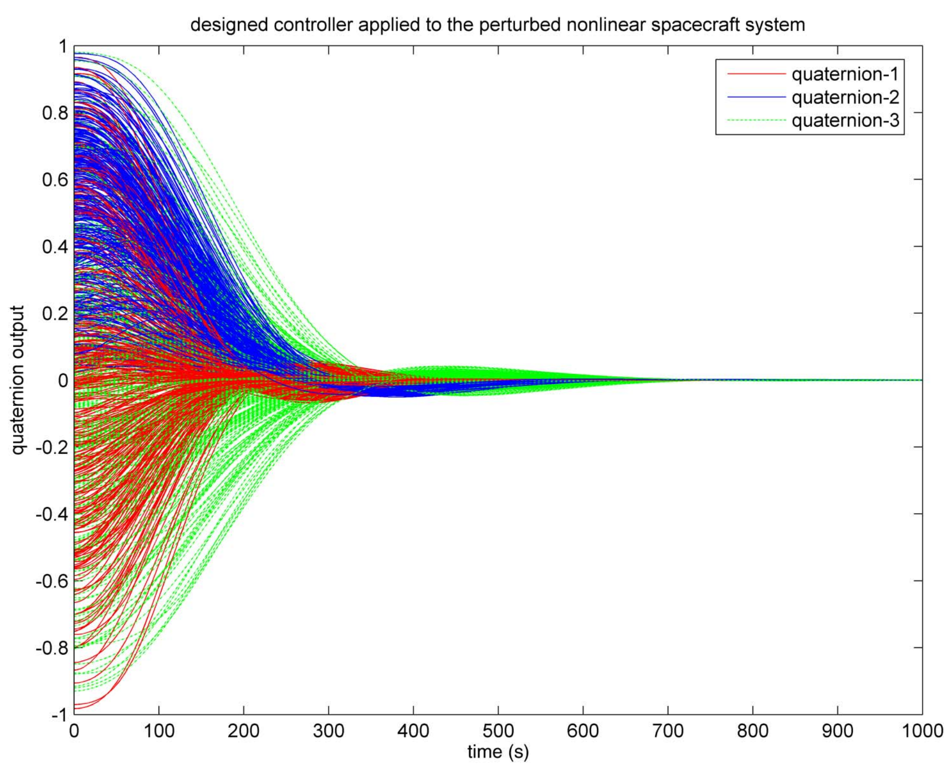

Fig. 3. Monte Carlo runs of quaternion response of the nonlinear system with nondiagonal inertia matrix

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