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Quaternion based model for momentum biased nadir pointing spacecraft

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In this paper, we propose a quaternion based spacecraft model that describes the nadir pointing spacecraft with momentum wheel under gravity gradient torque disturbance. This state space model uses only three components of the quaternion. From this nonlinear model, we derive a linearized state space model for the spacecraft system. We show that unlike all existing quaternion models, this linearized state space model is fully controllable. Therefore, all modern control system design methods can be directly applied to the attitude control system design for momentum biased nadir pointing spacecraft.

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1. Introduction

The quaternion based model has several advantages over the Euler angle based model. For example, the quaternion based model is uniquely defined because it does not depend on rotational sequence, while a Euler angle based model can be different for different rotational sequences. Therefore, a Euler angle based model is not uniquely defined, is sometimes inconvenient, and may be error-prone if different groups of people are working on the same project. In practice, an agreement has to be reached between different design groups working on the same project. Another attractive feature of quaternion based model is that a full quaternion model does not have any singular point in any rotational sequence. Therefore, quaternion model-based control design methods have been discussed in a number of papers. In [6], Lyapunov function was used to design model-independent control law, model-dependent control law, and adaptive control law. In [1,5], Lyapunov functions were used to design control systems under the restriction of control input saturation. Though Lyapunov function is a powerful tool in global stability analysis, obtaining a control law and the associated Lyapunov function for the nonlinear systems is postulated by intuition, as noted in [3]. Moreover, these designs are focused on the global stability but not focused on the performance of the control system. In [3,8], quaternion based linear error dynamics are adapted to get desired performance for the attitude control system using classical frequency domain methods. Since state space time domain design methods, such as optimal control and pole assignment, are more attractive than the classical frequency domain design methods, in [9], Zhou and Colgren derived a linearized state space model and investigated modern design methods. Their analysis shows that the linearized state space representation of the full quaternion model using all four components of quaternion is uncontrollable. Therefore, pole assignment can only be achieved in controllable subspace in the linearized state space quaternion model using all four components of quaternion. Moreover, the stability of the linearized closed loop system is unknown because an uncontrollable eigenvalue is on the imaginary axis. Another restriction in the existing quaternion modeling and controller design methods is that the investigation was focused on inertia pointing spacecraft without using reaction wheel while many low earth orbit spacecrafts are nadir pointing and use momentum wheel.

In this short paper, we propose a quaternion based model for nadir pointing spacecraft with momentum wheel. This is a more general model than inertia pointing spacecraft without a momentum wheel discussed in many literatures. This model includes three important features of many low orbit nadir pointing spacecrafts: (1) an additional term for the momentum wheels is incorporated to the nonlinear dynamic equations, (2) the local vertical local horizontal frame is used as the reference frame and the rotation between local vertical local horizontal frame and inertia frame is considered in the model similar to the treatment in [4] for Euler angle based models, and (3) gravity gradient torque which is a dominant and predictable disturbance for low orbit spacecraft is included to make the model more accurate. We will show that by using only vector component of the quaternion, the linearized spacecraft model is fully controllable. Therefore, the derived model is not only more accurate compared to existing full quaternion spacecraft model, but it is also easier to be used in controller design because all modern state space control system design methods can be used directly. The stability of the designed closed-loop spacecraft system is guaranteed because the linearized control system is fully controllable. The cost of using only three

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components of the quaternion in the model is that, similar to Euler angle representation, the reduced model has a singular point at $\alpha = \pm\pi$, where α is the rotation angle around the rotation axis. However, this singular point is the farthest point to $\alpha = 0$ where the linearization is carried out. Therefore, the model and designed controller work well in practice.

The rest of the paper is organized as follows. Section 2 presents the nonlinear model of the momentum biased nadir pointing spacecraft system with a term of the general disturbance torque. Section 3 derives the linearized model for the momentum biased nadir pointing spacecraft with the gravity gradient disturbance which is the dominant and predictable disturbance torque for low earth orbit spacecrafts. The conclusions are summarized in Section 4.

2. The system equations

Let $\times$ denote the cross product of any two 3-dimensional vectors $x = [x_1, x_2, x_3]^T$ and $y = [y_1, y_2, y_3]^T$. Let $\Omega(x)$ be a skew-symmetric matrix function of x defined by

$$\Omega(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{1}$$

The cross product of $x \times y$ can then be represented by a matrix multiplication $\Omega(x)y$, i.e.,

$$x \times y = \Omega(x)y.$$

It is well known that the dynamic equation of motion of a momentum biased spacecraft is described as follows:

$$\begin{aligned} J\dot{\omega}_I &= -\omega_I \times (J\omega_I + H) + T_d + u \\ &= -\Omega(\omega_I)(J\omega_I + H) + T_d + u \end{aligned} \tag{2}$$

where

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

is the inertia matrix, $\omega_I = [\omega_{I1}\ \omega_{I2}\ \omega_{I3}]^T$ is the angular velocity vector of the spacecraft with respect to an inertial frame, represented in the spacecraft body frame, vector T_d is the total effected disturbance torque due to gravitational, aerodynamic, solar radiation, and other environmental torques in body frame, vector $u = [u_1\ u_2\ u_3]^T$ is the control torques in body frame, vector $H = [h_1\ h_2\ h_3]^T = [0\ h_2\ 0]^T$ is the angular momentum of the momentum wheel.

For a nadir pointing spacecraft, the attitude of the spacecraft is represented by the rotation of the spacecraft body frame relative to the local vertical and local horizontal (LVLH) frame. Therefore we will represent the quaternion and spacecraft body rate in terms of the rotations from spacecraft body frame relative to LVLH frame. Let ω be the body rate with respect to LVLH frame represented in the body frame, $\omega_{lvlh} = [0\ \omega_0\ 0]^T$ be the orbit (and LVLH frame) rate with respect to the inertial frame, represented in LVLH frame. Let A_l^b represent the transformation matrix from LVLH frame to the spacecraft body frame. Then, ω_I can be expressed by

$$\begin{aligned} \omega_I &= \omega + A_l^b \omega_{lvlh} \\ &= \omega + \omega_{lvlh}^b \end{aligned} \tag{3}$$

where ω_{lvlh}^b is the LVLH frame rate with respect to the inertial frame, represented in the body frame. $\dot{\omega}_I$ is given by

$$\begin{aligned} \dot{\omega}_I &= \dot{\omega} + \dot{A}_l^b \omega_{lvlh} + A_l^b \dot{\omega}_{lvlh} \\ &= \dot{\omega} - \omega \times A_l^b \omega_{lvlh} \\ &= \dot{\omega} - \omega \times \omega_{lvlh}^b \end{aligned} \tag{4}$$

where we assume that $\dot{\omega}_{lvlh}$ is small and can be neglected.[1] Using Eqs. (3) and (4), we can rewrite Eq. (2) as

$$\begin{aligned} J\dot{\omega} &= J(\omega \times \omega_{lvlh}^b) - \omega \times (J\omega) - \omega \times (J\omega_{lvlh}^b) - \omega_{lvlh}^b \times (J\omega) \\ &\quad - \omega_{lvlh}^b \times (J\omega_{lvlh}^b) - \omega \times H - \omega_{lvlh}^b \times H + T_d + u \\ &= f(\omega, \omega_{lvlh}^b, H) + T_d + u. \end{aligned} \tag{5}$$

Let

$$q = [q_0, q_1, q_2, q_3]^T = [q_0\ \ \vec{q}^{\,T}]^T = \left[\cos\left(\frac{\alpha}{2}\right), \hat{e}^T \sin\left(\frac{\alpha}{2}\right)\right]^T$$

be the quaternion representing the rotation of the body frame relative to LVLH frame, where $\hat{e}$ is the unit length rotational axis and α is the rotation angle about $\hat{e}$. Therefore (see [2])

$$A_l^b = \begin{bmatrix} 2q_0^2 - 1 + 2q_1^2 & 2q_1q_2 + 2q_0q_3 & 2q_1q_3 - 2q_0q_2 \\ 2q_1q_2 - 2q_0q_3 & 2q_0^2 - 1 + 2q_2^2 & 2q_2q_3 + 2q_0q_1 \\ 2q_1q_3 + 2q_0q_2 & 2q_2q_3 - 2q_0q_1 & 2q_0^2 - 1 + 2q_3^2 \end{bmatrix}.$$

Since (see [8])

$$\begin{aligned} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} &= \frac{1}{2} \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \end{aligned} \tag{6}$$

using the fact that $q_0 = \sqrt{1 - q_1^2 - q_2^2 - q_3^2}$, we have

$$\begin{aligned} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} &= \frac{1}{2} \begin{bmatrix} \sqrt{1 - q_1^2 - q_2^2 - q_3^2} & -q_3 & q_2 \\ q_3 & \sqrt{1 - q_1^2 - q_2^2 - q_3^2} & -q_1 \\ -q_2 & q_1 & \sqrt{1 - q_1^2 - q_2^2 - q_3^2} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \\ &= \frac{1}{2} Q\omega = g(q_1, q_2, q_3, \omega). \end{aligned} \tag{7}$$

It is easy to verify

$$\begin{aligned} \det(Q) &= \det \begin{pmatrix} \sqrt{1 - q_1^2 - q_2^2 - q_3^2} & -q_3 & q_2 \\ q_3 & \sqrt{1 - q_1^2 - q_2^2 - q_3^2} & -q_1 \\ -q_2 & q_1 & \sqrt{1 - q_1^2 - q_2^2 - q_3^2} \end{pmatrix} \\ &= \frac{1}{\sqrt{1 - q_1^2 - q_2^2 - q_3^2}}. \end{aligned} \tag{8}$$

Hence Q is always a full rank matrix except for $\alpha = \pm\pi$. This means that unless $\alpha = \pm\pi$, the kinematics equation of motion using quaternion can be simplified from (6) to (7) and $\omega = 2Q^{-1}[\dot{q}_1, \dot{q}_2, \dot{q}_3]^T$.

The main advantages of using (7) instead of (6) are as follows: (a) the system dimension is reduced from 7 to 6, yielding a simpler

[1] This assumption is true as long as the orbit eccentricity is small, i.e., the orbit is close to a circle.

model, (b) the linearized system is controllable, (c) the stability analysis can be directly conducted based on the linearized system (there is no uncontrollable unstable pole, see [9]), and (d) all closed loop eigenvalues can be assigned to any position by appropriate control law because the linearized system is controllable.

3. Linearized system

It is difficult to design controller directly from the nonlinear spacecraft system model described by (5) and (7). The common practice is to design the controller using a linearized system and then check if the designed controller works for the original nonlinear system. For a nadir pointing spacecraft system, we need the closed loop spacecraft system to meet the following conditions: (a) the spacecraft body rate with respect to LVLH frame is as small as possible, ideally, $\omega = 0$; and (b) the spacecraft body frame is aligned with LVLH frame with an error as small as possible, ideally, $q_1 = q_2 = q_3 = 0$. Since the rotation axis length is always 1, this implies that the rotation angle $\alpha = 0$. Therefore the linearized model is the first order model of Taylor expansion of the nonlinear system (5) and (7) about $\omega = 0$ and $q_1 = q_2 = q_3 = 0$. By using quaternion representation of A_l^b, assuming J is almost diagonal (which is almost always true in real spacecraft designs), and neglecting high order terms of q_1, q_2 and q_3, we have the following relations:

$$\omega_{lvlh}^b = A_l^b \omega_{lvlh} = \begin{bmatrix} 2q_1q_2 + 2q_0q_3 \\ 2q_0^2 - 1 + 2q_2^2 \\ 2q_2q_3 - 2q_0q_1 \end{bmatrix} \omega_0 \Bigg|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} \cong \begin{bmatrix} 2q_3 \\ 1 \\ -2q_1 \end{bmatrix} \omega_0, \tag{9}$$

$$-\omega_{lvlh}^b \times \left(J\omega_{lvlh}^b\right)\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} \cong -\omega_0^2 \begin{bmatrix} 2(J_{22} - J_{33})q_1 \\ 0 \\ 2(J_{22} - J_{11})q_3 \end{bmatrix}, \tag{10}$$

and

$$-\omega_{lvlh}^b \times H\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} = -\omega_0 \begin{bmatrix} 2h_2q_1 \\ 0 \\ 2h_2q_3 \end{bmatrix}. \tag{11}$$

Using (9), (10), and (11), by direct calculation, we have

$$\frac{\partial f}{\partial \omega}\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} = -J\Omega\left(\omega_{lvlh}^b\right) + \Omega\left(J\omega_{lvlh}^b\right) - \Omega\left(\omega_{lvlh}^b\right)J + \Omega(H), \tag{12}$$

$$\frac{\partial f}{\partial q}\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} = \frac{\partial}{\partial q}\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} \left(-\omega_{lvlh}^b \times \left(J\omega_{lvlh}^b\right) - \omega_{lvlh}^b \times H\right)$$
$$= \begin{bmatrix} 2\omega_0^2(J_{33} - J_{22}) - 2h_2\omega_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\omega_0^2(J_{11} - J_{22}) - 2h_2\omega_0 \end{bmatrix}, \tag{13}$$

$$\frac{\partial g}{\partial \omega}\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} = \frac{1}{2} I_3, \tag{14}$$

$$\frac{\partial g}{\partial q}\Big|_{\substack{\omega \approx 0 \\ q_1 \approx q_2 \approx q_3 \approx 0}} = 0_3 \tag{15}$$

where I_3 is a (3×3)-dimensional identity matrix, 0_3 is a (3×3)-dimensional zero matrix. (12) can be simplified further as follows:

$$J\Omega\left(\omega_{lvlh}^b\right) = \begin{bmatrix} -J_{13}\omega_0 & 0 & J_{11}\omega_0 \\ -J_{23}\omega_0 & 0 & J_{21}\omega_0 \\ -J_{33}\omega_0 & 0 & J_{31}\omega_0 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & J_{11}\omega_0 \\ 0 & 0 & 0 \\ -J_{33}\omega_0 & 0 & 0 \end{bmatrix}, \tag{16}$$

$$\Omega\left(J\omega_{lvlh}^b\right) = \begin{bmatrix} 0 & -J_{32}\omega_0 & J_{22}\omega_0 \\ J_{32}\omega_0 & 0 & -J_{12}\omega_0 \\ -J_{22}\omega_0 & J_{12}\omega_0 & 0 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & J_{22}\omega_0 \\ 0 & 0 & 0 \\ -J_{22}\omega_0 & 0 & 0 \end{bmatrix}, \tag{17}$$

$$\Omega\left(\omega_{lvlh}^b\right)J = \begin{bmatrix} J_{31}\omega_0 & J_{32}\omega_0 & J_{33}\omega_0 \\ 0 & 0 & 0 \\ -J_{11}\omega_0 & -J_{12}\omega_0 & -J_{13}\omega_0 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & J_{33}\omega_0 \\ 0 & 0 & 0 \\ -J_{11}\omega_0 & 0 & 0 \end{bmatrix}, \tag{18}$$

$$\Omega(H) = \begin{bmatrix} 0 & 0 & h_2 \\ 0 & 0 & 0 \\ -h_2 & 0 & 0 \end{bmatrix}. \tag{19}$$

Therefore

$$\frac{\partial f}{\partial \omega}\Big|_{\substack{\omega = 0 \\ q_1 = q_2 = q_3 = 0}} = \begin{bmatrix} 0 & 0 & (-J_{11} + J_{22} - J_{33})\omega_0 + h_2 \\ 0 & 0 & 0 \\ (J_{11} - J_{22} + J_{33})\omega_0 - h_2 & 0 & 0 \end{bmatrix}. \tag{20}$$

For many applications, we need to model disturbance torque T_d in the linearized model. For low earth orbit spacecraft, aerodynamic torque and gravity gradient torque are the dominant disturbance torques. It is difficult to model the aerodynamic torque because it is related to solar activity, geomagnetic index, spacecraft geometry, spacecraft attitude, spacecraft altitude, and many other factors, but it is known that the gravity gradient torque can be modeled by (see [4])

$$T_{gg} = \begin{bmatrix} 3\omega_0^2(J_{33} - J_{22})\phi \\ 3\omega_0^2(J_{33} - J_{11})\theta \\ 0 \end{bmatrix} \tag{21}$$

where ϕ and θ are roll and pitch Euler angles. For small Euler angles (see [7]), $\phi = 2q_1$ and $\theta = 2q_2$, this gives

$$T_{gg} = \begin{bmatrix} 6\omega_0^2(J_{33} - J_{22})q_1 \\ 6\omega_0^2(J_{33} - J_{11})q_2 \\ 0 \end{bmatrix}. \tag{22}$$

Assuming $T_d = T_{gg}$, and combining Eqs. (5), (13), (14), (15), (20), and (22), we have the quaternion based linearized spacecraft system described by

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{11} & J_{12} & J_{13} \\ 0 & 0 & 0 & J_{21} & J_{22} & J_{23} \\ 0 & 0 & 0 & J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ f_{41} & 0 & 0 & 0 & 0 & f_{46} \\ 0 & f_{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & f_{63} & f_{64} & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ u_x \\ u_y \\ u_z \end{bmatrix} \tag{23}$$

where $f_{41} = 8(J_{33} - J_{22})\omega_0^2 - 2h_2\omega_0$, $f_{46} = (-J_{11} + J_{22} - J_{33})\omega_0 + h_2$, $f_{64} = -f_{46}$, and $f_{52} = 6(J_{33} - J_{11})\omega_0^2$, $f_{63} = 2(J_{11} - J_{22})\omega_0^2 - 2h_2\omega_0$; or equivalently by

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{11} & J_{12} & J_{13} \\ 0 & 0 & 0 & J_{21} & J_{22} & J_{23} \\ 0 & 0 & 0 & J_{31} & J_{32} & J_{33} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ f_{41} & 0 & 0 & 0 & 0 & f_{46} \\ 0 & f_{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & f_{63} & f_{64} & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0_3 \\ J^{-1} \end{bmatrix} u \tag{24}$$

where u is a 3-dimensional control vector. It is straightforward to check that the linearized spacecraft model is fully controllable. Therefore, all modern control design methods can be applied directly, and the designed linear system is guaranteed to be stable.

4. Conclusion

In this short paper, we have developed a reduced quaternion model for momentum biased nadir pointing spacecraft. The gravity gradient disturbance torque is also included in the model. Unlike the full quaternion model studied by other researchers, this model is fully controllable. This makes it possible to use all modern controller design methods, such as optimal control design method (LQR) and robust pole assignment method, to design the spacecraft attitude control systems.

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References

[1] J. Boskovic, S. Li, R. Mehra, Robust adaptive variable structure control of spacecraft under control input saturation, Journal of Guidance, Control and Dynamics 16 (1) (2001) 14–22.
[2] J.B. Juipers, Quaternions and Rotation Sequences: A Primer with Applications to Orbits, Aerospace, and Virtual Reality, Princeton University Press, Princeton and Oxford, 1998.
[3] R. Paielli, R. Bach, Attitude control with realization of linear error dynamics, Journal of Guidance, Control and Dynamics 16 (1) (1993) 182–189.
[4] M.J. Sidi, Spacecraft Dynamics and Control: A Practical Engineering Approach, Cambridge University Press, 1997.
[5] R. Wallsgrove, M. Akella, Globally stabilizing saturated control in the presence of bounded unknown disturbances, Journal of Guidance, Control and Dynamics 28 (5) (2005) 957–963.
[6] J. Wen, K. Kreutz-Delgado, The attitude control problem, IEEE Transactions on Automatic Control 36 (10) (1991) 1148–1161.
[7] J. Wertz (Ed.), Spacecraft Attitude Determination and Control, Kluwer Academic Publishers, Dordrecht, Holland, 1978 (reprint 1999).
[8] B. Wie, Vehicle Dynamics and Control, AIAA Education Series, AIAA, Reston, VA, 1998.
[9] Z. Zhou, R. Colgren, Nonlinear spacecraft attitude tracking controller for large non-constant rate commands, International Journal of Control 78 (March 2005) 311–325.